RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 47, NUMBER 4 APRIL 1993

Charge independence and charge symmetry breaking interactions and the Coulomb energy anomaly in isobaric analog states

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(Received 27 May 1992)

Effects of CIB (charge independence breaking) and CSB (charge symmetry breaking) interactions on the Coulomb displacement energies of isobaric analog states are investigated for ^{48}Ca, ^{90}Zr, and ^{208}Pb. The mass number dependence of the Coulomb energy anomalies is well explained when we employ CIB and CSB interactions which reproduce the differences of the scattering lengths as well as those of the effective ranges of low energy nucleon-nucleon scattering.

PACS number(s): 21.10.Sf, 21.30.+y, 21.60.Jz, 24.80.Dc

Effects of charge symmetry breaking (CSB) interactions on the Coulomb energy anomaly [1] have been recently studied in mirror nuclei with the masses $A=16\pm1$ and $A=40\pm1$ [2]. The calculated Coulomb energy differences (CED) have been found to agree with the experimental values within 1% (2%) accuracy for $A=17$ (41) when CSB interactions due to $\rho-\omega$ and $\pi-\eta$ meson mixings are included in addition to various other corrections [2]. The effects of the CSB interactions have also been studied in Ref. [3].

Here, we extend our studies to heavier nuclei such as ^{48}Ca, ^{90}Zr, and ^{208}Pb. The anomaly of Coulomb displacement energies (CDE) is noticed in the isobaric analog states (IAS) of these nuclei (^{48}Ca, ^{88}Sr, and ^{208}Pb) [4]. Deviations of the experimental CDE from the theoretical ones obtained by the Tamm-Dankoff approximation (TDA) with various corrections [4] become smaller for ^{48}Ca and ^{90}Zr than that of the ^{41}Sc-^{41}Ca pair, and even becomes negative for ^{208}Pb; the anomaly increases as the mass increases up to $A=41$, and begins to decrease as the mass increases more from $A=48$. The aim of the present paper is to study this peculiar A dependence of the Coulomb energy anomalies by taking into account the CSB and charge independence breaking (CIB) interactions.

Effects of CIB interactions have been studied in Coulomb energies of IAS of ^{49}Ca, ^{89}Sr, ^{139}Ba, and ^{209}Pb [5] and also those of $d_{3/2}$-shell nuclei [6] and $f_{7/2}$-shell nuclei [7]. The CIB interactions are found to play an important role in these nuclei. This is the first calculation to take into account both the effects of the CSB and CIB interactions on the CDE of isobaric analog states of ^{48}Ca, ^{90}Zr, and ^{208}Pb.

A simple estimation of the effects of CSB and CIB interactions can be done in the following way. In nuclei with neutron excess, the CDE is defined as the energy difference of the analog state from the parent state. Since the IAS is considered as the coherent proton-particle neutron-hole state,

$$|\mathrm{IAS}\rangle=|T,T_z=T-1\rangle=\sum_{\substack{j\in\text{excess}\\ \text{neutron orbits}}}\frac{1}{\sqrt{2T}}a_{jp}^{\dagger}a_{jn}|T,T_z=T\rangle\ . \tag{1}$$

The CDE due to the nuclear interactions is calculated to be

$$\begin{aligned}\Delta E_{\mathrm{CDE}}&=\langle T,T_z=T-1|H|T,T_Z=T-1\rangle-\langle T,T_z=T|H|T,T_z=T\rangle\\&=ZV_{pp}-NV_{nn}+(N-Z)V_{pn}\ ,\end{aligned} \tag{2}$$

where V_{pp}, V_{nn}, and V_{pn} denote the proton-proton, neutron-neutron, and proton-neutron interactions, respectively. The first and second terms are the interaction energy of the proton particle with the Z core protons and that of the neutron hole with the N core neutrons, respectively. The minus sign in the second term reflects the interaction of the hole with the particles. The third term represents the interaction of the proton particle with the excess neutrons. The interaction of the proton particle with the Z neutron particles in the closed core and that of the neutron hole with the Z proton particles in the same core cancel each other if there is no difference between the proton and the neutron wave functions in the same orbits. In general, this assumption holds accurately so that these contributions can be neglected.

It is interesting that Eq. (2) is written as follows:

$$\Delta E_{\mathrm{CDE}}=\frac{N+Z}{2}V_{\mathrm{CSB}}-(N-Z)V_{\mathrm{CIB}}\ , \tag{3}$$

where $V_{\mathrm{CSB}}=V_{pp}-V_{nn}$ and $V_{\mathrm{CIB}}=\frac{1}{2}(V_{pp}+V_{nn})-V_{pn}$. The nucleon-nucleon (*N-N*) interaction is assumed to be composed of the three parts, that is, a charge independent part (V_0), a CIB part (V_{CIB}), and a CSB part (V_{CSB});

$$V=V_0+\tfrac{1}{4}V_{\mathrm{CSB}}\{\tau_3(1)+\tau_3(2)\}$$
$$+\tfrac{1}{2}V_{\mathrm{CIB}}\{\tau_3(1)\tau_3(2)-\tfrac{1}{3}\tau(1)\cdot\tau(2)\}\ . \tag{4}$$

The last term is equivalent to the isotensor interaction. The type IV interaction [8], which has the isospin structure of $\tau_3(1)-\tau_3(2)$ or $\tau(1)\times\tau(2)$, is not considered here.

It is natural to think that the difference of the scattering lengths $\Delta a_{\mathrm{CSB}}=a_{pp}-a_{nn}$ is caused by the CSB interaction, while $\Delta a_{\mathrm{CIB}}=\frac{1}{2}(a_{pp}+a_{nn})-a_{np}$ is attributed to the CIB interaction. When we assume the proportionality between the strength of the interaction and the scattering length, the CDE in the analog state is related to the scattering lengths as follows:

$$\Delta E_{\mathrm{CDE}}\propto\frac{N+Z}{2}\Delta a_{\mathrm{CSB}}-(N-Z)\Delta a_{\mathrm{CIB}}\ . \tag{5}$$

The empirical values are $\Delta a_{\mathrm{CSB}}=1.5\pm0.3$ fm [9] and $\Delta a_{\mathrm{CIB}}=5.7\pm0.3$ fm [9] in the ^1S state. We can see from Eq. (5) that the CED in $N=Z$ nuclei increases as A becomes larger since the CIB interaction does not contribute. On the other hand, in $N\neq Z$ nuclei, both the CSB and CIB interactions become equally important. The CED and CDE given by Eq. (5) are proportional to 13, 31, −10, 11, and −95 for ^{17}F-^{17}O, ^{41}Sc-^{41}Ca, ^{48}Ca, ^{90}Zr, and ^{208}Pb, respectively. It is interesting to notice that the values of the CDE do not increase monotonically in the case of heavy mass asymmetric nuclei but decrease because of the CIB interaction. In Eq. (5), the differences of the effective ranges are neglected. In the following, we calculate the value ΔE_{CDE} by using realistic interactions and take into account both the effects of the differences in the scattering lengths and the effective ranges.

First, we calculate the CDE in finite nuclei from phenomenological V_{CSB} and V_{CIB} interactions which reproduce the low energy nucleon-nucleon (*N-N*) scattering parameters: Δa_{CSB}, Δa_{CIB} and the differences of the effective ranges, $\Delta r_{\mathrm{CSB}}=r_{pp}-r_{nn}$ and $\Delta r_{\mathrm{CIB}}=\frac{1}{2}(r_{pp}+r_{nn})-r_{pn}$. We use interactions of Yukawa form with radial cutoff. Low energy *N-N* scattering parameters in the ^1S channel are compiled in Refs. [9] and [10]. Miller *et al.* [9] gives $\Delta a_{\mathrm{CSB}}=1.5\pm0.3$ fm, $\Delta a_{\mathrm{CIB}}=5.7\pm0.3$ fm, $\Delta r_{\mathrm{CSB}}=0.10\pm0.12$ fm, and $\Delta r_{\mathrm{CIB}}=0.05\pm0.08$ fm. Machleidt [10] gives similar values: $\Delta a_{\mathrm{CSB}}=1.2\pm0.6$ fm, $\Delta a_{\mathrm{CIB}}=5.85\pm0.6$ fm, $\Delta r_{\mathrm{CSB}}=0.04\pm0.15$ fm, and $\Delta r_{\mathrm{CIB}}=0.07\pm0.12$ fm. We adopt the set of values given by Machleidt [10] to determine the interactions V_{pp}, V_{nn}, and V_{pn}. The parameters for the interaction are summarized in Table I. The radial cutoff at $r_C=0.45$ fm is used. Calculated CDE with this set of interactions are given in Table II for ^{48}Ca, ^{90}Zr, and ^{208}Pb. They are also shown in Figs. 1 and 2(a) together with the CED of $A=17$ and $A=41$ mirror pairs. The values of CDE and CED are obtained by taking matrix elements in the singlet even channel with use of the harmonic oscillator (HO) wave functions. The oscillator constant is taken to be $b=1.765$ fm for $A=17$, $b=1.90$ fm for $A=41$ and ^{48}Ca, $b=2.11$ fm for ^{90}Zr and $b=2.43$ fm for ^{208}Pb. The calculated CED and CDE in Table II satisfy qualitatively the simple relation (5). The quantitative difference is caused by the cutoff due to the short-range correlation and the effect of the differences in the effective ranges. When both the CSB and CIB interactions are taken into account, the A dependence of the observed anomaly of CDE and CED is well explained. The discrepancy between the observed and the calculated values is reduced to within 1–2 % of the observed CDE's which are 7.17 MeV for ^{48}Ca, 11.19 MeV for ^{90}Zr, and 18.83 MeV for ^{208}Pb.

The CDE are also calculated with the ρ-ω and π-η meson-exchange interactions [11,12] for the CSB part. We adopt the modified ρ-ω interaction [11] with the value $\beta=\{\kappa_s\kappa_v+(\kappa_s+\kappa_v)/2\}/2=1.21$ from the original one ($\beta=0.673$) [13] in order to obtain the empirical value of $\Delta a_{\mathrm{CSB}}=1.5$ fm. The π-η mixing potential [12] were not changed as well as the contribution from the *n-p* mass differences in the one-pion-exchange potential [3]. Δr changes from 0.018 to 0.016 fm when β is changed from 0.673 to 1.21 with the charge symmetric potential of Table I (taken to be V_{nn}) with $r_c=0.45$ fm. The π-η' meson-mixing interaction is not included here, nor is the *n-p* mass difference in the OPEP, since these contributions to the CDE are quite small (less than 5 keV) with the radial cutoff of relative motion at 0.5 fm. The π-η' mixing [12] contributes about $\frac{1}{3}$ as much as π-η mixing to the CED when there is no radial cutoff. When radial cutoff at $r_c=0.5$ fm is taken, the π-η' contributions get quite reduced: 1.12 keV for $A=17$ (16.47 keV if $r_c=0$ fm), 1.63 keV for $A=41$ (21.91 keV if $r_c=0$ fm), 1.95 keV for $A=48$ (26.38 keV if $r_c=0$ fm), 1.81 keV for

TABLE I. Parameters for the Yukawa interaction $V_0e^{-r/R_0}/(r/R_0)$ with the radial cutoff at $r_C=0.45$ fm and calculated scattering lengths a and effective ranges r in the $^1S_0$ channel.

	V_0 (MeV)	R_0 (fm)	a (fm)	r (fm)
pn	−597.67	0.646	−23.745	2.750
nn	−584.43	0.648	−18.801	2.801
pp	−527.06	0.669	−17.296	2.880

TABLE II. Calculated CDE from the CSB and CIB interactions with the Yukawa form given in Table I. The radial cutoff due to the short-range correlation is taken at $r_C=0.45$ fm.

Nucleus	b (fm)	CSB (keV)	CIB (keV)	Sum (keV)
^{17}O	1.765	186.7	0.0	187
^{41}Ca	1.90	220.8	0.0	221
^{48}Ca	1.90	270.8	−148.8	122
^{90}Zr	2.11	247.9	−106.2	142
^{208}Pb	2.43	239.6	−185.0	55

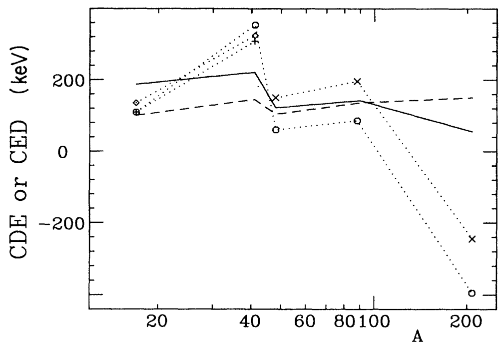

FIG. 1. Deviations of the experimental CDE and CED from the theoretical results including various contributions except those from the CIB and CSB interactions. The circles, the crosses, and the plusses correspond to the HF results with the Skyrme interactions SIII, SIV, and SG2, respectively, while the diamonds denote those of the Woods-Saxon wave functions. The solid and dashed lines show the calculated CDE and CED values from the phenomenological and the meson-exchange CSB and CIB interactions, respectively.

$A=90$ (26.29 keV if $r_c=0$ fm), and 2.15 keV for $A=208$.

As for the CIB interaction, we take into account the contributions from the $\pi^{\pm}-\pi^0$ mass difference in the one-pion-exchange potential (OPEP) [9,14], those from the $\rho^{\pm}-\rho^0$ mass difference in one-rho-exchange potential [14] and those from the 2π exchange [15,16]. These CIB interactions reproduce about 70% of the observed $\Delta_{\rm CIB}$ [9,10]. The calculated results of the CDE are shown in Table III and also in Figs. 1 and 2(b). Matrix elements in both the singlet even and triplet odd channels are taken into account with the use of the HO wave functions as well as the HF (SG2 [17]) wave functions with the radial cutoff at 0.5 fm. The two sets of wave functions, HO and HF, give rise to the difference of the calculated CDE by about 4%, 8%, and 20% for ^{48}Ca, ^{90}Zr, and ^{208}Pb, respectively. The negative contribution from the meson-exchange CIB interaction is not large enough compared with that from the phenomenological interaction to reproduce the observed CDE. It suggests the need of other contributions that reproduce properly the observed $\Delta a_{\rm CIB}$, for example, the contribution from the γ-π exchange potential, which is not taken into account in the present study because of the difficulty in quantitative evaluation [9,10,18].

The A dependence of the contributions from the meson-exchange CSB interaction is somewhat different from that obtained by the phenomenological Yukawa interaction. The contributions increase monotonically as A increases in the case of the meson-exchange interaction while they decrease slightly as A increases over $A=48$ in the case of the phenomenological interaction. The reason for this difference can be attributed to the difference of the effective range $\Delta r_{\rm CSB}$ between the two interactions. The phenomenological Yukawa interaction reproduces

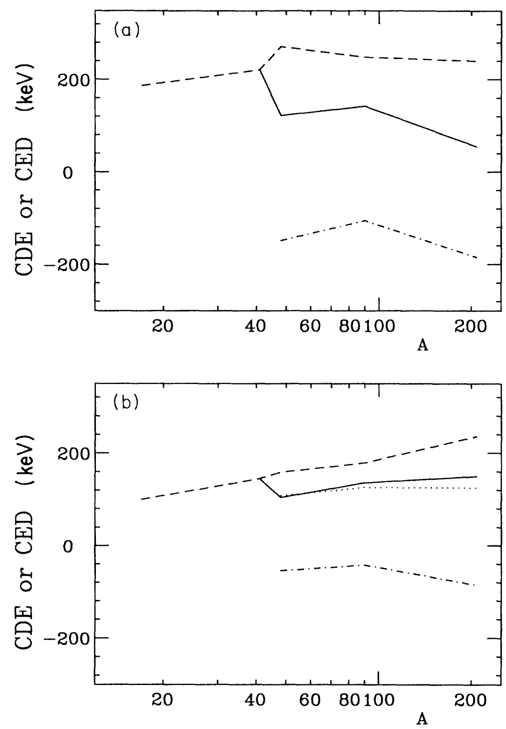

FIG. 2(a) Calculated CDE and CED from the CSB and CIB interactions of Yukawa-form with the cutoff at $r_C=0.45$ fm. The dashed and the dash-dotted lines shows results of the CSB and CIB interactions, respectively, while the solid line corresponds to the sum of the two contributions. The harmonic oscillator wave functions are used in the calculations. The solid line is the same as that in Fig. 1. (b) Calculated CDE and CED from the CSB and CIB interactions due to meson exchanges. The HF wave functions (obtained by the SG2 force) are used. The dashed and dash-dotted lines show results of the CSB and the CIB interactions, respectively, while the solid line corresponds to the sum of the two contributions. Results with HO wave functions are also shown by the dotted line. The solid line is the same as the dashed line in Fig. 1.

TABLE III. Calculated CDE by the meson-exchange interaction. Upper and lower rows refer to those of HO and HF-SG2 wave functions, respectively.

	CSB			CIB				
Nucleus	ρ-ω	π-η	Sum	$\pi^{\pm}$-π^0	$\rho^{\pm}$-ρ^0	2π	Sum	Total
^{49}Ca HO	136.3	25.2	161.5	−39.4	−3.5	−10.7	−53.6	108
HF	133.3	25.3	158.6	−40.3	−3.6	−10.7	−54.6	104
^{90}Zr HO	139.2	25.1	164.3	−28.4	−2.4	−7.9	−38.7	126
HF	150.7	28.0	178.7	−31.0	−2.6	−8.7	−42.3	136
^{208}PbHO	175.3	28.2	203.5	−56.0	−3.5	−18.6	−78.1	125
HF	202.6	33.1	235.7	−60.9	−2.9	−22.2	−86.0	150

the empirical value $\Delta r_{\text{CSB}} \approx 0.1$ fm while the meson-exchange interaction gives only $\Delta r_{\text{CSB}} = 0.02$ fm. The large difference in the ranges between the phenomenological V_{pp} and V_{nn} results in the sign change of $V_{\text{CSB}}(r)$ around $r = 1.5$ fm; it becomes negative at $r > 1.5$ fm. When negative contributions from larger radial part become important, as in heavy nuclei, the CSB contributions do not increase monotonically any more as A increases. Although there is also a sign change around $r = 1.4$ fm in the π-η mixing interaction, this effect is absent in the case of meson-exchange interactions since the magnitude of the negative value of $V_{\pi\text{-}\eta}(r)$ at $r > 1.4$ fm is so small compared to the phenomenological interaction that A dependence of the contributions from $V_{\text{CSB}}(r)$ is dominated by the ρ-ω mixing interaction which has no sign change. The interaction $V_{\text{CIB}}(r)$ changes sign at larger radial distance than in $V_{\text{CSB}}(r)$ in both the phenomenological (at $r \approx 2.6$ fm) and the meson-exchange (at $r \approx 2.9$ fm) cases. The contributions from large r with opposite sign are not so important as in V_{CSB} for both cases. The meson-exchange interaction gives the same value of $\Delta r_{\text{CIB}} = 0.09$ fm as given by the phenomenological Yukawa interaction. The A dependence of the contributions from V_{CIB} is, therefore, similar in both cases though the magnitude is smaller for the meson-exchange interaction.

Finally, we add a few comments. Δr_{CSB} around 0.04 ± 0.15 fm of Ref. [10] seems more favorable to get correct CED in the $A = 3$ system [19]. Δr_{CSB} around 0.10 ± 0.12 fm, on the other hand, is better for obtaining favorable A dependence of the CDE for heavy nuclei as ^{208}Pb. We pursued a possibility to use an exponential interaction Ve^{-r/R_V} for the CSB interaction which would give $\Delta a_{\text{CSB}} = 1.5$ fm and $\Delta r_{\text{CSB}} = 0.04$ fm. We found that it was impossible to get $\Delta r_{\text{CSB}} > 0.024$ fm even if we changed the range R_V down to 0.05 fm. (Parameter search was done in the range $R_V = 0.05-2.0$ fm.) When $R_V = 0.25$ fm is taken, we obtain $\Delta r_{\text{CSB}} = 0.02$ fm as can be expected from the original ρ-ω interaction. This exponential form is, essentially, of no difference from the original ρ-ω form (there is no sign change), and cannot lead to an improvement of the A dependence of the CSB contributions. The sum of Yukawa's with different ranges is essential to obtain Δr_{CSB} as large as 0.04–0.10 fm.

In summary, we have shown that the mass number dependence of the discrepancies of the CDE between the experimental and the theoretical values can be explained when we take into account the contributions from both the CSB and CIB interactions which reproduce properly the differences in both the scattering lengths (Δa) and effective ranges (Δr). The phenomenological CIB interaction gives a large negative contribution to the CDE and a good estimate to explain the anomaly of the previous calculations. On the other hand, the values obtained by the meson-exchange CIB interactions are almost half of those calculated by the phenomenological ones. This is caused by the difference in the scattering lengths Δa_{CIB} obtained by the two interactions. The difference in the effective ranges $\Delta r_{\text{CSB}} \approx 0.1$ fm is important for the behavior of the A dependence of the CDE due to the negative contribution of the V_{CSB} at the larger radial part in heavier nuclei. The meson-exchange interactions, which give small $\Delta r_{\text{CSB}} \approx 0.02$ fm, lead to a different A dependence of the CDE than that due to the Yukawa-type interactions.

We would like to express our thanks to O. Morimatsu and T. Cheon for useful comments. This work was financially supported in part by the Institute for Nuclear Study, the University of Tokyo, and also by the Research Center for Nuclear Physics, Osaka University. Numerical calculations have been performed by using M-880/310 at the Computer Center of the University of Tokyo and Sun-4 at the Physics Department of Nihon University.

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