Correlation symmetries in sequential three-body final state reactions

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The alpha-particle decay angular correlations for intermediate $^{15}N^*$ excited states in the reaction $^7Li(^{12}C,^{15}N^* \to \alpha+^{11}B(g.s.))\alpha$ are found to exhibit symmetry about 90° relative to the beam axis in the reaction plane, in the barycentric coordinates of the decay. The limiting case necessary to produce this symmetry also yields azimuthal symmetry. It is therefore sometimes unnecessary to measure out-of-plane correlations in order to integrate over all angles for the purpose of determining particle decay branching ratios. We use the observed symmetry to establish $\Gamma_\alpha/\Gamma = 0.60\pm0.04$ for the 12.56 MeV excited state of ^{15}N (absolute uncertainty is ~15%).

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Particle decay angular correlation symmetries have been discussed for a number of special cases [1–3] for the purpose of easing the extraction of spectroscopic information about nuclear excited states. These earlier papers provide important background for the present work which describes a method for extracting particle decay partial widths for intermediate states formed in heavy-ion direct reactions. We describe the correlation symmetry observed and its implications from the general theory of angular correlations and also discuss the experimental conditions under which the method may find application.

We consider a three-body final state which proceeds by sequential two-body decay through an intermediate state designated by

$$a+b \to c+\lambda \to c+(d+e).$$

Figure 1 defines the intrinsic spins (s_i) and orbital angular momenta (ℓ_i) and the appropriate magnetic projections (μ_i, and m_i, respectively), and the channel spins (S_i) for the reaction. The total angular momentum of the intermediate state λ is J_λ. The subscript letters refer to particles, while the subscript numbers, 1 and 2, refer to the first and second decay in the sequence.

The general expression for the differential cross section for a three-body final state nuclear reaction is given in center of mass coordinates by

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_2} = \sum_{\mu_a\mu_b\mu_c\mu_d\mu_e} \frac{|A_{\mu_c\mu_d\mu_e,\mu_a\mu_b}|^2}{(2s_a+1)(2s_b+1)}, \quad (1)$$

where the energy interval dE_2 is an interval in the relative energy between particles of the second binary decay, i.e., particles d and e. The reaction amplitudes referred to a z axis along the incident beam direction ($\mathbf{k}$) are expressed in the partial wave form using the uncoupled S matrix as

$$A_{\mu_c\mu_d\mu_e,\mu_a\mu_b}(\Omega_1,\Omega_2,E_2) = -i(\pi^{\frac{1}{2}}k)^{-1} \sum_{\ell,\ell_1 m_1} \sum_{\ell_2 m_2} \hat{\ell}\langle \ell_2 m_2 \ell_1 m_1 s_c\mu_c s_d\mu_d s_e\mu_e \mid S \mid \ell 0 s_a\mu_a s_b\mu_b\rangle Y_{\ell_1 m_1}(\Omega_1) Y_{\ell_2 m_2}(\Omega_2), \quad (2)$$

where $\hat{\ell} = \sqrt{2\ell+1}$. The angles $(\theta_j,\phi_j) = \Omega_j$ are the polar angles of the final state wave vectors $\mathbf{k}_j$, which are canonically related to the Jacobi vectors $\mathbf{r}_j$, given by

$$\mathbf{r}_1 = \mathbf{r}_c - [(m_d\mathbf{r}_d + m_e\mathbf{r}_e)/(m_d+m_e)], \quad \mathbf{r}_2 = \mathbf{r}_d - \mathbf{r}_e.$$

All these quantities are derived from the defining velocity addition diagram illustrated in Fig. 2 for $\phi_1 = \phi_2 = 0$.

If we now assume that the reaction proceeds sequentially through an isolated resonance, λ, in the subsystem $(d+e)$ then to a good approximation [4] the amplitude of Eq. (2) can be factorized into formation (F) and decay (D) terms for a resonance of angular momentum J_λ and magnetic projection μ_λ as

$$A_{\mu_c\mu_d\mu_e,\mu_a\mu_b} = \sum_{\mu_\lambda} D^{\mu_d\mu_e}_{J_\lambda\mu_\lambda}(\Omega_2,E_2) F^{\mu_a\mu_b\mu_c}_{J_\lambda\mu_\lambda}(\Omega_1), \quad (3)$$

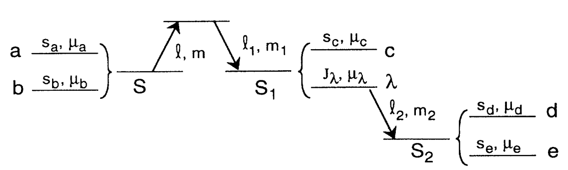

FIG. 1. Definition of angular momentum notation for the sequential reaction $a+b \to c+\lambda \to c+d+e$.

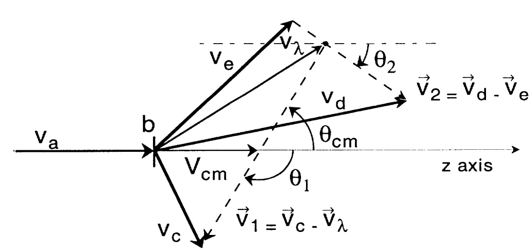

FIG. 2. Velocity addition diagram in the reaction plane, $\phi_1 = \phi_2 = 0°$, defining the decay angles θ_1 and θ_2, and the decay relative velocities and hence the Jacobi vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.

wherein

$$F_{J_\lambda \mu_\lambda}^{\mu_a \mu_b \mu_c}(\Omega_1) = -i(\pi^{\frac{1}{2}} k)^{-1} \sum_{\ell, \ell_1 m_1} \hat{\ell} Y_{\ell_1 m_1}(\Omega_1) \star \langle \ell_1 m_1 J_\lambda \mu_\lambda s_c \mu_c \mid S \mid \ell 0 s_a \mu_a s_b \mu_b \rangle, \quad (4)$$

and using the standard vector coupling coefficients in the notation of Rose [5],

$$D_{J_\lambda \mu_\lambda}^{\mu_d \mu_e}(\Omega_2, E_2) = (2\pi)^{-\frac{1}{2}} \sum_{\ell_2 m_2} \sum_{S_2} \frac{g^*_{\ell_2 S_2 J_\lambda}}{E_\lambda - E_2 + i\Gamma_\lambda/2} C(s_d s_e S_2, \mu_d \mu_e \mu_2) C(\ell_2 S_2 J_\lambda, m_2 \mu_2 \mu_\lambda) Y_{\ell_2 m_2}(\Omega_2), \quad (5)$$

with

$$\Gamma_\lambda = \sum_i \mid g_{\lambda_i} \mid^2 = \sum_i \Gamma_{\lambda_i}. \quad (6)$$

Equation (6) is merely an expression for the total width of the intermediate state, λ, written as a sum of partial widths for different possible decay channels, i.

Equation (1) can now be rewritten as a triple differential cross section evaluated in the energy region near the isolated resonance energy, $E_2 \sim E_\lambda$, and as an angular correlation function, W, expressed in the density matrix formalism:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_2} = W(\Omega_1, \Omega_2, E_2) = \mathrm{Tr}\{\rho(\Omega_1)\varepsilon(\Omega_2)\}. \quad (7)$$

The formation and decay sectors of the matrix elements are given, respectively, as

$$\rho_{\mu_\lambda \mu'_\lambda} = \sum_{\mu_a \mu_b \mu_c} \frac{\left(F_{J_\lambda \mu_\lambda}^{\mu_a \mu_b \mu_c}\right)\left(F_{J_\lambda \mu'_\lambda}^{\mu_a \mu_b \mu_c}\right)^*}{(2s_a+1)(2s_b+1)},$$

$$\varepsilon_{\mu'_\lambda \mu_\lambda} = \varepsilon^*_{\mu_\lambda \mu'_\lambda} = \sum_{\mu_d \mu_e} \left(D_{J_\lambda \mu_\lambda}^{\mu_d \mu_e}\right)\left(D_{J_\lambda \mu'_\lambda}^{\mu_d \mu_e}\right)^*. \quad (8)$$

The conservation of angular momentum along the beam direction, and z axis of Fig. 2, leads to the relations

$$\mu'_\lambda - \mu_\lambda = m_1 - m'_1 = m'_2 - m_2. \quad (9)$$

A well-known special case of Eq. (9), which was referred to in Litherland and Ferguson [1], is when either $\mathbf{k}_1$ or $\mathbf{k}_2$ is parallel to the z axis, for which $\mu_\lambda = \mu'_\lambda$, since either $m_1 = 0 = m'_1$ or $m_2 = 0 = m'_2$, respectively [1]. For this situation the correlation function would contain only diagonal elements of the density matrix and would be written as

$$W_D(\Omega_1, \Omega_2, E_2) = \sum_{\mu_\lambda} \rho_{\mu_\lambda \mu_\lambda}(\Omega_1) \varepsilon_{\mu_\lambda \mu_\lambda}(\Omega_2, E_2). \quad (10)$$

We will return to this diagonal approximation of the correlation function after rewriting the general correlation function in terms of the spin tensors associated with formation and decay, which are defined by

$$\rho_{kq}(\Omega_1) = \sum_{\mu_\lambda} (-1)^{J_\lambda - \mu'_\lambda} C(J_\lambda J_\lambda k, \mu_\lambda - \mu'_\lambda q) \rho_{\mu_\lambda \mu'_\lambda},$$

$$\varepsilon_{kq}(\Omega_2, E_2) = \sum_{\mu_\lambda} (-1)^{J_\lambda - \mu'_\lambda} C(J_\lambda J_\lambda k, \mu_\lambda - \mu'_\lambda q) \varepsilon^*_{\mu_\lambda \mu'_\lambda}. \quad (11)$$

The alternative general expression is then

$$W(\Omega_1, \Omega_2, E_2) = \sum_{kq} \rho_{kq}(\Omega_1) \varepsilon^*_{kq}(\Omega_2, E_2), \quad (12)$$

where ε^*_{kq} is given explicitly for the decay of the resonant state λ by

$$\varepsilon^*_{kq}(\Omega_2, E_2) = \sum_{\ell_2 \ell'_2 S_2} (-1)^{J_\lambda + S_2} \frac{\hat{J}^2_\lambda \hat{\ell}_2 \hat{\ell}'_2}{2\pi(4\pi)^{\frac{1}{2}} \hat{k}} \frac{g^*_{\ell_2 S_2 J_\lambda} g_{\ell'_2 S_2 J_\lambda}}{(E_\lambda - E_2)^2 + \Gamma^2_\lambda/4} C(\ell_2 \ell'_2 k, 000) W(\ell_2 J_\lambda \ell'_2 J_\lambda; S_2 k) Y_{kq}(\Omega_2). \quad (13)$$

The approximation of the dominance of diagonal terms which gave us Eq. (10), $\mu_\lambda = \mu'_\lambda$, now implies that $q = 0$ in Eq. (13), so that after integrating over the resonance we obtain an alternative form of Eq. (10) written as

$$\langle W_D(\Omega_1, \Omega_2, E_2) \rangle_{E_2} = \sum_k \rho_{ko}(\Omega_1) \langle \varepsilon^*_{ko}(\Omega_2, E_2) \rangle_{E_2},$$

with

$$\langle \varepsilon^*_{ko}(\Omega_2, E_2) \rangle_{E_2} = \frac{\hat{J}^2_\lambda}{4\pi} \sum_{\ell_2 \ell'_2 S_2} (-1)^{J_\lambda + S_2} g^*_{\ell_2 S_2 J_\lambda} g_{\ell'_2 S_2 J_\lambda} / \Gamma_\lambda \hat{\ell}_2 \hat{\ell}'_2 C(\ell_2 \ell'_2 k, 000) W(\ell_2 J_\lambda \ell'_2 J_\lambda; S_2 k) P_k(\cos\theta_2). \quad (14)$$

When the sequential decay takes place through an isolated resonant state of definite parity, π_λ, and decays into particles d and e also of definite parity, $\pi_d \pi_e$, then the decay spin tensor, ε^*_{kq}, is nonzero for only even values of the index k, and from the angular momentum coupling the maximum k value cannot exceed the minimum of $2J_\lambda$ and $2\ell_2(\max)$.

The importance of the diagonal approximation for the density matrices lies in the fact that the resulting correlation function is independent of the azimuthal decay angle, ϕ_2, and that it is symmetric about $\theta_2 = \pi/2$ in the reaction plane. The result of the diagonal approximation can be valid at values of $\mathbf{k}_1$ and $\mathbf{k}_2$ which are not collinear with the z axis, since the formation and decay density matrices, $\rho_{\mu_\lambda \mu'_\lambda}$ and $\varepsilon_{\mu'_\lambda \mu_\lambda}$, involve only positive definite diagonal terms in the summation and therefore the diagonal elements can, under certain circumstances, dominate the character of the general correlation function. A useful test of the dominance of diagonal terms is to observe the symmetry about $\theta_2 = \frac{\pi}{2}$ in the in-plane correlation function. When this symmetry is observed, then the particle decay branching ratio, given by

$$\frac{\Gamma_{\lambda,d+e}}{\Gamma_\lambda} = \sum_{\ell_2 S_2} \mid g_{\ell_2 S_2 J_\lambda} \mid^2 /\Gamma_\lambda, \tag{15}$$

can be obtained from the ratio of experimental quantities, the angle integrated double differential cross section, $d^2\sigma/d\Omega_1 d\Omega_2$, measured at a specific value of θ_1, divided by the production cross section for the intermediate state λ, measured at the same angle, θ_1. Taking advantage of the azimuthal isotropy, the branching ratio could be written as

$$\frac{\Gamma_{\lambda,d+e}}{\Gamma} = \frac{2\pi \int_{-1}^{+1} d(\cos\theta_2) d^2\sigma/d\Omega_1 d\Omega_2}{d\sigma_\lambda/d\Omega_1}$$
$$= \frac{2\pi \int_{-1}^{+1} d(\cos\theta_2) \langle W_D(\Omega_1,\Omega_2,E_2)\rangle_{E_2}}{\langle W_D(\Omega_1)\rangle}, \tag{16}$$

wherein $\langle W_D(\Omega_1)\rangle$ involves all decay channels.

We have measured the double differential cross sections and the production cross sections for the reaction ^{7}Li + ^{12}C $\to \alpha_1$ + ^{15}N* $\to \alpha_1 + \alpha_2$ + ^{11}B(g.s.), for a number of intermediate states of ^{15}N. The production cross sections are determined by bombardment of natural carbon targets with ^{7}Li beam particles at 52.5 MeV and detecting the primary alpha particles, α_1, over a laboratory angular range of 5° to 35° by a pair of small angle E-dE detector telescopes. For the angular correlation measurements we use the inverse reaction, bombarding isotopically separated ^{7}Li targets with a 90 MeV ^{12}C beam. In each case the coupled operation of the Florida State University FN-Tandem accelerator and superconducting LINAC is used. For the correlation measurements the ^{11}B particles are identified in a 1 cm diameter, x-y position sensitive, E-dE silicon detector telescope at a center angle of 8° from the beam direction. Alpha particles are detected in coincidence with the ^{11}B nuclei in a 1 cm × 5 cm horizontal position sensitive detector centered at 27°. The calibration of these detectors, a discussion of the general method, and other experimental and analytical details are discussed in previous publications [6–8]. A spectrum of the obtained relative energies, E_2, between the alpha particle and ^{11}B decay products of the intermediate nucleus ^{15}N, is shown in Fig. 3. The events shown here represent the total data summed over a large range of phase space, θ_1 vs θ_2, and do not fully represent the highest energy resolution available in a smaller range [9].

To illustrate the method of branching ratio calculation when the angular correlation is symmetric about $\theta_2 = \pi/2$, we have selected the well isolated state in ^{15}N at E_x=12.56 MeV. The events representing this state are binned in 10° intervals in the broad-ranged decay angle θ_2 and in a 7° window about a specific angle, $\theta_{\rm c.m.} = \pi - \theta_1$, at which we had previously measured

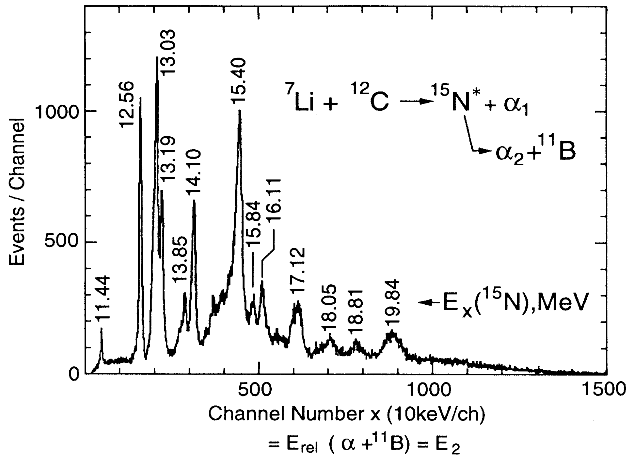

FIG. 3. Spectrum of the relative energies between the ^{15}N* decay products, α + ^{11}B(g.s.), for an excitation energy dependent broad range in the phase space, θ_1 vs θ_2. The ^{12}C bombarding energy is 90 MeV.

the production cross section. These yields are corrected for coincidence detection efficiency by use of the Monte Carlo code BEAST [7], and then they are converted to the double differential cross section vs θ_2.

In this case we have $s_d^\pi = 0^+, s_e^\pi = 3/2^-$, and the tabulated [10] value for the ^{15}N excited state is $J_\lambda^\pi = 9/2^+$. The relative angular momentum in the decay of ^{15}N* is therefore $\ell_2 = 3$ or 5, and the maximum value of k cannot exceed 8. Near threshold the higher ℓ_2 value is often greatly suppressed, and that effect here would make $k \le 6$. An equivalent form of Eq. (14), for the purpose of fitting the measured correlation function is simply a Legendre series,

$$\left.\frac{d^2\sigma}{d\Omega_1 d\Omega_2}\right|_{\theta_1} = \sum_{k=0,\text{even}}^{k_{\max}} a_k P_k(\cos\theta_2), \tag{17}$$

and then the branching fraction is given by

$$\frac{\Gamma_\alpha}{\Gamma} = \left(\frac{4\pi a_0}{d\sigma_\lambda/d\Omega_1}\right)_{\theta_1}. \tag{18}$$

For the detector geometry used, correlation data could be assembled over a broad angular range in θ_2 for the decay of the 12.56 MeV excited state at two fairly widely separated central values of $\theta_{\rm c.m.}$. These data, for $\theta_{\rm c.m.}$ ~39° and ~48° are shown in Figs. 4 and 5, respectively, along with fitted Legendre series. The immediately striking feature of the data is their remarkable symmetry about 90°. We have described each correlation function with a Legendre series with $k_{\max}$ values of 2, 4, 6, and 8. The results of the fitting procedure, and the values of the production cross sections and obtained branching fractions, are listed in Table I. The curves shown in Figs. 4 and 5 are for $k_{\max}$ values which result in the lowest two values of χ^2 per degree of freedom. The average values of the branching fraction for these best two values of χ^2/D are $\Gamma_\alpha/\Gamma = 0.58 \pm 0.06$ for $\theta_{\rm c.m.}$ ~39°, and $\Gamma_\alpha/\Gamma = 0.62 \pm 0.06$ for $\theta_{\rm c.m.}$ ~48°. The uncertainties quoted here

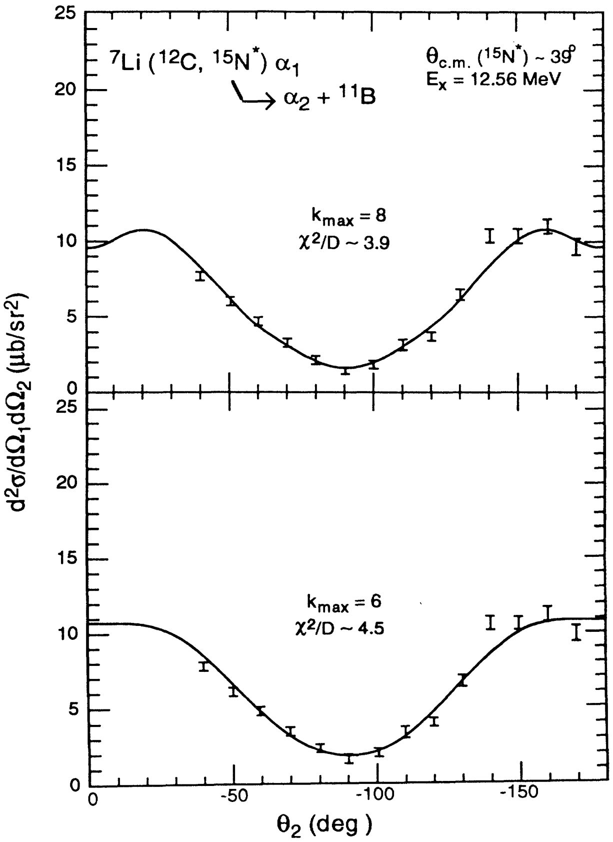

FIG. 4. Double differential cross section for alpha-particle decay of the intermediate state of $^{15}N^*$ at an excitation of 12.56 MeV. The value of $\theta_1 \sim -141°$ and $E(^{12}C) = 90$ MeV. The curves represent best fits of Eq. (16) to the data with k for the lowest two values of χ^2/D. (Note $\theta_{c.m.} = \pi - \theta_1$ of Fig. 2 corresponds to θ_1 of Ref. [3].)

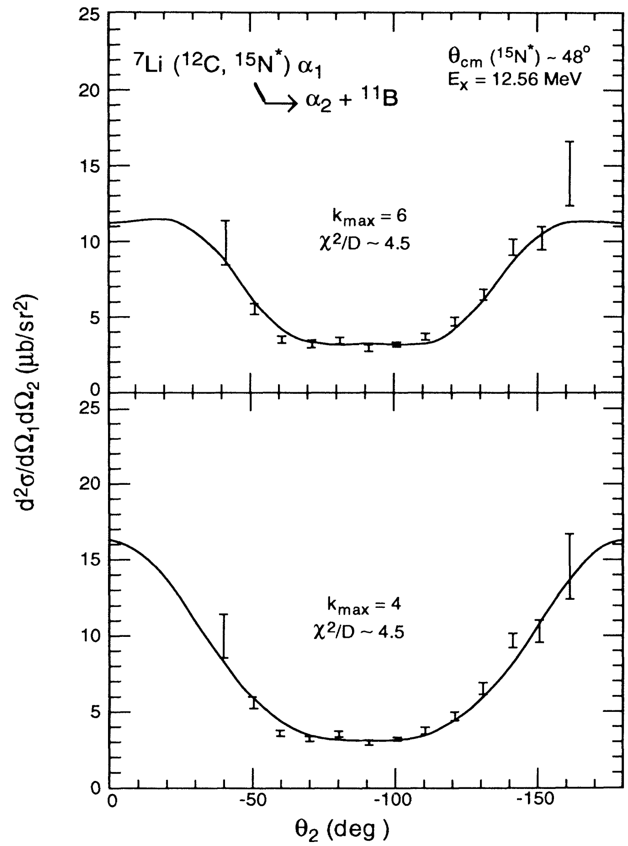

FIG. 5. Double differential cross section for alpha-particle decay of the intermediate state of $^{15}N^*$ at an excitation of 12.56 MeV. The value of $\theta_2 \sim -132°$ and $E(^{12}C) = 90$ MeV. See caption of Fig. 4.

and in Table I are statistical only, therefore showing the relative uncertainties. Absolute uncertainties in both single and double differential cross sections are the order of 10%. The same two targets and detector geometries were used for all single and double differential cross section measurements, respectively, so these relative uncertainties are appropriate for comparison to the two different branching fraction values obtained for center-of-mass angles of 39° and 48°. The agreement of these two values of Γ_α/Γ is further evidence of the validity of the symmetry method. We conclude that for the 12.56 MeV state of ^{15}N, $\Gamma_\alpha/\Gamma = 0.60\pm0.04$ with an absolute uncertainty of 15%.

It would be very useful if we could establish conditions under which this much simplified correlation, symmetry about $\theta_2 = \pi/2$, would take place. By far the majority of particle decay angular correlation studies have been concentrated either at or near the Litherland and Ferguson (LF) geometries of $\mathbf{k}_1$ or $\mathbf{k}_2$ parallel to the beam axis, or in investigations of the deviations from symme-

TABLE I. Results of the correlation fitting procedure using Eq. (17).

$\theta_{c.m.}$	k_{max}	χ^2/D	a_0 (μb/sr^2)	$d\sigma/d\Omega_1$ (μb/sr)	Γ_α/Γ
~39°	2	5.8	5.348 ± 0.085	115 ± 11	0.584 ± 0.057
	4	4.9	5.282 ± 0.085		0.577 ± 0.056
	6	4.5	5.337 ± 0.085		0.583 ± 0.057
	8	3.9	5.338 ± 0.085		0.583 ± 0.057
~48°	2	8.4	5.470 ± 0.11	121 ± 11	0.568 ± 0.053
	4	4.5	6.044 ± 0.162		0.628 ± 0.060
	6	4.5	5.839 ± 0.199		0.606 ± 0.059
	8	5.2	6.048 ± 0.251		0.628 ± 0.063

try for significant deviations from the LF geometries, for example, Refs. [2] and [3]. In both Refs. [2] and [3], the DWBA (distorted wave Born approximation) is demonstrated to be a fairly good predictor of an observed shift from symmetry about $\theta_2 = \pi/2$ for angular shifts of $\mathbf{k}_1$ from parallel or antiparallel to the z axis of $\leq 20°$, for reactions which are well matched in angular momentum transfer. For large values of θ_1, a DWBA calculation for the matched reaction, $^{12}C(^6Li,d)^{16}O^{6+}_{21.8}(\alpha)^{12}C$(g.s.), is found to have near symmetry about $\theta_2 = \pi/2$ for any $\theta_1 > 120°$ (see Fig. 9, Ref. [11]). At least one calculation, however, results in near symmetry about $\theta_2 = \pi/2$ for a wide range in $\theta_1 \leq 30°$, for the mismatched and hypothetical reaction $^{12}C + {}^{18}O \rightarrow {}^{12}C + {}^{18}O^* \rightarrow {}^{12}C + \alpha + {}^{14}C$, where $^{18}O^*$ is a $J^\pi = 4^+$ state at zero excitation (see Fig. 8(d), Ref. [2]). In the present experiment there is good angular momentum matching for the state at $E_x = 12.56$ MeV with $J^\pi = 9/2^+$, and in addition the symmetry seems to persist for a number of states near this excitation energy [9]. Considering the available data, there seems to be no comprehensive statement we can make which might predict the conditions for which a symmetry about $\theta_2 = \pi/2$ would occur.

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