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Extraction of total cross section data for the $\pi^- p \rightarrow \pi^+ \pi^- n$ reaction

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Total cross section extractions are sometimes based on extrapolations of in-plane data to unobserved regions of phase space. The total cross section can then only be derived if assumptions are made about the distributions of the independent phase space variables. In the case of the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ an out-of-plane measurement, performed by our group, indicates that these assumptions made by some experimenters are partially not supported by the data.

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The $(\pi,2\pi)$ reaction on the nucleon has attracted growing interest in recent years because it allows a study of nonlinear πN and $\pi\pi$ couplings. These nonlinearities are a testing ground of low energy QCD, where the fundamental chiral symmetry and chiral symmetry breaking can be studied [1]. The nonlinearities are most important near threshold. Obviously an accurate extraction of the amplitude is only possible from high quality cross section data.

In the isospin channel $\pi^- p \rightarrow \pi^+ \pi^- n$ there are three different methods of measurement. (i) Kinematically complete measurements covering major parts of the phase space for both pions, such as the bubble chamber experiments of Refs. [2,3]; as is known these experiments suffer from poor statistics. (ii) Single arm measurements detecting only the π^+ with a very high momentum and angular acceptance [4]. Here a good π^+ identification is used as a reaction trigger. Since the π^+ is the only observed particle, its azimuthal distribution is isotropic and only the full polar angle and momentum distribution has to be examined. The experiments showed that θ_{π^+} is also distributed nearly isotropically. (iii) Two arm measurements identifying the reaction via a coincident detection of both pions in the same plane as, for example, in Refs. [5–7]. In this case one has to make additional assumptions on the angular and momentum distributions of the outgoing particles to extract total cross section data.

This article refers only to these last kinds of measurements and especially to the elementary reaction measured in the OMICRON experiment. Here, the total cross section is determined by integration over the invariant mass of the outgoing pions, i.e.,

$$\sigma_{\text{tot}} = \int \frac{d\sigma}{dm_{\pi^+\pi^-}} dm_{\pi^+\pi^-} \, .$$

This method is based on the assumption that the invariant mass $m_{\pi^+\pi^-}$ is the only one out of the five independent kinematical variables which deviates from phase space. The other four variables are assumed to be distributed like phase space. The experimenters justified this assumption from in-plane data only [5] (which have been corrected for acceptance).

An out-of-plane experiment like ours provides the opportunity to test this assumption. Our experiment was done at the Paul-Scherrer Institute and investigated the isospin channel $\pi^- p \rightarrow \pi^+ \pi^- n$ at incoming kinetic energies of 218–330 MeV (this region is part of the OMICRON energy range). The experimental setup and results are described in Refs. [8] and [9].

The essential features of this experiment are the following: (i) kinematically complete measurement; (ii) good statistics [e.g., 35 000 $(\pi,2\pi)$ events at an incoming energy of 284 MeV]; (iii) coverage of 4 sr, i.e., $\frac{1}{3}$ of the total solid angle; (iv) measurement of in-plane as well as out-of-plane events. This last feature especially allows testing of the distribution of the four $m_{\pi^+\pi^-}$-independent kinematical variables.

It must be pointed out that the polar angle of the π^+ is fixed in the laboratory system to $50°\pm10°$ by the apparatus. Therefore, if the distribution of at least one kinematical variable is unlike phase space, the assumption of the OMICRON group on the phase space behavior of the four independent final state variables (except of the invariant mass $m_{\pi^+\pi^-}$) can be rejected. If all distributions are like phase space, no statement on the truth of this assumption at other polar angles of the π^+ can be made.

A possible choice of five independent variables spanning phase space are, besides the invariant mass $m_{\pi^+\pi^-}$, the solid angle of the neutron Ω_n in the overall c.m. system and the solid angle of the incoming π^- in the (π^+,π^-) c.m. system $\Omega^*_{\pi^-_{\text{in}}}$ [all quantities in this (π^+,π^-) c.m. system are denoted by an asterisk]. [In the overall c.m. system the direction of the neutron is opposite the direction of the (π^+,π^-) c.m. system.] The coordinate system used here is defined by the beam axis in z direction and the outgoing π^+ in the x-z plane; the polar angles are determined with respect to the z axis and the azimuthal angles with respect to the π^+, i.e., $\phi_{\pi^+}=0°$ by definition.

In order to test the assumption of a phase-space-like behavior of the $m_{\pi^+\pi^-}$-independent variables, we choose an incoming energy of 284 MeV. Closer to threshold one expects a more phase-space-like behavior. The distribution of the azimuthal angle of the neutron ϕ_n (where $d\Omega_n = d\phi_n d[\cos(\theta_n)]$) for different invariant masses $m_{\pi^+\pi^-}$ is shown as a scatter plot in Fig. 1(a). This plot shows an acceptance-corrected distribution divided by phase space; i.e., the area of the squares is proportional to the square of the matrix elements. The distribution of ϕ_n is significantly enhanced near 0° for all values of the invariant mass $m_{\pi^+\pi^-}$. All other angles are strongly suppressed against phase space. The squared matrix elements for a fixed invariant mass of the pions deviate by a factor of 30. Thus, the ϕ_n distribution deviates strongly from phase space.

This feature already showed up in the results presented in Fig. 2(a) of Ref. [8]: Here, the dependence of the triple differential cross section $d^3\sigma/d\Omega_{\pi^+}d\Omega_{\pi^-}dp_{\pi^+}$ of the azimuthal angle of the outgoing π^- was investigated. This feature can be easily understood. Remember that the incoming π^- together with the π^+ defined the x-z plane. A strong preference of ϕ_{π^-} near 180° was observed, i.e., the π^- prefers the x-z plane. Thus, also the neutron has to prefer the x-z plane. This agrees with the statement $\phi_n \approx 0°$.

At this point we have to make some remarks about the graphical representation of our results in Figs. 1 and 2. (i) The regions of the scatter plot without any squares are either not covered by the apparatus or are beyond the phase space limits. (ii) It has to be emphasized that the area of the squares (and not their length) is proportional to the square of the matrix elements. (iii) The statistical error (which is not indicated in the figures) originates not only from the experimental statistics (35 000 events), but also from the Monte Carlo simulations [500 000 generated $(\pi,2\pi)$ events]. The total statistical error is on the average about 10%. However, there are regions (especially near the phase space limits) where only very few Monte Carlo events appeared. Here, the uncertainty typically rises up to 20% (in very rare cases even up to 100%). (iv) The systematic error (mainly arising from the spectrometer acceptance) is about 14% [9].

The distribution of the polar angle of the neutron [$\cos(\theta_n)$; see Fig. 1(b)] shows a characteristic structure, however, a less pronounced deviation from phase space. Nevertheless, the squared matrix elements for a fixed invariant mass of the pions deviate typically by a factor of 3.

In contrast, the $\cos(\theta^*_{\pi^-_{\rm in}})$ distribution, shown in Fig. 1(c) (where $d\Omega^*_{\pi^-_{\rm in}} = d\phi^*_{\pi^-_{\rm in}} d[\cos(\theta^*_{\pi^-_{\rm in}})]$), also shows a rather strong deviation from the phase space for different values of the invariant mass $m_{\pi^+\pi^-}$. The values deviate typically by a factor of 5.

Summarizing we find that all three phase space variables deviate from phase space: while ϕ_n and $\cos(\theta^*_{\pi^-_{\rm in}})$ show a strong deviation, $\cos(\theta_n)$ deviates only moderately. (Note that $\phi^*_{\pi^-_{\rm in}}$ is fixed by definition.) Thus, for one possible choice of $m_{\pi^+\pi^-}$-independent kinematical variables it is demonstrated that these variables are not distributed like phase space. Therefore an extrapolation to regions of phase space not covered by the apparatus in order to extract the total cross section is not reasonable. Otherwise one ends up with incalculable errors. It must be stressed that this result can only be obtained by an

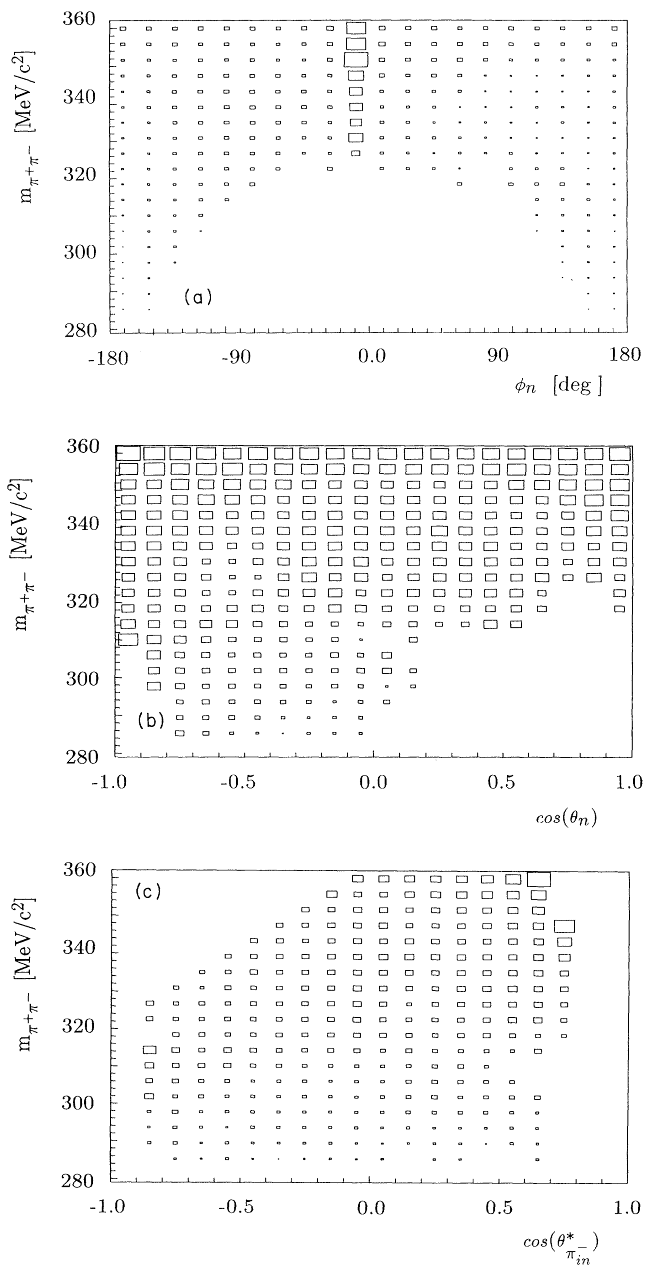

FIG. 1. Scatter plot of the distribution of (a) the azimuthal angle ϕ_n and (b) the polar angle $\cos\theta_n$ of the neutron in the overall c.m. system and (c) the polar angle $\cos\theta^*_{\pi^-_{\rm in}}$ of the incoming π^- in the (π^+,π^-) c.m. system for different invariant masses $m_{\pi^+\pi^-}$. The distributions are acceptance corrected and divided by phase space. The incoming energy is 284 MeV. (Regions without dots are not covered by the apparatus or are beyond the phase space limits.)

out-of-plane experiment like ours.

In spite of these shortcomings the total cross section extracted from in-plane experiments can have a value in the range expected. This is possible if the different regions of the phase space, covered by the apparatus added together, lead to an averaging process. This also shows up in the ϕ_{π^-} and θ_{π^-} dependence of the triple differential cross section, shown in Fig. 2 of Ref. [8]: While events with both pions going into the same direction are very unlikely, back-to-back events are strongly preferred. With special regard to the OMICRON apparatus, covering azimuthal angles of $\phi_{\pi^-}=0°\pm30°$ and $180°\pm30°$, respectively [10], this averaging process leads to a reasonable result.

Using other $m_{\pi^+\pi^-}$-independent variables as in Ref. [6], a similar deviation from phase space can be observed. Besides the invariant mass of the pion-pion and pion-neutron system the polar angle Θ (defined between the axis perpendicular to the production plane of the three outgoing particles and the incoming π^-) and the azimuthal angle Φ (defined with respect to the neutron) are chosen.

The distribution of the invariant (π^+,n) mass, $\cos(\Theta_{\pi^-})$, and Φ_{π^-} is shown for different invariant (π^+,π^-) masses in Figs. 2(a), 2(b), and 2(c), respectively. Here again, strong deviations from phase space can be seen (this behavior is indicated already in Fig. 10 of Ref. [6]). The squared matrix elements deviate typically by factors of 5–8. This choice of variables therefore leads to the same problems as mentioned above.

The sharp maximum at $\phi_n=0°$ of Figs. 1(a) is not reproduced at $\cos(\Theta_{\pi^-_{\rm in}})=0$ in Fig. 2(b) (which corresponds to the same kinematical situation). This is due to the angular binning of 18° in Fig. 1(a) and the fact that there is no direct correspondence of the angular binnings in the two coordinate systems. Thus, a bin in Fig. 2(b) at $\cos(\Theta_{\pi^-_{\rm in}})=0$ contains several bins of Fig. 1(a) at $\phi_n=0°$.

So far only the incoming energy of 284 MeV was discussed. To get an idea on the energy dependence of these phase space deviations also other energies were regarded: At 330 MeV incoming energy, the deviations from phase space become even more important, while at 246 MeV first preliminary data indicate smaller deviations.

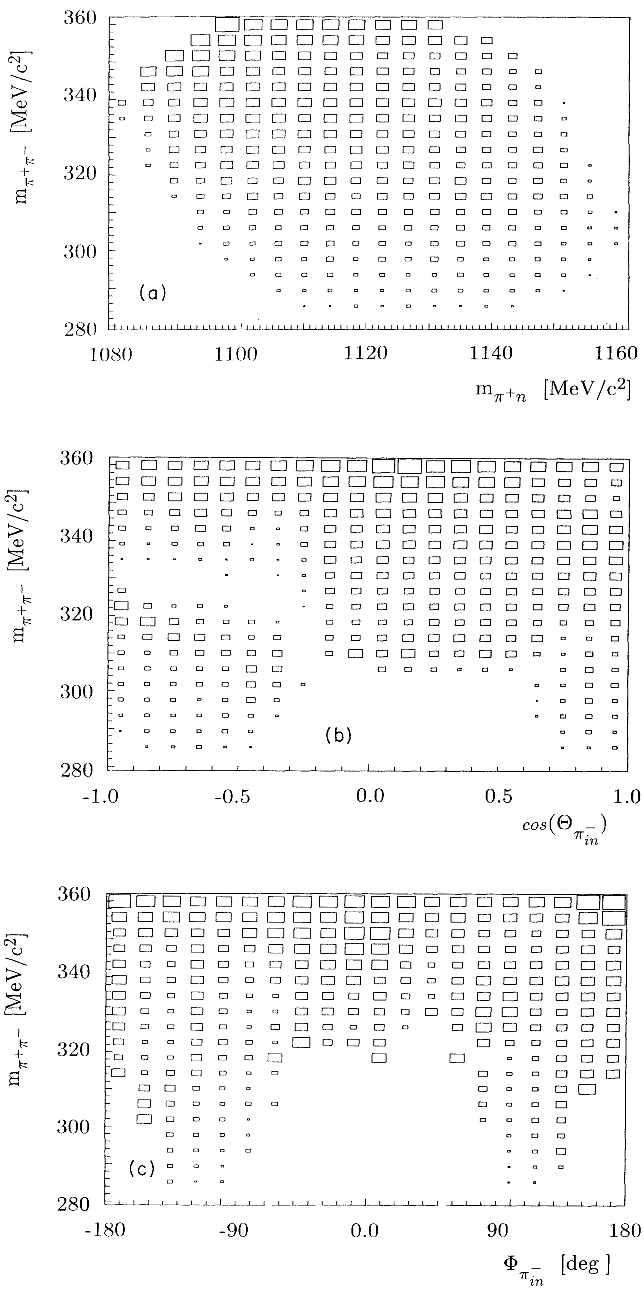

FIG. 2. Scatter plot of the distribution of (a) the invariant mass m_{π^+n}, (b) the polar angle $\cos\Theta_{\pi^-_{\rm in}}$, and (c) the azimuthal angle $\Phi_{\pi^-_{\rm in}}$ of the incoming π^- for different invariant masses $m_{\pi^+\pi^-}$. The coordinate system is chosen as in Ref. [6]. The distributions are acceptance corrected and divided by phase space. The incoming energy is 284 MeV.

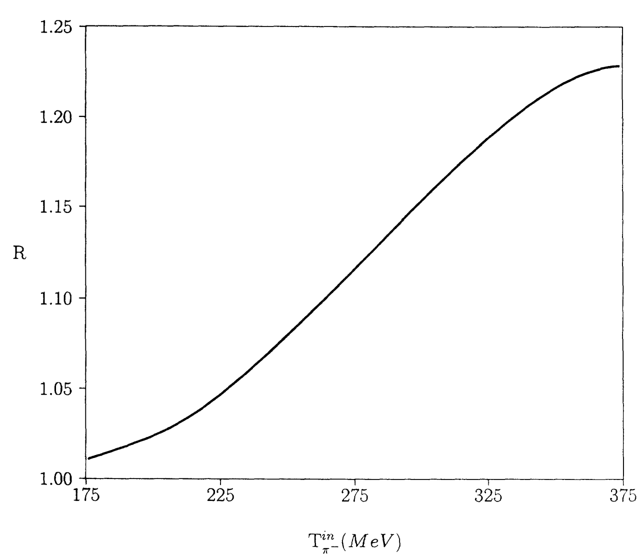

FIG. 3. Ratio R of the "true" model cross section to the "extrapolated" vs the incoming energy.

More quantitative insight into the energy dependence of these deviations can be gained by a model calculation. The model [11] is based on Weinberg's chiral Lagrangian and is capable of describing the exclusive data for the triple differential cross section (mentioned above) at the investigated energies, as well as other more inclusive data from other experiments. Therefore, the model seems suitable to estimate the uncertainty of the total cross section arising from the phase space deviations: We estimate the ratio R of the "true" model cross section to the "extrapolated" model cross section based on the acceptance of the OMICRON apparatus. This estimate is performed as follows. Integration over the phase space covered by the OMICRON apparatus; scaling of the obtained cross section to the full phase space ("extrapolated" cross section); and integration over the full phase space ("true" cross section). The ratio R is plotted in Fig. 3 versus the incoming energy. As expected, the uncertainties are small near threshold, but exceed the systematic error (about 10% according to Ref. [5]) already at 250 MeV.

Summarizing the preceding discussion, our arguments indicate that an extraction of total cross sections by extrapolation of in-plane measurements to nonobserved regions of phase space may lead to large systematic errors. An estimate of these uncertainties is only possible if out-of-plane data are included. The same arguments may also concern the total cross section measurements of other isospin channels of this $(\pi,2\pi)$ reaction and measurements on nuclei [7], where some model calculations [12] showed nonisotropical angular distributions.

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