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Induced pseudoscalar axial current in polarized nuclear β decay

S. Ying
Department of Physics, FM-15, University of Washington, Seattle, Washington 98195
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In this paper we present a study of all spin-dependent observables (without observing the neutrinos), which are even under time reversal, for the $1^+ \to 0^+$ $\beta^{\pm}$ decay transitions. Terms of the hadronic response function up to order κ/M and $\kappa^2 R^2$ in the momentum transfer κ are included. The completeness of the parametrization of the decay rates of the present approach is demonstrated. Albeit the results obtained in this paper contain a complete set of one-body weak current form factors (except for the scalar form factor F_S), particular attention is paid to the possible extraction of the induced pseudoscalar form factor g_P from observables in which charged lepton polarizations are measured. The theoretical uncertainties and the experimental feasibility are studied to some extent. It is shown that g_P can in principle be determined in precision β decay experiments where the charged lepton polarizations are measured.

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I. INTRODUCTION

The induced pseudoscalar part of the matrix elements of the nuclear axial vector current operator provides a window through which strong interaction effects manifest themselves in weak interaction processes. Partial conservation of axial vector current (PCAC) [1, 2] requires that there is a pion pole in the pseudoscalar part of the matrix elements of the hadronic axial vector current operator between single nucleon states and that the pseudoscalar form factor g_P is related to the pion and pion-nucleon interaction parameters through

$$g_P(q^2) = -2\frac{g_{\pi N} f_\pi}{q^2 - m_\pi^2}, \tag{1}$$

with the pion-nucleon coupling constant $g_{\pi N} = 13.5$ and the pion decay constant $|f_\pi| = 93.3$ MeV defined through the PCAC relation

$$\partial^\mu A_\mu^a(x) = m_\pi^2 f_\pi \pi^a(x). \tag{2}$$

$a = 1,2,3$ is an isospin index. The induced pseudoscalar form factor is the only one that changes rapidly with q^2 at low momentum transfers due to the low lying pion pole. It is sensitive to certain dynamical effects in strong interaction systems [3, 4]. The possible dynamical quenching of g_P in heavy nuclei [5] has not been established conclusively and can be checked by independent experiments. Whether or not the PCAC relation, Eq. (1), holds for a single nucleon in free space[1] or inside light nuclei is not a settled problem [5].

If the assumption that there is an underlying spontaneously broken chiral symmetry in the strong interaction vacuum is made, the PCAC relation immediately follows. The acceptance of QCD as the theory of strong interaction leads, almost conclusively, to the statement that the phenomenologically successful PCAC relationship originates from an approximate chiral symmetry (explicitly broken by small current quark masses) spontaneously broken down due to the strong interaction in the QCD vacuum. Further, a model recently studied [7] suggests a new possibility of realizing the spontaneous chiral symmetry breaking in a massless fermionic system, which, among other observable effects, results in a modification in a baryonic system of the relationship between g_P and the pion and pion-nucleon interaction parameters expressed by Eq. (1). Since full QCD has not been solved and there exists possible deviations of the value of g_P from its PCAC one in experimental observations, a precise determination of g_P can provide more definite information concerning the structure of a nucleon.

It will be demonstrated in the following sections that g_P is not accessible in semileptonic weak interactions involving electrons when the charged lepton polarizations are not observed partly due to our uncertainties in nuclear structure. To date g_P has been measured only in muon capture (MC) and radiative muon capture (RMC) experiments [5, 8–10]. We shall demonstrate that spin-dependent observables in β decay experiments provide means to determine g_P in a model independent way.

In Sec. II, the hadronic axial vector current operator is separated into two parts by isolating the pion dominated pseudoscalar piece. In Sec. III, the general polarization observables without observing the neutrinos in the β decay processes corresponding to a $1^+ \to 0^+$ transition is obtained up to order κ/M and $\kappa^2 R^2$, with κ the magnitude of the three-momentum transfer, M the nucleon mass, and R a typical size of a nucleus. In Sec. IV, the theoretical uncertainties are discussed. The possibility of extracting g_P from alignment observables is investigated. Section V is a summary.

[1] For a single nucleon in free space, the world average of recent measurements of the value of g_P is quite close to the PCAC one [6]. However, each individual measurement that contributes to the average has errors larger than 40%.

II. WEAK CURRENTS AND TRANSITION MATRIX ELEMENTS

The matrix elements of the nuclear axial vector current operator can be written as[2]

$$\langle f_h | A_\mu^{(\pm)}(x) | i_h \rangle = \langle f_h | \partial_\mu \hat{\Gamma}^{(\pm)5}(x) + \tilde{A}_\mu^{(\pm)}(x) | i_h \rangle, \quad (3)$$

where the pion-dominated pseudoscalar piece is written as $\partial_\mu \hat{\Gamma}^{(\pm)5}(x)$ and the remaining part is denoted by $\tilde{A}_\mu^{(\pm)}(x)$. $\hat{\Gamma}^{a5}(x)$ can be related to the pion field operator near the pion pole (i.e., for certain matrix elements) as

$$\hat{\Gamma}^{a5}(x) = f_\pi \pi^a(x). \quad (4)$$

Equation (3) is not manifestly gauge invariant. In the literature [11], a gauge invariant version of Eq. (3) is usually used, namely,

$$\langle f_h | A_\mu^{(\pm)}(x) | i_h \rangle = \langle f_h | (\partial_\mu \pm ieA_\mu^{\rm em}) \hat{\Gamma}^{(\pm)5}(x) + \tilde{A}_\mu^{\prime(\pm)}(x) | i_h \rangle, \quad (5)$$

where $A_\mu^{\rm em}$ is the electromagnetic vector field inside the system. The differences in the first term (namely, the pseudoscalar contribution) of Eqs. (3) and (5) are compensated by the remaining second term of the same pair of equations if a full gauge invariant calculation is performed. Albeit to achieve such a compensation can be a difficult task in a practical computation, the uncertainty resulting from these two ways of separating out the pseudoscalar part from the matrix elements of the axial vector current operator can be studied by comparing the results of these two choices. We develop our formulas using Eq. (3) in the following and will point out the differences between these two approaches whenever it is necessary.

To first order in the weak interaction coupling constant G ($\approx 10^{-5}/M^2$), the weak transition matrix (T matrix) is

$$T_{\rm fi} = \frac{G\cos\theta_C}{\sqrt{2}} \int d^4x \left\{ \langle f_l | j^{(-)\mu}(x) | i_l \rangle \langle f_h | V_\mu^{(+)}(x) + A_\mu^{(+)}(x) | i_h \rangle + \text{H.c.} \right\}, \quad (6)$$

where θ_C is the Cabbibo angle and $\langle f_l | j^{(\mp)\mu}(x) | i_l \rangle$ is a matrix element of the leptonic weak current. The pseudoscalar contribution is therefore

$$T_{\rm fi}^{\rm ps} = \frac{G\cos\theta_C}{\sqrt{2}} \int d^4x \left\{ \langle f_l | i\partial_\mu j^{(-)\mu}(x) | i_l \rangle \langle f_h | i\hat{\Gamma}^{(+)5}(x) | i_h \rangle + \text{H.c.} \right\}, \quad (7)$$

where an integration by part has been performed. The matrix element of the divergence of the leptonic weak current $\langle f_l | i\partial_\mu j^{(\mp)\mu}(x) | i_l \rangle$ for the charged current reaction can be evaluated most simply by using the Dirac equation for the leptonic wave function. In the case of β decay, the divergencies of the leptonic weak currents are for β^- decay:

$$i\partial_\mu \langle e\bar{\nu}_e | j^{(-)\mu}(x) | 0 \rangle = -V^{(-)}(x)\bar{\psi}_e(x)\gamma^0(1-\gamma^5)\phi_{\bar{\nu}_e}(x) - m\bar{\psi}_e(x)(1-\gamma^5)\phi_{\bar{\nu}_e}(x), \quad (8)$$

for β^+ decay:

$$i\partial_\mu \langle e^+\nu_e | j^{(+)\mu}(x) | 0 \rangle = V^{(+)}(x)\bar{\phi}_{\nu_e}(x)\gamma^0(1-\gamma^5)\psi_{e^+}(x) + m\bar{\phi}_{\nu_e}(x)(1+\gamma^5)\psi_{e^+}(x). \quad (9)$$

Here m is the electron mass and $V^{(\mp)}$ is the Coulomb potential for the charged leptons. It can be seen that in these cases, the divergencies of the leptonic weak currents are combinations of the time component of the original leptonic weak current multiplied by the Coulomb potential and a scalar part with only right- (left-) handed electron (positron) contributions. The property of the latter piece can be utilized to extract g_P from β decay experiments.

In the standard model the charged weak currents pertain only to the left- (right-) handed electron (positron) in the limit that the electron mass vanishes. For a finite electron mass, except for the leptonic weak current that couples to the pseudoscalar piece of the hadronic axial vector currents, the matrix element for the right- (left-) handed electron (positron), which we refer to as *rare* polarization in the following, are suppressed by m/ϵ, with ϵ the energy of the charged lepton. The terms in Eqs. (8) and (9) that are proportional to the Coulomb potentials can be absorbed into the hadronic axial charge operators, since they couple to the time component of the leptonic weak currents. The other terms in Eqs. (8) and (9) that are proportional to m are not suppressed further (in the power expansion in terms of m) for *rare* polariza-

[2] In a many-nucleon system, there is another term of the form $n_\mu n \cdot \partial \hat{\Gamma}^{(\pm)5'}(x)$ generated by the medium [4], where n_μ is a timelike vector that determines the medium's motion and satisfies $n^2 = 1$. The effective single-nucleon form factor g_P can be different from the one in the muon capture experiments, since the charged leptons in β decay are relativistic whereas the muon in muon capture experiments is nonrelativistic. The value of g_P extracted from β decay experiments does not directly correspond to the value of g_P obtained from muon capture experiments. The relation between these two is a dynamical problem which deserves a separate treatment in a more detailed study.

tion states; they provide g_P a chance to manifest itself in β decay processes when the polarization of the charged lepton is detected. The unpolarized β decay rates for nuclei are dominated by allowed matrix elements, whereas the matrix elements of the induced pseudoscalar part of the hadronic axial vector current operator are of recoil order; they are suppressed by a factor of ϵ/M. Thus, it is still practically difficult to extract g_P from the β decay of unpolarized nuclei by measuring *rare* polarization decay rates. In the following, we reveal the fact that in the β decay of a polarized nucleus, in which charged lepton polarization are detected, g_P can appear as the same order as other leading order terms.

The T matrix, Eq. (6), can be written as

$$\langle f\,|H_W|\,i\rangle = \frac{G\cos\theta_C}{\sqrt{2}}\int\frac{d^3q}{(2\pi)^3}\left[\langle f_h\,|\tilde{J}^{(+)}_\mu(-\mathbf{q})|\,i_h\rangle\langle f_l\,|j^{(-)\mu}(\mathbf{q})|\,i_l\rangle + \langle f_h\,|i\hat{\Gamma}^{5(+)}(-\mathbf{q})|\,i_h\rangle\langle f_l\,|j^{(-)}_s(\mathbf{q})|\,i_l\rangle + \text{H.c.}\right]. \quad (10)$$

Operators and functions with argument $\mathbf{q}$ are the Fourier transform of the corresponding objects in coordinate space, i.e.,

$$f(\mathbf{q}) = \int d^3x\, e^{i\mathbf{x}\cdot\mathbf{q}} f(\mathbf{x}, t=0) \quad (11)$$

and the divergencies of the leptonic weak charged currents are

$$j^{(\pm)}_s(x) = i\partial^\mu j^{(\pm)}_\mu(x). \quad (12)$$

The tilded hadronic weak currents are

$$\tilde{J}^{(\pm)0}(-\mathbf{q}) = \rho^{(\pm)0}(-\mathbf{q}) + \rho^{(\pm)5}_{\text{eff}}(-\mathbf{q}), \quad (13)$$

$$\tilde{J}^{(\pm)i}(-\mathbf{q}) = V^{(\pm)i}(-\mathbf{q}) + \tilde{A}^{(\pm)i}(-\mathbf{q}), \quad (14)$$

where the effective axial charge density operator is

$$\rho^{(\pm)5}_{\text{eff}}(-\mathbf{q}) = \tilde{\rho}^{(\pm)5}(-\mathbf{q}) + \int\frac{d^3q'}{(2\pi)^3}V(\mathbf{q}'-\mathbf{q})i\hat{\Gamma}^{(\pm)5}(-\mathbf{q}'), \quad (15)$$

with $\tilde{\rho}^{(\pm)5} = \tilde{A}^{(\pm)0}$ and $V(\mathbf{q}) = Ze^2/|\mathbf{q}|^2$ if screening and nuclear finite-size effects are not considered. The piece in $\rho^{(\pm)5}_{\text{eff}}(-\mathbf{q})$ involving $V(\mathbf{q})$ is from the terms that are proportional to $V^{(\pm)}$ in Eqs. (8) and (9). Had we taken Eq. (5) instead of Eq. (3), the term proportional to $V^{(\pm)}$ in Eq. (15) would have been absent. This is the only formal difference resulting from the choices between Eqs. (3) and (5) so far. If the Coulomb distortion [except for the term in Eq. (15)] is neglected, the leptonic matrix elements can be written as

$$\langle f_l\,|j^{(\mp)}_\mu(\mathbf{q})|\,i_l\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}'+\mathbf{q})l^{(\mp)}_\mu(\mathbf{k},\mathbf{k}'), \quad (16)$$

$$\langle f_l\,|j^{(\mp)}_s(\mathbf{q})|\,i_l\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}'+\mathbf{q})l^{(\mp)}_s(\mathbf{k},\mathbf{k}'), \quad (17)$$

with

$$\begin{aligned} l^{(-)}_\mu(\mathbf{k},\mathbf{k}') &= \bar{u}_e(\mathbf{k})\gamma_\mu(1-\gamma^5)v_{\bar{\nu}}(\mathbf{k}'),\, l^{(-)}_s(\mathbf{k},\mathbf{k}') \\ &= m\bar{u}_e(\mathbf{k})(1-\gamma^5)v_{\bar{\nu}}(\mathbf{k}'), \end{aligned} \quad (18)$$

$$\begin{aligned} l^{(+)}_\mu(\mathbf{k},\mathbf{k}') &= \bar{u}_\nu(\mathbf{k}')\gamma_\mu(1-\gamma^5)v_{e^+}(\mathbf{k}),\, l^{(+)}_s(\mathbf{k},\mathbf{k}') \\ &= m\bar{u}_\nu(\mathbf{k}')(1+\gamma^5)v_{e^+}(\mathbf{k}). \end{aligned} \quad (19)$$

The charged lepton (neutrino) three-momentum is denoted by $\mathbf{k}$ ($\mathbf{k}'$). The δ functions in Eqs. (16) and (17) ensure, when the Coulomb distortion is not taken into account, that the momentum transfer at the weak interaction vertex is controlled by external lepton momenta. Albeit the Coulomb distortion effects can be included in a systematic manner [12] in the present approach (see Appendix B), we shall not consider them here as a first step.

Next, we expand the hadronic weak currents in terms of their multipoles [13]. Using Wigner's D functions,

$$i\hat{\Gamma}^5(-\mathbf{q}) = 4\pi\sum_{J=0}^{\infty} i^J\sqrt{\frac{2J+1}{4\pi}}\sum_{m=-J}^{J} D^{J*}_{0,m}(\hat{\mathbf{q}})\hat{S}^5_{J,m}(\kappa), \quad (20)$$

$$\tilde{J}^0(-\mathbf{q}) = 4\pi\sum_{J=0}^{\infty} i^J\sqrt{\frac{2J+1}{4\pi}}\sum_{m=-J}^{J} D^{J*}_{0,m}(\hat{\mathbf{q}})\hat{C}_{J,m}(\kappa), \quad (21)$$

$$\tilde{J}^3(-\mathbf{q}) = -4\pi\sum_{J=0}^{\infty} i^J\sqrt{\frac{2J+1}{4\pi}}\sum_{m=-J}^{J} D^{J*}_{0,m}(\hat{\mathbf{q}})\hat{L}_{J,m}(\kappa), \quad (22)$$

$$\tilde{J}^\lambda(-\mathbf{q}) = -4\pi\sum_{J=1}^{\infty} i^J\sqrt{\frac{2J+1}{4\pi}}\sum_{m=-J}^{J} D^{J*}_{\lambda,m}(\hat{\mathbf{q}})\hat{T}^\lambda_{J,m}(\kappa), \quad (23)$$

where the isospin indices $(\pm)$ on the operators are suppressed and $\hat{T}^\lambda_{J,m} = \hat{T}^{\text{el}}_{J,m} + \lambda\hat{T}^{\text{mag}}_{J,m}$. Here $\lambda = \pm 1$ is the helicity label and $D^J_{m,m'}(\hat{\mathbf{q}})$ is a shorthand notation for $D^J_{m,m'}(\phi,\theta,-\phi)$ with (θ,ϕ) the polar angle of $\hat{\mathbf{q}}$. Conventional definitions for the multipole operators $\hat{C}_{Jm}$, $\hat{L}_{Jm}$, $\hat{T}^{\text{el}}_{Jm}$ and $\hat{T}^{\text{mag}}_{Jm}$ are adopted (see, e.g., Refs. [13, 14]). The label $\hat{C}_{Jm}$ is used in place of $\hat{M}_{Jm}$ of Ref. [13], and $\hat{S}^5_{Jm}$ is defined as

$$\hat{S}^5_{Jm} = \int d^3x\, j_J(\kappa x)Y_{Jm}(\hat{\mathbf{x}})i\hat{\Gamma}^5(x)|_{t=0}, \quad (24)$$

where the phase convention $Y^*_{lm} = (-1)^m Y_{l-m}$ is adopted.

With the Bjorken and Drell convention for Dirac matrices [15], the matrix elements of the hadronic weak currents between single-nucleon states are of the following form:

$$\langle p\,|V^{(+)\mu}(-\mathbf{q})|\,n\rangle = (2\pi)^3\delta(\mathbf{p}_p-\mathbf{p}_n-\mathbf{q})\bar{u}(\mathbf{p}_p)\left(F_1^{(+)}\gamma^\mu + F_2^{(+)}\frac{iq_\nu}{2M}\sigma^{\mu\nu}\right)u(\mathbf{p}_n), \tag{25}$$

$$\langle p\,|\tilde{A}^{(+)\mu}(-\mathbf{q})|\,n\rangle = (2\pi)^3\delta(\mathbf{p}_p-\mathbf{p}_n-\mathbf{q})\bar{u}(\mathbf{p}_p)\left(g_A^{(+)}\gamma^\mu\gamma^5 + g_T^{(+)}\frac{iq_\nu}{2M}\sigma^{\mu\nu}\gamma^5\right)u(\mathbf{p}_n), \tag{26}$$

$$\langle p\,|i\hat{\Gamma}^{(+)5}(-\mathbf{q})|\,n\rangle = -(2\pi)^3\delta(\mathbf{p}_p-\mathbf{p}_n-\mathbf{q})g_P^{(+)}\bar{u}(\mathbf{p}_p)\gamma^5 u(\mathbf{p}_n), \tag{27}$$

where form factors for the charge lowering transition (i.e., the $p \to n$ transition) are obtained by interchanging proton and neutron quantum numbers and replacing the form factors with a "+" superscript by a "–" superscript. Hermiticity and time reversal invariance restrict all "+" form factors except $g_T^{(+)}$ to be the same as the corresponding "–" form factors, which is consistent with charge symmetry and conserved vector current (CVC), whereas $g_T^{(-)} = -g_T^{(+)}$. This latter equality corresponds to a maximum violation of charge symmetry (or isospin rule) [13, 16]. It should be noted that nuclear charge-symmetry-breaking effects can mimic the time reversal invariant part of the second class currents in β decay experiments. CVC predicts that the weak form factors for the vector currents are related to those of the electromagnetic form factors [17], namely,

$$F_1^{(\pm)}(q^2=0) = F_1^V(q^2=0) = 1, \tag{28}$$

$$F_2^{(\pm)}(q^2=0) = F_2^V(q^2=0) = \mu_p - \mu_n - 1 = 3.706. \tag{29}$$

$g_A^{(\pm)}(q^2=0) = -1.26$. The value of the other two form factors $g_P^{(\pm)}$ and $g_T^{(\pm)}$ are still poorly known from direct experimental observations. The existence of $g_T^{(\pm)}$ indicates a violation of G parity; the corresponding induced current is called a second-class current [18]. Time reversal invariance can be violated when $g_T^{(\pm)} \neq 0$ and charge symmetry is preserved. In this case, the second-class current form factor $g_T^{(\pm)}$ acquires a phase that violates time reversal invariance (see, e.g., Ref. [12]). The time reversal invariant part of $g_T^{(\pm)}$ has been studied in nuclear β decay and it was found to be $|g_T^{(\pm)}| \leq 0.25|g_A^{(\pm)}|$ [19, 20]. Information on the possible time reversal violating part of $g_T^{(\pm)}$ is still scarce.[3] PCAC predicts the value of $g_P^{(\pm)}$ to be given by Eq. (1) for a single nucleon. In a light nucleus, g_P is expected to follow the PCAC value. In a many-nucleon system, $g_P^{(\pm)}$ is expected to be quenched from its "bare" value given by Eq. (1) [5]. The value of g_P seems to be systematically larger than the PCAC one in muon capture experiments involving light nuclei [5, 8–10]. Independent experiments can be carried out to confirm these deviations.

III. $1^+ \to 0^+$ TRANSITION: NEUTRINOS NOT OBSERVED

When a first-order approximation in G is taken, the differential rate for a β decay transition with detection of the final state polarization is

$$dW = 2\pi\delta(E_f - E_i + \Delta)\mathrm{tr}(\rho_i H_W^\dagger \rho_f H_W)d\xi, \tag{30}$$

where ρ_i is the initial state density operator, ρ_f is the final state polarization density operator, H_W is the weak transition Hamiltonian, E_i and E_f are the initial and final state energies of the nuclei involved, respectively, Δ is the maximum energy of the charged lepton, and $d\xi$ is an infinitesimal phase space element of the final states. The trace is limited to the initial and final polarization subspaces.

Following Eq. (10), Eq. (30) has the following form:

$$dW = 2\pi\delta(E_f - E_i + \Delta)\frac{G^2\cos^2\theta_C}{2}\left[U(l_s l_s^*) + Z_S^\mu(l_s l_\mu^*)_S + Z_A^\mu(l_s l_\mu^*)_A + W^{\mu\nu}(l_\mu l_\nu^*)\right]d\xi, \tag{31}$$

where the leptonic tensors are

$$(l_a l_b^*) = \mathrm{tr}\rho_f l_a l_b^* \quad (a,b\,=\,s \text{ or } 0,1,2,3), \tag{32}$$

$$(l_s l_\mu^*)_S = \frac{1}{2}(l_s l_\mu^* + l_\mu l_s^*), \tag{33}$$

$$(l_s l_\mu^*)_A = \frac{1}{2}(l_s l_\mu^* - l_\mu l_s^*), \tag{34}$$

and the hadronic response functions are

$$U = \mathrm{tr}\rho_i(i\hat{\Gamma}^{(\pm)5})^\dagger(i\hat{\Gamma}^{(\pm)5}), \tag{35}$$

$$Z_S^\mu = \mathrm{tr}\rho_i\left[(i\hat{\Gamma}^{(\pm)5})^\dagger\tilde{J}^\mu + \tilde{J}^{\mu\dagger}(i\hat{\Gamma}^{(\pm)5})\right], \tag{36}$$

$$Z_A^\mu = \mathrm{tr}\rho_i\left[(i\hat{\Gamma}^{(\pm)5})^\dagger\tilde{J}^\mu - \tilde{J}^{\mu\dagger}(i\hat{\Gamma}^{(\pm)5})\right], \tag{37}$$

$$W^{\mu\nu} = \mathrm{tr}\rho_i\tilde{J}^{\mu\dagger}\tilde{J}^\nu. \tag{38}$$

The traces are over leptonic polarization states in Eq. (32). In Eqs. (35)–(38), they are over the hadronic polarization states.

[3] The time reversal invariance test in the β decay process in the $A = 8$ system is sensitive [12, 17] to the time reversal invariance violation part of g_T.

To justify the idea presented in Sec. II, we consider one of the simplest cases of a $1^+ \to 0^+$ $\beta^\pm$ transition. The differential decay rate for this transition depends on relatively fewer hadronic matrix elements, since only $\Delta J = 1$ operators are allowed. An example of a $1^+ \to 0^+$ weak transition is the $\beta^\pm$ decay processes of the $A = 12$ system, which has been studied both theoretically and experimentally [19–22] as a test of CVC and the existence of second-class currents. Nevertheless, some of the conclusions of this paper do not depend on the fact that a specific transition has been chosen.

The angular momentum of the parent nucleus is one. The hadronic polarization density matrix of Eq. (30) can be decomposed into irreducible tensors of the spatial rotation group,

$$\rho_i = \rho_0 + \rho_1 + \rho_2, \tag{39}$$

where $\rho_0 = \frac{1}{3}$ transforms as a scalar under rotation, ρ_1, which determines the polarization of the parent nucleus, transforms as a vector under rotation, and ρ_2, which determines the alignment of the parent nucleus, transforms as an irreducible second-rank tensor under rotation. The final state polarization matrix for charged leptons is expressed in terms of the matrix [15]

$$P = \frac{1}{2}(1 - \gamma^5 \not{S}), \tag{40}$$

which projects out pure polarization states. In the rest frame of the charged lepton $S^\mu(0) = (0, \mathbf{S}(0))$ and the unit vector $\mathbf{S}(0)$ points in the direction of the charged lepton quantization. In the frame where the electron (positron) has three-momentum $\mathbf{k}$, $S^\mu(\mathbf{k})$ is obtained by a Lorentz transformation:

$$\begin{aligned} S^0(\mathbf{k}) &= \frac{k}{m}\mathbf{S}_\parallel(0), \\ \mathbf{S}_\parallel(\mathbf{k}) &= \frac{\epsilon}{m}\mathbf{S}_\parallel(0), \\ \mathbf{S}_\perp(\mathbf{k}) &= \mathbf{S}_\perp(0), \end{aligned} \tag{41}$$

where $\mathbf{S}_\parallel$ is the component of $\mathbf{S}$ parallel to $\mathbf{k}$ and $\mathbf{S}_\perp$ is the component of $\mathbf{S}$ perpendicular to $\mathbf{k}$.

All of the dynamical response functions for a $1^+ \to 0^+$ weak transition depend on the reduced matrix elements of five multipole operators, namely, C_1^5, L_1^5, E_1^5, M_1, and S_1^5, which are functions of the momentum transfer $\kappa = |\mathbf{q}|$. For low momentum transfer reactions, we expand the momentum dependent multipoles in powers of κ keeping only the leading order ones. Up to order $O(\kappa/M)$ and $O(\kappa^2 R^2)$, the general results of the expansion are

$$S_1^5 = -i\frac{\kappa}{2M}g_P\frac{1}{\sqrt{3}}A'_2, \tag{42}$$

$$C_1^5 = -i\frac{\kappa}{2M}\left[[g_A + x_c(\kappa)g_P \mp \tilde{g}_T]\frac{1}{\sqrt{3}}A''_2 + g_A\frac{2}{3}A_3\right], \tag{43}$$

$$L_1^5 = i\left(g_A \pm \frac{\Delta}{2M}g_T\right)\frac{1}{\sqrt{3}}\left[A_2 - \frac{1}{6}\frac{\kappa^2}{M^2}A_7 + \frac{\sqrt{2}}{15}\frac{\kappa^2}{M^2}A_8\right], \tag{44}$$

$$E_1^5 = i\left(g_A \pm \frac{\Delta}{2M}g_T\right)\frac{1}{\sqrt{3}}\left[\sqrt{2}A_2 - \frac{\sqrt{2}}{6}\frac{\kappa^2}{M^2}A_7 - \frac{1}{15}\frac{\kappa^2}{M^2}A_8\right], \tag{45}$$

$$M_1 = i\frac{\kappa}{2M}G_M^V\sqrt{\frac{2}{3}}A'''_2 - i\frac{\kappa}{3M}F_1^V A_4. \tag{46}$$

$\tilde{g}_T$ is not necessarily the same as g_T. A nonrelativistic one-body approximation to the weak current operator (see Appendix A) results in the relations

$$A_2 = A'_2 = A''_2 = A'''_2 \tag{47}$$

and

$$\tilde{g}_T = g_T = g_T^{(+)} = -g_T^{(-)}. \tag{48}$$

In addition, the dimensionless static multipole amplitudes in terms of one-body operators are

$$A_2 = \left\langle 0 \left\| \sum \tau^{(\pm)}\mathcal{Y}_{10}\cdot\sigma \right\| 1 \right\rangle, \tag{49}$$

$$A_3 = \left\langle 0 \left\| \sum \tau^{(\pm)} r Y_1 \sigma\cdot\nabla \right\| 1 \right\rangle, \tag{50}$$

$$A_4 = \left\langle 0 \left\| \sum \tau^{(\pm)} r \mathcal{Y}_{11}\cdot\nabla \right\| 1 \right\rangle, \tag{51}$$

$$A_7 = M^2\left\langle 0 \left\| \sum \tau^{(\pm)} r^2 \mathcal{Y}_{10}\cdot\sigma \right\| 1 \right\rangle, \tag{52}$$

$$A_8 = M^2\left\langle 0 \left\| \sum \tau^{(\pm)} r^2 \mathcal{Y}_{12}\cdot\sigma \right\| 1 \right\rangle \tag{53}$$

where the summation is over all single nucleons in the system, and the vector spherical harmonic is defined as

$$\mathcal{Y}_{jlm} = \sum_s \langle lm - s; 1s|jm\rangle Y_{lm-s}\epsilon_s, \tag{54}$$

$$\epsilon_\pm = \mp\frac{1}{\sqrt{2}}(\epsilon_1 \pm i\epsilon_2), \quad \epsilon_0 = \epsilon_3. \tag{55}$$

It will be shown in Sec. IV that the assumptions made in Eqs. (47) and (48) do not affect the generality of the resulting differential decay rates. The term in C_1^5 proportional to $x_c(\kappa)$ is from the Coulomb term in the effective axial charge operator [Eq. (15)] with $x_c(\kappa)$ related to the Coulomb potential through

$$x_c(\kappa) = \int_0^\infty \frac{d\kappa'\kappa'^2}{(2\pi)^2}v_1(\kappa,\kappa'), \tag{56}$$

$$v_1(\kappa,\kappa') = \int_{-1}^1 d(\cos\theta)\cos\theta V(\mathbf{q}' - \mathbf{q}), \tag{57}$$

where θ is the angle between $\mathbf{q}$ and $\mathbf{q}'$ and $\kappa' = |\mathbf{q}'|$.

The polarized differential decay rate of the β decay processes can be written as a sums of products of irreducible hadronic response functions, which are bilinear sums of the reduced matrix elements of the weak multipole operators, and certain kinematical functions possessing definite symmetry properties [12]. The results for the specific case of a $1^+ \to 0^+$ weak transition considered in this paper are given briefly in Appendix B. After writing the kinematical functions in Cartesian form, keeping terms that are of order $O(\kappa/M)$ and $O(\kappa^2 R^2)$ or lower, integrating over the direction of the neutrino momentum, it is straightforward, though tedious, to find the differential decay rate

$$dW = dW_0\left\{ \alpha^{(0)} + P\left(\alpha^{(1)}\hat{\mathbf{n}}\cdot\frac{\mathbf{k}}{\epsilon} + a\beta^{(1)}\hat{\mathbf{n}}\cdot\mathbf{S}_\perp(0)\right)\right.$$
$$\left. +AT_2(\hat{\mathbf{n}}) : \left(\alpha^{(2)}\frac{\mathbf{k}}{\epsilon}\frac{\mathbf{k}}{\epsilon} + a\beta^{(2)}\frac{\mathbf{k}}{\epsilon}\mathbf{S}_\perp(0)\right)\right\}, \tag{58}$$

where $\hat{\mathbf{n}}$ is a unit vector in the direction of the parent nuclear polarization, $T_2^{ij}(\hat{\mathbf{n}}) = \hat{n}_i\hat{n}_j - \delta_{ij}/3$, the tensorial contraction $T_2(\hat{\mathbf{n}}) : \mathbf{AB} = T_2^{ij}(\hat{\mathbf{n}})A_iB_j$, ϵ is the energy of the charged lepton, and dW_0 is the leading unpolarized rate (i.e., the rate determined by the leading Gamow-Teller matrix element A_2 only). The quantity $-1 \le a \le 1$ is defined as

$$a = \frac{t_+ - t_-}{t_+ + t_-}. \tag{59}$$

t_+ and t_- are introduced to account for the ability of the charged lepton detector to distinguish between spin parallel and antiparallel charged lepton states relative to its quantization axis $\mathbf{S}$. $t_{+/-}$ is the probability of an charged lepton with spin parallel (antiparallel) to $\mathbf{S}(\mathbf{k})$ to be registered by the detector. The response functions $\alpha^{(0)}$, $\alpha^{(1)}$, $\beta^{(1)}$, $\alpha^{(2)}$, and $\beta^{(2)}$ have the following forms:

$$\alpha^{(0)} = \left[1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M \mp 2f_T - \frac{\Delta}{M}\eta_7\right) - \frac{\epsilon}{3M}\left(f_C^5 \mp 2f_M \pm f_T - 2\frac{\Delta}{M}\eta_7\right)\right]\left(1 \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right)$$
$$+\frac{\epsilon}{3M}\left(f_C^5 \pm 2f_M \pm f_T + \frac{4\sqrt{2}}{9}\frac{\Delta}{M}\eta_8\right)\left(\frac{k^2}{\epsilon^2} \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right) + \frac{m^2}{3M}f_P\left(\frac{\Delta-\epsilon}{\epsilon} \pm a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right)$$
$$-\frac{2\epsilon^2}{3M^2}\left[\eta_7\left(1 \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right) + \frac{2\sqrt{2}}{9}\eta_8\left(\frac{k^2}{\epsilon^2} \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right)\right], \tag{60}$$

$$\alpha^{(1)} = \mp\frac{\epsilon^2}{k^2}\left[1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M \mp 2f_T - \frac{\Delta}{M}\eta_7\right)\right.$$
$$\left.-\frac{\epsilon}{3M}\left(f_C^5 \mp 2f_M \pm f_T - 2\frac{\Delta}{M}\eta_7\right) - \frac{2}{3}\frac{\epsilon^2}{M^2}\eta_7 + \frac{\sqrt{2}}{30}\frac{k^2}{M^2}\eta_8\right]\left(\frac{k^2}{\epsilon^2} \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right)$$
$$-\frac{\epsilon}{M}f_M\left(1 \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right) - \frac{\epsilon}{3M}(\Delta-\epsilon)\frac{m^2}{k^2}f_P a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)$$
$$\pm\frac{\sqrt{2}}{10}\frac{\epsilon^2}{M^2}\eta_8\left[\left(\frac{k^2}{\epsilon^2} \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right) - \frac{10}{9}\left(\frac{\Delta-\epsilon}{\epsilon}\right)\left(1 \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right)\right], \tag{61}$$

$$\beta^{(1)} = \frac{m}{\epsilon}\left[1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M \mp 2f_T - \frac{\Delta}{M}\eta_7\right) - \frac{\epsilon}{3M}\left(f_C^5 \mp 2f_M \pm f_T - 2\frac{\Delta}{M}\eta_7\right) - \frac{2}{3}\frac{\epsilon^2}{M^2}\eta_7 + \frac{\sqrt{2}}{30}\frac{k^2}{M^2}\eta_8\right]$$
$$+\frac{m}{3M}f_P\left(\frac{3k^2}{2\epsilon} + \epsilon - \Delta\right), \tag{62}$$

$$\alpha^{(2)} = -\frac{\epsilon^2}{k^2}\left[\frac{\epsilon}{2M}\left(f_C^5 \mp f_M \pm f_T + \frac{\sqrt{2}}{45}\frac{\Delta}{M}\eta_8\right)\frac{\sqrt{2}}{90}\frac{\epsilon^2}{M^2}\eta_8\right]\left(\frac{k^2}{\epsilon^2} \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right) \mp \frac{m^2}{2M}\frac{\epsilon}{k}f_P a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)$$
$$-\frac{\sqrt{2}}{2}\frac{\epsilon^2}{M^2}\eta_8\left(1 \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0)\right), \tag{63}$$

$$\beta^{(2)} = \pm\frac{m}{2M}\left(f_C^5 \mp f_M \pm f_T + \frac{\sqrt{2}}{45}\frac{\Delta}{M}\eta_8\right) \mp \frac{\sqrt{2}}{90}\frac{m\epsilon}{M^2}\eta_8 \mp \frac{m\epsilon}{2M}f_P, \tag{64}$$

where

$$f_P = -\frac{g_P}{g_A}, \tag{65}$$

$$f_M = -\frac{G_M^V}{g_A} + \sqrt{\frac{2}{3}}\frac{F_1^V}{g_A}\mathrm{Re}\left(\frac{A_4}{A_2}\right), \tag{66}$$

$$f_C^5 = 1 - \langle x_c\rangle f_P + \frac{2}{\sqrt{3}}\mathrm{Re}\left(\frac{A_3}{A_2}\right), \tag{67}$$

$$f_T = -\frac{g_T}{g_A}, \tag{68}$$

$$\eta_7 = \mathrm{Re}\left(\frac{A_7}{A_2}\right), \tag{69}$$

$$\eta_8 = \mathrm{Re}\left(\frac{A_8}{A_2}\right), \tag{70}$$

and

$$\langle x_c\rangle = \frac{1}{2}\int_{-1}^{1} d(\cos\phi)x_c(\kappa), \tag{71}$$

with ϕ the angle between the neutrino and charged lepton momentum. f_C^5 is the axial charge matrix element (except for the term proportional to f_T) C_1^5 divided by the Gamow-Teller matrix element and f_M is the magnetic dipole matrix element M_1 divided by the Gamow-Teller matrix element.

In the following, we discuss several special cases of the differential decay rate, Eq. (58), with regard to the possibility of extracting g_P in a $1^+ \to 0^+$ transition.

A. Differential decay rate in the absence of charged lepton polarization measurements

If the charged lepton detector does not discriminate between the spin up and down [relative to quantization axis $\mathbf{S}(\mathbf{k})$] charged leptons, then $a = 0$. In this case, the differential decay rate for the emission of relativistic charged leptons (i.e., $k \gg m$) is

$$dW_a = dW_0\left\{\alpha_a^{(0)} + P\alpha_a^{(1)}\hat{\mathbf{n}}\cdot\frac{\mathbf{k}}{\epsilon} + A\alpha_a^{(2)}T_2(\hat{\mathbf{n}}) : \frac{\mathbf{k}}{\epsilon}\frac{\mathbf{k}}{\epsilon}\right\}, \tag{72}$$

with

$$\alpha_a^{(0)} = 1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M + \frac{m^2}{\epsilon}f_P \mp 2f_T - \frac{\Delta}{M}\eta_7\right) + \frac{4\epsilon}{3M}\left[\pm f_M - \frac{m^2}{4\epsilon}f_P + \frac{\Delta}{2M}\left(\eta_7 + \frac{2\sqrt{2}}{9}\eta_8\right)\right] - \frac{2\epsilon^2}{3M^2}\left(\eta_7 + \frac{2\sqrt{2}}{9}\eta_8\right), \tag{73}$$

$$\alpha_a^{(1)} = \mp\left\{ 1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M \mp 2f_T - \frac{\Delta}{M}\eta_7\right) - \frac{\epsilon}{3M}\left[f_C^5 \mp 5f_M \pm f_T - 2\frac{\Delta}{M}\left(\eta_7 + \frac{1}{3\sqrt{2}}\eta_8\right)\right] - \frac{2\epsilon^2}{3M^2}\left(\eta_7 + \frac{4\sqrt{2}}{15}\eta_8\right)\right\}, \tag{74}$$

$$\alpha_a^{(2)} = -\frac{\epsilon}{2M}\left(f_C^5 \mp f_M \mp f_T + \frac{\sqrt{2}}{45}\frac{\Delta}{M}\eta_8\right) - \frac{22\sqrt{2}}{45}\frac{\epsilon^2}{M^2}\eta_8. \tag{75}$$

From the PCAC value of g_P given by Eq. (1), it is easy to show that the term proportional to f_P in $\alpha_a^{(0)}$ is practically undetectable. Since even if the experiments could be done to an arbitrary accuracy, the magnitude of nuclear physics uncertainties in f_C^5, f_M, η_7, and η_8 are comparable to, or larger than the $\frac{m^2}{\epsilon}f_P$ term; these uncertainties would render a reliable extraction of f_P from the data extremely difficult, if not impossible.

B. Differential decay rates when the charged lepton longitudinal polarization is measured

Choose the quantization axis of the electron (positron) spin to be antiparallel (parallel) to its three-momentum $\mathbf{k}$. The *rare* polarized charged leptons emitted in the decay are selected by using an idealized detector that is highly efficient to the spin down charged leptons and is totally unresponsive for spin up ones, namely, $t_- = 1$, $t_+ = 0$, and $a = -1$. Then, for relativistic charged leptons, we find

$$\mathbf{S}_\perp(0) = 0, \tag{76}$$

$$1 \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0) = 1 - \frac{k}{\epsilon} \simeq \frac{m^2}{2\epsilon^2}, \tag{77}$$

$$\frac{k^2}{\epsilon^2} \mp a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0) = \frac{k}{\epsilon}\left(\frac{k}{\epsilon} - 1\right) \simeq -\frac{m^2}{2\epsilon^2}, \tag{78}$$

$$\pm a\frac{\mathbf{k}}{\epsilon}\cdot\mathbf{S}(0) = \frac{k}{\epsilon} \simeq 1. \tag{79}$$

The differential rate for the *rare* polarization decay is

$$dW_b = dW_0 \left\{ \alpha_b^{(0)} + P\alpha_b^{(1)} \hat{\mathbf{n}} \cdot \frac{\mathbf{k}}{\epsilon} + A\alpha_b^{(2)} T_2(\hat{\mathbf{n}}) : \frac{\mathbf{k}}{\epsilon}\frac{\mathbf{k}}{\epsilon} \right\}, \tag{80}$$

with

$$\alpha_b^{(0)} = \frac{m^2}{2\epsilon^2} \left\{ 1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M + 2\epsilon f_P \mp 2f_T - \frac{\Delta}{M}\eta_7 \right) - \frac{2\epsilon}{3M}\left[f_C^5 \pm f_T + \frac{\Delta}{M}\left(-\eta_7 + \frac{2\sqrt{2}}{9}\eta_8\right)\right] + \frac{2\epsilon^2}{3M^2}\left(-\eta_7 + \frac{2\sqrt{2}}{9}\eta_8\right)\right\}, \tag{81}$$

$$\alpha_b^{(1)} = \pm\frac{m^2}{2\epsilon^2} \left\{ 1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M - 2\epsilon f_P \mp 2f_T - \frac{\Delta}{M}\eta_7 \right) - \frac{\epsilon}{3M}\left[f_C^5 \pm f_M - 2\epsilon f_P \pm f_T + 2\frac{\Delta}{M}\left(-\eta_7 + \frac{1}{3\sqrt{2}}\eta_8\right)\right] + \frac{2\epsilon^2}{3M^2}\left(-\eta_7 + \frac{\sqrt{2}}{15}\eta_8\right)\right\}, \tag{82}$$

$$\alpha_b^{(2)} = \frac{m^2}{2\epsilon^2}\left[\frac{\epsilon}{2M}\left(f_C^5 \mp f_M - 2\epsilon f_P \pm f_T + \frac{\sqrt{2}}{45}\frac{\Delta}{M}\eta_8 \right) - \frac{23\sqrt{2}}{45}\frac{\epsilon^2}{M^2}\eta_8 \right]. \tag{83}$$

In this case, for an aligned parent nucleus, $\alpha_b^{(2)}$, which contains f_P, can be determined. Using the PCAC value for g_P at $q^2 = 0$, we find $2\epsilon f_P \sim f_C^5$ or f_M for $\epsilon > 5$ MeV. In addition, the f_P term has a different ϵ dependence from f_C^5 and f_M, which permits one to distinguish between these terms. Compared to the dominant polarization (i.e., spin up) differential decay rate, which is dW_a [Eq. (72), with the omission of the terms containing f_P], the *rare* transition rate dW_b is suppressed by a factor of $m^2/2\epsilon^2 \sim 10^{-2} - 10^{-4}$. However, compared to the leading term for the dominant polarization $\alpha_a^{(0)}$, the contribution corresponding to $\alpha_b^{(2)}$ is suppressed by a factor of $m^2/4M\epsilon \sim 10^{-4} - 10^{-6}$.

C. Differential decay rate when the charged lepton transverse polarizations are measured

The transverse polarizations of the charged lepton are of two kinds: (1) $\mathbf{S}_\perp(0)$ may lie in the plane formed by the parent nuclear polarization axis $\hat{\mathbf{n}}$ and charged lepton three-momentum $\mathbf{k}$, and (2) $\mathbf{S}_\perp(0)$ is perpendicular to the plane formed by $\hat{\mathbf{n}}$ and $\mathbf{k}$. The differential decay rate for the second case is trivial because both $\hat{\mathbf{n}} \cdot \mathbf{S}_\perp(0)$ and $T_2(\hat{\mathbf{n}}) : \frac{\mathbf{k}}{\epsilon}\mathbf{S}_\perp(0)$ vanish so that the differential decay rate is identical to dW_a. Only the first case is of interest; the differential decay rate for the emission of relativistic charged leptons is

$$dW_c = dW_0 \left\{ \alpha_c^{(0)} + P\left(\alpha_c^{(1)} \hat{\mathbf{n}} \cdot \frac{\mathbf{k}}{\epsilon} + a\beta_c^{(1)} \hat{\mathbf{n}} \cdot \mathbf{S}_\perp(0) \right) + AT_2(\hat{\mathbf{n}}) : \left(\alpha_c^{(2)} \frac{\mathbf{k}}{\epsilon}\frac{\mathbf{k}}{\epsilon} + a\beta_c^{(2)} \frac{\mathbf{k}}{\epsilon}\mathbf{S}_\perp(0) \right) \right\}, \tag{84}$$

with

$$\alpha_c^{(0)} = \alpha_a^{(0)}, \tag{85}$$

$$\alpha_c^{(1)} = \alpha_a^{(1)}, \tag{86}$$

$$\beta_c^{(1)} = \frac{m}{\epsilon}\left[1 + \frac{\Delta}{3M}\left(f_C^5 \mp 2f_M - \epsilon f_P \mp 2f_T - \frac{\Delta}{M}\eta_7 \right) - \frac{\epsilon}{3M}\left(f_C^5 \mp 2f_M - \frac{5}{2}\epsilon f_P \pm f_T - 2\frac{\Delta}{M}\eta_7 \right) + \frac{2\epsilon^2}{3M^2}\left(-\eta_7 + \frac{\sqrt{2}}{20}\eta_8\right)\right], \tag{87}$$

$$\alpha_c^{(2)} = \alpha_a^{(2)}, \tag{88}$$

$$\beta_c^{(2)} = \pm\frac{m}{\epsilon}\left[\frac{\epsilon}{2M}\left(f_C^5 \mp f_M - \epsilon f_P \pm f_T + \frac{\sqrt{2}}{45}\frac{\Delta}{M}\eta_8 \right) - \frac{\sqrt{2}}{90}\frac{\epsilon^2}{M^2}\eta_8 \right]. \tag{89}$$

In this case, it is still favorable to determine g_P from alignment observables, since all terms in $\alpha_c^{(2)}$ and $\beta_c^{(2)}$ are of recoil order. Compared to the leading term $\alpha_c^{(0)}$, $\beta_c^{(2)}$ is suppressed by a factor of $m/M \sim 10^{-3}$. Compared to the spin independent alignment response function $\alpha_c^{(2)}$, $\beta_c^{(2)}$ is suppressed by a factor around 1–0.03.

IV. THEORETICAL UNCERTAINTIES AND EXPERIMENTAL FEASIBILITY

The results given by Eqs. (60)–(64) provide a complete parametrization for the differential decay rates up to order κ/M and $(\kappa R)^2$ and before considering Coulomb corrections to the charged lepton wave functions. This statement can be verified by the observation that the only essential assumptions made in deriving these results are given by Eqs. (47) and (48), which are valid if, albeit not necessary, a nonrelativistic one-body approximation to the weak current operators is made. The explicit expressions given by the right-hand side of Eqs. (49)–(53) have never been used in the computation of the differential decay rates. The assumptions made in Eqs. (47) and (48) do not affect the generic nature of the results, provided certain renormalizations of the single-nucleon weak form factors are made. Let us define A_2, which is proportional to the Gamow-Teller matrix element, by Eqs. (44) and (45). That A_2, A_7, and A_8 are identical in Eqs. (44) and (45) is guaranteed by angular momentum theory. The $g_P A_2'$ term in S_1^5 can be rewritten as $\tilde{g}_P A_2$ with $\tilde{g}_P$ the renormalized pseudoscalar form factor of a nucleon inside the system, namely,

$$\tilde{g}_P = g_P \frac{A_2'}{A_2}. \tag{90}$$

This reasoning can also be applied to the A_2''' term in M_1 [see Eq. (46)] by defining a renormalized G_M^V. The difference between A_2'' and A_2 in C_1^5 [see Eq. (43)] can be absorbed into A_3 (it in fact defines A_3, which could be different for β^+ and β^- decays if $g_T \neq 0$). Finally, the difference between g_T and $\tilde{g}_T$ can also be absorbed into A_3 [see Eq. (43)]. This completes our proof that the results given in this paper provide a complete parametrization of the differential decay rates for the $1^+ \to 0^+$ weak transition. If the effects of the Coulomb distortion of the charged leptonic wave function are considered, more terms involving single-nucleon form factors and η's are expected to enter the differential decay rates. These terms are suppressed by $Z\alpha = Z/137$. They will not be investigated here.

Equation (90) shows that we should interpret the value of g_P extracted from the experimental data carefully, since it contains nuclear structure effects not related to the change of the single-nucleon properties inside the system. Let us write A_2' as $A_2 + \Delta A_2$, then

$$\tilde{g}_P = g_P(1 + y), \tag{91}$$

with $y = \Delta A_2/A_2$. In order to know g_P, one has to know y, which is a parameter that cannot be determined from the β decay data. Model calculations have to be performed to obtain the value of y. This brings in nuclear model dependences. The magnitude of the uncertainties in y can only be assessed in more detailed studies.

Two different ways of introducing the pionic part of the hadronic axial vector current operators represented by Eqs. (3) and (5) have been discussed in Sec. II. These two approaches differ in a term proportional to $\langle x_c \rangle$ in f_C^5 [see Eq. (67)]. The difference resulting from Eq. (3) and Eq. (5) is a technical one due to the approximate computational procedure adopted. The "physical" quantity f_C^5 does not depend on the choices, so there are corresponding terms in A_3 that render the decay rates independent of the choices. It is discussed above that f_C^5 can be determined by fitting experimental data. On the other hand, if the value of f_C^5 is to be computed from a microscopic theoretical model, which is not expected to be fully complete, the term resulting from $\langle x_c \rangle$ can be utilized to estimate the uncertainties related to the treatment of gauge invariance as far as the g_P measurement is concerned. The differential decay rates obtained from "gauge invariant" way of introducing the $\hat{\Gamma}^5(x)$ term are obtained by dropping all the terms proportional to $\langle x_c \rangle$ or putting $x_c = 0$. Therefore the theoretical uncertainties associated with the approximate treatment of Eq. (3) or Eq. (5) can be measured by the contribution of $\langle x_c \rangle$ to the differential decay rates. Figure 1 shows the numerical value of $\langle x_c \rangle$ as a function of ϵ. $\langle x_c \rangle f_P$ is of order 10^{-2}, which indicates that it is an unimportant source of uncertainty. It is hard to separate $\langle x_c \rangle f_P$ term from meson exchange corrections to f_C^5 (basically an axial charge matrix element), electromagnetic corrections to the hadronic wave functions, etc., which are contained in the $\eta_3 \equiv A_3/A_2$ term of f_C^5.

One of the terms in $\alpha_b^{(2)}$ or $\beta_c^{(2)}$ involving η_8 has the same ϵ dependence as the term involving f_P. A rough order of magnitude estimate of η_8 [see Eqs. (51) and (52)] indicates that this term is of the same order as the f_P term. In order to separate the contribution of f_P to the ϵ^2 dependent terms in $\alpha_b^{(2)}$ or $\beta_c^{(2)}$ from that of η_8, η_8 should be determined in some other ways. To make a

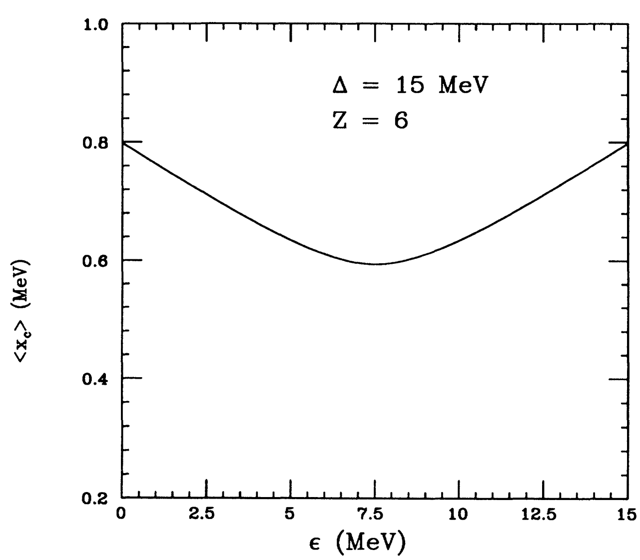

FIG. 1. Coulomb effects on the contribution of g_P to the differential decay rates in the $Z = 6$ system. $\langle x_c \rangle$ is defined through Eqs. (56) and (71). A typical value of $\Delta = 15$ MeV is used. For an unscreened point charge Coulomb potential, Eq. (56) diverges at $\kappa' = \infty$ as well as at $\kappa' = \kappa$. The nuclear size effects and screening effects have to be taken into account. We use a modified Coulomb potential in momentum space of the form $V(\mathbf{q}) = Ze^2 \exp(-|\mathbf{q}|^2/b^2)/(|\mathbf{q}|^2 + \mu^2)$ with $b = 100$ MeV and $\mu = 0.001$ MeV. The result depends very little on μ.

model independent determination of η_8, one can study the alignment observables in cases where the charged lepton polarization is not detected. Equation (75) shows that the ϵ^2 term of $\alpha_a^{(2)}$ depends only on η_8. So, a careful study of the alignment observables in these cases should in principle determine the value of η_8.

In certain systems, the contribution from η_8 can be small. The $A = 12$ system, which has been studied carefully as a system to test CVC [19], can serve as an example. Figure 9 of Ref. [18] shows that the nonlinearity of $\alpha_a^{(2)}$ is very small for this particular transition. Thus we can conclude that compared to the f_P contribution to the ϵ^2 dependent terms, the η_8 contribution is less important in such a system.

The current discrepancies between the PCAC value and the values of g_P extracted from muon capture experiments in light nuclei are of order 20–100 % (see, e.g., Refs. [5–10]). If these discrepancies are true, a not very precise determination of η_8 (say, within 10–20 %) can confirm these discrepancies.

In Fig. 2, $\frac{2\epsilon^2}{m^2}\alpha_b^{(2)}$ is plotted against ϵ, with arbitrary chosen "typical" values $f_C^5 = 2.0$, $f_M = 3.5$, $f_T = 0$, and $\Delta = 15$ MeV. The solid line is obtained with $\eta_8 = 25 \sim (RM)^2$ and the magnitude of the PCAC value of $f_P \approx 0.1$ MeV^{-1}. The dashed line is obtained with $\eta_8 = 0$ and the PCAC value of f_P. The dotted line is obtained with $\eta_8 = 25$ and $f_P = 0$. It can be seen that the effects of a 100% modification of g_P results in a change in $\alpha_b^{(2)}$ around 100% at the highest energy of the charged leptons. The effects of η_8 is about 20%. At higher ϵ, the relative (to the allowed) counting rates are very small due to the $\frac{m^2}{\epsilon}$ suppression factor. In addition, the total counting rates are small at both ends of ϵ due to the phase space suppression. Thus not only the magnitude of the total counting rates but also the relative counting rates due to g_P are very small at higher energies.

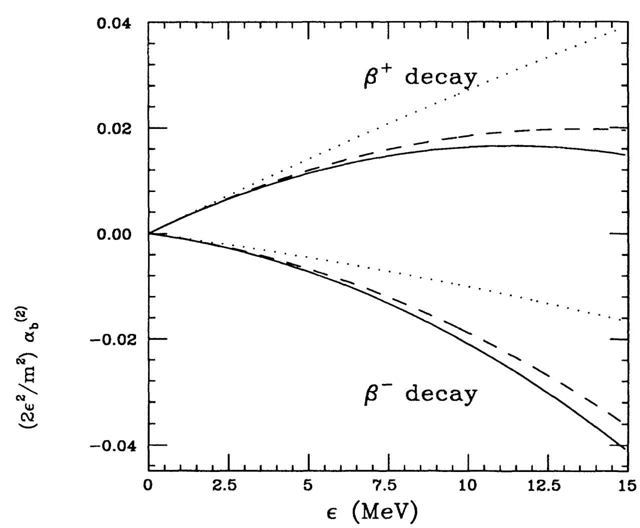

FIG. 2. The dependence of $\frac{2\epsilon^2}{m^2}\alpha_b^{(2)}$ on ϵ, with arbitrary chosen "typical" values $f_C^5 = 2.0$, $f_M = 3.5$, $f_T = 0$, and $\Delta = 15$ MeV. The solid line is obtained with $\eta_8 \sim (RM)^2 \approx 25$ and the magnitude of the PCAC value of $f_P \approx 0.1$ MeV^{-1}. The dashed line is obtained with $\eta_8 = 0$ and the magnitude of the PCAC value of f_P. The dotted line is obtained with $\eta_8 = 25$ and $f_P = 0$.

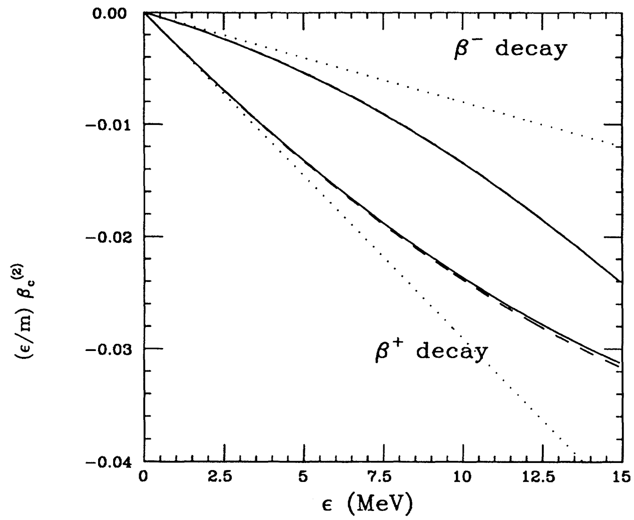

FIG. 3. The dependence of $\frac{\epsilon}{m}\beta_c^{(2)}$ on ϵ, with the same values of f_C^5, f_M, f_T, and Δ as those in Fig. 2. The solid line is obtained with $\eta_8 = 25$ and the magnitude of the PCAC value of f_P. The dashed line is obtained with $\eta_8 = 0$ and the magnitude of the PCAC value of f_P. The dotted line is obtained with $\eta_8 = 25$ and $f_P = 0$.

In Fig. 3, $\frac{\epsilon}{m}\beta_c^{(2)}$ is plotted against ϵ, with the same values of f_C^5, f_M, f_T, and Δ as those in Fig. 2. The solid line is obtained with $\eta_8 = 25$ and the magnitude of the PCAC value of f_P. The dashed line is obtained with $\eta_8 = 0$ and the PCAC value of f_P. The dotted line is obtained with $\eta_8 = 25$ and $f_P = 0$. Again, a 100% change in g_P results in about a 100% change in $\beta_c^{(2)}$, especially at higher charged lepton energies. The effects of η_8 is extremely small for $\beta_c^{(2)}$ due to the small coefficients in front of it. In addition, the relative counting rates are not suppressed at large ϵ for $\beta_c^{(2)}$ compared to the absolute decay rates.

Finally, the effects of the Coulomb distortion of the charged lepton wave function, which are suppressed by $Z/137$, are not expected to change the value and the ϵ dependence of $\alpha_b^{(2)}$ and $\beta_c^{(2)}$ significantly for light nuclei.

V. SUMMARY

We have studied the differential β decay rate for $1^+ \to 0^+$ weak transitions. The results show that for observables from aligned parent nuclei, the contribution of the induced pseudoscalar term (g_P) can be as large as the leading one-body terms. The different energy dependence of the g_P term allows it to be separated from other nuclear matrix elements. The structure for the differential decay rates given by Eq. (58) remains unchanged even if exchange current and/or other correction contributions are included, provided a proper change (renormalization) of single-nucleon form factors and matrix elements is made. In conclusion we believe that the experimental determination of some of these matrix elements and of g_P can be carried out.

Whether to determine g_P from β decay experiments in which longitudinal polarizations of the charged leptons are measured or from β decay experiments in which the transverse polarizations of the charged leptons are measured depends on the experimental techniques available. Case C is favored due to its larger relative counting rate, smaller η_8 contamination, and relatively easier experimental determination of the transverse polarization of the charged leptons. The small counting rates in determining g_P requires precision experiments.

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APPENDIX A: NONRELATIVISTIC REDUCTION OF THE HADRONIC WEAK ONE-BODY CURRENTS

The nonrelativistic reduction of Eqs. (25)–(27) for many-nucleon systems to leading $1/M$ is found to be

$$\rho^{(\pm)}(\mathbf{x}) = F_1^V \sum_{i=1}^{A} \tau^{(\pm)}(i)\delta(\mathbf{x}-\mathbf{r}_i), \tag{A1}$$

$$\begin{aligned}\tilde{\rho}^{(\pm)5}(\mathbf{x}) = {} & \mp\frac{ig_T}{2M}\sum_{i=1}^{A}\tau^{(\pm)}(i)\sigma(i)\cdot\nabla\delta(\mathbf{x}-\mathbf{r}_i)\\ & +g_A\sum_{i=1}^{A}\tau^{(\pm)}(i)\sigma(i)\cdot\left[\frac{\mathbf{p}_i}{2M}\delta(\mathbf{x}-\mathbf{r}_i)\right.\\ & \left.+\delta(\mathbf{x}-\mathbf{r}_i)\frac{\mathbf{p}_i}{2M}\right],\end{aligned} \tag{A2}$$

$$\begin{aligned}\tilde{J}^{(\pm)}(\mathbf{x}) = {} & F_1^V\sum_{i=1}^{A}\tau^{(\pm)}(i)\left[\frac{\mathbf{p}_i}{2M}\delta(\mathbf{x}-\mathbf{r}_i)+\delta(\mathbf{x}-\mathbf{r}_i)\frac{\mathbf{p}_i}{2M}\right]\\ & +\frac{G_M^V}{2M}\sum_{i=1}^{A}\tau^{(\pm)}(i)\nabla\times\sigma(i)\delta(\mathbf{x}-\mathbf{r}_i),\end{aligned} \tag{A3}$$

$$\tilde{A}^{(\pm)}(\mathbf{x}) = \left(g_A \pm \frac{\Delta}{2M}g_T\right)\sum_{i=1}^{A}\tau^{(\pm)}(i)\sigma(i)\delta(\mathbf{x}-\mathbf{r}_i), \tag{A4}$$

$$i\hat{\Gamma}^{(\pm)5}(\mathbf{x}) = i\frac{g_P}{2M}\sum_{i=1}^{A}\tau^{(\pm)}(i)\sigma(i)\cdot\nabla\delta(\mathbf{x}-\mathbf{r}_i), \tag{A5}$$

where Δ is the maximum energy of the electron or positron and $G_M^V = F_M^V + 1 = 4.706$.

APPENDIX B: SOME DETAILS ON THE DIFFERENTIAL DECAY RATES

The unpolarized differential decay rates are found to be [12]

$$dW^{(0)} = 2\pi\delta(E_f - E_i + \Delta)\frac{G^2\cos^2\theta_C}{2}\left(4\pi\sum_{i=1}^{5}R_i^{(0)}K_i^{(0)} + (\text{higher order})\right)d\xi, \tag{B1}$$

where "higher order" stands for terms that are beyond $O(\kappa/M)$ or $O(\kappa^2R^2)$. $K_i^{(0)}$ $(i = 1, \ldots, 5)$ are kinematical functions:

$$K_1^{(0)} = \frac{1}{\sqrt{3}}\sqrt{4\pi}[Y_1(\hat{\mathbf{q}})\otimes(l_s\mathbf{l}^*)_S]_0, \tag{B2}$$

$$K_2^{(0)} = \frac{1}{\sqrt{3}}\sqrt{4\pi}[Y_1(\hat{\mathbf{q}})\otimes(l_0\mathbf{l}^*)_S]_0, \tag{B3}$$

$$K_3^{(0)} = \frac{1}{\sqrt{3}}\sqrt{4\pi}[Y_0(\hat{\mathbf{q}})\otimes[\mathbf{l}\otimes\mathbf{l}^*]_0]_0, \tag{B4}$$

$$K_4^{(0)} = \frac{1}{\sqrt{3}}\sqrt{4\pi}[Y_1(\hat{\mathbf{q}})\otimes[\mathbf{l}\otimes\mathbf{l}^*]_1]_0, \tag{B5}$$

$$K_5^{(0)} = \frac{1}{\sqrt{3}}\sqrt{4\pi}[Y_2(\hat{\mathbf{q}})\otimes[\mathbf{l}\otimes\mathbf{l}^*]_2]_0, \tag{B6}$$

where $[\phi_{l_1}\otimes\psi_{l_2}]_{jm}$ denotes the Clebsch-Gordan coupling between two angular momentum states with angular momentum l_1 and l_2 to form a new state with angular momentum j, the $(l_0\mathbf{l}^*)_{S/A}$ that appear below are defined in the same way as $(l_s\mathbf{l}^*)_{S/A}$ [see Eqs. (33) and (34)]. The dynamical response functions $R_i^{(0)}$ $(i = 1, \ldots, 5)$ are

$$R_1^{(0)} = -\frac{2}{\sqrt{3}}\mathrm{Re}(S_1L_1^{5*}), \tag{B7}$$

$$R_2^{(0)} = -\frac{2}{\sqrt{3}}\mathrm{Re}(C_1^5L_1^{5*}), \tag{B8}$$

$$R_3^{(0)} = -\frac{1}{3}(|L_1^5|^2 + |E_1^5|^2 + |M_1|^2), \tag{B9}$$

$$R_4^{(0)} = -\sqrt{\frac{2}{3}}\mathrm{Re}(M_1E_1^{5*}), \tag{B10}$$

$$R_5^{(0)} = \frac{\sqrt{2}}{3}|L_1^5|^2 - \frac{1}{3\sqrt{2}}|E_1^5|^2 - \frac{1}{3\sqrt{2}}|M_1|^2, \quad \text{(B11)}$$

where S_1^5, C_1^5, L_1^5, E_1^5, and M_1 are the reduced matrix elements of $\hat{S}^5_{Jm}$, $\hat{C}^5_{Jm}$, $\hat{L}^5_{Jm}$, $\hat{T}^{\rm el5}_{Jm}$, and $\hat{T}^{\rm mag}_{Jm}$ with $J=1$, respectively, e.g., $S_1^5 \equiv \langle J_f \,|\hat{S}_1^5||\, J_i\rangle$ and

$$\langle J_f m_f \,|\hat{S}^5_{Jm}|\, J_i m_i\rangle = (-1)^{J_i - J - m_f}\begin{pmatrix} J_f & J & J_i \\ m_f & m & -m_i \end{pmatrix}\langle J_f \,||\hat{S}^5_J||\, J_i\rangle. \quad \text{(B12)}$$

The differential decay rate with polarization is given by

$$dW^{(1)} = 2\pi\delta(E_f - E_i + \Delta)\frac{G^2\cos^2\theta_C}{2}\left(4\pi\sum_{i=1}^{6} R_i^{(1)}K_i^{(1)} + (\text{higher order})\right)d\xi, \quad \text{(B13)}$$

where the kinematical functions $K_i^{(1)}$ $(i = 1,\ldots,6)$ are

$$K_1^{(1)} = -\sqrt{2\pi}P[\hat{\mathbf{n}} \otimes [Y_1(\hat{\mathbf{q}}) \otimes (l_s\mathbf{l}^*)_A]_1]_0, \quad \text{(B14)}$$

$$K_2^{(1)} = -\sqrt{2\pi}P[\hat{\mathbf{n}} \otimes [Y_1(\hat{\mathbf{q}}) \otimes (l_0\mathbf{l}^*)_A]_1]_0, \quad \text{(B15)}$$

$$K_3^{(1)} = -\sqrt{2\pi}P[\hat{\mathbf{n}} \otimes [Y_0(\hat{\mathbf{q}}) \otimes [\mathbf{l} \otimes \mathbf{l}^*]_1]_1]_0, \quad \text{(B16)}$$

$$K_4^{(1)} = -\sqrt{2\pi}P[\hat{\mathbf{n}} \otimes [Y_1(\hat{\mathbf{q}}) \otimes [\mathbf{l} \otimes \mathbf{l}^*]_0]_1]_0, \quad \text{(B17)}$$

$$K_5^{(1)} = -\sqrt{2\pi}P[\hat{\mathbf{n}} \otimes [Y_1(\hat{\mathbf{q}}) \otimes [\mathbf{l} \otimes \mathbf{l}^*]_2]_1]_0, \quad \text{(B18)}$$

$$K_6^{(1)} = -\sqrt{2\pi}P[\hat{\mathbf{n}} \otimes [Y_2(\hat{\mathbf{q}}) \otimes [\mathbf{l} \otimes \mathbf{l}^*]_1]_1]_0. \quad \text{(B19)}$$

Here $\hat{\mathbf{n}}$ is a unit vector pointing in the direction of the parent nuclear polarization, $P = a_+ - a_-$ with a_m $(m = 0, \pm 1)$ the statistical population of the magnetic states of the parent nucleus, and the dynamical response functions $R_i^{(1)}$ (i=1,...,6) are

$$R_1^{(1)} = -\sqrt{2}\,\mathrm{Re}(S_1^5 E_1^{5*}), \quad \text{(B20)}$$

$$R_2^{(1)} = -\sqrt{2}\,\mathrm{Re}(C_1^5 E_1^{5*}), \quad \text{(B21)}$$

$$R_3^{(1)} = -\frac{1}{2\sqrt{3}}(|E_1^5|^2 + |M_1|^2) - \sqrt{\frac{2}{3}}\mathrm{Re}(L_1^5 E_1^{5*}), \quad \text{(B22)}$$

$$R_4^{(1)} = \sqrt{\frac{2}{3}}\mathrm{Re}(M_1 E_1^{5*}), \quad \text{(B23)}$$

$$R_5^{(1)} = -\sqrt{\frac{3}{5}}\mathrm{Re}(M_1 L_1^{5*}) - \sqrt{\frac{2}{15}}\mathrm{Re}(M_1 E_1^{5*}), \quad \text{(B24)}$$

$$R_6^{(1)} = \frac{1}{\sqrt{6}}(|E_1^5|^2 + |M_1|^2) - \sqrt{\frac{1}{3}}\mathrm{Re}(L_1^5 E_1^{5*}). \quad \text{(B25)}$$

The differential decay rate corresponding to alignment is given by

$$dW^{(2)} = 2\pi\delta(E_f - E_i + \Delta)\frac{G^2\cos^2\theta_C}{2}\left(4\pi\sum_{i=1}^{5} R_i^{(2)}K_i^{(2)} + (\text{higher order})\right)d\xi, \quad \text{(B26)}$$

where the kinematical functions $K_i^{(2)}$ $(i = 1,\ldots,5)$ are

$$K_1^{(2)} = \sqrt{\pi}A[T_2(\hat{\mathbf{n}}) \otimes [Y_1(\hat{\mathbf{q}}) \otimes (l_s\mathbf{l}^*)_S]_2]_0, \quad \text{(B27)}$$

$$K_2^{(2)} = \sqrt{\pi}A[T_2(\hat{\mathbf{n}}) \otimes [Y_1(\hat{\mathbf{q}}) \otimes (l_0\mathbf{l}^*)_S]_2]_0, \quad \text{(B28)}$$

$$K_3^{(2)} = \sqrt{\pi}A[T_2(\hat{\mathbf{n}}) \otimes [Y_1(\hat{\mathbf{q}}) \otimes [\mathbf{l} \otimes \mathbf{l}^*]_1]_2]_0, \quad \text{(B29)}$$

$$K_4^{(2)} = \sqrt{\pi}A[T_2(\hat{\mathbf{n}}) \otimes [Y_2(\hat{\mathbf{q}}) \otimes [\mathbf{l} \otimes \mathbf{l}^*]_0]_2]_0, \quad \text{(B30)}$$

$$K_5^{(2)} = \sqrt{\pi}A[T_2(\hat{\mathbf{n}}) \otimes [Y_2(\hat{\mathbf{q}}) \otimes [\mathbf{l} \otimes \mathbf{l}^*]_2]_2]_0. \quad \text{(B31)}$$

The second rank irreducible tensor $T_2(\hat{\mathbf{n}})_{ij} = \hat{n}_i\hat{n}_j - \delta_{ij}/3$, $A = 1 - 3a_0$, and the dynamical response functions are

$$R_1^{(2)} = -\frac{4}{3}\sqrt{\frac{3}{5}}\mathrm{Re}(S_1^5 L_1^{5*}) - \sqrt{\frac{6}{5}}\mathrm{Re}(S_1^5 E_1^{5*}), \quad \text{(B32)}$$

$$R_2^{(2)} = -\frac{4}{3}\sqrt{\frac{3}{5}}\mathrm{Re}(C_1^5 L_1^{5*}) - \sqrt{\frac{6}{5}}\mathrm{Re}(C_1^5 E_1^{5*}), \quad \text{(B33)}$$

$$R_3^{(2)} = \sqrt{\frac{3}{5}}\mathrm{Re}(M_1 L_1^{5*}) + \sqrt{\frac{2}{15}}\mathrm{Re}(M_1 E_1^{5*}), \quad \text{(B34)}$$

$$R_4^{(2)} = -\frac{1}{3\sqrt{2}}|E_1^5|^2 + \frac{\sqrt{2}}{3}|L_1^5|^2 - \frac{1}{3\sqrt{2}}|M_1|^2, \quad \text{(B35)}$$

$$R_5^{(2)} = -\frac{5}{3\sqrt{14}}|E_1^5|^2 + \frac{2}{3}\sqrt{\frac{2}{7}}|L_1^5|^2 + \frac{1}{\sqrt{7}}\mathrm{Re}(L_1^5 E_1^{5*}) + \frac{\sqrt{14}}{6}|M_1|^2. \quad \text{(B36)}$$

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