N_pN_n scheme and the saturation of collectivity in the $A \approx 170$ and 230 regions

M. Saha and S. Sen
Saha Institute of Nuclear Physics, Calcutta 700 064, India
(Received 27 July 1992)

It is shown that the well known phenomenon of the saturation in the $B(E2;0_1^+ \rightarrow 2_1^+)$, as well as the $E_{2_1^+}$ values near midshell in the even rare-earth and actinide nuclei, can be reproduced in the N_pN_n scheme through a very simple parametrization in terms of the maximum number of valence protons and neutrons available in the major shells under consideration. This parametrization leads to a product $(N_pN_n)_{\text{eff}}$ which is found to have a more universal character as a structure variable than the usual N_pN_n.

PACS number(s): 21.60.Ev, 23.20.Gq, 27.70.+q, 27.90.+b

The N_pN_n scheme, originally proposed by Casten [1], has been found to be very useful [2–7] in understanding the qualitative behavior of the development of collectivity in different mass regions. In this scheme, the nuclear systematics are parametrized in terms of the product of the valence protons and neutrons (particles or holes, whichever are less) N_pN_n, or a related quantity $P = N_pN_n/(N_p+N_n)$ instead of the usual N, Z, or A. In a recent publication, Casten *et al.* [8] pointed out the existence of identical bands in a number of even-A rare-earth nuclei, differing by as many as 24 mass units, but having equal or nearly equal values of their N_pN_n product. The striking similarity between the transition energies in pairs of nuclei (e.g., ^{158}Dy-^{170}Hf) having an identical N_pN_n product and $|N_p - N_n|$ values indicates the possibility of obtaining a quantitative description of the collective variables, such as moments of inertia, within the framework of the N_pN_n scheme. In a recent Rapid Communication [9], we have shown that, in the rare-earth region, the moments of inertia can be expressed as a very simple function of valence proton and neutron numbers, i.e.,

$$\mathcal{I}_k \propto [f(N_pN_n)]^{1/k}, \quad k = 1,2 ,$$

where

$$\mathcal{I}_1 = 3\hbar^2/E_2, \quad \mathcal{I}_2 = 4\hbar^2/(E_4 - 2E_2) ,$$

and

$$f(N_pN_n) = N_pN_n(N_p+N_n) .$$

In subsequent discussions, the function $f(N_pN_n)$ will be denoted as the structure factor (SF). E_2 and E_4 denote the excitation energies of the 2_1^+ and 4_1^+ states, respectively. Using these expressions, the relative E_2 values of all the nuclei in the isotopic range $66 \le Z \le 74$ and $90 \le N \le 98$ could be reproduced within 0–10% accuracy. But this simple functional dependence cannot reproduce E_2 values of the nuclei near midshell ($N \approx 100$–106), and the deviation of the calculated values from the experimental data shows a gradual worsening as the midshell nuclei are approached. This is not unexpected in view of the known fact of saturation of collectivity near midshell, as evidenced through the systematics of the experimental E_2 and $B(E2)$ data in this region (Figs. 1, 2). Casten *et al.* [10] explained this behavior in terms of weakening of the quadrupole component of the valence n-p interaction strength near the midshell due to poor overlap of dissimilar neutron-proton orbits. From the systematics of the calculated [9] and experimental E_2 values, it appeared to us that this weakening of the n-p interaction might be of gradual nature.

To incorporate this feature into the N_pN_n scheme, we introduce a quantity, SP (saturation parameter), defined as

$$\text{SP} = [1 + \text{SF}/(\text{SF})_{\text{max}}] ,$$

were $(\text{SF})_{\text{max}}$ is evaluated with the maximum number of valence protons and neutrons available within the major shells under consideration. For the rare-earth nuclei $N \ge 90$, the proton and neutron shells are defined by $Z = 50$–82 and $N = 82$–126. So the maximum values of N_p, N_n and SF are 16, 22, and 13 376, respectively. In our earlier work, it was found that using a modified function $(\text{SF})_{\text{mod}} = \text{SF} \times (\text{SP})^{-1}$ for the calculation of E_2, the saturation effect near the midshell region could be approximately reproduced.

The purpose of the present Brief Report is to show that this simple parametrization of the saturation effect within the N_pN_n scheme opens up several new possibilities.

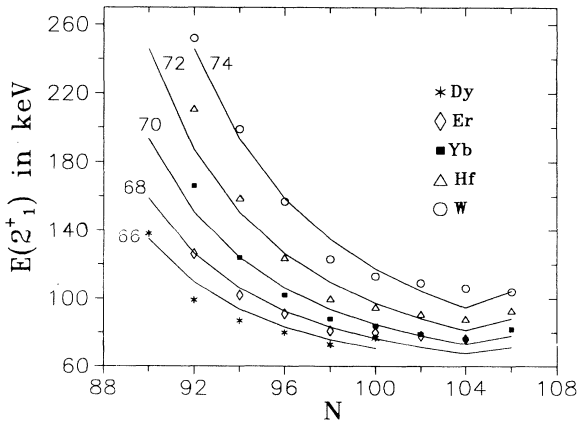

FIG. 1. Calculated (smooth curves) and experimental E_2 in the rare-earth nuclei.

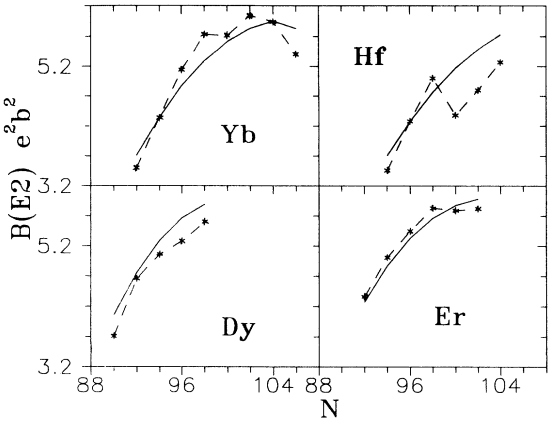

FIG. 2. Calculated (smooth curves) and experimental $B(E2)$ in the rare-earth nuclei.

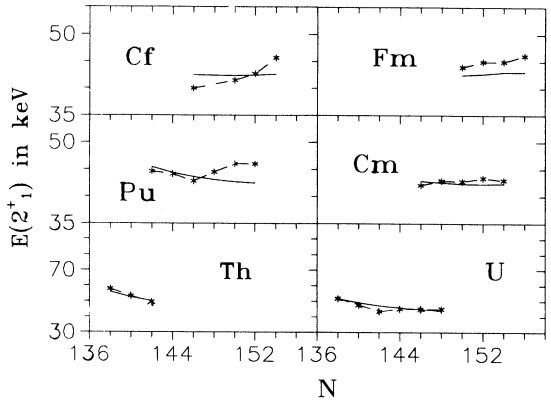

FIG. 3. Calculated (smooth curves) and experimental E_2 in the actinide nuclei.

Specifically, it enables us (i) to make quantitative predictions of the E_2 and $B(E2;0^+\rightarrow 2^+)$ values in the rare-earth and actinide regions, and (ii) to obtain a universal description of the development of collectivity in different mass regions in terms of a quantity $(N_pN_n)_{\text{eff}}$, to be defined later, instead of N_pN_n.

An analysis of the available excitation energies and lifetime data in the rare-earth region shows that the following relations,

(i) $\mathcal{I}_1 \propto \text{SF}\times(\text{SP})^{-1.5}$,

(ii) $B(E2)\propto \text{SF}\times(\text{SP})^{-2.25}$

are able to reproduce experimental E_2 [11] and $B(E2)$ values [12] in almost all the rare-earth nuclei having $66\leq Z\leq 74$ and $90\leq N\leq 106$ (^{156}Dy-^{180}W) with reasonable accuracy (Figs. 1, 2). The proportionality factors are determined by fitting E_2 and $B(E2)$ data in ^{164}Yb. In fact, out of 39 nuclei considered in this region, E_2 values are reproduced with 0–5% accuracy in 22 cases, 5–11% in 14 cases, and $>11\%$ in three $N=90$ nuclei, ^{158}Er, ^{160}Yb, and ^{162}Hf. Similarly, out of 25 available $B(E2)$ data, 14 values are reproduced with 0–5% accuracy, 9 with 5–10% accuracy, and 2 with $>10\%$ accuracy. Not only the absolute values, but also the systematics of E_2 and $B(E2)$ with increasing N are well reproduced for all the isotopes. The $B(E2)$ data in ^{172,174,176}Hf deviate considerably from the predicted values, but their ratios are well reproduced.

Encouraged by this result, we have carried out a similar analysis in the actinide region. If one looks at the systematics of E_2 in this region (Fig. 3), it will appear that this energy changes rather slowly with change in valence proton and neutron number. On the contrary, the saturation in $B(E2)$ values near midshell seems to be less dramatic than that observed in the rare-earth region. An analysis of the experimental data in 29 nuclei in this region with $88\leq Z\leq 100$ and $138\leq N\leq 156$ (i.e., ^{226}Ra-^{256}Fm) shows that the expressions

(i) $\mathcal{I}_1 \propto (\text{SF})^{1/2}\times(\text{SP})^{-2.5}$,

(ii) $B(E2)\propto (\text{SF})^{1/2}\times(\text{SP})^{-1.5}$

produce reasonable agreement with the experimental E_2 and $B(E2)$ data (Figs. 3, 4). The proportionality factors are determined by fitting E_2 and $B(E2)$ data in ^{230}U. So far as the agreement with experimental E_2 is concerned, 20 cases are reproduced with an accuracy of 0–5% and the rest within 10%. Although the absolute values are reproduced with reasonable accuracy, the systematic trends of E_2 with increasing N are not reproduced in Cf and Pu nuclei. Out of 14 available $B(E2)$ data in U, Pu, Cm, and Cf nuclei, all are reproduced within 5% accuracy. However, the agreement is very bad in ^{226,228}Ra. In case of Th nuclei, the systematics is well reproduced but not the absolute values.

In short, it may be said that the factor

$$[1+f(N_pN_n)/f(N_pN_n)_{\max}]^{-n}$$

can take into account, within N_pN_n scheme, the empirically observed saturation in E_2 as well as in $B(E2)$ values in deformed rare-earth and actinide nuclei near midshell. A convenient parametrization of this effect in the present form indicates that the reduction in the integrated quadrupole n-p interaction strength arising from reduced spatial overlaps of the valence neutron-proton orbits takes place in a gradual manner as the valence particles are

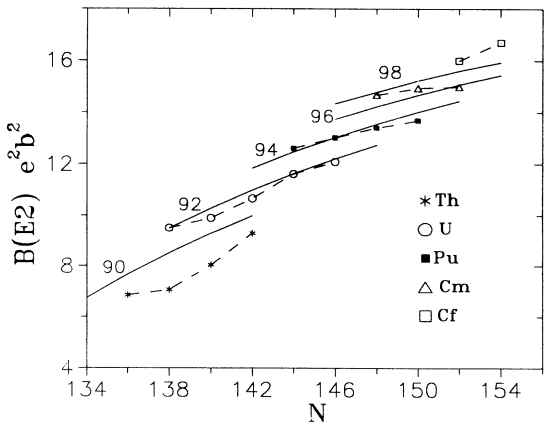

FIG. 4. Calculated (smooth curves) and experimental $B(E2)$ in the actinide nuclei.

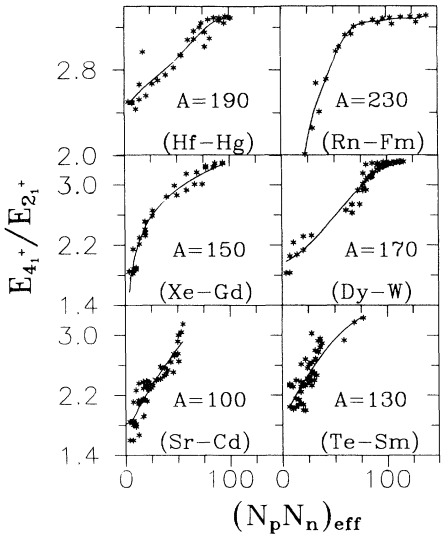

FIG. 5. R_4 plots in six mass regions.

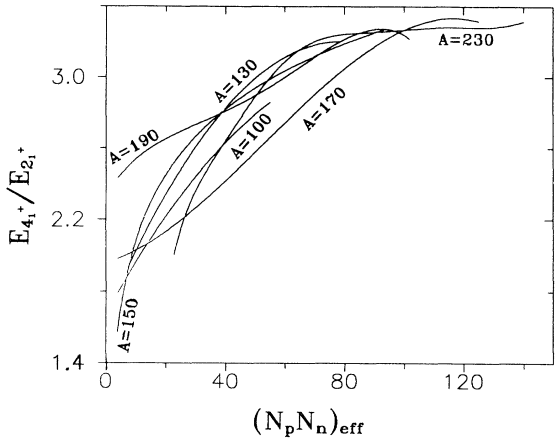

FIG. 6. Smooth curves of Fig. 5.

filling up the available orbits within a major shell.

In order to see whether this saturation parameter is specific to rare-earth and actinide regions only or if it has a more general validity, we have studied the changes in the ratio $R_4 = E_{4_1^+}/E_{2_1^+}$ for six transitional regions between $A \approx 100$ and 230 as a function of the parameter

$$(N_pN_n)_{\text{eff}} = (N_pN_n)[1 + N_pN_n/(N_pN_n)_{\max}]^{-1.5} .$$

We have chosen this simpler form of the parameter to facilitate the comparison of our results with those obtained earlier by Casten *et al.* [1–3] using the product N_pN_n or P. The R_4 ratios plotted against this parameter for all six regions $A = 100$, 130, 150, 170, 190, and 230 are shown in Fig. 5. In this calculation, the proton and neutron shells are taken following an earlier prescription [3]. Specifically, in the $A = 100$ (150) regions, the proton shells are defined by $Z = 38–50$ (50–64) for $N < 60$ (90), and by normal $Z = 28–50$ and 50–82 otherwise. In the $A = 190$ region a neutron gap at $N = 114$ is used for $Z = 78–80$.

To compare the different regions in Fig. 5, smooth curves have been drawn through the data points. For low R_4 values within each region, some structure is observed. This observation is similar to those observed by Casten *et al.* [3]. However, for our present purpose, this structure is not very important. These six smooth curves as functions of $(N_pN_n)_{\text{eff}}$ are shown in Fig. 6. The most remarkable result obtained in the present scheme is near coalescence of the curves for $A = 100$, 130, 150, and 230. If we compare our results with those obtained by Casten *et al.* using the parameter P, it is seen that all the special features mentioned in their work are reproduced in the present scheme, i.e., the horizontal compression and onset of deformation within the same range of $(N_pN_n)_{\text{eff}}$, regardless of mass region. In fact, each region passes through the transitional value $R_4 \approx 3.0$ around $(N_pN_n)_{\text{eff}} \approx 60$, which expresses the more universal character of $(N_pN_n)_{\text{eff}}$ as a structure variable, rather than the usual N_pN_n product.

A preliminary analysis of the data in $A = 130$ and 150 regions indicates the general applicability of the present scheme in predicting E_2 and $B(E2)$ values. In conclusion, it may be said that the inclusion of a suitably parametrized form of the saturation effect in the N_pN_n scheme adds a new dimension to it, in understanding the development of collectivity in different mass regions on a qualitative as well as quantitative basis. It may also be useful in studying the effective boson number problem [13,14] in the interacting boson model formalism. The success of the present scheme in reproducing the experimental E_2 and $B(E2)$ values within a reasonable accuracy suggests that the orbit dependence of the n-p interaction in any given shell is rather weak and gradual in nature. This feature is also reflected in the fact that the universal character of the development of collectivity in different mass regions is better revealed when R_4 is plotted against $(N_pN_n)_{\text{eff}}$ rather than the usual N_pN_n. Specificity of the orbitals in different major shells, and the resulting strength of the n-p interactions, are reflected in different values of the proportionality factors and the powers of the structure factor (SF) and the saturation parameter (SP) required to fit the experimental data in the $A \approx 170$ and 230 nuclei. Only detailed theoretical studies in different mass regions can throw light into this aspect of the problem. Energy and lifetime measurements in those rare-earth and actinide even-A nuclei, where experimental data do not exist, and remeasurement of lifetime in ^{172,174,176}Hf, are also suggested for further experimental verification of the present approach.

[1] R. F. Casten, Phys. Lett. **152B**, 145 (1985).
[2] R. F. Casten, Phys. Rev. Lett. **54**, 1991 (1985).
[3] R. F. Casten, D. S. Brenner, and P. E. Haustein, Phys. Rev. Lett. **58**, 658 (1987).
[4] H. Mach, Phys. Lett. B **185**, 20 (1987).
[5] D. Bonatsos, J. Phys. G **14**, 569 (1988).
[6] D. Bucurescu *et al.*, Phys. Lett. B **229**, 321 (1989).
[7] A. S. Mowbray *et al.*, Phys. Rev. C **42**, 1126 (1990).

[8] R. F. Casten *et al.*, Phys. Rev. C **45**, R1413 (1992).
[9] M. Saha and S. Sen, Phys. Rev. C **46**, R1587 (1992).
[10] R. F. Casten, K. Heyde, and A. Wolf, Phys. Lett. B **208**, 33 (1988).
[11] M. Sakai, At. Data Nucl. Data Tables **31**, 399 (1984); P. C. Sood, D. M. Headly, and R. K. Sheline, *ibid.* **47**, 89 (1991).
[12] S. Raman *et al.*, At. Data Nucl. Data Tables **36**, 1 (1987).
[13] O. Scholten, Phys. Rev. C **28**, 1783 (1983).
[14] S. T. Hsieh *et al.*, J. Phys. G **12**, L167 (1986).