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Effect of breakup reactions on the fusion of a halo nucleus

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We discuss the effect of breakup on heavy-ion fusion reactions induced by a halo nucleus ^{11}Li based on a semiclassical method. Our formula leads to a smaller effect of breakup than that calculated in a recent paper using unitarity. We show that although the large enhancement of the fusion cross section is moderated by the breakup, the halo nucleus ^{11}Li still leads to a larger fusion cross section than the other Li isotopes. This trend is especially significant at low energies, but occurs also at energies near the Coulomb barrier where experimental study is feasible.

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Secondary beams are rapidly opening a new field of research concerning the structure and reactions of unstable nuclei, in particular, of very neutron-rich nuclei. A number of novel features on the structure have already been revealed such as the existence of a neutron halo in several nuclei near the neutron dripline and the characteristic low-energy peaks, e.g., the so-called soft dipole mode, in the strength function of various multiple transitions [1]. We have shown in previous papers that these exotic features lead to a large enhancement of the fusion cross section in heavy-ion collisions at energies near and below the Coulomb barrier if a halo nucleus is used as the projectile [2,3]. Extended study on the effect of breakup reactions is, however, needed in order to make a reliable estimate of the fusion cross section, because a halo nucleus is expected to be easily broken. This paper is addressed to this question. A related study has been taken up by Hussein *et al.* [4]. As we point out later however, their approach overestimates the effect of breakup.

For simplicity we incorporate the effect of breakup through a local dynamic polarization potential $V_{\rm DPP}$. The fusion cross section is then given by

$$\sigma_{\rm CF}=\frac{\pi}{k^2}\sum_l(2l+1)P_F^{(0)}(l)\exp\left[2\int_{r_{0l}}^{\infty}W(r)\frac{dr/v(r)}{\hbar}\right], \tag{1}$$

where $P_F^{(0)}(l)$ is the fusion probability in the absence of breakup, k is the wave number in the entrance channel, $W(r)$ is the imaginary part of $V_{\rm DPP}$, r_{0l} is the distance of closest approach for the l wave, and $v(r)$ is the local velocity.

A simply way to justify Eq. (1) is obtained by the semiclassical theory of the three turning point scattering problem, where the wave function of the elastic scattering for a given partial wave $\psi(r)$ is given in terms of the action integrals between classical turning points and the phase shift in the external region [5,6]. We denote two turning points at the fusion barrier as r_1 and r_2, r_1 being the most external one, and the one inside the potential pocket as r_3. The wave propagation matrix method in Ref. [6] then gives the following expression for the asymptotic behavior of $\psi(r)$ as $r\to\infty$:

$$\psi(r)\sim A(1\;\;1)\begin{bmatrix} e^{-iS_{32}} & 0\\ 0 & e^{iS_{32}}\end{bmatrix} e^{\pi\epsilon}\begin{bmatrix} N(i\epsilon) & 1\\ 1 & \bar{N}(i\epsilon)\end{bmatrix} \times\begin{bmatrix} e^{-i\delta_1} & 0\\ 0 & e^{i\delta_1}\end{bmatrix}\begin{bmatrix} I_c(r)\\ O_c(r)\end{bmatrix}, \tag{2}$$

where

$$S_{ij}=\int_{r_i}^{r_j}\left[\frac{2\mu}{\hbar^2}[E-V(r)]\right]^{1/2}dr, \tag{3a}$$

$$\epsilon=\frac{i}{\pi}\int_{r_1}^{r_2}\left[\frac{2\mu}{\hbar^2}[E-V(r)]\right]^{1/2}dr=\frac{i}{\pi}S_{12}, \tag{3b}$$

$$N(i\epsilon)=\frac{\sqrt{(2\pi)}}{\Gamma(\frac{1}{2}+i\epsilon)}e^{-(1/2)\pi\epsilon+i\epsilon\ln(\epsilon/e)}, \tag{3c}$$

$$\bar{N}(i\epsilon)=\frac{\sqrt{(2\pi)}}{\Gamma(\frac{1}{2}-i\epsilon)}e^{-(1/2)\pi\epsilon-i\epsilon\ln(\epsilon/e)}, \tag{3d}$$

$$\delta_1=\int_{r_1}^{\infty}\left[\frac{2\mu}{\hbar^2}[E-V(r)]\right]^{1/2}dr-\int_{r_c}^{\infty}\left[\frac{2\mu}{\hbar^2}[E-U_c(r)]\right]^{1/2}dr. \tag{3e}$$

Here μ is the reduced mass in the entrance channel. U_c is the sum of the point Coulomb and the centrifugal potentials and r_c is the turning point in the Coulomb scattering. $I_c(r)$ and $O_c(r)$ are the asymptotic expressions of the incoming and the outgoing waves, respectively. The first and the third (2×2) matrices give the gain of

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phase when the particles travel through the potential pocket and the asymptotic region, respectively. The product of $e^{\pi\epsilon}$ and the second (2×2) matrix describes the change of the wave function when the particles traverse the potential barrier. The (1×2) vector $(1\ \ 1)$ guarantees the regular boundary condition at the origin. A is the normalization constant. Details can be seen in Refs. [5] and [6].

In describing heavy-ion fusion reactions, one often assumes that there is very strong absorption inside the Coulomb barrier and adopts the incoming wave boundary condition at somewhere inside the potential pocket. Under such circumstances, Eq. (2) can be rewritten as

$$\psi(r)\sim C_0(1\ \ 0)e^{\pi\epsilon}\begin{pmatrix} N(i\epsilon) & 1 \\ 1 & \bar{N}(i\epsilon)\end{pmatrix} \times\begin{pmatrix} e^{-i\delta_1} & 0 \\ 0 & e^{i\delta_1}\end{pmatrix}\begin{pmatrix} I_c(r) \\ O_c(r)\end{pmatrix} . \tag{4}$$

C_0 is the amplitude of the incoming wave to the left of the potential barrier, i.e., at r_2. The amplitude of the incoming wave at $r\to\infty$ is given by

$$C_\infty^{(-)}=C_0 e^{\pi\epsilon}N(i\epsilon)e^{-i\delta_1} . \tag{5}$$

We assume that the potential $V(r)$ consists of the bare potential $U(r)$, which is given, for example, by the double folding procedure, and the dynamic polarization potential associated with the breakup process $V_{\mathrm{DPP}}(r)$ whose imaginary part is denoted by $W(r)$. Using the semiclassical expression of the phase shift δ_1 given by Eq. (3e) and the properties of the barrier transmission factor N [5,6], one can easily establish the relationship between the fusion probability in the presence of breakup $P_F(l)$ and that in the absence of the breakup $P_F^{(0)}(l)$. To the leading order of $W(r)$, the result reads

$$P_F(l)=P_F^{(0)}(l)\ \exp\left[2\int_{r_{0l}}^{\infty}W(r)\frac{dr/v(r)}{\hbar}\right] . \tag{6}$$

We have assumed that $W(r)$ exists only outside of the most external turning point r_1, and also ignored the effect of the real part of $V_{\mathrm{DPP}}(r)$. Equation (6) proves Eq. (1).

It should be remarked that the argument of the exponential function in Eq. (6) is half of that in the paper of Hussein *et al.* [4], which bases the derivation of the formula on an argument of unitarity. The difference originates from the fact that Ref. [4] determines the factor to reduce the fusion probability by considering the effect of breakup not only in the entrance channel, but also in the exit channel. Our point is that the breakup only in the entrance channel should be taken into account in the evaluation of the fusion cross section. The unitarity argument also gives Eq. (6) if one treats the fusion and the breakup processes simultaneously.

An important problem is how to determine the dynamic polarization potential. In this paper, we follow the prescription of Refs. [4] and [7], partly because we wish to compare our results with theirs, and to highlight essential physical features rather than to perform a fully rigorous calculation for comparison with the experimental data. We start from the Feshbach formalism of the dynamic polarization potential [8]

$$V_{\mathrm{DPP}}=\left\langle\phi_0\left|V_c\frac{1}{E^{(+)}-T-U-H_0}V_c\right|\phi_0\right\rangle , \tag{7}$$

where ϕ_0 is the wave function of the ground state of the colliding nuclei in the entrance channel. We consider only the motion of the halo neutrons as for the intrinsic motion. Furthermore, we take a dineutron cluster model for the halo neutrons [2,3,9]. In the examples to be discussed later in this paper, the intrinsic motion is, therefore, the relative motion between the dineutron cluster in the halo part of ^{11}Li and the ^{9}Li core. The corresponding Hamiltonian is H_0. V_c is the coupling Hamiltonian to cause the breakup of dineutron cluster. We introduce the adiabatic approximation and set H_0 equal to zero. This is motivated by the fact that the transition strength for various transition operators is concentrated at energies close to the threshold of breakup [10]. We assume that the coupling Hamiltonian is separable. We convert the originally nonlocal dynamic polarization potential into the so-called trivially equivalent local potential. The final expression of the dynamic polarization potential for the partial wave l then reads

$$V_{\mathrm{DPP},l}^{(\mathrm{TELP})}=-\frac{2\mu}{\hbar^2}\frac{1}{k}F(r)\frac{1}{R_l(r)}\left\{if_l(r)\int_0^\infty dr'f_l(r')F(r')R_l(r')+\left[g_l(r)\int_0^r dr'f_l(r')F(r')R_l(r') +f_l(r)\int_r^\infty dr'g_l(r')F(r')R_l(r')\right]\right\} . \tag{8}$$

$R_l(r)$ is the solution of the Schrödinger equation including the dynamic polarization potential. The wave functions $f_l(r)$ and $g_l(r)$ are the regular and the irregular solutions of

$$(E-T_l-U)\begin{pmatrix} f_l \\ g_l\end{pmatrix}=0 \tag{9}$$

with the asymptotic boundary conditions

$$f_l\sim\sin(\phi_l) ,\quad g_l\sim\cos(\phi_l) \tag{10}$$

with

$$\phi_l=kr-\eta\ln(2kr)-\tfrac{1}{2}l\pi+\sigma_l+\delta_l , \tag{11}$$

where η, ϕ_l, and δ_l are the Coulomb parameter, the Coulomb phase shift, and the nuclear phase shift, respectively. They have been introduced to represent the Green's function $1/(E^{(+)}-T-U)$ [11]. $F(r)$ is the form factor of the breakup reaction. We assume that it is given by

$$F(r)=F_0 e^{-r/\alpha} , \tag{12}$$

$$\alpha=(\hbar^2/2\mu_H S_{2n})^{1/2} , \tag{13}$$

where μ_H is the reduced mass of the two valence neutrons in ^{11}Li against the ^{9}Li core, and S_{2n} is their separation energy. One should note that the form factor of breakup has a longer range for smaller separation energy S_{2n}. Therefore, it has a very long range property in heavy-ion collisions induced by a halo nucleus.

We now apply our formalism to the collisions of ^{11}Li with ^{208}Pb. As in Ref. [2], we calculate the bare potential by the double folding procedure by assuming the Michigan three-range Yukawa interaction for the nucleon-nucleon force. A difference is that in this paper we do not replace the interaction between the ^{9}Li core and the target by the phenomenological Akyüz-Winther potential as has been done in Ref. [2]. We choose the magnitude of the breakup form factor $F_0=4.9$ MeV following Ref. [4] and $\alpha=6.1$ fm corresponding to the experimental value of the separation energy $S_{2n}=0.34$ MeV [12]. Similar to Ref. [4], we approximate $R_l(r)$ by $f_l(r)$, and ignore the terms in square brackets in Eq. (8), and furthermore approximate $f_l(r)$ by the regular Coulomb wave function $F_l(r)$, which is numerically calculated.

Figure 1 shows thus calculated fusion probability for each partial wave at six bombarding energies near and below the Coulomb barrier, which is 25.9 MeV. The solid lines connect the results of our calculations using Eq. (6), while the dotted lines the results obtained by the same method as in Ref. [4]. The fusion probability without breakup $P_F^{(0)}(l)$ was calculated in the same way as in Refs. [2] and [3]. For a given incident energy, $P_F^{(0)}(l)$ is a decreasing function of the angular momentum l, while the reduction factor arising from the breakup, i.e., the second factor on the right-hand side of Eq. (6), is an increasing function. This is why the fusion probability at energies near and above the Coulomb barrier is not a monotonic function of l.

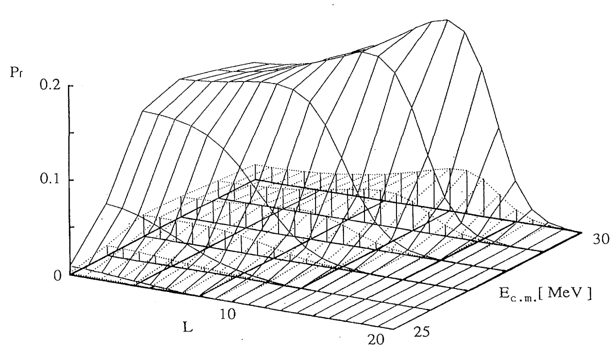

FIG. 1. Fusion probability for each partial wave in the collisions of ^{11}Li with ^{208}Pb. The solid lines were calculated following Eq. (6) by taking the effect of breakup only in the entrance channel, while the dotted lines by the same method as in Ref. [4], which includes the effect of breakup in the exit channel as well.

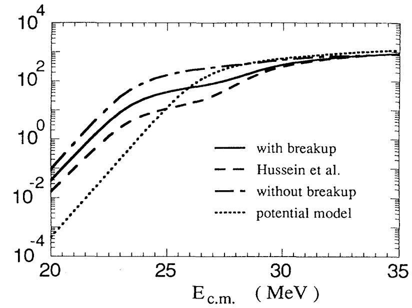

FIG. 2. Excitation function of the fusion cross section in units of mb in the collisions of ^{11}Li with ^{208}Pb. The dotted line is the results in the potential model. The other lines take the effect of the coupling of the relative motion to the soft dipole mode into account. The dot-dashed line ignores the effect of breakup. The solid line takes its effect into account following Eq. (6), while the dashed line by the same method as in Ref. [4].

Figure 2 shows the excitation function of the fusion cross section. The dotted line is the results of the potential model. It takes the lowering of the fusion barrier due to the neutron halo into account. The dot-dashed line corresponds to the results reported in Refs. [2] and [3]. It includes the effect of the coupling of the relative motion of the colliding nuclei to the soft dipole mode in ^{11}Li. We have updated the separation energy S_{2n} of the halo neutrons in ^{11}Li from 0.2 MeV used in Refs. [2] and [3] to 0.34 MeV in this paper. The solid line is the result of the present paper, which takes the effect of breakup into account. It was obtained following Eq. (1) and by adding the cross section for ^{11}Li to fuse with the target after it was broken into ^{9}Li and two neutrons. The dashed line was calculated in the same way as in Ref. [4]. In obtaining the dot-dashed, the solid, and the dashed lines, the partial waves were summed up to $l=20$. Figures 1 and 2 clearly show that the breakup reduces the fusion cross section. However, the amount of reduction is about one order smaller than that given by the method in Ref. [4], though details depend on the incident energy.

In Fig. 3, we have decomposed our final fusion cross section into two components. The dashed line is the fusion cross section in the entrance channel, i.e., the cross section for ^{11}Li to fuse with ^{208}Pb without breakup and to form a compound nucleus ^{219}At. The dotted line is the

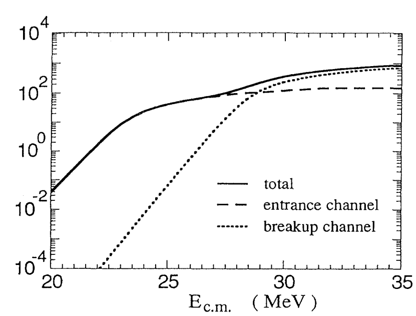

FIG. 3. Decomposition of the fusion cross section of ^{11}Li with ^{208}Pb into those in the entrance and in the breakup channels. The dashed and the dotted lines correspond to the former and the latter, respectively. The unit is mb.

fusion cross section in the breakup channel, i.e., the cross section for ^{11}Li to fuse after it was broken into ^{9}Li and dineutrons and to form ^{217}At. It would be very interesting and informative to confirm this separation experimentally.

Another interesting issue is whether one can experimentally observe a clear difference in the fusion cross section when the halo nucleus is used as the projectile compared to the cases of the other isotopes. In this connection, Fig. 4 compares the theoretically predicted fusion cross section in the collisions of various Li isotopes with ^{208}Pb at low energies. The line with open circles is the same as the solid line in Fig. 2. It represents the fusion cross section of ^{11}Li with ^{208}Pb. It takes the lowering of the fusion barrier due to the neutron halo, the enhancement due to the coupling to the soft dipole mode of ^{11}Li, and effect of breakup into account. The excitation functions of the fusion cross section of the other four Li isotopes with ^{208}Pb are denoted by the dot-dashed, the dotted, the solid, and the dashed lines in the order of increasing mass number. They were calculated by using the Wong formula [13]. The partial wave summation gives almost the same results except at very high energies. The fusion cross section gradually increases with the mass of the Li isotope when it is varied from 6 to 9. It then becomes much larger when the halo-nucleus ^{11}Li is used as the projectile. There is an experimental proposal to confirm this trend [14]. A difficulty is that the beam intensity is extremely low for the halo-nucleus ^{11}Li. Therefore, it may not be practical to compare the fusion cross section at too low energies. In this connection, it is very interesting to observe the enhancement of the fusion cross section for the ^{11}Li-induced reaction over the other cases for a wide range of bombarding energies including those close to the Coulomb barrier, where the fusion cross section is fairly large.

In summary, we have presented a semiclassical formula of the fusion cross section which takes the effect of breakup into account via a local dynamic polarization potential. By applying the formalism to the fusion of the exotic halo nucleus ^{11}Li with ^{208}Pb, we showed that the effect is not so large as that claimed in a recent paper [4], and that a halo nucleus is still expected to lead to a much larger fusion cross section than the other isotopes. The enhancement is predicted to occur for a wide range of bombarding energies, and becomes especially large at low energies. Also, we showed that the fusion cross section is dominated by that in the entrance channel at low energies, and that in the breakup channel at high energies. These theoretical findings should supply useful information to experimental study of heavy-ion fusion reactions with neutron-rich beams, though further study is needed on the mechanism of the breakup reactions and also on the effect of transfer reactions on the fusion cross section in order to attain a more quantitative theoretical prediction of the fusion cross section.

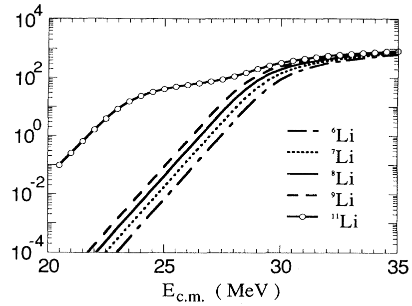

FIG. 4. Comparison of the fusion cross section of Li isotopes with ^{208}Pb. The unit is mb. The line with open circles is that for the ^{11}Li-induced reactions. The other lines are for the other isotopes as designated in the figure.

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