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Reconciling deformation parameters from fusion with those from Coulomb excitation

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The effects of deformation on sub-barrier fusion were first demonstrated almost fifteen years ago. Since that first analysis, it has become generally accepted that quadrupole deformation parameters deduced from fusion are significantly lower than those deduced from Coulomb excitation. Following a detailed analysis of the excitation function for ^{154}Sm + ^{16}O we find the excitation function is best fitted with deformation parameters which are consistent with Coulomb excitation data when a sufficiently complete model is used. The shape of the deduced barrier distribution strongly favors the inclusion of a positive hexadecapole moment.

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Heavy ion fusion cross sections at energies near and below the Coulomb barrier can be orders of magnitude higher than those expected from penetration of a single one-dimensional barrier. The importance of deformation in producing such enhancement has been beautifully demonstrated [1–4] using spherical ^{16}O projectiles on a series of samarium targets, which range from the spherical, closed shell nucleus, ^{144}Sm, to the permanently deformed ^{154}Sm; the sub-barrier enhancement increases rapidly with neutron number. The analyses [1,3–5] of these cross sections, in terms of a range of barriers arising from different orientations of permanently deformed target nuclei, were only partially successful. The cross sections could be reproduced but the quadrupole deformation parameters, β_2, required for ^{154}Sm, for example, were considerably smaller than the values extracted from Coulomb excitation [6–13] or inelastic α-scattering [14] measurements. Possible reasons for the discrepancy were discussed in Refs. [1, 5]. More recently higher precision measurements for the reaction ^{154}Sm +^{16}O were made and the analysis also gave a small value of β_2 for ^{154}Sm [15]. The β_2 parameters extracted from three different analyses of fusion data for ^{154}Sm + ^{16}O all lie in the range 0.20 to 0.23, considerably smaller than values obtained from Coulomb excitation measurements which range from 0.26 [6] to 0.34 [13]. This latter range is largely due to different parametrizations of the nuclear density distribution, to which the deformation parameters are sensitive (see Ref. [12], for example).

This Rapid Communication shows that the agreement between the values from fusion is fortuitous; two of the analyses used approximations which lead to spuriously low values while the other suffered from insufficient data. We show that a careful theoretical analysis of fusion data, for the deformed nucleus ^{154}Sm, yields deformation parameters which are indeed consistent with those from other measurements.

Of the three sets of analysis, the earliest [1, 5] used the most precise treatment of the effects of deformation for the reactions $^{148-154}$Sm + ^{16}O. However, the parameters defining the nuclear potential, excluding deformation effects, could not be defined unambiguously due to a lack of data at high energies, where the effects of deformation are negligible. For this reason the reaction ^{148}Sm + ^{16}O was used to provide a spherical reference potential. Initially ^{148}Sm was assumed to be spherical, yielding $\beta_2 = 0.2$ for ^{154}Sm. Even when ^{148}Sm was assumed to have a small permanent deformation, $\beta_2 = 0.13$ [the root-mean-square (rms) value from Coulomb excitation] the extracted deformation of ^{154}Sm was still small. This would be expected if the effective deformation of the vibrational ^{148}Sm were underestimated. Treating a vibrational nucleus as deformed tends to do this because the rms deformation ignores the largest surface excursions which produce the lowest barriers. Also, as recognized in [5], the barrier distributions are different [16]: the deformed one is less symmetric giving smaller weights to the lower barriers. Both effects reduce the importance of the low barriers and thus underestimate the effective deformation for ^{148}Sm, giving a low β_2 for ^{154}Sm.

Subsequently, following measurements of fusion for ^{144,147,149}Sm + ^{16}O [3, 4], deformation parameters were extracted using the Wong model [17]. Wong was the first to derive simple analytical expressions, using a perturbative approach, to quantify the effects of nuclear deformation on fusion and these have been used for many years. The model gives the barriers for a deformed nucleus as a function of the angle, θ, between its symmetry axis and the beam direction. It requires a knowledge of the average height, V_B, of the interaction barrier, its curvature, $\hbar\omega$, the fusion radius, R_B, and β_2. The values of V_B and R_B in Refs. [3, 4] were obtained by fits to the high energy cross sections for each of the Sm targets. The low energy data for the ^{144}Sm target were then fitted with $\hbar\omega$ and β_2 as free parameters, resulting in a β_2 value for ^{144}Sm consistent with zero and $\hbar\omega = 3.9$ MeV. The data for the heavier targets were then fitted with $\hbar\omega$ fixed at this value and β_2 as the only free parameter. The resultant deformation parameter for ^{154}Sm was $\sim$0.2.

The Wong prescription used in Refs. [3, 4] takes the radius of the nuclear potential to be $R(\theta) = R_0[1+\beta_2 Y_2(\theta)]$ but makes the assumption that the radial position of the resulting potential barrier is independent of θ and the same as that for the spherical problem. The barrier height is then taken to be the sum of the deformed Coulomb and nuclear potentials at this radius. Figure 1 shows the extreme potentials for ^{154}Sm + ^{16}O with $\theta = 0°$ and $90°$ as well as the potential for the spherical case. Clearly the Wong model leads to "barriers" which are lower than the true maxima in the potentials. More importantly, because the potential falls much more rapidly inside the barrier than outside, the Wong approximation considerably overestimates the range of barriers. It is then necessary to use a smaller deformation parameter to reduce this range of barriers in order to give an optimum fit to the experimental data. As indicated above, this reduction gives a value of β_2 outside the range from Coulomb excitation but in agreement with the earlier more detailed analysis of Refs. [1, 5].

Recently we determined the fusion barrier distribution for ^{154}Sm +^{16}O [15] from new high precision data. We interpreted this distribution in terms of the quadrupole deformation of ^{154}Sm, with the fusion radius now dependent on the orientation of the target nucleus. However, we only used the monopole term of the Coulomb interaction in that analysis. The best fit to these new data was then obtained with $\beta_2 = 0.22$.

The monopole interaction is not adequate for large deformations and the Coulomb potential for a deformed charged distribution should have been used. This is derived from the general formulas of Ref. [18] and, to first order in β_2, is given by

$$V_C(r,\theta) = (Z_1 Z_2 e^2/r)[1 + 3\beta_2 Y_2(\theta) R_2^2/5r^2], \qquad (1)$$

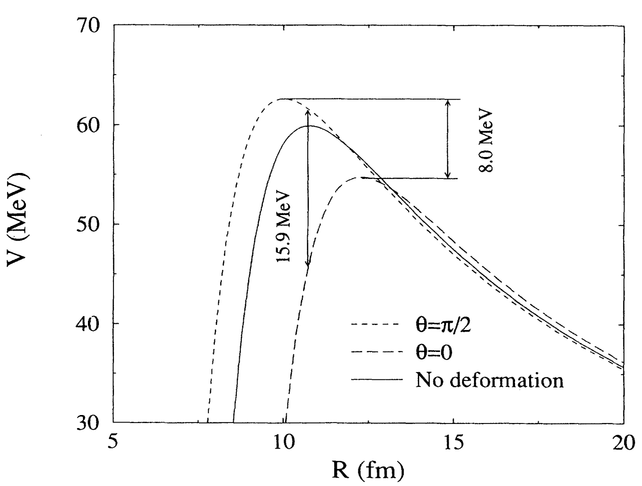

FIG. 1. Calculated potentials for ^{154}Sm + ^{16}O with no deformation and with the samarium target nucleus ($\beta_2 = 0.376$) oriented at $\theta = 0°$ and $90°$ to the beam direction. The "barriers" from the Wong prescription, indicated at 10.7 fm, are generally lower than the true maxima but, more importantly, extend to considerably lower values for small θ.

where Z_2 and R_2 are the atomic number and mean radius of the target and the projectile is assumed to be spherical. It is easy to show that the use of this potential instead of the spherical one gives essentially the same distribution of barrier heights if $\beta_2(1 - 3R_2/5R_B)$ is equal to the β_2 value obtained using the spherical Coulomb potential. For ^{154}Sm + ^{16}O, $R_2/R_B \sim 0.6$, implying that the β_2 we previously extracted was about 64% of the value expected using a more realistic deformed Coulomb potential.

Thus three different analyses of the ^{154}Sm + ^{16}O fusion data have produced low values of β_2. Two of them made approximations which demonstrably lead to spuriously low values. The other made no such approximations but probably underestimated the effective deformation of ^{148}Sm, used as a reference.

Our new data [15] are sufficiently extensive and accurate to fit the ^{154}Sm cross sections directly, without reference to any other system. We have therefore performed calculations of the orientation dependent barriers, without the previous approximations, and including the effects of angular momentum, $L\hbar$. We have used the potential

$$V(r,\theta) = V_C(r,\theta) + V_N(r,\theta) + V_L(r), \qquad (2)$$

where $V_C(r,\theta)$ is the Coulomb potential including quadrupole and hexadecapole terms and expanded to order β_2^2 and β_4. The nuclear potential, $V_N(r,\theta)$, is essentially taken to have an exponential tail and may therefore be simply parametrized by its surface diffuseness, a, and potential strength at some radius. For convenience we also parametrize the potential in terms of the resulting barrier height, B_0, for the spherical case. The exponent in $V_N(r,\theta)$ contains a factor $R(\theta)/a$, with $R(\theta) = R_2[1 + \beta_2 Y_2(\theta) + \beta_4 Y_4(\theta)]$, to account for the angular dependence of the radius of the deformed ^{154}Sm. Coulomb and nuclear radii were evaluated using a radius parameter of 1.06 fm. In Eq. (2) V_L is the usual angular momentum barrier. The barrier height, $B(L,\theta)$, position, and curvature are calculated numerically for each L and target nucleus orientation. The fusion cross section may then be calculated using

$$\sigma = \Sigma_L \int_0^{\pi/2} \sigma(L,\theta) \sin\theta d\theta, \qquad (3)$$

where

$$\sigma(L,\theta) = \frac{\pi \lambda\!\!\!^{-2}(2L+1)}{1 + \exp[2\pi(B(L,\theta) - E)/\hbar\omega(L,\theta)]}. \qquad (4)$$

In practice the integration in Eq. (3) was performed using the eigenchannel formalism of Ref. [19]. The results converge for any number of eigenchannels greater than six, corresponding to a standard coupled-channels calculation including states up to 10^+ in the rotational band of ^{154}Sm; twenty eigenchannels were used in our calculations.

The high precision data of Ref. [15] were fitted using this model. Approximate values of the nuclear parameters, a and B_0, were obtained by fitting only the cross sections for bombarding energies above 70 MeV, where the calculated cross sections are not expected to be sensitive to the deformation parameters. Subsequently these parameters were readjusted in conjunction with the deformation parameters β_2 and β_4 to optimize the fit to the complete excitation function.

Initially β_4 was set to zero and β_2 was varied, giving an optimum fit with $\beta_2 = 0.376$. This is considerably larger than that from all earlier fusion analyses and also larger than other nonfusion estimates which vary from 0.26 [6] to 0.34 [13]. The quality of the fit with $\beta_4 = 0$ is sensitively illustrated in Fig. 2(a) where the shapes of the $d^2(E\sigma)/dE^2$ distributions, related to the distribution of barrier heights [20], obtained from experiment and theory are compared. There are clear systematic differences in shape, over a large energy range, which cannot be accounted for using only a quadrupole deformation; also the lowest energy cross sections are dramatically underestimated.

The inclusion of a β_4 deformation greatly improves the fit to the cross sections. Now the χ^2 per point, in fitting the cross sections, is about 8 compared with 21 with $\beta_4 = 0$. The change in shape of the barrier distribution, shown in Fig. 2(b), reduces the obvious systematic discrepancies in Fig. 2(a). There appears to be some structure in the distribution around 60 MeV which cannot be explained in terms of the nuclear shape and which we are unable to reproduce.

The optimum fits are obtained with β_2 and β_4 values

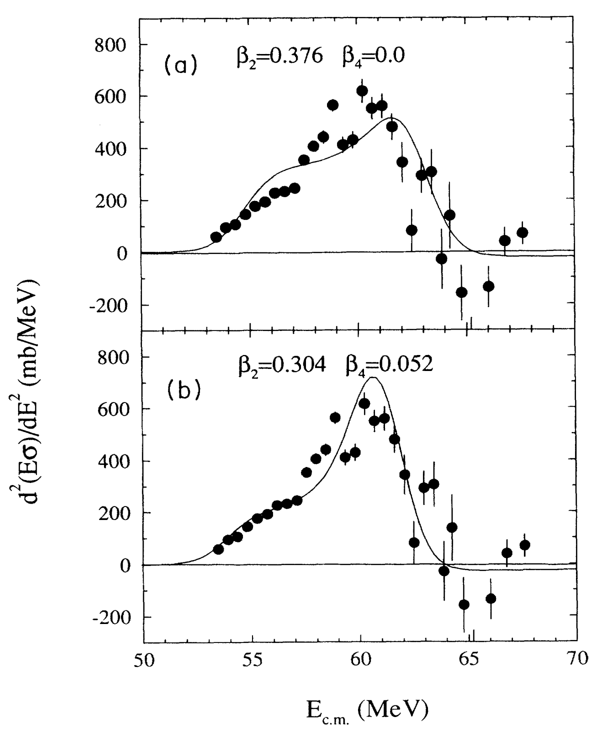

FIG. 2. The curvature of $E\sigma$ vs E extracted from the measured excitation function (full circles) for ^{154}Sm + ^{16}O, compared with the theoretical values using (a) $\beta_2 = 0.376$ and $\beta_4 = 0.0$ and (b) $\beta_2 = 0.304$ and $\beta_4 = +0.052$. Large, systematic differences in (a) are reduced by the inclusion of the *positive* hexadecapole deformation.

of 0.304 and 0.052, respectively. The value of β_2 is now consistent with other nonfusion measurements while the β_4 value is within the range of previously published values, 0.044 [10] to 0.13 [6]. This range of deformation parameters results, in large degree, from the choice of the Coulomb radius parameter; analyses of Coulomb excitation data [9, 11, 12] which use a radius parameter of $\sim$1.10, close to our value of 1.06, all yield values of β_2 close to 0.30 and β_4 near 0.11. The inelastic scattering data of Ref. [14] give $\beta_2 = 0.29$ and $\beta_4 = 0.06$, when adjusted to this same radius parameter. The agreement between these results and our new fusion analysis is remarkable but on the basis of this one fusion measurement it is not clear to what degree it is fortuitous.

However, the sensitivity of the deformation parameters to the choice of radius is small in our model; a 10% increase in radius gives $\sim$5% decrease in the β's. Thus it is clear that the interpretation of fusion for this reaction does require much larger deformation parameters than those previously published.

The nuclear potential which gives the optimum fit has a value of -8.53 MeV at 10.5 fm and a diffuseness of $a =$ 1.27 fm. This gives $B_0 = 59.50$ MeV and the resulting $\hbar\omega$ of the spherical barrier at $L = 0$ is 3.1 MeV. The value of $a = 1.27$ fm is higher than that obtained from elastic scattering data [21]. However, a double-folded potential for heavy ions [22] does produce a rather deep potential with an effective surface diffuseness of the same order of that obtained in our analysis.

Previous measurements for this reaction were made with insufficient detail and would not have been sensitive to the β_4 deformation. It is interesting to note, however, that the use of the Wong model, with only a quadrupole deformation, produces a distribution of barrier heights more resembling that of Fig. 2(b) than Fig. 2(a). The reason for this can be seen in Fig. 1. The asymmetry of the potential causes the range of "barriers" from the Wong model to be compressed above the spherical one and extended below, similar to the effect of including a β_4 deformation.

The remaining discrepancies evident in Fig. 2(b) still appear to be systematic and this may reflect the inadequacy of the model at this 3% level. There are several effects resulting from transfer reactions and projectile excitation, for example, which may affect the theoretical distribution but which are more difficult to take into account.

In summary, we have fitted the fusion excitation function for ^{154}Sm $+^{16}$O with a model which uses a distribution of fusion barriers arising from the static deformation of the target nucleus. The best fit to the data requires the inclusion of a positive hexadecapole deformation and the magnitudes of both β_2 and β_4 are consistent with values extracted from other reactions. The remarkable agreement obtained may be somewhat fortuitous given the uncertainties in the analyses of all data. However, it is clear that the values extracted in this work are much more realistic than the much smaller β_2 values previously published. These low values resulted from the limited data available and approximations in estimating the effects of deformation on the fusion barrier distributions.

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