Gamow-Teller matrix elements for two-neutrino double β decay within a second quasi-random-phase approximation

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A second quasi-random-phase approximation (QRPA) procedure is employed to perform calculations of the $M_{\rm GT}^{2\nu}$ matrix elements for several $\beta\beta$ emitters. It is found that higher-order QRPA (HQRPA) corrections display a weak dependence on particle-particle (pp) strength $g_{\rm pp}$ and, in the physical region (around $g_{\rm pp}=1.0$), they become important by comparison with QRPA predictions. It turns out that a further investigation of the HQRPA corrections could be illuminating to obtain more stable and reliable values for $M_{\rm GT}^{2\nu}$ matrix elements in this region, where they are very sensitive to the pp strength.

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In the last decade the double-β-decay phenomenon has been the subject of many experimental as well as theoretical investigations [1–7]. The interest in this field is mainly related to the neutrinoless double beta decay ($0\nu\beta\beta$) mode since its discovery would imply that (i) lepton number is not conserved; (ii) ν_e is a Majorana particle with a nonvanishing mass and/or there is a small right-handed component in the weak interaction. Reliable predictions for upper limits of neutrino mass and right handedness of the weak interaction are possible only if we have reliable calculations for the nuclear matrix elements (ME) which enter the half-life formulas. Since there are no experimental data to confirm the existence of the $0\nu\beta\beta$ decay mode, a good test for nuclear ME would be to use them for computing the two-neutrino double beta decay ($2\nu\beta\beta$) rates and then to compare the predictions to the existing data [8–11]. One of the many-body methods which were successfully used to compute the nuclear ME in double β decay is the proton-neutron (QRPA) (*pn* QRPA) [12–16]. In the frame of this approach the agreement with the experiment, for the predicted $\beta\beta$ half-lives, was achieved only when pp interactions are taken into account [14–17]. However, the drawback of these calculations is the fact that the Gamow-Teller (GT) ME ($M_{\rm GT}^{2\nu}$) are functions very sensitive on $g_{\rm pp}$, especially in the physical region. An attempt to improve the calculations has been to use a projected-QRPA procedure in order to restore the particle number and spin symmetries [17]. Another one has been to go beyond QRPA and to include HQRPA corrections in computing the GT ME [18].

The present work reports the results of the computation of $M_{\rm GT}^{2\nu}$ ME for several $\beta\beta$ emitters, performed in the frame of a second QRPA-type approach. This means that we used improved QRPA wave functions (by including HQRPA corrections) but we kept the HFB vacuum unchanged. The procedure is described in more detail in references [18]. Here we have extended the calculations to several $\beta\beta$ emitters in order to reach a final conclusion about the influence of HQRPA corrections upon the GT ME. We have shown that these corrections are functions slightly sensitive to the $g_{\rm pp}$ parameter and, in the physical region, they become important by comparison with QRPA predictions. To describe the properties of the parent, daughter, and intermediate nuclei involved in the $\beta\beta$ decay process we used a many-body Hamiltonian with a one body term describing the independent motion of the nucleons in a Woods-Saxon potential including Coulomb corrections and a nucleon-nucleon residual interaction taken as G matrices $\langle (ab)J|G|(cd)J\rangle$, $(J=0,1,2)$ calculated with Bonn OBEP. In our numerical calculations the single-particle space is restricted to two full shells. This truncation requires a renormalization of the two-body matrix elements which can be achieved by multiplying the G matrices

$$\langle (ab)J=0|G_\tau|(cd)J=0\rangle \ ,$$

$\tau=p,n$ by strength constants as follows: (i) for pairing interaction $(J=0)$ by $g_{\rm pp}^{\tau}$; (ii) for dipole interaction $(J=1)$ by $g_{\rm ph}^{(1)}$ if b, d are hole states and by $g_{\rm pp}^{(1)}$ when b,d are particle states; (iii) for quadrupole interaction $(J=2)$ by $g_{\rm ph}^{(2)}$ and $g_{\rm pp}^{(2)}$ both for the proton and neutron system. Accordingly to the previous experience for such type of calculations these constants are not far from unity and are fixed in the following way: $g_{\rm pp}^{(\tau)}$ are chosen so that the experimental odd-even mass differences are reproduced. $g_{\rm ph}^{(1)}$ is fitted so that the position of the GT resonance for the intermediate odd-odd nucleus be reproduced and $g_{\rm ph}^{(2)}$ is determined by fitting the experimental energy of the first 2^+ state for the parent and daughter nuclei. $g_{\rm pp}^{(2)}$ is set equal to unity while $g_{\rm pp}^{(1)}$ is varied from 0.0 to 1.15. In the frame of the QRPA approach phonon states of a nucleus are described by applying the corresponding operators:

$$\Gamma_{s\mu}^{+}(l)=\sum_k [x_l^s(k)A_{s\mu}^{\dagger}(k)-Y_l^s(K)A_{s-\mu}(k)(-1)^{s+\mu}] \qquad (1)$$

to the QRPA vacuum state. The index s takes on the

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values 1 or 2 for particle-unlike or particle-like QRPA approaches. X and Y are the upwards and backwards QRPA amplitudes while $A^{\dagger}_{s\nu}(k)$ are the pair operators and are defined as two quasiparticle tensors of rank s. The summation index k stands for the (p,n) pair states or for (pp) and (nn) pair states for particle-unlike or particle-like QRPA approaches, respectively. As is well known, QRPA can be viewed as the first term in a boson expansion of certain quasiparticle operator combinations. To introduce HQRPA corrections we have developed the quasiparticle pair operators $A^{\dagger}_{s\mu}(k)$ in terms of QRPA boson operators $\Gamma^{\dagger}_{s\mu}(k)$ and retained the next order beyond QRPA in the series expansion. In this way the properties of the initial, intermediate and final nuclei involved in the $\beta\beta$ decay are described by the following multiphonon states.

(1) Initial (i):

$$|0\rangle_i \ , \quad {}_i\Gamma^{\dagger}_{2\mu}(k)|0\rangle_i \ ; \tag{2}$$

(2) intermediate (int):

$$ {}_i\Gamma^{\dagger}_{1\mu}(k)|0\rangle_i \ , \quad ({}_i\Gamma^{\dagger}_{1\mu_1}(k_1)\,{}_i\Gamma^{\dagger}_{2\mu_2}(k_2))_{1\mu}|0\rangle_i \ ; \tag{3a}$$

$$ {}_f\Gamma^{\dagger}_{1\mu}(k)|0\rangle_f \ , \quad ({}_f\Gamma^{\dagger}_{1\mu_1}(k_1)\,{}_f\Gamma^{\dagger}_{2\mu_2}(k_2))_{1\mu}|0\rangle f \ ; \tag{3b}$$

(3) final (f):

$$|0\rangle_f \ , \quad {}_f\Gamma^{\dagger}_{2\mu}(k)|0\rangle_f \ . \tag{4}$$

The two sets of multiphonon states which describe the intermediate nucleus come from two different QRPA procedures applied for the initial and final nuclei, respectively. The $2\nu\beta\beta$ decay rate for the transition $0^+_i \to 0^+_f$ has the expression:

$$M_{\mathrm{GT}} = \sum_{k,m} \frac{{}_f\langle 0\|\beta^{\dagger}\|km\rangle_f \langle km|km'\rangle_i \langle km'\|\beta^{\dagger}\|0\rangle_i}{E_{\mathrm{int}} - E_f + mc^2 + Q_{\beta\beta}/2} = \sum_{k=1,2} M^{(k)}_{\mathrm{GT}} \ , \tag{5}$$

where $k=1$ and 2 means one- and two-boson states, respectively. In order to calculate the ME of the transition operators $\beta^{\dagger}$ from (5), we have also to expand their QRPA expressions in terms of Γ boson operators. Thus, for the $\beta^{\dagger}$ operator one obtains:

$$\beta^{\dagger}_{\mu} = \sum_k [B^{(10)}_k \Gamma^{\dagger}_{1\mu}(k) + B^{(01)}_k \Gamma_{1-\mu}(k)(-1)^{1-\mu}] + \sum_{k_1,k_2} [B^{(20)}_{k_1k_2}(\Gamma^{\dagger}_1(k_1)\Gamma^{\dagger}_2(k_2))_{1\mu} + B^{(02)}_{k_1k_2}(\Gamma_1(k_1)\Gamma_2(k_2))_{1\mu}]$$
$$+ \sum_{k_1,k_2} [B^{(12)}_{k_1k_2}(\Gamma^{\dagger}_1(k_1)\Gamma_2(k_2))_{1\mu} + B^{(21)}_{k_1k_2}(\Gamma^{\dagger}_2(k_2)\Gamma_1(k_1))_{1\mu}] \ . \tag{6}$$

The coefficients B were obtained by requiring that the ME of the $\beta^{\dagger}_{\mu}$ operators, in the boson basis, be identical to those corresponding to the rhs of Eq. (6) [20]. If, in the expansion (6), one keeps only the linear terms, one find the QRPA boson image of the fermion $\beta^{\dagger}_{\mu}$ operators. We have neglected in (6) the terms of the type $(\Gamma^{\dagger}_1\Gamma_2)_{1\mu}$ since they have a vanishing effect on the transition $0^+_i \to 0^+_f$. We have also neglected all three boson state contributions for the following reasons: (i) if one takes into account only two-boson-state contributions to the HQRPA corrections, the sum rules which single beta strengths must fulfill very accurate in the frame of QRPA $\sum_l [\langle \beta^+ \rangle^2_l - \langle \beta^- \rangle^2_l] = 3(N-Z)$, are violated only about 1%. (ii) One can easily show [21] that the expression (6) of the β operators still commutes with the expansions of particle number and isospin operators taken in the same approximation as (6). If we add three boson state contributions, the communtation relations mentioned above no longer hold and we could have an additional source of spurious contributions during the computation. Accordingly with the previous results from boson expansion theory these three boson state contributions should be at most of the same order of magnitude as two boson state ones and hence do not modify qualitatively our conclusion about the influence that HQRPA corrections could have on GT ME. Our numerical results refer to six $\beta\beta$ transitions: ^{76}Ge$\to$^{76}Se, ^{110}Cd$\to$^{110}Pd, ^{116}Cd$\to$^{116}Sn, ^{128}Te$\to$^{128}Xe, ^{130}Te$\to$^{130}Xe, ^{136}Ba$\to$^{136}Xe. The $M^{2\nu}_{\mathrm{GT}}(g_{\mathrm{pp}})$ functions are plotted in Figs. 1–3: the upper curves (which drop steeply with respect to g_{pp}) represent the pnQRPA result of the computation of these ME while the lower curves represent HQRPA corrections as functions of g_{pp}. One can see that these corrections are functions more stable on this parameter. In spite of the small

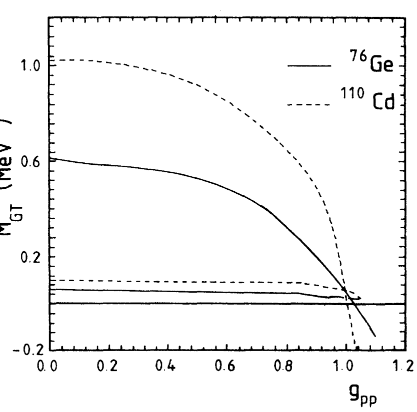

FIG. 1. The GT ME M_{GT} are plotted as functions of $g^{(1)}_{\mathrm{pp}}$ for ^{76}Ge (solid lines) and ^{110}Cd (dashed lines) cases. The upper curves (which drop steeply with respect to g_{pp}) represent the pnQRPA result while the lower curves represent the HQRPA corrections. One can see that these corrections are functions more stable on this parameter and become important around $g_{\mathrm{pp}}=1$. The inclusion of them shifts the zero point of the $M^{2\nu}_{\mathrm{GT}}$ functions, computed in the frame of QRPA, to the region where the QRPA procedure collapses.

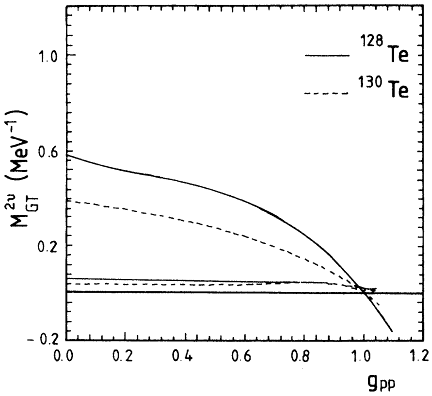

FIG. 2. The same as Fig. 1 but for ^{128}Te and ^{130}Te cases.

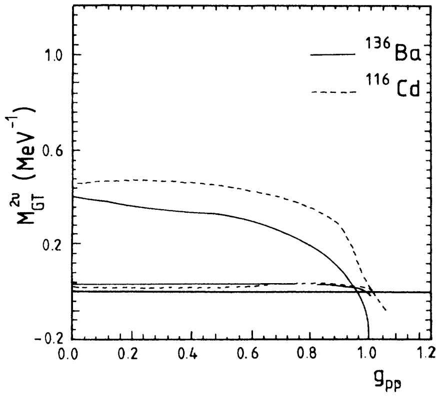

FIG. 3. The same as Fig. 1 but for ^{136}Ba and ^{116}Cd cases.

values of the HQRPA corrections, they become important by comparison with QRPA values just around $g_{pp}=1.0$, where the desired suppression for M_{GT} is acquired, for all the transitions which are studied. By including them one gains stability in the computation of the GT ME since the zero point of the function $M^{2\nu}_{GT}(g_{pp})$ is shifted to the region where QRPA procedure collapses. This happens because in our formalism are included not only proton-neutron interactions (as in the standard *pn*QRPA) but also proton-proton and neutron-neutron interactions (through HQRPA corrections). These particle-like interactions are also present in extended QRPA-type theories [19]. Concluding, a further investigation of the HQRPA corrections could be illuminating for reliable predictions of the GT ME. For example as an extension of our formalism one can take also into account three boson state contributions by using a projection procedure in order to avoid the spurious states contributions which violate the single beta sum rules by about 20%. Some work is in progress on this line. On the other hand, additional meaningful correlations could be included if an extended QRPA approach [19] would be employed where, beside the higher-order correlations to the QRPA wave functions, an improved HFB vacuum is also used.

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