Distortion of two-pion interferometry by multipion correlations

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Multipion correlations arising from the symmetrization of the n-pion wave function affect the extracted information from two-pion correlation measurements. The influence of multipion correlations on a sample of like-pion pairs can be expressed as a multipion correlation factor, the distribution of which offers good sensitivity to the multipion correlation effect. Analyses of the multipion correlation factor for two Bevalac streamer chamber data samples of $2.1A$ GeV Ne + Pb and $1.8A$ GeV Ar + Pb collisions show that the multipion correlation effect in the former sample is greater than in the latter. This result mainly arises from the fact that the pion source for Ne projectiles is smaller than for Ar projectiles. The residual correlations in the reference sample are related to the multipion correlation factor in multipion events, which can be expressed as a residual correlation factor. The influence of multipion correlations on two-pion interferometry analyses arises from the ratio of the residual correlation factor to the multipion correlation factor.

I. INTRODUCTION

The study of Bose-Einstein correlations among identical pions in high energy heavy ion collisions provides information about the space-time structure, degree of coherence, and the dynamics of the region where the pions were produced [1–3]. In a multipion event, there are multipion correlations among the identical pions. Two-pion interferometry investigates only the two-body correlation between identical pions. In order to reveal the properties of nuclear matter at progressively higher temperature and density, heavy ion collisions should be studied at increasing bombarding energy and the number of identical pions in each event becomes higher. Therefore, it becomes more and more important to investigate the dependence of multipion correlations on parameters of the pion source, to study the influence of multipion correlations on two-pion correlations, and to develop a pion interferometry technique which involves the multipion correlation effect [4–12]. Furthermore, two-pion interferometry results depend on the reference (background) sample. In the usual pion interferometry analyses, the reference sample is constructed by selecting pions from different events with the same pion multiplicity, and there are residual correlations in the reference sample [2, 13, 14]. The multipion correlations, the residual correlations, and the influence of these effects on two-pion interferometry are the subject of this investigation.

In this paper we concentrate on the effect of multipion correlations at Bevalac energies. It is more difficult to study multipion correlations in very energetic heavy ion collisions since the simple space-time interpretation of pion interferometry often breaks down due to strong correlations between spatial and momentum coordinates [15–19] and the computation time for a Monte Carlo simulation of the event grows exponentially with pion multiplicity. Nevertheless, our work presented here is a first step in this direction.

The content of the present paper is as follows: In Sec. II, the effect of the multipion correlations on the sample of like-pion pairs in multipion events is discussed, which can be expressed as a multipion correlation factor. In Sec. III, we study the dependence of the residual correlation in the reference sample on the multipion correlation factor, and investigate the influence of multipion correlations on two-pion interferometry analyses. In Sec. IV, the data for $2.1A$ GeV Ne + Pb and $1.8A$ GeV Ar + Pb at the Bevalac streamer chamber are analyzed, and the multipion correlation effects in these two experiments are compared. Finally, the conclusions are given in Sec. V.

In the following, assuming a Gaussian space-time source [20], the two-pion correlation function takes the form

$$C_2(q, q_0) = 1 + \lambda \exp(-q^2 R^2/2 - q_0^2 \tau^2/2)\,, \qquad (1)$$

where q and q_0 are the relative momentum and the relative energy of the two pions, and λ is the coherence factor for two-pion correlations. Because our Bevalac streamer chamber data are not sensitive to the lifetime [3, 21–23], we take $\tau = 0$ for simplicity in our analyses.

II. THE INFLUENCE OF MULTIPION CORRELATIONS ON THE SAMPLE OF LIKE-PION PAIRS

Denoting the identical pion multiplicity of an event by n, the n-pion correlation function in the event can be defined as

$$C_n(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n) = \frac{P_n(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n)}{P(\mathbf{p}_1)P(\mathbf{p}_2)\cdots P(\mathbf{p}_n)}, \quad (2)$$

where $P_n(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n)$ is the probability of observing n identical pions of momenta $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$ in the event, and $P(\mathbf{p}_i)$ is the single-pion-inclusive distribution without the effect of identical pion correlations.

Assuming that the probability of observing two identical pions of momenta $\mathbf{p}_1, \mathbf{p}_2$ in an n-pion event is $P_{2/n}(\mathbf{p}_1, \mathbf{p}_2)$, the two-pion correlation function in the event is defined as

$$C_{2/n}(\mathbf{p}_1, \mathbf{p}_2) = \frac{P_{2/n}(\mathbf{p}_1, \mathbf{p}_2)}{P(\mathbf{p}_1)P(\mathbf{p}_2)}, \quad (3)$$

where $P_{2/n}(\mathbf{p}_1, \mathbf{p}_2)$ is related to $P_n(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n)$ by

$$P_{2/n}(\mathbf{p}_1, \mathbf{p}_2) \propto \int \cdots \int d\mathbf{p}_3 \cdots d\mathbf{p}_n P_n(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n). \quad (4)$$

According to the definition of the correlation function Eq. (3), and using the relative momentum q of two pions as a variable in two-pion interferometry analyses, the two-pion correlation function in an n-pion event is obtained from

$$C_{2/n}(q) \propto \frac{\mathrm{Cor}_{2/n}(q)}{\mathrm{Uncor}(q)}, \quad (5)$$

where

$$\mathrm{Cor}_{2/n}(q) = \int d\mathbf{p}_1 \int d\mathbf{p}_2 P_{2/n}(\mathbf{p}_1, \mathbf{p}_2)\delta(|\mathbf{p}_1 - \mathbf{p}_2| - q) \quad (6)$$

is the distribution of correlated-pion pairs in terms of q, and

$$\mathrm{Uncor}(q) = \int d\mathbf{p}_1 \int d\mathbf{p}_2 P(\mathbf{p}_1)P(\mathbf{p}_2)\delta(|\mathbf{p}_1 - \mathbf{p}_2| - q) \quad (7)$$

is the distribution of uncorrelated-pion pairs in terms of q. In particular, Eq. (1) represents the two-pion correlation function $C_{2/2}(q)$ in a two-pion event, which is also denoted by $C_2(q)$ from Eq. (2).

From Eq. (5), the two-pion correlation function in an n-pion event can be written as

$$C_{2/n}(q) = C_2(q)\overline{C}_{n-2}(q), \quad (8)$$

where

$$\overline{C}_{n-2}(q) = \kappa \frac{\mathrm{Cor}_{2/n}(q)}{\mathrm{Cor}_{2/2}(q)}, \quad (9)$$

and

$$\mathrm{Cor}_{2/2}(q) = \int d\mathbf{p}_1 \int d\mathbf{p}_2 P_2(\mathbf{p}_1, \mathbf{p}_2)\delta(|\mathbf{p}_1 - \mathbf{p}_2| - q), \quad (10)$$

which is the distribution of correlated-pion pairs in a two-pion event, and κ is a normalization constant. We call $\overline{C}_{n-2}(q)$ the multipion correlation factor, which reflects the influence of multipion correlations on the two-pion correlation function [for $n = 2$, $\overline{C}_{n-2}(q) = 1$].

The Metropolis approach is the standard Monte Carlo technique which allows one to generate an ensemble of multibody configurations according to some probability density. Using this approach, Monte Carlo events with multipion correlations can be generated [8, 24]. In our Monte Carlo simulations, the probability of multipion correlations is calculated using the Ryser-Wilf-Nijenhuis (RWN) algorithm [8, 25], and the pion source is completely incoherent. Figure 1 shows the variation of $\overline{C}_{n-2}(q)$ with source radius R and temperature T and with pion multiplicity, for the Monte Carlo simulated events, where Figs. 1(a), (b), and (c) show the multipion correlation factor for eight-pion, five-pion, and three-pion events, respectively. The total number of correlated-pion pairs for every kind of event is 2×10^5, and the single-pion-inclusive distribution is simulated according to the Boltzmann distribution. In Figs. 1(a), (b), and (c), the open triangle symbol corresponds to the pion source parameters $T = 50$ MeV, $R = 3.0$ fm; the solid dot symbol corresponds to the pion source parameters $T = 80$ MeV, $R = 3.0$ fm; and the open square symbol corresponds to

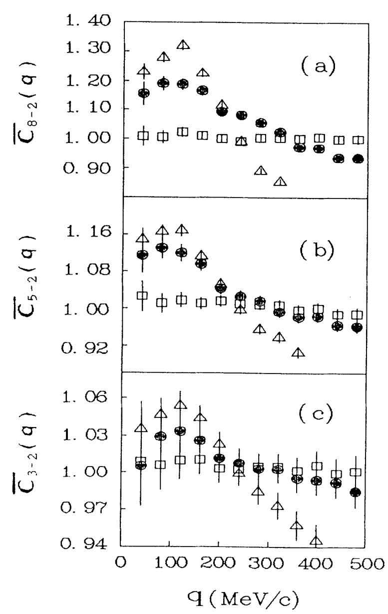

FIG. 1. Multipion correlation factors $\overline{C}_{n-2}(q)$ for Monte Carlo multipion events; (a) for eight-pion events, (b) for five-pion events, and (c) for three-pion events. The symbols △, •, and □ correspond to the following pion source parameters: ($T = 50$ MeV, $R = 3.0$ fm), ($T = 80$ MeV, $R = 3.0$ fm), and ($T = 70$ MeV, $R = 5.5$ fm).

the pion source parameters $T = 70$ MeV, $R = 5.5$ fm.

From Fig. 1 it can be seen that $\overline{C}_{n-2}(q) \sim 1$, and the influence of multipion correlations can be neglected when the radius of a pion source is large and the pion multiplicity is low. Conversely, when the radius of a pion source is small and the pion multiplicity is high, the multipion correlation effect becomes important. In such a case, the distribution of $\overline{C}_{n-2}(q)$ is greater than unity in the low relative momentum region, and indicates that the influence of multipion correlations makes the number of correlated-pion pairs increase in the low relative momentum region and decrease in the high relative momentum region. Because the correlation between two like pions becomes stronger when $q \to 0$, the influence of multipion correlations becomes relatively weaker. Under conditions of the same pion multiplicity, the multipion correlation factor is affected more by the radius than by the temperature of the pion source. The results of previous theoretical analyses pointed to the multipion correlation effect increasing with the inverse of R^2T [5–7, 9]. For a partially coherent pion source, because the coherence factors for each order of correlation are less than unity, the multipion correlation effect is less than for an incoherent source.

III. THE INFLUENCE OF MULTIPION CORRELATIONS ON TWO-PION INTERFEROMETRY

In Sec. II, we discussed the influence of multipion correlations on the sample of like-pion pairs, which can be represented by the multipion correlation factor, while the reference sample Uncor(q), Eq. (7), which is called the ideal reference sample, does not contain any Bose-Einstein correlations. When carrying out a two-pion interferometry analysis, the reference sample is usually constructed by mixing pions selected from different events with the same pion multiplicity, which can be expressed as [14]

$$\mathrm{Uncor}'(q) = \int d\mathbf{p}_1 \int d\mathbf{p}_2 \; P'(\mathbf{p}_1)P'(\mathbf{p}_2)\delta(|\mathbf{p}_1 - \mathbf{p}_2| - q)\,, \tag{11}$$

where $P'(\mathbf{p})$ is the single-pion-inclusive distribution obtained from n-pion events,

$$\begin{aligned} P'(\mathbf{p}) \propto P(\mathbf{p}) &\int d\mathbf{p}' P(\mathbf{p}')C_{2/n}(\mathbf{p},\mathbf{p}') \\ = P(\mathbf{p}) &\int d\mathbf{p}' \; P(\mathbf{p}')C_2(\mathbf{p},\mathbf{p}')\overline{C}_{n-2}(\mathbf{p},\mathbf{p}'). \end{aligned} \tag{12}$$

From Eq. (11) and Eq. (12) it can be seen that there are residual correlations in the reference sample Uncor$'(q)$, which are directly related to the multipion correlation factor in an n-pion event. Let

$$\overline{\mathrm{Uncor}}(q) = \frac{\mathrm{Uncor}'(q)}{\mathrm{Uncor}(q)}\,, \tag{13}$$

we call $\overline{\mathrm{Uncor}}(q)$ the residual correlation factor, which reflects residual correlations in the reference sample Uncor$'(q)$.

In two-pion interferometry analyses for multipion events, the correlation function is obtained from the ratio of the sample $\mathrm{Cor}_{2/n}(q)$ of like pion pairs to the reference sample Uncor$'(q)$, which is different from $C_2(q)$. From Eq. (5), Eq. (8), and Eq. (13), it can be obtained that

$$C_2(q) \propto \alpha \frac{\mathrm{Cor}_{2/n}(q)}{\mathrm{Uncor}'(q)}\,, \tag{14}$$

where

$$\alpha = \frac{\overline{\mathrm{Uncor}}(q)}{\overline{C}_{n-2}(q)}\,. \tag{15}$$

We see that the influence of multipion correlations on two-pion interferometry analyses arises from both the multipion correlation factor $\overline{C}_{n-2}(q)$ and the residual correlation factor $\overline{\mathrm{Uncor}}(q)$.

Figures 2(a) and (b) show the comparison of these two correlation factors $\overline{\mathrm{Uncor}}(q)$ (solid dot symbol) and $\overline{C}_{n-2}(q)$ (open triangle symbol) in eight-pion and five-pion Monte Carlo events. A total of 2×10^5 correlated-pion pairs were used to construct $\overline{C}_{n-2}(q)$ for each kind of event, and the number of pion pairs in each reference sample used to construct $\overline{\mathrm{Uncor}}(q)$ is about 20 times the number of correlated-pion pairs. The difference between these two correlation factors in the low q region increases with the pion multiplicity.

References [2] and [13] deal with two-pion interferometry analyses for experimental data from the Bevalac's Janus spectrometer. By weighting the events appropriately, Zajc *et al.* and Chacon *et al.* removed the residual correlation effect arising from two-pion correlations in the usual reference sample. Because of the small acceptance of the Janus spectrometer, their analyses were based on events in which no more than two pions per

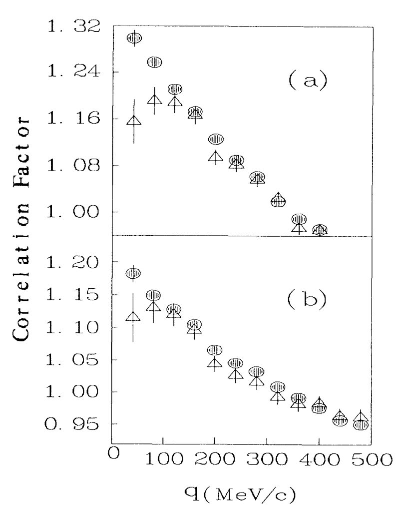

FIG. 2. The comparison of $\overline{\mathrm{Uncor}}(q)$ ($\bullet$) and $\overline{C}_{n-2}(q)$ ($\triangle$) for Monte Carlo multipion events, with $T = 80$ MeV, $R = 3.0$ fm; (a) for eight-pion events, and (b) for five-pion events.

TABLE I. Two-pion interferometry results for Monte Carlo multipion events.

		5π event		8π event	
	T (MeV)	80	70	80	70
Pion source Parameters	R (fm)	3.0	5.5	3.0	5.5
	λ	1	1	1	1
Fitted results (a)	R (fm)	3.16±0.07	5.63±0.26	3.24±0.07	5.44±0.26
	λ	0.85±0.04	0.94±0.09	0.80±0.04	0.91±0.08
Fitted results (b)	R (fm)	3.01±0.06	5.59±0.21	2.91±0.08	5.61±0.21
	λ	0.96±0.06	1.03±0.10	0.92±0.05	1.02±0.08

event were detected. From Eq. (12) it can be seen that the difference in the residual correlations between the reference samples for n-pion events and two-pion events is only the factor $\overline{C}_{n-2}$. Thus, using a method similar to that in Refs. [2] and [13], the residual effects due to multipion correlations in the reference sample can be removed. Furthermore, if we weight the reference sample to remove the residual correlations using the multipion correlation factor [9], both the influence of the residual correlations in the reference sample and the multipion correlations in the sample of correlated-pion pairs can be eliminated from two-pion interferometry results. In the Appendix, the expression $\overline{C}_{n-2}(q)$ for calculating weights is given.

Table I lists the two-pion interferometry results for multipion events generated by Monte Carlo. The numbers of five-pion and eight-pion events are approximately 2×10^4 and 7.2×10^3, respectively. The number of correlated-pion pairs for each kind of event is 2×10^5. The fitted results in row (a) are uncorrected. Although the multipion correlation factor for the sample of correlated-pion pairs and the residual correlation factor for the reference sample can partially cancel each other, the difference between them increases in the low relative momentum region when the pion source has a small radius and the pion multiplicity is high. In such a case, the effect of multipion correlations makes the fitted results deviate significantly from the real parameters of the pion source. The fitted results in row (b) of Table I were obtained after correcting for the residual correlations in the reference sample as well as for the multipion correlations in the sample of correlated-pion pairs according to the method described above.

IV. ANALYSES OF EXPERIMENTAL DATA

The data analyzed in this paper come from 2.1A GeV Ne + Pb and 1.8A GeV Ar + Pb central collisions at the Bevalac streamer chamber [3, 5, 6]. In order to remove the effect of electron contamination of the π^- sample, a momentum cut $p_{\rm lab} \geq 100$ MeV/c has been imposed [3, 5, 6]. In our experimental data, the number of correlated-pion pairs from events with a single specific value for the π^- multiplicity is limited. In order to compare the multipion correlation effect in experimental events with adequate statistics, we extend Eq. (9) and define

$$\overline{C'}_{n-2}(q) = \kappa \frac{\sum_{i=n}^{n'} \mathrm{Cor}_{2/n}(q)}{\sum_{j=2}^{m} \mathrm{Cor}_{2/j}(q)} \quad (n \geq m), \tag{16}$$

where the sum extends over some range of π^- multiplicity. The Monte Carlo results indicate that $\overline{C'}_{n-2}(q)$ and $\overline{C}_{n-2}(q)$ for various pion source parameters have the same pattern of variation. The Gamow factor [1–3] for Coulomb interaction between two correlated pions cancels in $\overline{C'}_{n-2}(q)$.

Let

$$\langle M_\pi \rangle_{\rm pair} = \sum_{M_\pi=m_1}^{m_2} M_\pi N(M_\pi) \Big/ \sum_{M_\pi=m_1}^{m_2} N(M_\pi) \ , \tag{17}$$

where $N(M_\pi)$ denotes the number of correlated-pion pairs provided by all the events with a given π^- multiplicity M_π, $[m_1, m_2]$ is a certain range of M_π, and $\langle M_\pi \rangle_{\rm pair}$ is the average pion multiplicity of the subset of correlated-pion pairs in the range $[m_1, m_2]$. Table II gives the number of events, the number of correlated-pion pairs, and the value of $\langle M_\pi \rangle_{\rm pair}$ for different ranges of π^- multiplicity in our two experimental data samples.

Figure 3 and Fig. 4 show the variation of $\overline{C'}_{n-2}(q)$ for these two experimental samples. In Fig. 3, referring to Eq. (16), the sum range of j is [2,5], the sum ranges of i are $[n, n']$=[5,8] (corresponding to the solid dot symbol in Fig. 3) and $[n, n']$=[8,17] (corresponding to the

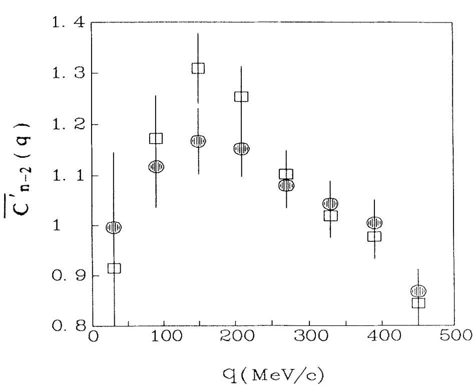

FIG. 3. The variation of $\overline{C'}_{n-2}(q)$ for 2.1A GeV Ne + Pb. Referring to Eq. (16), the sum range of j is [2,5], and the sum ranges of i are [5,8] (corresponding to the symbol •) and [8,17] (corresponding to the symbol □).

TABLE II. The number of events, the number of correlated-pion pairs, and the value of $\langle M_\pi \rangle_{\text{pair}}$ in our two experimental samples.

	2.1A GeV Ne + Pb			1.8A GeV Ar + Pb		
Multiplicity interval	$2 \le M_\pi \le 5$	$5 \le M_\pi \le 8$	$8 \le M_\pi \le 17$	$2 \le M_\pi \le 8$	$8 \le M_\pi \le 12$	$12 \le M_\pi \le 20$
Number of events	4271	2117	1124	1596	1635	928
Number of like-pion pairs	9858	19528	25648	15809	46579	52676
$\langle M_\pi \rangle_{\text{pair}}$	4.02	6.63	10.30	6.67	10.24	14.36

open square symbol in Fig. 3). In Fig. 4, the sum range of j is [2,8], and the sum ranges of i are $[n, n']$=[8,12] (corresponding to the solid dot symbol in Fig. 4) and $[n, n']$=[12,20] (corresponding to the open square symbol in Fig. 4). Comparing the distribution of $\overline{C'}_{n-2}(q)$ in Fig. 3 and in Fig. 4, it can be seen that the multipion correlation effect in the 2.1A GeV Ne + Pb collisions is greater than that in the 1.8A GeV Ar + Pb collisions. This result mainly arises from the fact that the π^- source in the former sample is smaller than in the latter. Furthermore, we can also see that the difference between the distributions of the multipion correlation factor for different ranges of π^- multiplicity is larger in Fig. 3 than in Fig. 4.

For these two experiments, Table III gives the two-pion interferometry results after imposition of the Gamow correction factor for π^--π^- Coulomb repulsion [1–3]. In this table, the fitted results in row (a) are obtained by using the usual reference sample, and the fitted results in row (b) are obtained after removing the residual correlations in the reference sample and the multipion correlations in the sample of correlated-pion pairs. In row (b), the two independent initial parameters R_0 and λ_0 are needed to calculate the weights iteratively (see the Appendix). Because the residual correlation factor for the reference sample and the multipion correlation factor for the sample of correlated-pion pairs can partially cancel each other, the results in rows (a) and (b) are the same within the statistical errors of our experimental data. The fitted results in row (c) of Table III are obtained after correction only for the residual correlations. Because of the influence of multipion correlations in the sample of correlated-pion pairs, the number of correlated-pion pairs in the low q region increases, hence the two-pion correlation effect is enhanced, and the fitted R becomes smaller. The fitted results in row (d) of Table III are obtained after correction only for the multipion correlations in the sample of correlated-pion pairs. Because the residual correlations make the number of pion pairs in the reference sample increase in the low q region, the fitted R becomes larger. In these two cases, the fitted results have a relatively large deviation from the fitted results in rows (a) and (b).

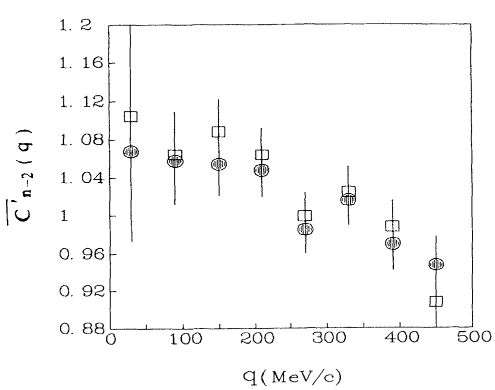

FIG. 4. The variation of $\overline{C'}_{n-2}(q)$ for 1.8A GeV Ar + Pb. Referring to Eq. (16), the sum range of j is [2,8], and the sum ranges of i are [8,12] (corresponding to the symbol •) and [12,20] (corresponding to the symbol □).

V. CONCLUSIONS

It has been shown over the past decade that multipion interferometry [4–12] offers a powerful tool to check assumptions about two-particle correlations, and to test the findings of two-pion interferometry analyses. However, the study of multipion Bose-Einstein interference has been restricted to a few lower orders because of an excessive number of computations as the correlation order increases, and poor statistics. In this paper, we propose an alternative approach which reveals the multipion correlation effects in Bevalac streamer chamber data samples and offers a means to correct the distortion of two-pion analyses caused by multipion interference.

The influence of multipion symmetrization on two-pion interferometry analyses is attributed to the ratio of the residual correlation factor to the multipion correlation factor. Because these two correlation factors can partially cancel each other in data analyses, the distortion of two-pion interferometry caused by multipion correlations cannot be identified within the experimental accuracy of our streamer chamber data. Nevertheless, the distributions of multipion correlation factors $\overline{C}_{n-2}(q)$ or $\overline{C'}_{n-2}(q)$ offer good sensitivity to multipion correlation effects. Comparing 2.1A GeV Ne+Pb and 1.8A GeV Ar+Pb, we find a larger multipion effect for the Ne beam than for the Ar beam, which is a consequence of a smaller pion source radius with the lighter projectile.

TABLE III. The fitted results for experimental data.

		2.1A GeV Ne + Pb		1.8A GeV Ar + Pb	
	Multiplicity interval	$2 \le M_\pi \le 8$	$9 \le M_\pi \le 17$	$2 \le M_\pi \le 11$	$12 \le M_\pi \le 20$
Fitted results (a)	R (fm)	3.24±0.41	3.91±0.72	5.21±0.52	5.28±0.60
	λ	0.90±0.22	0.68±0.28	0.80±0.16	0.99±0.20
Fitted results (b)	R (fm)	3.25±0.44	3.63±0.60	5.22±0.55	5.81±0.62
	λ	0.87±0.23	0.84±0.24	0.76±0.16	0.99±0.20
Fitted results (c)	R (fm)	2.85±0.39	2.06±0.31	4.85±0.51	5.12±0.58
	λ	0.86±0.26	0.47±0.16	0.82±0.15	1.04±0.20
Fitted results (d)	R (fm)	3.98±0.70	6.18±1.43	6.15±0.90	7.55±0.98
	λ	0.82±0.28	0.35±0.12	0.80±0.19	0.72±0.20

The distortion of two-pion interferometry from multipion interference depends on the difference between the multipion correlation factor and the residual correlation factor in the low relative momentum region, which increases when the pion source has a small radius and the pion multiplicity is high. We propose a procedure for correcting the distortion from multipion interference in two-pion interferometry. Now the emphasis in the heavy ion field is shifting more to CERN SPS and RHIC energies where up to several hundred identical pions might be detected in a single event. The effect of multiparticle Bose-Einstein correlations is an obvious and important subject in this energy regime.

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APPENDIX A: THE EXPRESSION FOR CALCULATING WEIGHTS

From Eq. (8), and using Eq. (5) and Eq. (6), we have

$$\overline{C}_{n-2}(q) \propto \frac{1}{C_2(q)\mathrm{Uncor}(q)} \int d\mathbf{p}_1 \int d\mathbf{p}_2 \; P_{2/n}(\mathbf{p}_1, \mathbf{p}_2)\delta(|\mathbf{p}_1 - \mathbf{p}_2| - q), \tag{A1}$$

substituting Eq. (4) into Eq. (A1) , and using Eq. (2), it follows that

$$\overline{C}_{n-2}(q) = \frac{\kappa'}{C_2(q)\mathrm{Uncor}(q)} \int d\mathbf{p}_1 P(\mathbf{p}_1) \int d\mathbf{p}_2 P(\mathbf{p}_2)\delta(|\mathbf{p}_1 - \mathbf{p}_2| - q) \int d\mathbf{p}_3 P(\mathbf{p}_3) \cdots \int d\mathbf{p}_n P(\mathbf{p}_n) C_n(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n). \tag{A2}$$

Using the explicit expression for the high-order correlation function, $C_n(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n)$, for a Gaussian source derived with the graphical method [5, 6], the integration on the right side of Eq. (A2) can be completed in principle. Up to the fourth order of correlation, $\overline{C}_{n-2}(q)$ can be written as

$$\overline{C}_{n-2}(q) \approx \kappa' \bigg\{ 1 + \frac{1}{I_0(q)} \bigg[2n_1\lambda I_1(q) + 2n_2\lambda I'_1(q) + 2n_1\xi I_2(q) + \frac{2}{3}(n+2)n_2\xi I'_2(q) + n_2\lambda^2 I_3(q) + \frac{1}{4}(n^2 - n - 4)n_2\lambda^2 I'_3(q) + 6n_2\eta I_4(q) + \frac{1}{2}(n^2 - n - 12)\eta I'_4(q) \bigg] \bigg\}, \tag{A3}$$

where

$$I_0(q) = C_2(q)\mathrm{Uncor}(q)\,,$$

$$n_1 = n - 2, \qquad n_2 = (n-2)(n-3)/2\,,$$

and κ' is a normalization constant; λ, ξ, and η are the coherence factors for two-pion, three-pion, and four-pion correlations, respectively.

In Eq. (A3), the term $\lambda I_1(q)$ reflects the average effects of the two-pion correlations between one pion of the sampled pion pair from an n-pion event and one other pion in the event; the term $\lambda I'_1(q)$ reflects the average effects of the two-pion correlations, which exclude the pions of the sampled pair; the term $\xi I_2(q)$ reflects the average effects of the pure triplet correlations among the two pions of the sampled pion pair combined with one other pion; the term $\xi I'_2(q)$ reflects the average effects of the other pure triplet correlations; the term $\lambda^2 I_3(q)$ reflects the average effects of the two-pion pair correlations between the sampled pion pair and another possible pion pair from the remaining pions; the term $\lambda^2 I'_3(q)$ reflects the average effects of the other two-pion pair correlations; the term $\eta I_4(q)$ reflects the average effects of the pure quadruplet correlations among the two pions of the sampled pion pair combined with two other pions; and the term $\eta I'_4(q)$ reflects the average effects of the other pure quadruplet correlations.

Because the fraction of particle phase space which corresponds to maximal correlations becomes smaller with increasing number of particles, it is more difficult to observe maximal Bose-Einstein correlations for large multiplets. Monte Carlo calculation shows that the values of $I_m(q)$ and $I'_m(q)$ $(m = 1,2,3,4)$ averaged over q decrease with increasing m. On the other hand, the contribution of each order of correlation to $\overline{C}_{n-2}(q)$ in Eq. (A3) also depends on the coefficient of each corresponding term, which increases with the multiplicity of the event. In the analyses of our 4π streamer chamber data, it is adequate to take into account the correlations up to the fourth order and to neglect the higher-order terms.

Because λ, ξ, and η are related [5,6], only two independent initial values R_0 and λ_0 for the source radius and the coherence factor are needed to calculate the weights. These values remain stable within the statistics errors [2, 13] after one to three iterations.

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