

Transport model with quasipions

L. Xiong,^{(1,2),*} C. M. Ko,⁽²⁾ and V. Koch⁽¹⁾

⁽¹⁾*Department of Physics, State University of New York, Stony Brook, New York 11794*

⁽²⁾*Cyclotron Institute and Physics Department, Texas A&M University, College Station, Texas 77843*

(Received 6 July 1992)

We extend the normal transport model to include the medium effect on pions by treating them as quasiparticles. The property of the quasipion is determined using the delta-hole model. Modeling heavy-ion collisions at intermediate energies with the new transport equations, we find that it leads to an enhanced production of pions with low kinetic energies. This gives a plausible explanation for the observed enhancement of soft pions in the Bevalac experiments.

PACS number(s): 25.75.+r

I. INTRODUCTION

Pion production from heavy-ion collisions has been studied extensively in the past. Early studies were based on the intranuclear cascade model [1, 2]. More recently, the transport model that takes into account both nucleon-nucleon collisions and the nuclear mean-field potential (normally called the Vlasov-Uehling-Uhlenbeck or Boltzmann-Uehling-Uhlenbeck model [3]) have been extended to include the pion degree of freedom [4–7]. In these studies, the pion is produced from delta decay and can be absorbed by a nucleon via the process $\Delta \leftrightarrow N\pi$, which is responsible for the collision term in the transport equation for the pion [8]. In deriving the transport equation, the pion collision term is obtained from the imaginary part of its self-energy. In nuclear medium, the pion self-energy is modified by the strong p -wave pion-nucleon interaction. This not only affects the production and absorption of the pion but also leads to a nonvanishing real part of the pion self-energy. The latter contributes to the mean-field potential for the pion so it will not propagate as a free particle as usually assumed in the transport model and used extensively in the literature.

On the other hand, the pion self-energy in the nuclear medium has been studied before in the delta-hole model [9, 10]. It has been found that the in-medium pion dispersion relation consists of a lower pion branch and an upper delta-hole branch and that the pion branch becomes softened in the matter. The softened pion dispersion relation in the nuclear medium has been shown to lead to enhanced nucleon-nucleon interaction cross section [11–13].

The pion dynamics in dense matter can be studied via dilepton production from pion-pion annihilation [14–16]. With a softened pion spectrum in the medium, an enhanced production of dileptons with invariant masses around twice the pion mass has been predicted. The change of pion property in the medium also affects the

property of the rho meson [17] which can be investigated from the dilepton invariant mass spectrum as well. Experiments are being carried out at the Bevalac by the DLS Collaboration [18–20] and are also planned at SIS [21] to test these predictions.

Another interesting phenomenon about pions in heavy-ion collisions is the enhancement of low energy pions that has been observed in experiments carried out at Bevalac [22], the Brookhaven AGS [23], and CERN [24, 25]. Three suggestions have been proposed to explain this enhancement. One is that the low-energy pions are mostly produced from delta decays at the later stage of the collision [7]. In another explanation [26, 27], it is argued that although the thermal equilibrium is achieved in heavy-ion collisions, the chemical processes are not equilibrated, and this leads to a finite positive chemical potential for pions. From the Bose-Einstein distribution, pions with energy close to the chemical potential are therefore greatly enhanced. Finally, it has been suggested [28] that a soft pion dispersion relation in the nuclear medium would affect the pion energy distribution in heavy-ion collisions, leading to an enhancement of soft pions because pions lose their kinetic energies in climbing out of the momentum-dependent potential well.

In the transport model, both delta decays and chemical nonequilibrium have already been included. It is thus of interest to take into account the modified pion dispersion relation in the transport model. Although thermal equilibrium may not be reached in heavy-ion collisions at Bevalac and SIS energies of about 1 GeV/nucleon, the medium effect on the pion dispersion relation can be included via the local-density approximation as in the treatment of the mean-field potential for the nucleon. As shown later, the delta-hole polarization of the nuclear matter is mainly a density effect if the nucleon kinetic energy is less than the mass difference between delta and nucleon, i.e., about twice the pion mass. For heavy-ion collisions at Bevalac and SIS energies, this condition is fulfilled most of the time except in the initial approaching stage of the collision when the system is best described by two colliding Fermi spheres. Due to the strong final-state interaction for the pion, the approaching stage of the collision is not expected to play an important role in

*Present address: Department of Physics, State University of New York, Stony Brook, NY 11794.

the final pion energy spectrum.

This paper is organized as follows. In Sec. II, we review the delta-hole model for calculating the pion dispersion relation in the nuclear medium. The in-medium delta width is discussed in Sec. III. In Sec. IV, the propagation of the quasipion in the nuclear medium is described. In Sec. V, the pion spectrum from the collision of two La nuclei at an incident energy of 1.35 GeV/nucleon is calculated with the new transport model and is compared with the experimental data from the Bevalac. Finally, the summary is given in Sec. VI.

II. PION DISPERSION RELATION IN THE NUCLEAR MEDIUM

The pion dispersion relation is modified in the nuclear medium because of the strong p -wave pion-nucleon interaction. This has been studied before [9, 10] but for completeness we shall describe it briefly. The self-energy of a pion with energy ω and momentum k due to the Δ -hole polarization is shown in Fig. 1 and is given by

$$\Pi_0 = k^2 \chi \approx \frac{8}{9} \left(\frac{f_\Delta}{m_\pi} \right)^2 k^2 \rho_N \frac{\omega_R}{\omega^2 - \omega_R^2} e^{-2k^2/b^2}, \quad (1)$$

with

$$\omega_R = \frac{k^2}{2m_\Delta} + m_\Delta - m_N. \quad (2)$$

In the above, m_π , m_N , and m_Δ are the pion, nucleon, and delta masses, respectively; $f_\Delta \sim 2$ is the $\pi N\Delta$ coupling constant; ρ_N is the nuclear density; and $b \sim 7m_\pi$ is the cutoff of the $\pi N\Delta$ form factor. In deriving Eq. (1) the kinetic energy of the nucleons has been neglected in comparison with ω_R of Eq. (2), so the pion self-energy becomes proportional to the nuclear density. This is valid for both normal nuclear matter and the dense matter formed in heavy-ion collisions at about 1 GeV/nucleon.

But one needs to include the short-range Δ -hole interaction and this is usually done via the Migdal parameter $g' \sim 0.6$. The resulting pion self-energy becomes

$$\Pi = k^2 \frac{\chi}{1 - g'\chi}. \quad (3)$$

The pion dispersion relation in the medium is thus given by

$$\omega^2 = k^2 + m_\pi^2 + \Pi(\omega, k). \quad (4)$$

In Fig. 2, we show the pion dispersion relation in the nuclear medium. The curve labeled by ω_F is the pion dis-

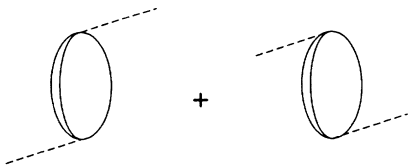


FIG. 1. Delta-hole polarization. The pion, delta, and nucleon hole are represented by the dashed line, the bar, and the solid curve, respectively.

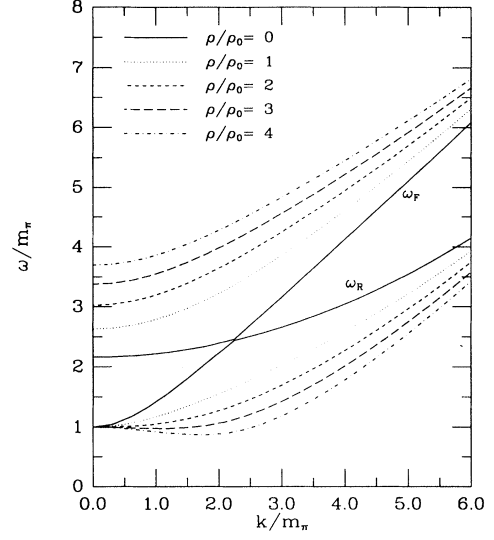


FIG. 2. Pion dispersion relation in the nuclear medium at different densities.

ersion relation in free space while the curve labeled by ω_R is the energy of the Δ -hole state as given by Eq. (2). The $\pi N\Delta$ interaction mixes the pion with the Δ -hole state and leads to two branches of eigenstates. They are normally called the pion branch and the Δ -hole branch and are shown by those curves in the lower and upper parts of the figure, respectively, for different nuclear matter densities. We see that the pion branch becomes softened as the nuclear matter density increases. In particular, the pion spectrum given by the pion branch shows a minimum once the nuclear density is higher than four times the normal density. On the other hand, the Δ -hole branch becomes stiffer when its coupling to the pion is taken into account.

Both pion and Δ -hole branches consist of the pion component. At a given momentum k , the probability $S(k, \rho_N)$ that a state in Fig. 2 is a pion can be determined from the residue of the pion propagator, i.e.,

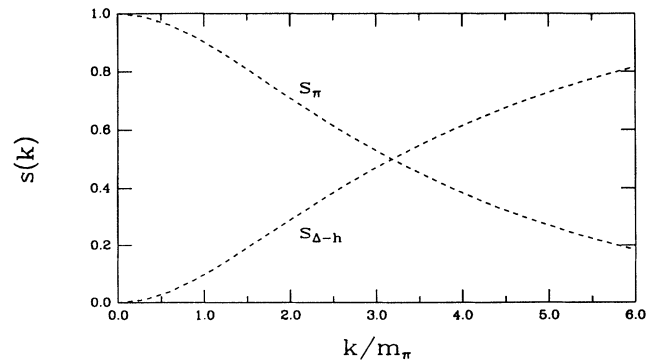


FIG. 3. Probabilities S_π and $S_{\Delta-h}$ of the pion component in the pion branch and the Δ -hole branch, respectively, as functions of the pion momentum k at three times normal nuclear density.

$$S(k, \rho_N) = \frac{1}{1 - \partial\Pi(\omega, \mathbf{k})/\partial\omega^2}. \quad (5)$$

In Fig. 3, the probabilities S_π and $S_{\Delta-h}$ of the pion component in the pion branch and Δ -hole branch, respectively, are shown as functions of the pion momentum k for nuclear matter at three times normal density. It is seen that at a given momentum their sum is exactly one as expected.

The nucleon particle-hole polarization is normally neglected in calculating the pion self-energy. Its contribution is small because of the smaller πNN coupling constant than $\pi N\Delta$, the smaller nucleon's spin and isospin

$$\begin{aligned} \Gamma(m_\Delta, \rho) &= \frac{1}{2m_\Delta} \int \frac{d^3\mathbf{p}_N}{(2\pi)^3 2E_N} \frac{d^3\mathbf{k}d\omega}{(2\pi)^3} \delta(\omega^2 - \mathbf{k}^2 - m_\pi^2 - \Pi(\omega, \mathbf{k})) |\mathcal{M}|^2 (2\pi)^4 \delta^{(3)}(\mathbf{p}_N + \mathbf{k}) \delta(\sqrt{m_N^2 + \mathbf{k}^2} + \omega - m_\Delta) \\ &= \sum_i \frac{\pi \mathbf{k}_i^2}{(2\pi)^2 m_N m_\Delta \omega_i} S(k_i, \rho_N) |\mathcal{M}|^2 \left| \frac{d\omega_i}{d\mathbf{k}_i} \right|^{-1}. \end{aligned} \quad (6)$$

The summation in Eq. (6) is over the pion component in both the pion and delta-hole branches. A delta in the nuclear medium can decay to either one of the two branches of the pion spectrum, and its total width is given by the sum of the two partial widths.

The invariant matrix element for delta decay to a nucleon and a pion in the delta rest frame is

$$|\mathcal{M}|^2 = \frac{1}{4} \left(\frac{f_{\pi N\Delta}}{m_\pi} \right)^2 \text{Tr}[P^{\mu\nu}(p_\Delta) P_N(p_N)] k_\mu k_\nu, \quad (7)$$

where P_Δ , P_N , and k_μ are the four-momenta of the delta, nucleon, and pion, respectively; and the factor $\frac{1}{4}$ comes from the average over the delta spin. Using the well-known nucleon and delta projection operators,

$$P_N(p_N) = \not{p}_N + m_N \quad (8)$$

and

$$\begin{aligned} P^{\alpha\beta}(p_\Delta) &= (\not{p}_\Delta + m_\Delta) \left(g^{\alpha\beta} - \frac{2p_\Delta^\alpha p_\Delta^\beta}{3m_\Delta^2} - \frac{\gamma^\alpha \gamma^\beta}{3} \right. \\ &\quad \left. + \frac{p_\Delta^\alpha \gamma^\beta - p_\Delta^\beta \gamma^\alpha}{3m_\Delta} \right), \end{aligned} \quad (9)$$

respectively, we have

$$|\mathcal{M}|^2 = \frac{4}{3} \left(\frac{f_{\pi N\Delta}}{m_\pi} \right)^2 m_N m_\Delta \mathbf{k}^2. \quad (10)$$

This expression needs to be corrected by the vertex correction due to the delta-hole polarization [29] and the result is

$$|\mathcal{M}|^2 = \frac{4}{3} \left(\frac{f_{\pi N\Delta}}{m_\pi} \right)^2 m_N m_\Delta \left(\frac{\mathbf{k}}{1 - g'\chi} \right)^2. \quad (11)$$

In Fig. 4, we show as a function of delta mass the partial decay widths of a delta in the nuclear medium to the pion branch and the delta-hole branch, and also

degeneracies, and the larger suppression by local field corrections.

III. DELTA WIDTH IN THE NUCLEAR MEDIUM

Because of the change of the pion spectrum in nuclear medium, the delta width is expected to be modified as well. In Ref. [29], the self-consistent delta width has been calculated in a hot dense nuclear matter and is found to be much larger than its value in free space. So the process $\Delta \leftrightarrow \pi N$ is expected to occur more frequently in the dense matter formed in heavy-ion collisions.

For a delta of mass m_Δ decaying into a nucleon and a pion with four momenta (E_N, \mathbf{p}_N) and (ω, \mathbf{k}) , respectively, its width is given by

the total width. The partial decay width to the delta-hole branch, given by the top panel of Fig. 4, becomes nonzero above a threshold delta mass which increases with nuclear density due to the stiffening of the delta-hole branch in the nuclear medium. Since the probability of the pion component in the delta-hole branch becomes larger with

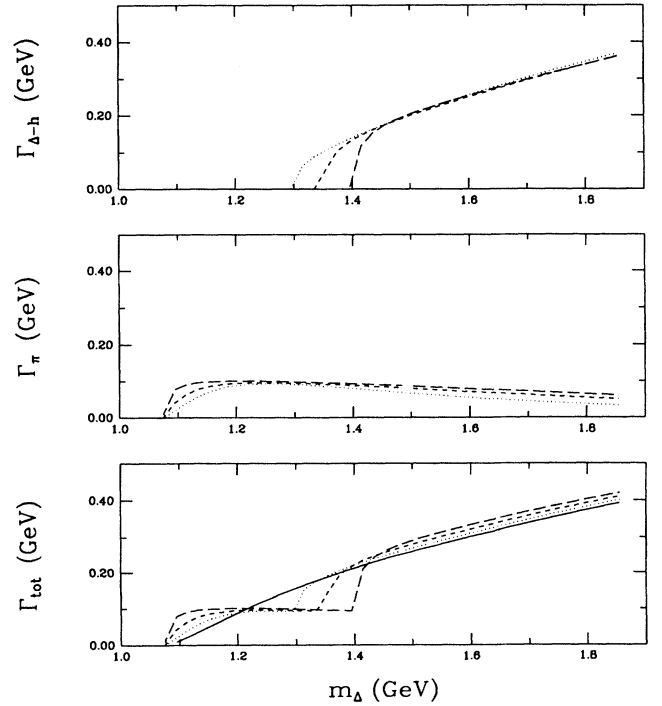


FIG. 4. Delta decay width in the medium. Top panel: partial width to the delta-hole branch; middle panel: partial width to the pion branch; bottom panel: total width. Dotted, short-dashed, and long-dashed curves correspond to nuclear densities at ρ_0 , $2\rho_0$, and $3\rho_0$, respectively. The solid curve in the bottom panel is the delta width in free space.

increasing pion momentum (see Fig. 3), the partial decay width to the delta-hole branch is seen to increase with the delta mass.

For deltas with small masses, they can only decay to the pion branch because of energy-momentum conservation. As seen in the middle panel of Fig. 4, the partial decay width of these deltas to the pion branch is enhanced at higher densities as a result of the softening of the pion branch with increasing nuclear density. As the delta mass becomes larger, both the decay matrix element and phase space increase because of the increasing pion momentum. The probability of the pion component in the pion branch decreases, however, with the pion momentum (see Fig. 3), and the resulting delta partial decay width to a pion turns out almost independent of the delta mass.

The total delta width, given by the sum of the two partial widths and shown in the bottom panel of Fig. 4, is thus enhanced in dense nuclear medium for small delta masses and reduced for medium delta masses. For large delta masses it remains almost unchanged from its width in free space shown by the solid curve.

In calculating the delta width, we do not include the decay of delta into the delta-hole state as this effect has already been included in the collision term of the transport model via the process $N\Delta \rightarrow N\Delta$, except that its cross section is assumed to be similar to the $NN \rightarrow NN$ cross section in free space. In more realistic studies, one needs to include the medium effect on these cross sections as well [12, 13].

IV. PION DYNAMICS IN THE NUCLEAR MEDIUM

Pions are created from delta decays in the transport model. Since the delta is treated as an unpolarized particle in the model, both pion and nucleon are emitted isotopically in its rest frame. For a delta of mass m_Δ and momentum \mathbf{p}_Δ , the momentum of the pion \mathbf{k} can be found from the relation

$$\sqrt{(\mathbf{p}_\Delta - \mathbf{k})^2 + m_N^2} + \omega(\mathbf{k}, \rho) = \sqrt{m_\Delta^2 + \mathbf{p}_\Delta^2}. \quad (12)$$

To treat the propagation of a quasipion, we use the test particle method as in the normal transport model. Then the motion of the test particle is governed by the classical equations of motion, i.e.,

$$\frac{d\mathbf{r}}{dt} = \frac{d\omega}{d\mathbf{k}}, \quad (13)$$

$$\frac{d\mathbf{k}}{dt} = -\nabla\omega. \quad (14)$$

A quasipion can be absorbed by a nucleon to form a delta and destroyed through the process $N\Delta \rightarrow NN$. To check the condition of a collision involving a quasipion with a nucleon, we need to transform the two particles to their center-of-mass frame and find the closest point of approach as in the normal transport model [3]. To carry out this transformation conveniently, we introduce the effective pion momentum k^* and energy ω^* ,

$$k^* = k - k_0 \quad \text{and} \quad \omega^* = \omega - u, \quad (15)$$

so the quasipion satisfies the dispersion relation

$$\omega = \sqrt{(k - k_0)^2 + m_0^2} + u, \quad (16)$$

which is similar to that of a free particle. In the above, m_0 is the effective pion mass in the medium. In this way, a quasipion moves like a relativistic particle with the group velocity determined by the effective momentum and energy,

$$\frac{d\omega}{dk} = \frac{k - k_0}{\omega - u} = \frac{k^*}{\omega^*}. \quad (17)$$

The three parameters k_0 , u , and m_0 are density dependent and are determined from fitting the true pion dispersion relation in Fig. 2. We find that the above parametrization of the pion dispersion relation is satisfactory for densities up to three times normal nuclear density and for pion momentum up to four times the pion mass. Both are sufficient for heavy ion collisions at energies around 1 GeV/nucleon. The three parameters are plotted in Fig. 5 for both the pion branch (solid curves) and the delta-hole branch (dashed curves).

The velocity of the center-of-mass frame for the quasipion and the nucleon is then determined by the effective pion energy and momentum, i.e., $\beta = (\mathbf{k}^* + \mathbf{P}_N)/(\omega^* + E_N)$, where \mathbf{k}^* has the same direction as \mathbf{k} as the nuclear medium is assumed to be locally isotropic. The energy and momentum conservation in the scattering is, however, governed by the real energy and momentum of the

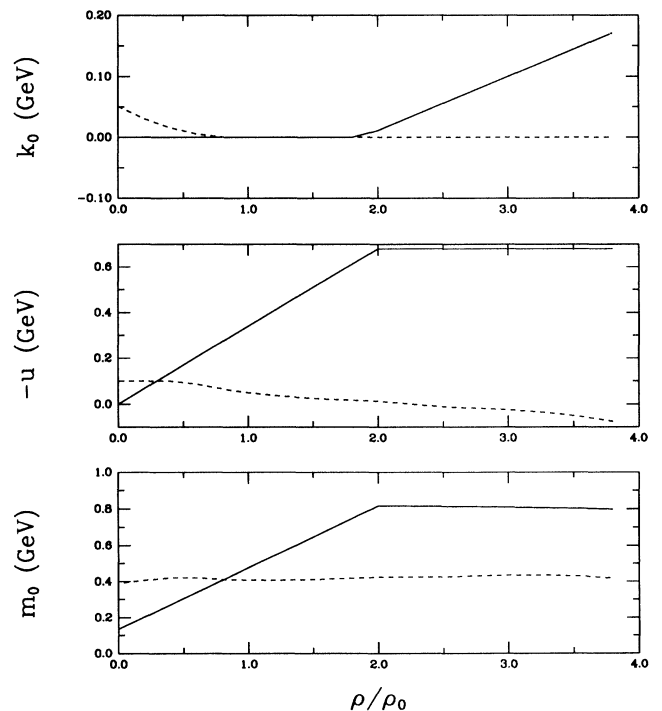


FIG. 5. The effective energy, momentum, and mass parameters for the quasipion in the pion branch (solid curves) and the delta-hole branch (dashed curves).

quasipion.

The mass of a delta from a pion-nucleon scattering is calculated from the invariant mass of the pion-nucleon system, i.e.,

$$m_{\Delta} = \sqrt{(E_N + \omega)^2 - (\mathbf{p}_N + \mathbf{k})^2}. \quad (18)$$

Since a quasipion can sometimes have a momentum that is larger than its energy, the resulting delta mass can be, although with a very small probability, smaller than the sum of the nucleon and pion masses. If the real part of the delta self-energy in the nuclear medium is also included in the transport model, then the mass of these low-mass deltas would increase with decreasing density and becomes larger than $m_N + m_{\pi}$ when the system freezes out. Since we have not included this effect in the present study, we do not allow in the transport model the formation of deltas with mass less than $m_N + m_{\pi}$.

For both low and high energy quasipions, their pion component is large and will eventually become free pions as the nuclear density decreases. However, for some intermediate energy pions, the delta-hole component dominates. These quasipions will approach the free delta-hole state as the nuclear density decreases and cannot materialize as free pions. Since there is no delta-hole state at zero density, these particles are thus unphysical. This difficulty will not appear in a more complete theory which takes into account the damping of the delta-hole states in nuclear matter. Then, these delta-hole-like quasipions will be either absorbed by the nuclear medium or make transitions to the pionlike states. Since only a very small fraction of the quasipions are delta-hole-like, we have not attempted to carry out such a calculation. Instead, we simply put these delta-hole-like quasipions on shell at freeze out by changing their momenta but keeping their energies the same.

V. THE PION SPECTRUM

We have applied the transport model, with and without the medium effect, to calculate the pion spectrum from La+La collision at 1.35 GeV/nucleon. In the calculation, we have incorporated the correct treatment of the detailed balance relation [30] for the process $NN \leftrightarrow N\Delta$ by taking into account the finite delta width. This modification turns out to be significant in increasing the delta absorption and reducing thus the final pion yield.

For comparison with the experimental data [22] that are obtained with the central trigger, we have thus carried out the transport model calculations at zero impact parameter. For the nuclear equation of state, we use the popular Skyrme parametrization with a compressibility of 380 MeV [3]. It is well known that the pion yield is only marginally sensitive to the nuclear equation of state. With a softer equation of state, corresponding to a nuclear compressibility of 200 MeV, the pion yield increases by only about 15% [4]. The calculated pion kinetic energy spectra in the transverse direction with angles in the range $|\theta - 90^\circ| < 30^\circ$ in the nucleus-nucleus center-of-mass frame are shown in Fig. 6. The dotted histogram is obtained with the normal transport model, which treats

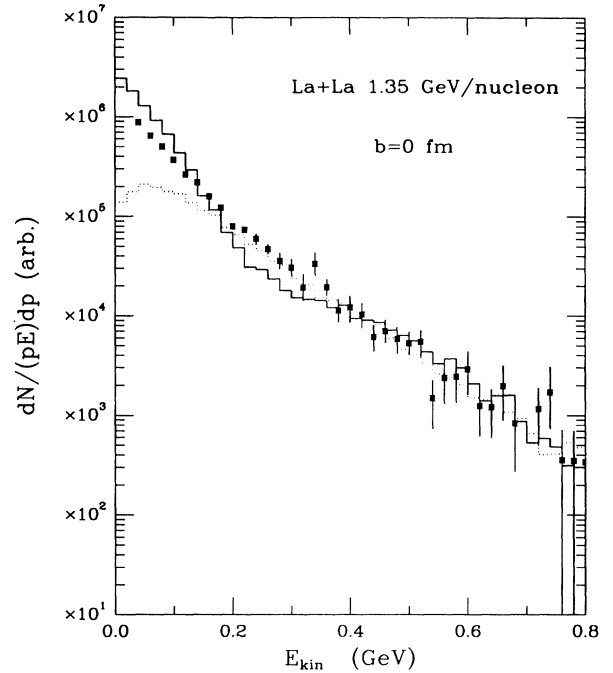


FIG. 6. Pion transverse kinetic energy spectrum ($|\theta - 90^\circ| < 30^\circ$) for collisions between two La nuclei at an incident energy of 1.35 GeV/nucleon. Dotted and solid histogram are theoretical results with free and in-medium pion dispersion relations, respectively. The experimental data are given by the solid squares.

pions as free particles; while the solid histogram is from the new transport model with quasipions. The experimental data are shown by the solid squares with error bars. The calculation with free pions already gives the correct slope for the high energy pion spectrum but fails to explain the low energy peak in the experimental data. In the calculation with quasipions, there is an enhancement of low energy pions while the result for high energy pions remains similar to that from the normal transport model. The enhancement of low energy pions is due to several mechanisms in the quasipion transport model. From delta decays in dense medium, quasipions in the pion branch generally carry less energy than free pions due to the strong p -wave attraction with the medium. With the increased delta width for small mass deltas, it is more probable to form small mass deltas, so that more soft pions are created from decays of these deltas. Also, as the quasipions propagate from a more dense region to less dense region, as most likely happened during heavy ion collisions, the momenta of the pions should decrease since the p -wave interaction reduces. This phenomenon can be interpreted straightforwardly from Eq. (14).

The resulting pion spectrum consists now of two slopes. The slope of the low energy component is smaller than the empirical one, and this leads to a slight overestimate of pions with kinetic energies less than 100 MeV and a corresponding underestimate of pions with kinetic energies between 100 and 300 MeV. Our results can be improved by treating the pion energy spectrum in the nuclear medium more realistically. In particular, the ef-

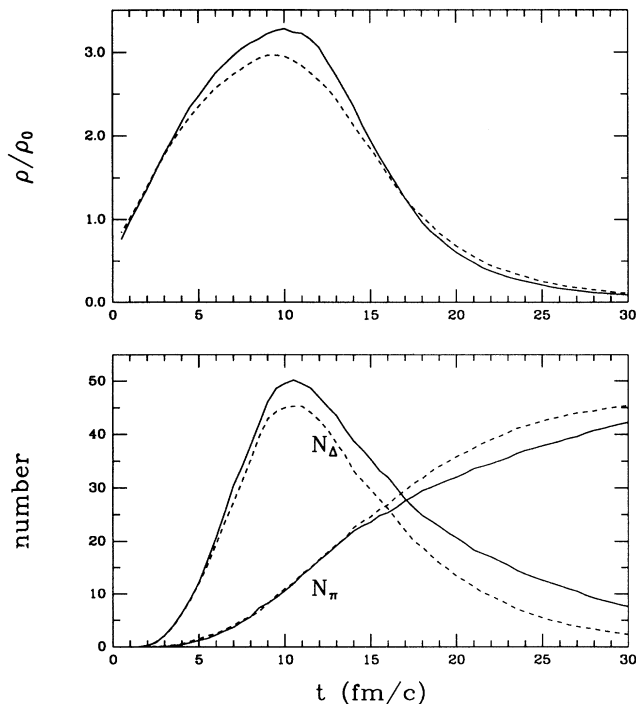


FIG. 7. The time evolution of the central density (top panel) and the pion and delta numbers (bottom panel). Dashed curves are with free pions while solid curves are with quasipions.

fect of the imaginary part of the pion self-energy on its real part needs to be included. As shown in [13, 29], this would lead to the disappearance of the delta-hole branch and the broadening of the pion branch. We expect that it would smooth out the pion spectrum obtained in the present study and lead to a better agreement with the experimental data. It is worthwhile to point out that due to more $\Delta \leftrightarrow N\pi$ reactions in the quasipion transport model, the final pion spectrum is more isotropic than that in the normal transport model. The number of pions in the transverse direction shown in Fig. 6 is thus larger in the transport model with quasipions than with free pions.

Finally we show in Fig. 7 the time dependence of the nuclear density in the central region as well as the evolu-

tion of the pion and delta numbers from both the normal transport model (given by dashed curves) and the new transport model with quasipions (given by solid curves). The central nuclear density is slightly higher in the transport model with quasipions than with free pions. In both cases, the sum of the pion and delta numbers becomes constant after about 16 fm/c. The final pion number is about 48 in both calculations and agrees with the experimentally measured [31] pion number of about 47.

We note that in the quasipion transport model, the delta number decreases slower with time than in the normal transport model. Most of these deltas have masses close to $m_N + m_\pi$ and decay thus slowly.

VI. SUMMARY

Because of the strong p -wave pion-nucleon interaction, the pion dispersion relation in dense matter is modified. Treating pions as quasiparticles, we have generalized the normal transport model to take into account via the local-density approximation the medium effects, which modify the pion kinetics, delta decay width, pion creation, and pion annihilation. Applying this model to heavy-ion collisions, we find that the inclusion of the medium effects on pion dynamics leads to enhanced production of low energy pions and can account for the soft pion puzzle observed in heavy-ion experiments. In this model, the three effects on soft pion production, delta decay, finite pion chemical potential, and the in-medium pion dispersion relation, are all included. The new transport model with quasipions thus provides a unified description of the pion dynamics in the nuclear medium.

ACKNOWLEDGMENTS

This work was supported in part by the Department of Energy under Contract No. DE-FG02-88Er40388 (L.X. and V.K.), the National Science Foundation under Grant No. PHY-8907986, and the Welch Foundation under Grant No. A-1110 (C.M.K. and L.X.). We would like to acknowledge many fruitful discussions with W. Bauer, B. A. Li, P. D. Danielewicz, and U. Mosel concerning pion production. One of the authors (C.M.K.) also thanks M. Asakawa for helpful discussions.

-
- [1] J. Cugnon, D. Kinet, and J. Vandermeullen, Nucl. Phys. **A379**, 553 (1982).
 - [2] Y. Kitazoe, M. Gyulassy, P. Danielewicz, H. Toki, Y. Yamamura, and M. Sano, Phys. Lett. B **138**, 341 (1984); Y. Kitazoe, M. Sano, H. Toki, and S. Nagamiya, *ibid.* **166**, 35 (1986).
 - [3] G. F. Bertsch and S. Das Gupta, Phys. Rep. **160**, 189 (1989).
 - [4] H. Kruse, B. V. Jacak, and H. Stöcker, Phys. Rev. Lett. **54**, 289 (1985).
 - [5] L. Xiong, Z. G. Wu, C. M. Ko, and J. Q. Wu, Nucl. Phys. **A512**, 772 (1990).
 - [6] Gy. Wolf, G. Batko, T. S. Biro, W. Cassing, and U. Mosel, Nucl. Phys. **A517**, 615 (1990).
 - [7] B. A. Li and W. Bauer, Phys. Lett. B **254**, 335 (1991); Phys. Rev. C **44**, 450 (1991).
 - [8] S. J. Wang, B. A. Li, W. Bauer, and J. Randrup, Ann. Phys. (N.Y.) **209**, 251 (1991).
 - [9] G. E. Brown and W. Weise, Phys. Rep. **22**, 279 (1975).
 - [10] B. Friemann, V. P. Pandharipande, and Q. N. Usmani, Nucl. Phys. **A372**, 483 (1981), and references therein.
 - [11] M. Gyulassy and W. Greiner, Ann. Phys. (N.Y.) **109**, 485 (1977).
 - [12] G. F. Bertsch, G. E. Brown, V. Koch, and B. A. Li, Nucl. Phys. **A490**, 745 (1988).
 - [13] G. E. Brown, E. Oset, M. Vicente Vacas, and W. Weise,

- Nucl. Phys. **A505**, 823 (1989).
- [14] C. Gale and J. Kapusta, Phys. Rev. C **35**, 2107 (1987).
- [15] L. H. Xia, C. M. Ko, L. Xiong, and J. Q. Wu, Nucl. Phys. **A485**, 721 (1988).
- [16] C. L. Korpa, L. Xiong, C. M. Ko, and P. J. Siemens, Phys. Lett. B **246**, 333 (1990).
- [17] Y. Asakawa, C. M. Ko, P. Lévai, and X. J. Qiu, Phys. Rev. C **46**, R1159 (1992).
- [18] G. Roche *et al.*, Phys. Rev. Lett. **61**, 1069 (1988); Phys. Lett. B **226**, 228 (1989).
- [19] C. Naudet *et al.*, Phys. Rev. Lett. **62**, 2652 (1989).
- [20] P. A. Seidel for the DLS collaboration, Nucl. Phys. **A525**, 299c (1991).
- [21] W. Kuehn and V. Metag (private communication).
- [22] G. Odyniec *et al.*, in Proceedings of the 8th High Energy Heavy Ion Study, edited by J. Harris and G. Woznick, LBL Report No. 24580, 1988, p. 215.
- [23] G. E. Brown, J. Stachel, and G. M. Welke, Phys. Lett. **B253**, 19 (1991).
- [24] H. Stroebele *et al.*, Z. Phys. C **38**, 89 (1988).
- [25] J. W. Harris for the NA38 collaboration, Nucl. Phys. **A498**, 133 (1989).
- [26] M. Kataja and P. J. Ruuskanen, Phys. Lett. B **243**, 181 (1990).
- [27] M. I. Gorenstein and S. N. Yang, Phys. Rev. C **44**, 2875 (1991); M. I. Gorenstein, S. N. Yang, and C. M. Ko, Chin. J. Phys. **30**, 543 (1992).
- [28] E. Shuryak, Phys. Rev. D **42**, 1764 (1990).
- [29] C. M. Ko, L. H. Xia, and P. J. Siemens, Phys. Lett. B **231**, 16 (1989).
- [30] P. D. Danielewicz and G. F. Bertsch, Nucl. Phys. **A533**, 712 (1991).
- [31] J. W. Harris *et al.*, Phys. Rev. Lett. **58**, 377 (1987).