Temperature and density effects on the one-pion exchange potential

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After considering the vacuum polarization and the radiative correction of the nucleon-nucleon-pion ($NN\pi$) system, the one-pion exchange potential with pseudovector coupling or pseudoscalar coupling at finite temperature and finite density is investigated. By using finite-temperature and density quantum field theory, we find that both the pseudovector and pseudoscalar effective coupling of $NN\pi$ decrease as temperature and/or density increase, and drop remarkably at high temperature regions and/or high density regions. At a critical temperature T_c or a critical density ρ_c, the effective $NN\pi$ coupling diminishes to zero and the one-pion exchange potential vanishes.

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It is generally accepted that the nucleon-nucleon (NN) interaction can be understood on the basis of the exchange of various mesons [1]. At large distances, the exchange of the pion meson gives the dominant contribution to the NN interaction. In the past few years, the behavior of nuclear matter under extreme conditions has attracted much attention [2–11]. This is partly due to the experiments of heavy-ion collisions which provide us with an opportunity for studying matter at high temperature as well as at high density, and partly due to address the equation of state for compact stellar objects such as supernova, neutron star, etc. In order to study the thermodynamical quantities of nuclear matter, it is essential to determine the temperature and density dependences on the NN interaction.

As a first step, we hope to investigate the temperature and density effects of the one-pion exchange potential (OPEP). In finite-temperature and -density quantum field theory, the quantum statistical effects of physical quantities can be calculated through the modification of particle propagators in hot and density media and by summing the same diagrams as that in the zero-temperature and -density quantum field theory [11–14]. In our previous work [11], after replacing the Feynman propagators of the nucleon and pion at zero temperature and density $G^0(k)=1/(\gamma\cdot k-m)$ and $\Delta_0(k)=1/(k^2-m_\pi^2)$ by the finite-temperature and -density propagators

$$G(k)=(\gamma\cdot k+m)\left[\frac{1}{k^2-m^2+i\epsilon}+\frac{i\pi}{E(k)}\delta(k_0-E(k))n_k\right.\\ \left.+\frac{i\pi}{E(k)}\delta(k_0+E(k))\bar{n}_k\right], \qquad (1)$$

where

$$E(k)=(\mathbf{k}^2+m^2)^{1/2},$$

$$n_k=\frac{1}{\exp[\beta(E(k)+\mu)]+1},$$

$$\bar{n}_k=\frac{1}{\exp[\beta(E(k)-\mu)]+1},$$

and

$$\Delta(k)=\frac{1}{k^2-m_\pi^2+i\epsilon}-2\pi i n_\pi(k)\delta(k^2-m_\pi^2) \qquad (2)$$

where $n_\pi=1/\exp[\beta\omega(k)-1]$, $\omega(k)=(\mathbf{k}^2+m_\pi^2)^{1/2}$, respectively, and summing the bubble diagrams [11] in the calculations of the vacuum polarization for the $NN\pi$ system in usual quantum field theory, we found that, because of the temperature and density dependences of the effective pion mass, the attractive interaction between the nucleon and nucleon becomes weaker as the temperature and/or density increases. But in Ref. [11], we focused our attention on the effective mass of the pion only.

It is well known that in quantum field theory not only the vacuum polarization but also the radiative correction have important effects on OPEP [15]. The renormalized $NN\pi$ coupling constant can be calculated from the three-line vertices graphs [15]. A complete calculation for the temperature and density effects on OPEP should include not only mass but also coupling constant corrections. The above concern has motivated us to extend our calculations to that with vertices corrections.

I. PSEUDOVECTOR COUPLING (PVC)

The Lagrangian density of nucleon and pion with pseudovector coupling interaction is

$$L=\frac{i}{2}\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-m\bar{\psi}\psi+\frac{1}{2}(\partial_{\mu}\phi)^2-\frac{1}{2}m_{\pi}^2\phi^2+L_I\ , \quad (3)$$

$$L_I=(g_v/m)\bar{\psi}\gamma^5\gamma^{\mu}\tau\psi\cdot\partial_{\mu}\phi\ , \quad (4)$$

where ψ and ϕ are the fields of nucleon and pion, and τ the isospin Pauli operator.

The effective coupling at finite temperature and finite density $\tilde{g}_v(\beta,\rho)$ for the three-lines vertices graphs correction is written as

$$-\gamma^5\gamma^{\mu}q_{\mu}\tau_i\tilde{g}_v(\beta,\rho)/m=-\gamma^5\gamma^{\mu}q_{\mu}\tau_i(g_v/m)-(g_v/m)\Lambda_v(p,p')\ , \quad (5)$$

where $\Lambda_v(p,p')$ is

$$\begin{aligned}-(g_v/m)\Lambda_v(p,p')(2\pi)^2\delta^4(p-q-p')=&\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}i\Delta(k)(-g_v/m)\gamma^5\gamma^{\mu}k_{\mu}\tau_i\\&\times(2\pi)^4\delta^4(p'-k-k'')iG(k')(-g_v/m)\gamma^5\gamma^{\nu}q_{\nu}\tau_j(2\pi)^4\delta(q+k''-k')\\&\times iG(k')(-g_v/m)\gamma^5\gamma^{\alpha}k_{\alpha}\tau_i(2\pi)^4\delta(k+k'-p)\ , \end{aligned}\quad (6)$$

where $G(k)$ and $\Delta(k)$ are given by Eqs. (1) and (2), respectively. From Eq. (5) we can obtain

$$-(g_v/m)\Lambda_v(p,p')=-(g_v/m)\Lambda_{v0}(p,p')(-g_v/m)\tilde{\Lambda}_{v(\beta,\rho)}(p,p')\ , \quad (7)$$

where $\Lambda_{v0}(p,p')$ is the contribution at zero-temperature divergence and can be calculated by introducing a form factor [16]. Since we are interested in the temperature and density effects, hereafter we only discuss the temperature and density dependence part $\tilde{\Lambda}_{v(\beta,\rho)}(p,p')$. Under the nonrelativistic limit $|\mathbf{p}|$, $|\mathbf{p}'|\ll m$, after some calculations we obtain

$$-(g_v/m)\tilde{\Lambda}_{v(\beta,\rho)}(p,p')=i(g_v/m)^3I\gamma\cdot q\gamma^5\tau_i\ , \quad (8)$$

where

$$\begin{aligned}I=&8m^3\int\frac{d|\mathbf{k}|}{(2\pi)^3}[E(k)-m]\frac{i\pi}{E(k)}n_k\frac{2m^2-2mE(k)-2|\mathbf{k}|^2-m_{\pi}^2}{[2m^2+2mE(k)-m_{\pi}^2]^2}\\&+8mi\pi\int\frac{|\mathbf{k}|^2d|\mathbf{k}|}{(2\pi)^3}n_k\frac{1}{2m^2-2mE(k)-m_{\pi}^2}+8m^3\int\frac{d|\mathbf{k}|}{(2\pi)^3}[E(k)+m]\frac{i\pi}{E(k)}\bar{n}_k\frac{2|\mathbf{k}|^2-2m^2-2mE(k)+m_{\pi}^2}{[2m^2+2mE(k)-m_{\pi}^2]^2}\\&-8mi\pi\int\frac{|\mathbf{k}|^2d|\mathbf{k}|}{(2\pi)^3}\bar{n}_k\frac{1}{2m^2+2mE(k)-m_{\pi}^2}\\&+4i\pi\int\frac{|\mathbf{k}|^2d|\mathbf{k}|}{(2\pi)^3}n_{\pi}(k)\frac{1}{\omega_k}\left[1+\frac{4m^2m_{\pi}^2(m_{\pi}^4+4\omega_k^2m^2)}{(m_{\pi}^4-4\omega_k^2m^2)}+\frac{8m^2\omega_k^2}{m_{\pi}^4-4\omega_k^2m^2}\right]\\&+\frac{8}{3}m^2\int\frac{|\mathbf{k}|^2d|\mathbf{k}|}{(2\pi)^3}\frac{i\pi}{E(k)}n_k\frac{1}{[2m^2-2mE(k)-m_{\pi}^2]^2}[6m^2-6mE(k)-3m_{\pi}^2+2|\mathbf{k}|^2]\\&+\frac{8}{3}m^2\int\frac{|\mathbf{k}|^2d|\mathbf{k}|}{(2\pi)^3}\frac{i\pi}{E(k)}\bar{n}_k\frac{1}{(2m^2-2mE(k)-m_{\pi}^2)^2}[6m^2-6mE(k)-3m_{\pi}^2+2|\mathbf{k}|^2]\\&+\frac{32}{3}m^2\int\frac{|\mathbf{k}|^4d|\mathbf{k}|}{(2\pi)^3}\frac{i\pi}{\omega_k}n_{\pi}(k)\frac{m_{\pi}^4+4m^2\omega_k^2}{(m_{\pi}^4-4m^2\omega_k^2)^2}\ .\end{aligned}\quad (9)$$

Substituting Eq. (8) into Eq. (5), we obtain

$$\tilde{g}_v(\beta,\rho)=g_v[1-i(g_v/m)^2I]\ . \quad (10)$$

Now let us turn to discuss OPEP. By using of the same consideration as in [8], after vertices corrections and summing the bubble diagrams in center-of-mass coordinates of the two-nucleon system, we find that OPEP at finite temperature and finite density is

$$V_v(r)=\frac{F\tilde{g}_v(\beta,\rho)}{2m}\frac{M_{\pi v}^3}{3\pi}[Z(x)S_{12}+Y(x)\sigma_1\cdot\sigma_2]\tau_1\cdot\tau_2\ , \quad (11)$$

where

$$\begin{aligned}&x=M_{\pi v}r,\quad Y(x)=e^{-x}/x\ ,\\&Z(x)=(1+3/x+3/x^2)Y(x)\ ,\\&F=[1+2(g_v/m)^2B]^{-1/2}\ ,\\&M_{\pi v}=Fm_{\pi}=\frac{m_{\pi}}{[1+2(g_v/m)^2B]^{1/2}}\ ,\end{aligned}\quad (12)$$

$$B=\frac{1}{\pi^2\beta^2}\int_0^\infty \frac{x^2dx}{\beta W}\left[\frac{1}{\exp[\beta(W-\mu)]+1}+\frac{1}{\exp[\beta(W+\mu)]+1}\right] . \tag{13}$$

The relation of density ρ and chemical potential μ is given by

$$\rho=\frac{f}{(2\pi)^3}\int d^3\mathbf{k}(n_k-\bar{n}_k) , \tag{14}$$

where $f=(2s+1)(2\tau+1)=4$ is the spin-isospin degeneracy. By using Eq. (14), we investigate the temperature effects on effective coupling and OPEP as well as density effects.

II. PSEUDOSCALAR COUPLING (PSC)

Now we are in the position to study the temperature and density effects on $NN\pi$ pseudoscalar coupling and corresponding OPEP. The interaction Lagrangian density is

$$L_I=-ig_s\bar{\psi}\gamma^5\boldsymbol{\tau}\psi\cdot\boldsymbol{\phi} . \tag{15}$$

The Feynman diagrams are the same as PVC, but with different Feynman rules. After some calculations, we obtain the effective coupling constant $\tilde{g}_s(\beta,\rho)$ with vertices correction as

$$\tilde{g}_s(\beta,\rho)=g_s(1-g_s^2I_s) , \tag{16}$$

where

$$I_s=\int\frac{d|\mathbf{k}|}{8\pi^2}\frac{n_k}{E(k)}\frac{1}{[2m^2-m_\pi-2mE(k)]^2}[2(m_\pi^2+m^2)E(k)^2-(4m^3-mm_\pi^2)E(k)-3m^2m_\pi^2+2m^4]$$
$$+\int\frac{d|\mathbf{k}|}{8\pi^2}\frac{\bar{n}_k}{E(k)}\frac{1}{[2m^2-m_\pi-2mE(k)]^2}[2(m_\pi^2+m^2)E(k)^2-(4m^3-mm_\pi^2)E(k)-3m^2m_\pi^2+2m^4]$$
$$-\int\frac{|\mathbf{k}|^2d|\mathbf{k}|}{(2\pi)^2}\frac{n_\pi(k)}{\omega_k}\left[\frac{1}{(m_\pi^2-2m\omega_k)^2}+\frac{1}{(m_\pi^2+2m\omega_k)^2}\right] . \tag{17}$$

After summing the bubble diagrams for the effective mass of the pion and taking the vertices corrections into account, the OPEP with PSC is obtained

$$V_s(r)=[H\tilde{g}_s(\beta,\rho)]^2\frac{M_{\pi s}}{4\pi}\frac{M_{\pi s}^2}{12m^2}[Z(\tilde{x})S_{12}+\tilde{Y}(\tilde{x})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \tag{18}$$

where $\tilde{x}=M_{\pi s}r$, $H=(1-g_s^2B/4m^2)^{-1/2}$, and $M_{\pi s}=H[m_\pi^2+g_s^2B]^{1/2}$.

Solving from Eqs. (9), (10), (12)–(14), (16)–(17), we get the finite-temperature and -density effective pseudovector and pseudoscalar couplings of $NN\pi$ and the effective OPEP with both mesons mass and vertices corrections. The numerical results are shown in Figs. 1–4. The parameters in our formulas are chosen as $g_v^2/4\pi=14.9$, $g_s^2/4\pi=13.0$, $m=938.3$ MeV, $m_\pi=138.0$ MeV.

The pseudovector effective coupling $\tilde{g}_v(\beta,\rho)$ against T curves for different densities are shown in Fig. 1. The similar curves, but for $\tilde{g}_s(\beta,\rho)$, are shown in Fig. 2. We find that the density dependences of $\tilde{g}_v(\beta,\rho)$ or $\tilde{g}_s(\beta,\rho)$ are very similar to that the of the temperature dependence.

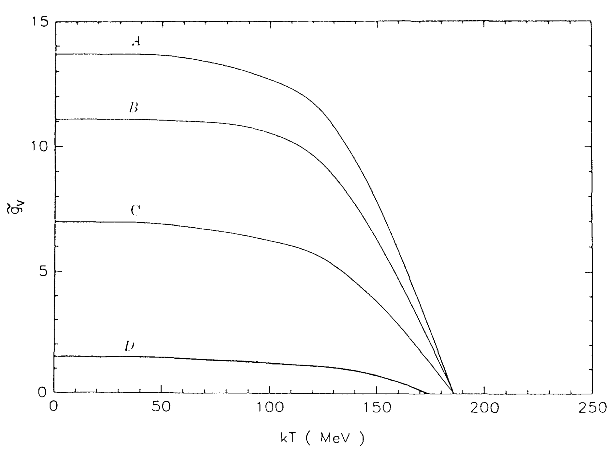

FIG. 1. The PVC $\tilde{g}_v$ vs kT curves for various densities: A, $\rho=1.00\times10^{-4}$ fm^{-3}; B, $\rho=0.100$ fm^{-3}; C, $\rho=0.170$ fm^{-3}; D, $\rho=0.250$ fm^{-3}.

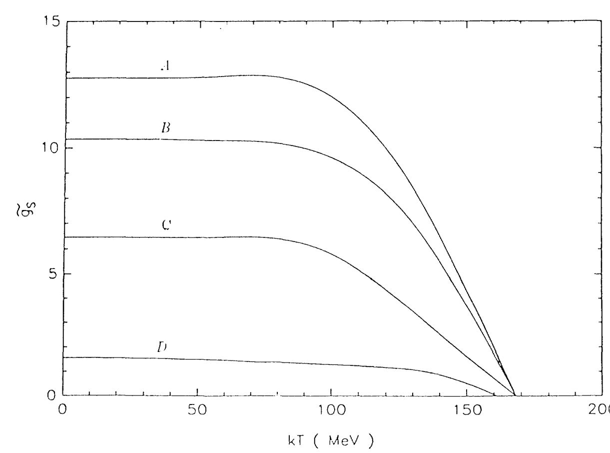

FIG. 2. Same as Fig. 1 except for PSC.

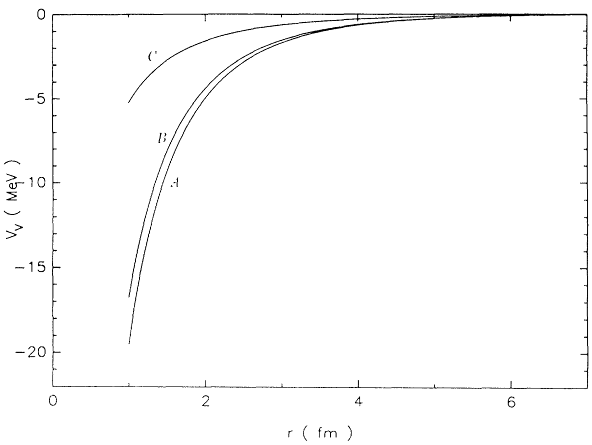

FIG. 3. The effective OPEP with PVC at various temperatures but fixed density $\rho = 0.100\ \mathrm{fm}^{-3}$: *A*, $kT = 50$ MeV; *B*, $kT = 100$ MeV; *C*, $kT = 160$ MeV.

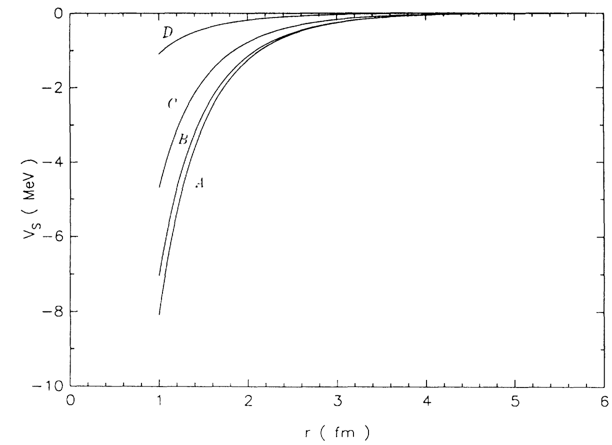

FIG. 4. Same as Fig. 3, except for PSC and *C*, $kt = 135$ MeV; *D*, $kT = 150$ MeV.

The effective coupling is almost unchanged in the low temperature region (with fixed density) or in the low density region (with fixed temperature), but changes considerably in the high temperature region or at high density. At a critical temperature T_c, $NN\pi$ coupling $\tilde{g}_v$ and $\tilde{g}_s$ drop to zero and the nucleon-nucleon-pion decouples. We find that the critical temperature $kT_c^v = 186$ MeV for PVC, $kT_c^s = 168$ MeV for PSC. As shown in Figs. 1 and 2, the critical temperatures are almost unchanged with density in low density regions. But in high density regions, the critical temperatures kT_c^v and kT_c^s will decrease as the density increases. For example, when $\rho = 0.250\ \mathrm{fm}^{-3}$, the critical temperatures become $kT_c^v = 175$ MeV and $kT_c^s = 161$ MeV. Similarly, if we draw the $\tilde{g}_v(\beta,\rho)$ vs ρ curves, we find that there is a critical density ρ_c^v or ρ_c^s at which the $NN\pi$ coupling $\tilde{g}_v$ (or $\tilde{g}_s$) equals zero. The values of $\rho_c^v = 0.320\ \mathrm{fm}^{-3}$ or $\rho_c^s = 0.340\ \mathrm{fm}^{-3}$ are almost independent on temperature in low temperature region ($kT \leq 100$ MeV). But in high temperature regions, for example, when the temperature increases to $kT = 150$ MeV, the critical density decreases to $\rho_c^v = 0.300\ \mathrm{fm}^{-3}$ and $\rho_c^s = 0.302\ \mathrm{fm}^{-3}$.

The OPEP curves at different temperature (with fixed density) for PVC and PSC are shown in Fig. 3 and Fig. 4, respectively. We find from these figures that the modulus of OPEP, namely, $|V_v(r)|$ [or $|V_s(r)|$ become small for fixed r as the temperature or density increases. The physical meaning of this result is that the attractive interaction between nucleon and nucleon at large distances becomes weaker because temperature or density plays a "repulsive" role. At critical temperature or critical density, $V_v(r)$ [or $V_s(r)$] approaches zero and the hadron gas phase transition takes place. These results, of course, are very reasonable.

It is of interest to compare the influences of the $NN\pi$ coupling $\tilde{g}_v$ (or $\tilde{g}_s$) with that of the effective pion mass $M_{\pi v}$ (or $M_{\pi s}$) upon OPEP. After comparing Fig. 3 in this paper with Fig. 4 in Ref. [11], we come to the conclusion that the most important temperature and density contributions on OPEP come from $\tilde{g}_v$ or $\tilde{g}_s$.

In summary, we should like to point out that influences of temperature and density on OPEP are very important. Our calculations show that the temperature and the density plays the same role. A complete theory which deals with the nuclear force in extreme condition must take both temperature and density effects into account.

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