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K^+ production far below the free nucleon-nucleon threshold in heavy-ion collisions

M. Belkacem,[(1)] E. Suraud,[(1)] and S. Ayik[(2)]

[(1)]*Grand Accélérateur National d'Ions Lourds, BP 5027, F14021 Caen CEDEX, France*

[(2)]*Tennessee Technological University, Cookeville, Tennessee 38505*

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The K^+ production cross section far below the free nucleon-nucleon production threshold is estimated in $^{12}C+^{12}C$ collisions using simulations of the Boltzmann and the Boltzmann-Langevin equations. Fluctuations increase substantially the cross sections especially at low beam energies. We also give an estimate of the K^+ yield in $^{42}Ca+^{42}Ca$ collisions at an incident energy of 90 MeV/nucleon in order to compare our result to recent experimental data obtained in a similar system by Julien *et al.*

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INTRODUCTION

Particle production in heavy-ion collisions at bombarding energies below the free nucleon-nucleon threshold is of great interest. It can be related to the time evolution of the nuclear density during the collision which, in turn, could provide information on the equation of state (EOS) of the nuclear matter under extreme conditions [1]. It could also help elucidate the question of whether particle production in heavy-ion collisions is a collective process or can still be explained by elementary baryon-baryon processes. K^+ mesons are particularly suitable, because, due to strangeness conservation, their interactions with the baryonic environment is much smaller than for other particles like pions.

In this work, assuming an incoherent production mechanism, we evaluate K^+ production cross sections at energies below the free nucleon-nucleon threshold in the framework of transport approaches based on the nuclear version of the Boltzmann equation (NBE) [2,3] and on the Boltzmann-Langevin equation (BLE) [4]. The BLE approach, which has recently been proposed by Ayik and Grégoire, goes beyond the average description of the Boltzmann equation by incorporating fluctuations due to high-order correlations. These correlations give rise to large fluctuations in the one-body momentum distribution, which may play a dominant role in particle production mechanism at subthreshold energies. Therefore, it is of great interest to study the particle production mechanism at subthreshold energies on the basis of the BLE approach. In order to investigate the influence of fluctuations, we carry out calculations using the BLE approach for K^+ production in $^{12}C+^{12}C$ collisions below threshold energies, and compare the results to those obtained with the NBE. We also estimate the K^+ production cross section far below the free nucleon-nucleon threshold and compare it to recent data obtained in the $^{36}Ar+^{48}Ti$ collision at 92 MeV/nucleon [5].

KAON PRODUCTION IN THE NUCLEAR BOLTZMANN APPROACH

We first recall the kaon production mechanism in the framework of the NBE approach [2]. In this model, the ensemble averaged one-body distribution function $f(\mathbf{r},\mathbf{p},t)$ is determined by the semiclassical transport equation

$$\left[\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\cdot\nabla_r-\nabla_r U\cdot\nabla_p\right]f(\mathbf{r},\mathbf{p},t)=K(f)\ , \qquad (1)$$

where $U[\rho\mathbf{r})]$ denotes the nuclear mean field, here assumed to be momentum independent, and the collision term is given by

$$K(f)=\int d^3p_2 d\Omega\,\sigma(\Omega)|\mathbf{v}_1-\mathbf{v}_2| \times\{f_{1'}f_{2'}(1-f)(1-f_2) -ff_2(1-f_{1'})(1-f_{2'})\}\ . \qquad (2)$$

In Eq. (2), $f_j=f(\mathbf{r},\mathbf{p}_j,t)$ and $\sigma(\Omega)$ is the in-medium nucleon-nucleon cross section. Kaons are produced predominantly in elementary baryon-baryon collisions,

$$B+B'\rightarrow B''+K+Y\ , \qquad (3)$$

where B,B', and B'' are either a nucleon N or a Δ, and Y represents either a Λ or a Σ hyperon. At beam energies above 100 MeV/nucleon, production channels with pions,

$$\pi\text{-}N\rightarrow K\text{-}Y \qquad (4)$$

in which π are coming from the Δ disintegration, are also taken into account.

Because of the small production cross section at energies below the free nucleon-nucleon threshold, we use the usual perturbative approach and evaluate the invariant K^+ production cross section according to (with obvious notations)

$$E_k\frac{d^3\sigma}{dp_K^3}=\sum_c\int 2\pi b\,db\int dt\int d\mathbf{r}\,d\mathbf{p}_B d\mathbf{p}_{B'}|\mathbf{v}_B-\mathbf{v}_{B'}|E_K\frac{d^3\sigma_C}{dp_K^3}(\mathbf{p}_B+\mathbf{p}_{B'}\rightarrow\mathbf{p}_K)\mathbf{Z} +\sum_{c'}\int 2\pi b\,db\int dt\int d\mathbf{r}\,d\mathbf{p}_\pi d\mathbf{p}_N|\mathbf{v}_\pi-\mathbf{v}_N|E_K\frac{d^3\sigma_{C'}}{dp_K^3}(\mathbf{p}_\pi+\mathbf{p}_N\rightarrow\mathbf{p}_K)\mathbf{Z}'\ . \qquad (5)$$

Here, $d^3\sigma_C/dp_K^3$ and $d^3\sigma_{C'}/dp_K^3$ are the elementary cross sections for channels c in baryon-baryon collisions [Eq. (3)] and for channels c' in pion-nucleon collisions [Eq. (4)], respectively. In Eq. (5), the factors Z and Z' are given by

$$\mathbf{Z}=f(\mathbf{r},\mathbf{p}_B,t)f(\mathbf{r},\mathbf{p}_{B'},t)[1-f(\mathbf{r},\mathbf{p}_{B''},t)], \quad \mathbf{Z}'=f(\mathbf{r},\mathbf{p}_\pi,t)f(\mathbf{r},\mathbf{p}_N,t) . \tag{6}$$

Equation (5) essentially represents the folding of the elementary cross section with the momentum distributions of the colliding particles in which the Pauli blocking of the final nucleon section by solving the NBE numerically using a pseudoparticle method [3]. In this approach, the K^+ production cross section can further be discretized as [6]

$$\begin{aligned} E_K\frac{d^3\sigma}{dp_K^3} = {} & \sum_C \sum_b 2\pi b\,\Delta b \frac{1}{N_{\rm ev}}\frac{1}{N_G}\sum_{\rm ev}\sum_{\substack{\rm coll\\ S>S_C}} \frac{1}{\sigma_{BB'}^{\rm tot}} E_K \frac{D^3\sigma_C}{dp_K^3}(\mathbf{p}_B+\mathbf{p}_{B'}\to\mathbf{p}_K)(1-f_{B''}) \\ & + \sum_{C'} \sum_b 2\pi b\,\Delta b \frac{1}{N_{\rm ev}}\frac{1}{N_G}\sum_{\rm ev}\sum_{\substack{\rm coll\\ S>S_{C'}}} \frac{1}{\sigma_{\pi N}^{\rm tot}} E_K \frac{d^3\sigma_{C'}}{dp_K^3}(\mathbf{p}_\pi+\mathbf{p}_N\to\mathbf{p}_K) , \end{aligned} \tag{7}$$

where S_C and $S_{C'}$ are the squares of the production threshold energies in the center of mass of the two colliding pseudoparticles for channels c and c', respectively, $N_{\rm ev}$ the number of generated events and N_G the number of pseudoparticles per physical nucleon in each event. For the elementary cross sections, we use the parametrizations of Randrup and Ko [7].

KAON PRODUCTION IN BOLTZMANN-LANGEVIN APPROACH

In the NBE model, by neglecting correlations, dynamics is treated in the independent binary collision approximation. As a result, this model can provide a basis for describing the average properties of one-body observables in situations where the effects of correlations are not important. In dynamical processes in which the correlations play a dominant role, the NBE model does not provide a realistic description, and it must be improved by including the effects of correlations. Subthreshold particle production may be an example for such a situation where the correlations play an important role. As a result of these correlations, the high-energy tails of the momentum distribution exhibit large fluctuations. For a proper description of the production cross section, the fluctuations in the one-body momentum distribution should be incorporated into the calculations.

Recently, an attempt was made by Ayik and Grégoire in order to improve the NBE model by incorporating the high-order correlations into the equation of motion in a stochastic approximation [4]. This gives rise to a stochastic transport equation for the fluctuating one-body distribution function $f_\lambda(\mathbf{r},\mathbf{p},t)$

$$\left[\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\cdot\nabla_r-\nabla_r U\cdot\nabla_p\right]f_\lambda(\mathbf{r},\mathbf{p},t) = K(f_\lambda)+\delta K_\lambda(\mathbf{r},\mathbf{p},t) \tag{8}$$

and it is referred to as the BLE. Here the collision term $K(f_\lambda)$ is the same as in Eq. (2), but expressed in terms of the fluctuating distribution function f_λ for the trajectory "λ," and δK_λ is the fluctuating collision term characterized by its correlation function

$$\overline{\delta K_\lambda(\mathbf{r},\mathbf{p},t)\delta K_\lambda(\mathbf{r}',\mathbf{p}',t')}=C(\mathbf{r},\mathbf{p},\mathbf{r}',\mathbf{p}')\delta(t-t') , \tag{9}$$

where the reduced correlation function $C(\mathbf{r},\mathbf{p},\mathbf{r}',\mathbf{p}')$ can be obtained explicitly in the weak-coupling limit [4].

On the basis of the BLE model, the invariant K^+ production cross section is evaluated in the same way as in Eq. (5), but the quantities Z and Z' are now ensemble averages over the events of the BLE

$$\begin{aligned} \mathbf{Z}\to\mathbf{Z}_{\rm BLE}&=\overline{f_\lambda(\mathbf{r},\mathbf{p}_B,t)f_\lambda(\mathbf{r},\mathbf{p}_{B'},t)[1-f_\lambda(\mathbf{r},\mathbf{p}_{B''},t)]} , \\ \mathbf{Z}'\to\mathbf{Z}'_{\rm BLE}&=\overline{f_\lambda(\mathbf{r},\mathbf{p}_\pi,t)f_\lambda(\mathbf{r},\mathbf{p}_N,t)} . \end{aligned} \tag{10}$$

For an accurate calculation of the cross section, a large number of simulations of the BLE is required. In our numerical calculations, we determine the solutions of the BLE using an approximate method developed in Ref. [8]. In this method, the fluctuating events of the BLE are determined by following fluctuations in a set of local multipole moments of the momentum distribution. Here, we consider a single moment, namely, the z component of the quadrupole moment of the momentum distribution, $Q_{ZZ}=3p_Z^2-p_X^2-p_Y^2$. Fluctuations in Q_{ZZ} are characterized by a diffusion coefficient $C_Q(t)$, which can be deduced from the microscopic correlation function $C(\mathbf{r},\mathbf{p},\mathbf{r}',\mathbf{p}')$ [Eq. (9)]

$$C_Q(t)=\int d\mathbf{p}_1 d\mathbf{p}_2 d\Omega\,\sigma(\Omega)|\mathbf{v}_1-\mathbf{v}_2|(\Delta Q)^2 \times f_1 f_2(1-f_{1'})(1-f_{2'}) \tag{11}$$

with $\Delta Q=Q_{ZZ}(\mathbf{p}_1)+Q_{ZZ}(\mathbf{p}_2)-Q_{ZZ}(\mathbf{p}_{1'})-Q_{ZZ}(\mathbf{p}_{2'})$. According to the method of Ref. [8], the events of the BLE are generated iteratively by first calculating the fluctuations in the local quadrupole moments during each time step, then by scaling the momentum distribution to a specific local quadrupole deformation while preserving energy and momentum (see Ref. [8] for details). We carry out calculations for the K^+ production cross section on the basis of these BLE simulations, where we use the same input quantities as employed in NBE calculations.

KAON PRODUCTION FAR BELOW THE NUCLEON-NUCLEON THRESHOLD

The K^+ production cross section far below the free nucleon-nucleon threshold is very sensitive to the high-energy tail of the momentum distribution. In order to carry out reliable numerical calculations, a huge number of BLE events is required. This is practically not feasible. To avoid this problem, we evaluate the cross section [Eq. (5)] by Monte Carlo integration, using as inputs the one-body phase-space distributions extracted from Boltzmann-Langevin simulations.

In the spirit of the direct numerical simulations of the BLE (see above), we consider that BLE events can be characterized by a single collective variable. We take the z component of the total quadrupole moment of the momentum distribution as the collective variable

$$f(\mathbf{r},\mathbf{p},t)\rightarrow f_Q(\mathbf{r},\mathbf{p},t) \tag{12}$$

and assume that the distribution function $\mathbf{P}(Q,t)$ of Q's is a Gaussian determined by the ensemble averaged value $\langle Q(t)\rangle$ and width $\sigma_Q(t)$ given by $\sigma_Q^2(t)=\langle Q(t)^2\rangle-\langle Q(t)\rangle^2$.

In heavy-ion collisions at bombarding energies below 400 MeV/nucleon, the number of deltas and pions created during the early stages of the collision is very small. Therefore in this first calculation, we retain only the contribution to the kaon production arising from the nucleon-nucleon channels. Furthermore, in the overlap region of the colliding nuclei, the one-body phase-space distribution is approximated as

$$f_Q(\mathbf{r},\mathbf{p})=C\langle\rho\rangle n_Q(\mathbf{p}) \ , \tag{13}$$

where $\langle\rho\rangle$ is the average $\mathbf{r}$ space density in the overlap region and $n_Q(\mathbf{p})$ the $\mathbf{p}$ space density. The normalization constant C is determined by mass conservation.

The K^+ production cross section [Eq. (5)] then becomes

$$E_K\frac{d^3\sigma}{dp_K^3}=C^4\int 2\pi b\, db\int dt\int d\mathbf{r}\langle\rho\rangle^2 d\mathbf{p}_N d\mathbf{p}_{N'}|\mathbf{v}_N-\mathbf{v}_{N'}| \times E_K\frac{d^3\sigma}{dp_K^3}(\mathbf{p}_N+\mathbf{p}_{N'}\rightarrow\mathbf{p}_K)\mathbf{Z} \tag{14}$$

with

$$\mathbf{Z}=\int dQ\,\mathbf{P}(Q,t)n_Q(\mathbf{p}_{N},t)n_Q(\mathbf{p}_{N'},t)[1-n_Q(\mathbf{p}_{N''},t)] \ . \tag{15}$$

According to Eq. (14), the production cross section is evaluated by folding the elementary cross section with a momentum distribution with a fixed quadrupole moment, and by integrating over all possible Q's allowed by energy conservation and weighted by the distribution $P(Q,t)$. We need to know the mean value $\langle Q(t)\rangle$, the width $\sigma_Q(t)$ of $\mathbf{P}(Q,t)$, and the momentum distributions $n_Q(\mathbf{p})$ with fixed values of the quadrupole moment. These quantities are extracted from numerical simulations of the BLE with typically 100 events and inserted in Eq. (14). For the sake of simplicity, the quadrupole space in Eq. (15) is divided into six bins, and, for each bin, we use the mean momentum distribution obtained with all the events which have their quadrupole moment in this bin. The remaining integrals in Eq. (14) are done numerically.

Using Eq. (14), we can also calculate the production cross section in the NBE approach. This is done by approximating the factor Z in Eq. (15) as a product of the average momentum distributions corresponding to the mean value of the quadrupole moment, $n_Q(\mathbf{p},t)\approx n_{\langle Q\rangle}(\mathbf{p},t)$.

RESULTS

In numerical calculations we use a simplified three parameters (t_0,t_3,γ) Skyrme interaction [9] which gives an incompressibility modulus of 200 MeV (soft mean field) or 380 MeV (stiff mean field). Such a schematic, momentum-independent interaction is realistic only in the low-energy domain we are aiming at [5]. At higher energies absolute values of cross sections are to be taken with caution but qualitative effects and comparison between theories make sense. The nucleon-nucleon cross section entering the collision integral is taken with its energy dependence [2]. In each generated event, computations are performed with 30 numerical pseudoparticles per physical nucleon and the collision integral is evaluated by means of the so-called full-ensemble technique [3].

As a first step, we compare our results obtained in a pure Boltzmann simulation (without fluctuations) to some existing theoretical data obtained with exactly the same inputs. In Table I, we show a comparison of our results with the results of Ref. [10] for the K^+ production cross section in ^{93}Nb+^{93}Nb collisions at 700 MeV/nucleon using both a soft and a stiff equation of state. One can see the very good agreement between our results and those of Ref. [10]. Comparison with the more realistic results of Ref. [1] will be reported in a subsequent work [11].

In a second step, we test our Monte Carlo integration of Eq. (14) by comparing its predictions with the results of the numerical simulation of the same equation in an energy range where direct simulations of the NBE or BLE are feasible. Numerical simulations have been performed with a soft mean field in the energy range 600–1000 MeV/nucleon. Only the nucleon-nucleon channel for K^+ production has been taken into account in order to compare the simulations with the Monte Carlo integration which contains neither deltas nor pions. The Monte Carlo integration is performed with phase-space distributions extracted from the simulations. Figure 1 shows the K^+ production cross section versus beam energy per nucleon obtained both with the numerical simulation and the Monte Carlo integration, and in both Boltzmann and Boltzmann-Langevin approaches. One

TABLE I. K^+ production cross section in ^{93}Nb+^{93}Nb collisions at 700 MeV/nucleon. The mean field is momentum independent.

	Our result	Aichelin and Ko [10]
σ_K (mb) Soft mean field	0.88	0.89±0.04
σ_K (mb) Stiff mean field	0.46	0.46±0.03

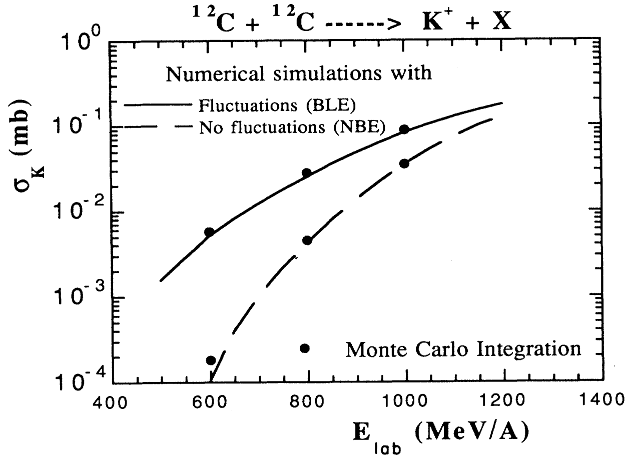

FIG. 1. Comparison between the numerical simulations of both the NBE (dashed line) and the BLE (solid line) approaches and the Monte Carlo integration. The K^+ production cross section in $^{12}C + ^{12}C$ collisions is plotted as a function of beam energy. The simulations are performed with a soft mean field.

can draw the following conclusions from this figure. (i) There is a very good agreement between the two calculations in both theories (with and without fluctuations) which is not surprising because they are just two different approaches of the same equation and the Monte Carlo integration used the distributions extracted from the simulations. (ii) The theory with fluctuations gives larger cross sections than those obtained with the Boltzmann approach, but the difference tends to disappear at high energy. (iii) In the case of NBE, there is a slight difference between the simulation and the Monte Carlo integration at low energy. This presumably reflects the lack of statistics for such small cross sections in direct simulations.

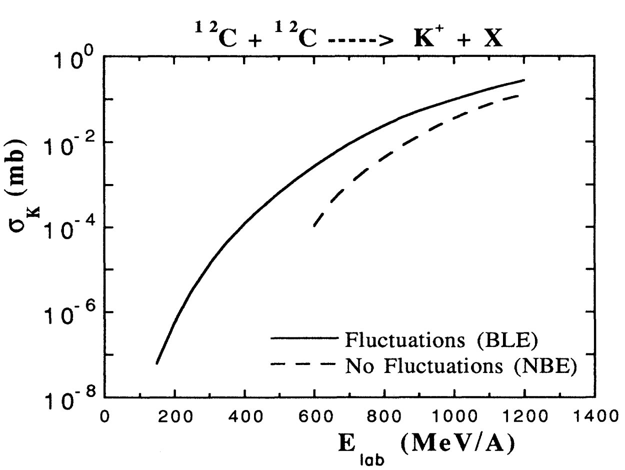

FIG. 2. Inclusive K^+ production cross section in $^{12}C + ^{12}C$ collisions versus the beam energy. Calculations have been done with the Monte Carlo integration using distributions of the NBE (dashed line) and of the BLE (solid line) simulations performed with a soft mean field.

TABLE II. K^+ production cross section in $^{42}Ca + ^{42}Ca$ collisions at an incident energy of 90 MeV/nucleon. The range of values obtained in the Monte Carlo method reflects the error bars of the Monte Carlo integration.

	Monte Carlo integration	Experimental data
σ_K (mb)	$(0.3-8.1)\times 10^{-7}$	$(2.4\pm 1.5)\times 10^{-7}$

At energies far below the free nucleon-nucleon threshold, the results obtained in Fig. 1 allow us to rely on the Monte Carlo method to calculate the K^+ production cross sections. We determine the mean value and the width of the Q distribution, and the momentum distribution for each Q bin by a simulation of a reasonable number of BLE events with a soft mean field and we calculate the cross section in Eq. (15) by numerical integration. The results are shown in Fig. 2 for $^{12}C + ^{12}C$ collisions as a function of bombarding energy. We also carry out a similar calculation for $^{42}Ca + ^{42}Ca$ collision at a bombarding energy of 90 MeV/nucleon, and estimate the K^+ production cross section. The estimated cross section, which is shown in Table II, is the same order of magnitude as the recent experimental data obtained in a similar system by Julien *et al.* [5] (our simulations are restricted to symmetric systems). Analysis of the timing of production of the particles in such a reaction reveals that most of them are produced in the early stages of the collision during the transient regime before thermalization of the system [8]. This points out the importance of nonequilibrium effects [11]. Note that the cross section for $^{42}Ca + ^{42}Ca$ collision at 90 MeV/nucleon vanishes when we use in the Monte Carlo method, momentum distribution extracted from the simulations of the NBE.

CONCLUSIONS

In this work, assuming an incoherent production mechanism, we have calculated the total K^+ production cross section in $^{12}C + ^{12}C$ collisions at very low bombarding energies (below and far below the free nucleon-nucleon threshold) in the framework of the BLE approach, and compared the results with those obtained in the NBE approach. At very low bombarding energies, direct numerical simulations of the BLE cannot give reliable results for production cross sections. Therefore, in order to estimate production cross sections far below the nucleon-nucleon threshold, we have developed an approximate method by parametrizing the fluctuations in terms of a single collective variable (quadrupole moment of the momentum distribution). This allows us to calculate the production cross section in terms of the one-body distribution functions extracted from the simulations of the NBE or the BLE. The major results of this exploratory investigation may be summarized as follows: (i) fluctuations substantially increase K^+ production cross sections at low beam energies; (ii) comparison with recent experimental data [5] indicates no contradiction with an incoherent production mechanism in a transport theory with fluctuations. More work is probably needed for a better understanding of the role of fluctuations on the particle production mechanism at far below threshold energies [11]. We should also point out that there are other effects which are also relevant for subthreshold kaon production. They include the momentum-dependent mean field [12], the medium effects [13] and kaon production from secondary interactions [14].

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