

Hexadecapole interacting boson approximation structure functions in neodymium isotopes

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Low-lying hexadecapole states in stable even-even neodymium isotopes have been investigated by means of inelastic electron scattering. Transition charge densities were extracted in a Fourier-Bessel analysis of the form factors. The analysis of the experimental results within the interacting *sdg*-boson model with only one *g* boson allowed the extraction of the radial shapes of the hexadecapole structure functions of three of the different boson-pair configurations (i.e., *dd*, *sg*, and *dg*) involved in the hexadecapole excitations.

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The structure of nuclear excited states can be investigated [1,2] by studying the radial shape of the transition charge density, which can be obtained from high-resolution inelastic electron scattering. Recently, the interpretation of the transition charge densities of two vibrational nuclei ¹¹⁰Pd and ¹¹⁰Cd [3] in the framework of the classic *sd*-interacting boson model (IBA) expanded with only one *g* boson has allowed the deduction of the structure functions of the main configurations involved in the excitation of the low-lying hexadecapole states. The function $\alpha_{dd}^{(4)}(r)$, related to a recoupling of two *d* bosons, was found to be very similar in shape to the second derivative (∂'') of the ground-state (g.s.) charge density. This is in agreement with collective models, which predict such a shape for two-phonon configurations. The *dd* configuration constitutes, in vibrational nuclei, the main component in the wave function of the 4_1^+ level, which belongs to the well-known two-quadrupole-phonon triplet. In the *sd*-IBA it is, of course, the only one allowed. The functions $\alpha_{sg}^{(4)}(r)$ and $\alpha_{dg}^{(4)}(r)$, related to the exchange of one *s* or *d* boson with one *g* boson, were instead found to be similar to the first derivative (∂') of the g.s. density, the shape predicted by collective models for one-phonon configurations. These configurations constitute the main components of higher 4^+ states, but can also influence the 4_1^+ wave function if a mixing among the different configurations is introduced in the Hamiltonian of the model. The *sg*, *dg*, and the second-order *gg* terms are the extra boson-pair configurations introduced in the IBA model when the hexadecapole degree of freedom, namely, the *g* boson, is added explicitly to the *sd* expansion.

The aim of this Rapid Communication is to extend the investigation of the radial shapes of the *sdg*-IBA hexadecapole structure functions to transitional nuclei. The even-even Nd isotopes constitute a good test case for such an investigation, since two transitions are involved

in this chain: a spherical-collective one and a vibrational-rotational one going, respectively, from $A=142$ to $A=146$ and from $A=146$ to $A=150$.

The transition charge densities in ^{144,146,148,150}Nd have been derived from the data of high-energy inelastic electron-scattering experiments performed at NIKHEF-K using the quadrupole-dipole-dipole magnetic spectrometer [4]. Results from these experiments on ^{144,146,150}Nd have been published elsewhere [5–7]. The effective transferred momentum in the experiments ranged from 0.5 to 2.8 fm⁻¹. Isotopically enriched targets of approximately 10 mg/cm² were used. An energy resolution of about 12 keV was achieved at the lowest incident electron energy (112 MeV), and of up to about 30 keV at the highest energy (450 MeV). The identification of the excited states observed was checked against the results of a recent investigation [8] of these nuclei performed with inelastic scattering of protons and deuterons. The transition charge densities were deduced through a Fourier-Bessel analysis of the cross sections, as detailed in Ref. [6]. The analysis accounted for the finite *q* range through a high-*q* constraint.

The values of the reduced transition probability, $B(E4)$, related to the transition charge density, $\rho_4(r)$, are shown versus the excitation energy in Fig. 1 as full circles. The isoscalar components of the same quantities evaluated from the inelastic scattering of protons and deuterons in Ref. [8] are also indicated in Fig. 1 by full squares. Because of the better resolution achieved in Ref. [8] many weak hexadecapole transitions could also be observed in that experiment. Figure 1 suggests that the strongest excitations have been observed in each nucleus in the present electron-scattering experiment and, furthermore, that the isovector component in the excitation of levels observed in both experiments is small. This last observation justifies the use of the isoscalar version

(IBA-1) of the IBA model. In order to simulate the fragmentation and spreading at higher excitation energies, data and theoretical predictions have been folded with a Gaussian of 200 keV width (full and dashed curves in Fig. 1, respectively).

The gross structure of the data in Fig. 1 can be nicely reproduced by the *sdg* approximation of the IBA model [9] with only one *g* boson added. In this case the Hamiltonian can be written as the sum of the *sd* part, the *g* one, and an interaction (*sd-g*) between these:

$$H_{sdg} = H_{sd} + \varepsilon_g (g^\dagger \bar{g})^{(0)} + \zeta [(g^\dagger s^\dagger)^{(4)} (\bar{d} \bar{d})^{(4)} + (d^\dagger d^\dagger)^{(4)} (\bar{g} \bar{g})^{(4)}]^{(0)}. \quad (1)$$

For the *sd* Hamiltonian, its multipole expansion [9] has been used. The *g* boson energy is denoted by ε_g . The last term in Eq. (1) is the leading term [3] of a more complete *g*-boson Hamiltonian [10].

The electromagnetic transition operators are given by the extension of the usual *sd* operators [9]:

$$\hat{Q} = (s^\dagger \bar{d} + d^\dagger \bar{s})^{(2)} + \chi_{dd}^{(2)} (d^\dagger \bar{d})^{(2)} + \chi_{dg}^{(2)} (g^\dagger \bar{d} + d^\dagger \bar{g})^{(2)} + \chi_{gg}^{(2)} (g^\dagger \bar{g})^{(2)}, \quad (2)$$

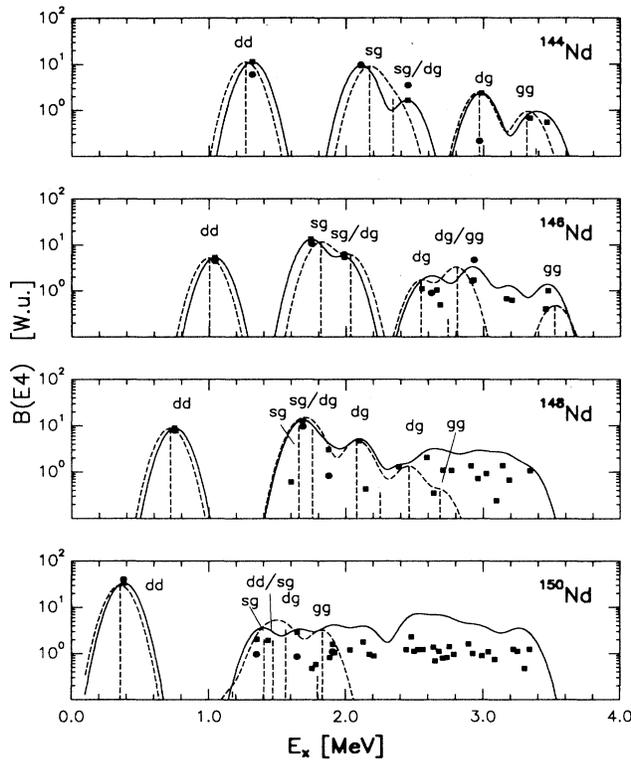


FIG. 1. Reduced transition probabilities, $B(E4)$, in Weisskopf units (definition of W.u. is the usual one with $r = 1.2A^{1/3}$ fm) for the excitation from the g.s. of the observed 4^+ states in the Nd isotopes. The data from the present experiment are denoted by full circles. The dashed vertical lines are the results of the IBA calculations described in the text. The full and dashed curves are obtained by folding the experimental data of Ref. [8] and the IBA evaluations with a Gaussian of 200 keV width. The *dd*, *sg*, *dg*, and *gg* labels on the dashed lines indicate their dominant boson-pair configuration.

$$\hat{H} = (d^\dagger \bar{d})^{(4)} + \chi_{sg}^{(4)} (g^\dagger \bar{s} + s^\dagger \bar{g})^{(4)} + \chi_{dg}^{(4)} (g^\dagger \bar{d} + d^\dagger \bar{g})^{(4)} + \chi_{gg}^{(4)} (g^\dagger \bar{g})^{(4)}. \quad (3)$$

In addition to the *sd* approximation, Eqs. (2) and (3) include the new *sg*, *dg*, and *gg* modes introduced by the *g* boson to form $L=2$ and 4 angular momenta. The multipole moments are related to the reduced transition probabilities through the usual polarization charges: $B(E2) = |e_2 \langle 2_f^+ || \hat{Q} || 0_{g.s.}^+ \rangle|^2$, $B(E4) = |e_4 \langle 4_f^+ || \hat{H} || 0_{g.s.}^+ \rangle|^2$.

In the previous equations the parameters introduced with the *g* boson act in the following way: ε_g shifts the excitation energies of the 4^+ states belonging to the *sg*, *dg*, and *gg* configurations with respect to the *dd* ones; the parameter ζ influences only slightly the position of the levels, but causes a mixing of the pure *sd* and *g* configurations and, at large values, the splitting of the pure (degenerate) *g* configurations. The $\chi_{dg}^{(2)}$, $\chi_{gg}^{(2)}$, $\chi_{sg}^{(4)}$, $\chi_{dg}^{(4)}$, and $\chi_{gg}^{(4)}$ polarization charges normalize the strength of the different states, but do not influence the sequence of excited levels.

Calculations have been performed in the *sdg*-IBA model with an existing set of *sd*-IBA-1 parameters [11] for the Nd isotopes to which the new parameters ε_g and ζ given in Table I have been added. In Ref. [11], the *sd* parameters were evaluated only for the three heaviest even-*A* Nd isotopes. For ^{144}Nd we have fitted the *sd* parameters by reproducing the excitation energies and strengths of the lowest states. The quadrupole strength evaluated with the *sd* approximation is not significantly altered by the addition of one *g* boson. In this paper we concentrate on the hexadecapole case which displays a richer strength distribution in the *g*-boson expansion. Most of this strength is concentrated in a few levels (dashed vertical lines in Fig. 1), each of which has a dominant boson-pair configuration, i.e., the *dd*, *sg*, *dg*, and *gg* ones labeled on the top of the dashed curves in Fig. 1.

The resulting parameters display a rather smooth dependence [12] on the product of the neutron and proton boson numbers. Here two remarks are in order.

TABLE I. IBA parameters used in the codes PHINTL and FBEML [9] for the *sdg*-analyses described in the text. The parameter CHQ has an SU(3) limiting value of $-\sqrt{35}/2$.

Parameter	Isotope	144	146	148	150
ε_d (keV)		816	743	605	369
PAIR (keV)		0.0	0.0	0.0	0.0
ELL (keV)		0.0	1.9	1	-0.4
QQ (keV)		-92	-71.4	-53.6	-38.9
OCT (keV)		-2.2	-0.5	0.7	1.1
HEX (keV)		2.2	2.7	3.7	4.5
CHQ		-1.1	-1.34	-2.24	-2.46
ε_g (keV)		1950	1650	1530	1200
ζ (keV)		300	300	100	10
e_4 ($e b^2$)		0.18	0.13	0.09	0.12
$\chi_{sg}^{(4)}$		0.09	0.17	0.30	0.10
$\chi_{dg}^{(4)}$		0.66	0.94	1.02	0.50
$\chi_{gg}^{(4)}$		26.7	30.1	43.4	210.

Firstly, the use of only one g boson can be justified by the results of Ref. [13] where it was concluded that in the ground state of ^{150}Nd the probability of a g boson is 10% at most, and that the reproduction of the multipole strength is obtained by a Hamiltonian intermediate between the $\text{SU}(3)$ sd and the $\text{SU}(3)$ sdg approximations. Secondly, the ratio $\varepsilon_g/\varepsilon_d$ (Table I) of the energies of the g and d bosons increases with isotope mass indicating an increasing collectivity for the quadrupole degree of freedom. This further results in a weaker interaction between the sd and g bosons, reflected in the strong decrease of ζ observed in the present work.

The parameters for the transition operator \hat{H} [Eq. (3)] were chosen on the basis of the following considerations. As discussed in Ref. [13], simple microscopic evaluations give values close to 1 for the parameters $\chi_{sg}^{(4)}$, $\chi_{dg}^{(4)}$, and $\chi_{gg}^{(4)}$. In the present analysis values of this order were found for $\chi_{dg}^{(4)}$, but the experiment clearly requires smaller $\chi_{sg}^{(4)}$ values. Very large $\chi_{gg}^{(4)}$ values are required to describe the experimental strength. Such a large $\chi_{gg}^{(4)}$ value could be due to a too limited model space, which may possibly still point to the need for more g bosons. In this case the contribution of the gg configuration in Fig. 1 is overestimated and therefore it is not reliably evaluated in the present analysis. However, it is clear from Fig. 1 that the hexadecapole strength of the other configurations is reasonably reproduced in an excitation energy with an upper limit of 3.0 MeV in ^{144}Nd and decreasing to an upper limit of 1.7 MeV for ^{150}Nd .

The extracted hexadecapole transition charge densities for the strongest transitions are shown in the first four columns of Fig. 2 with full points. The densities belonging to the same isotope are displayed in columns. The densities in general show roughly a ∂' shape with no clear

evidence for a ∂'' shape. However, the densities peak at different radial distances: those in the first row generally peak 0.5 fm more outwards. Moreover, some of the transition densities display a more complex structure involving a small inner peak. The experimental densities of the various isotopes in each of the rows in Fig. 2 are dominated, according to the IBA analysis performed, by one hexadecapole configuration: the dd , sg , and dg ones, in order of excitation energy.

The matrix elements evaluated from the sdg analysis and the experimental transition charge densities in Fig. 2 have been used to evaluate the radial shapes of the structure functions $\alpha_{dd}^{(4)}(r)$, $\alpha_{sg}^{(4)}(r)$, and $\alpha_{dg}^{(4)}(r)$. This is done by equating at several radii:

$$\langle 4_i^+ || \hat{\rho}_4^{\text{IBA}}(r) || 0_{\text{g.s.}}^+ \rangle = \rho_{4_i}^{\text{exp}}(r) \quad (4)$$

for each of the three experimental densities for every nucleus, with the r -dependent operator defined as

$$\hat{\rho}_4^{\text{IBA}}(r) = e_4 [(d^\dagger \bar{d})^{(4)} \alpha_{dd}^{(4)}(r) + \chi_{sg}^{(4)} (g^\dagger \bar{s} + s^\dagger \bar{g})^{(4)} \alpha_{sg}^{(4)}(r) + \chi_{dg}^{(4)} (g^\dagger \bar{d} + d^\dagger \bar{g})^{(4)} \alpha_{dg}^{(4)}(r)]. \quad (5)$$

The solutions resulting from this procedure depend on the relative sign of the different experimental densities. This, unfortunately, cannot be determined from electron-scattering cross sections. The solution adopted displays a ∂' shape for the three structure functions in all isotopes; in general, the surface peak of the $\alpha_{dd}^{(4)}(r)$ function for a given isotope is pushed 0.5 fm more outward compared to that of $\alpha_{sg}^{(4)}(r)$ and of $\alpha_{dg}^{(4)}(r)$. Variations smaller than 0.5 fm in the peak positions and a factor of 2 in the heights of the surface peaks are the only differences observed for a given structure function in any of the four

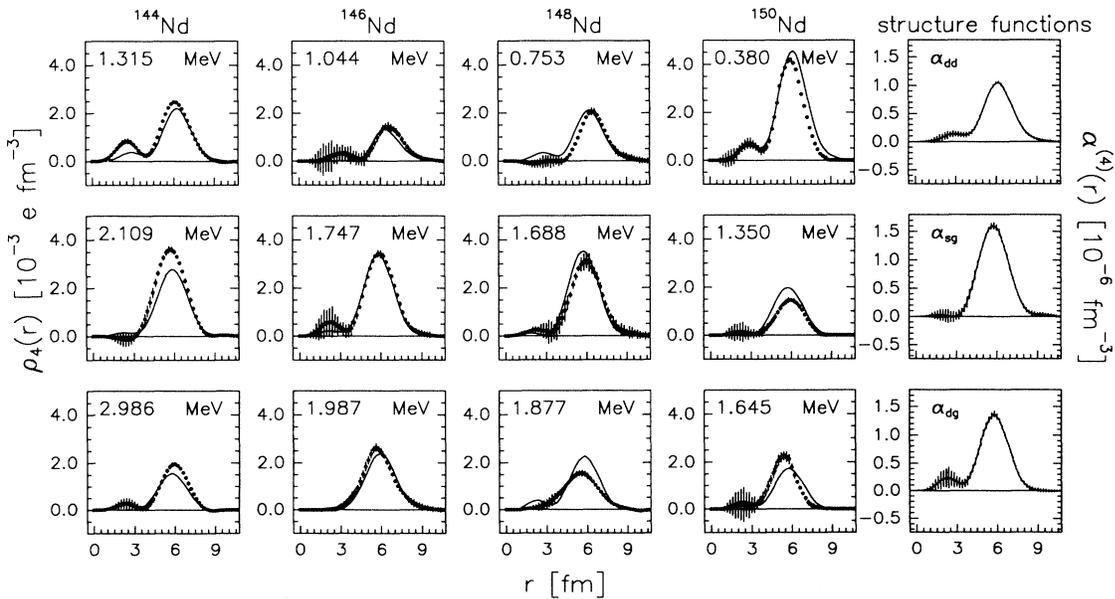


FIG. 2. Transition charge densities (data points) of some low-lying hexadecapole excitations in the four investigated Nd isotopes. The densities are labeled by the excitation energy (in MeV) of the relevant level. The densities belonging to the same isotope are given in one column. In the rightmost column the isotope averages of the extracted IBA hexadecapole boson structure functions $\alpha_{dd}^{(4)}$, $\alpha_{sg}^{(4)}$, and $\alpha_{dg}^{(4)}$ are displayed. The transition densities calculated from these averaged structure functions are displayed as full curves.

isotopes. A possible different solution of Eq. (4) for the $\alpha_{dd}^{(4)}(r)$ structure function, yielding a shape resembling a ∂'' shape, was found only in ^{146}Nd . However, the ∂'' outer peak is weak compared to the inner one and it has a small radial extension. This seems to be an unacceptable solution if one assumes that one set of structure functions should be capable of describing the hexadecapole transition densities in all Nd isotopes.

One set of structure functions, $\alpha_{dd}^{(4)}(r)$, $\alpha_{sg}^{(4)}(r)$, and $\alpha_{dg}^{(4)}(r)$, for all four isotopes was obtained by averaging the structure functions obtained for each different isotope. This could be done because of the small variations observed. These average structure functions are displayed in the fifth column of Fig. 2. The transition charge densities for all observed states calculated with these average structure functions (full curves in the first four columns of Fig. 2) still reproduce nicely the experimental densities.

In summary, transition charge densities with small differences in the radial shapes have been observed for the strongest hexadecapole excited levels of the even-even Nd isotopes with $A = 144-150$. The performed *sdg*-IBA calculations with only one *g* boson, which satisfactorily describe the experimental $B(E4)$ data with smoothly varying parameters, allow the extraction of three hexadecapole structure functions. All of these have a ∂' shape. The transition charge densities calculated with this set of hexadecapole structure functions reasonably reproduce the experimental data for all four isotopes.

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