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Spin-isospin SU(4) symmetry in *sd*- and *fp*-shell nuclei

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For all even-A nuclei in the *sd* shell we evaluate the overlap between several low-lying states, obtained by diagonalizing the realistic Wildenthal interaction, and the eigenstates of the SU(4) Casimir operator. We find that the $J = 0^+$ ground states of even-even nuclei near the middle of the shell have rather small overlaps (less than 0.5) with the lowest SU(4) Young tableaux, while the $J = 0^+$ and 1^+ lowest states of odd-odd nuclei have noticeably larger overlaps (0.6–0.7). We also find that the expansion in the SU(4) Young tableaux converges quite rapidly, and that the two or three lowest tableaux usually account for more than 90% of the nuclear wave function. We then extend the calculation to the *fp* shell and evaluate the overlaps between the $J = 0^+$ ground states obtained by diagonalizing a realistic interaction and the SU(4) eigenstates for even-even nuclei with maximum isospin. For the *fp* shell the overlaps are even smaller than in the *sd* shell. Since we observe such sizable SU(4) symmetry breaking effects in the relatively simple *sd* and *pf* nuclei, we are rather pessimistic about the prospects of using conclusions based on SU(4) in heavier, more complex nuclei.

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I. INTRODUCTION

The question whether nuclear states can be characterized by the quantum numbers of the spin-isospin symmetry group SU(4) has been raised many times during the 55 years since the original proposal of Wigner [1]. Even though it is clear that in most nuclei SU(4) symmetry is at best a rather crude approximation, various manifestations of the symmetry (or apparent symmetry) have been noted. So, Franzini and Radicati [2], and later Gaponov *et al.* [3] noted the effect of SU(4) symmetry on nuclear masses. The giant Gamow-Teller resonance [4], close in energy to the isobar analog state, is also compatible with SU(4) symmetry.

There is an obvious intimate relation between nuclear β decay and SU(4) symmetry. Since the Gamow-Teller operator is one of the group generators, it can connect only states belonging to the same SU(4) representation. Hence, the difference between the ft values of superallowed and "normal" allowed transitions follows naturally if SU(4) is only slightly broken.

Nuclear double-beta decay (with emission of two neutrinos) would also be forbidden in SU(4), at least in the closure approximation. Treating the symmetry violating parts of the interaction in perturbation theory leads to double-beta-decay matrix elements of the correct order of magnitude [5]. Moreover, when considering the total double Gamow-Teller strength, one finds that the SU(4) prediction [6] appears to be reasonable. Detailed numerical calculations in the *sd* shell [7] reveal a broad, but recognizable, "double Gamow-Teller" state near the double isobar analog state, with the total strength in agreement with the SU(4) prediction. The distributions of double Gamow-Teller strength for a few nuclei in the *sd* and *fp* shells have also been analyzed by Zheng *et al.* [8], and its relation to SU(4) symmetry breaking has been noted.

It has been sometimes argued that SU(4) symmetry violation is exaggerated by the theoretical treatment of nuclear-structure problems. In the extreme single-particle model, the spin-orbit splitting is the symmetry violating part. Pairing, in the standard BCS treatment involves only proton-proton and neutron-neutron interactions, and again violates spin-isospin symmetry. It is possible that a full calculation, which includes the total nuclear Hamiltonian, would, to some extent, "restore" the symmetry violated by the approximate treatment of the interaction. Again, in the *sd* shell one can, perhaps, see this tendency when comparing the β^+ strength derived from occupation numbers of individual subshells with the result of a full calculation [9]. The neutron-proton interaction further reduces the β^+ strength, in agreement with the "symmetry restoration" concept. (The same tendency in a much broader range of nuclei has been noted by Mairle *et al.* [10].)

SU(4) symmetry is clearly recognizable in the p-shell nuclei [11, 12]. However, it is well known that LS coupling is only gradually replaced by jj coupling as the nuclear mass increases and thus the p shell might not be a good testing ground for the validity of the spin-isospin symmetry. The *sd* shell is probably a better candidate. The nucleon number is small enough so that an exact shell-model diagonalization is possible, and good phenomenological interactions are available [13]. However, in the middle of the shell there are typically many (> 1000) states for each J, T, and thus one might hope that the states there are complicated enough to reveal general trends. An early test [14], based on the spectral distribution method, suggested that SU(4) is badly broken for realistic interactions. (However, the method might exaggerate the symmetry violation [15].) In the present work, we test the validity of SU(4) using nuclear states with good J, T quantum numbers and modern shell-model codes [16]. We also extend the calculation to Ca isotopes in the fp shell.

Our aim, therefore, is an assesment of goodness of SU(4) symmetry in nuclei with relatively many nucleons in valence shells. We expect that the tendencies that we find here will be even more pronounced in heavier nuclei, which are inaccessible to the exact shell-model treatment. Since we observe sizable SU(4) symmetry breaking even in the simpler sd- and pf-shell nuclei which we can handle, we are rather pessimistic about the prospects of using conclusions based on the SU(4) symmetry in even more complicated systems.

Alternatively, one can test the goodness of nuclear symmetry by classifying the effective Hamiltonian according to the irreducible tensor character of the corresponding symmetry group (see the review of Hecht [15] for a general description of the method). For the sd shell Vincent [17] derived the selection rules for the central and spin-orbit interactions, while Pluhař [18] and Draayer [19] have shown that the Kuo-Brown interaction (and some of its modifications) contain large symmetry conserving components. Since the question of goodness of SU(4) symmetry hinges, as we will show below, on the competition between the symmetry breaking one-body spin-orbit force and (largely) symmetry conserving two-body interaction, it is difficult to estimate the degree of symmetry breaking *a priori*. With this in mind, we chose to perform numerical evaluation of the overlaps with SU(4) eigenstates as described below.

II. EVALUATION OF OVERLAPS

We use the OXBASH shell-model code [16] with the Wildenthal interaction [13] and obtain the eigenvalues and eigenvectors of the lowest-lying states (typically five) in each nucleus. We then use the same computer code and produce the eigenstates of the SU(4) Casimir operator

$$C[\mathrm{SU}(4)] = \sum_{1,2} \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} + \boldsymbol{\tau_1} \cdot \boldsymbol{\tau_2} + \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}\ \boldsymbol{\tau_1} \cdot \boldsymbol{\tau_2}. \tag{1}$$

The eigenstates of C are degenerate; the degeneracy is equal to the number of states belonging to the given representation of SU(4), and having given angular momentum J and isospin T. One can evaluate this degeneracy as follows. First, using, e.g., the tables of McKay and Patera [20], one can identify the Young tableau [which we use to label the irreducible representations of the group SU(n)] corresponding to a given Casimir operator eigenvalue. Then, in the same tables, one finds the decomposition of SU(4) into SU(2) of spin and SU(2) of isospin, i.e., the possible S and T values (including possible degeneracies) for each irreducible representation of SU(4). The spatial part of the wave function can be classified, in the case of the nuclear sd shell, by the Young tableaux of the group SU(6) (6 is the maximum number of particles in an identical spin-isospin state, the sum of $2l_i+1$). In the fp shell, the spatial part can be classified by the irreducible representations of SU(10) for the same reason. Since the total wave function must be antisymmetric, the Young tableaux of SU(6) [or SU(10) for the fp shell] must have the associated shape of the corresponding Young tableaux of SU(4). Again, one can use the tables [20] to find the orbital angular momenta L and their degeneracy for the given Young tableaux of SU(6) and combine them with the S values to obtain the desired total angular momentum J.

Having calculated the Wildenthal interaction and SU(4) eigenstates, we then evaluate the direct overlaps between them and sum their squares over all degenerate SU(4) eigenstates belonging to the same Young tableau. [In practice the Lanczos diagonalization in our code is not particularly well suited to obtain eigenvalues of spectra that are manyfold degenerate. Therefore, small random numbers were added to the matrix of the Casimir operator, Eq. (1), in order to remove the degeneracy to a small degree.]

Another way to remove most of the degeneracies would be to diagonalize not only $C[\mathrm{SU}(4)]$ but also (with different coefficients) the Casimir operators of SU(3) and R(3) of space and SU(2) of spin. (For several sd-shell nuclei such a program has been carried out by McGrory [21].) Since our goal here is simply to test the goodness of SU(4) symmetry, we did not follow this path although it would be obviously useful when considering, e.g., Gamow-Teller transitions that must be diagonal not only in the SU(4) but also in the SU(3) and R(3) labels.

The Young tableau with the lowest eigenvalue of C, Eq. (1), for the even-even nuclei with $T = (N - Z)/2$ is $[T, T, 0]$ in the standard notation [22], or $(0, T, 0)$ in the

TABLE I. SU(4) overlaps of the $J^\pi = 0^+$ ground states of even-even nuclei. The upper entry is the overlap (in %) with the lowest Young tableau, and the lower entry (in parentheses) is the sum of the overlaps with the two lowest tableaux. (Occasionally two distinct Young tableaux have identical eigenvalues. In such a case we add the overlaps.)

Z/T	0	1	2	3	4	5
$_8$O	-	86.1	61.7	41.9	39.6	63.4
		(100.0)	(99.7)	(96.4)	(96.0)	(100.0)
$_{10}$Ne	87.1	56.0	39.8	37.9	50.1	78.6
	(98.6)	(88.1)	(85.0)	(86.7)	(91.5)	(100.0)
$_{12}$Mg	51.2	35.1	30.7	34.9	51.8	-
	(87.0)	(66.7)	(75.8)	(82.4)	(99.2)	-
$_{14}$Si	32.0	23.7	28.4	33.9	-	-
	(72.2)	(54.4)	(76.0)	(93.8)	-	-
$_{16}$S	30.5	31.4	34.7	-	-	-
	(70.4)	(68.8)	(94.1)	-	-	-
$_{18}$Ar	55.2	59.1	-	-	-	-
	(88.7)	(100.0)	-	-	-	-

notation of [20]. For odd-odd nuclei, the lowest Young tableau is $[1,1,0]$ [or (0,1,0) in the other notation] for $T=0$ and $[T+1,T,1]$ [or $(1,T-1,1)$] for $T \neq 0$. We list the overlaps with these Young tableaux in Tables I and II, and display them in Figs. 1 and 2.

SU(4) symmetry is clearly badly broken once we are dealing with nuclei more than a few nucleons away from closed shells. States belonging to the lowest Young tableau are obviously not a good substitute for the true nuclear state, particularly in even-even nuclei. In odd-odd nuclei the situation is a bit better, and we tentatively explain the bigger overlaps to the reduced effect of pairing in these nuclei. (One can, perhaps, see a hint of this tendency already in the work of French and Parikh [14].) We will show in the next section that the pairing interaction is one of the main SU(4) symmetry breaking influences.

On the other hand, the distribution of the overlaps is not random. The states belonging to the lowest Young tableau have 5–10 times bigger overlaps than an average state. Moreover, the overlaps are related to the value of the Casimir operator, as Fig. 3 for eight particles (holes) with $J=0^+$, $T=2$ and six particles (holes) with $J=1^+$, $T=0$ illustrates. This is a typical situation. One sees in Tables I and II that by including the first and second tableaux the differences between even-even and odd-odd nuclei largely disappear, and that a major part of the nuclear wave function is accounted for. This behavior suggest that an expansion in SU(4) states might be a sensible approach if one is interested in processes allowed under SU(4).

TABLE II. SU(4) overlaps of the lowest $J^\pi = 1^+$ or $J^\pi = 0^+$ (entries with *) states of odd-odd nuclei. The upper entry is the overlap (in %) with the lowest Young tableau, and the lower entry (in parentheses) is the sum of the overlaps with the two lowest tableaux. (Occasionally two distinct Young tableaux have identical eigenvalues. In such a case we add the overlaps.)

Z/T	0	1	2	3	4	5
$_9$F	94.9	84.6	83.0	72.1	77.8	96.1
	(100.0)	(91.3)	(96.7)	(92.1)	(95.6)	(100.0)
$_{11}$Na	69.9	66.3	68.2*	71.9	85.4	-
	(89.2)	(70.4)	(90.6*)	(91.8)	(97.1)	-
$_{13}$Al	69.8*	53.7*	68.4*	65.7	-	-
	(80.0*)	(59.9*)	(91.5*)	(89.0)	-	-
$_{15}$P	56.1*	41.6	55.6	-	-	-
	(64.0*)	(54.0)	(83.6)	-	-	-
$_{17}$Cl	65.6	62.5	-	-	-	-
	(72.7)	(7.4)	-	-	-	-
$_{19}$K	97.3	-	-	-	-	-
	(100.0)	-	-	-	-	-

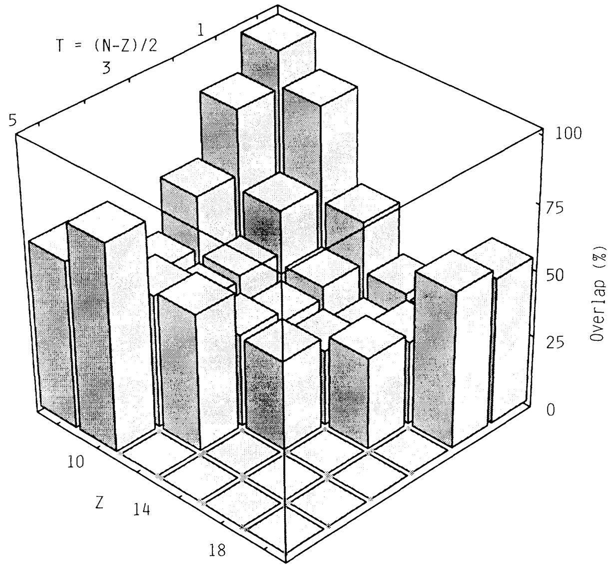

FIG. 1. Barchart representation of the overlaps between the $J^\pi = 0^+$ ground states of all even-even nuclei and the eigenstates of the SU(4) Casimir operator corresponding to its lowest eigenvalue and compatible with the given A, T, and J.

III. SYMMETRY VIOLATION AND RESTORATION

In order to trace the role of various components of nuclear mean field and residual interaction, we made a number of tests. We chose the case of eight nucleons in the *sd* shell and with $T=0,2$ and $J=0^+$ as suitably typical cases. Spin-orbit splitting is a major source of SU(4)

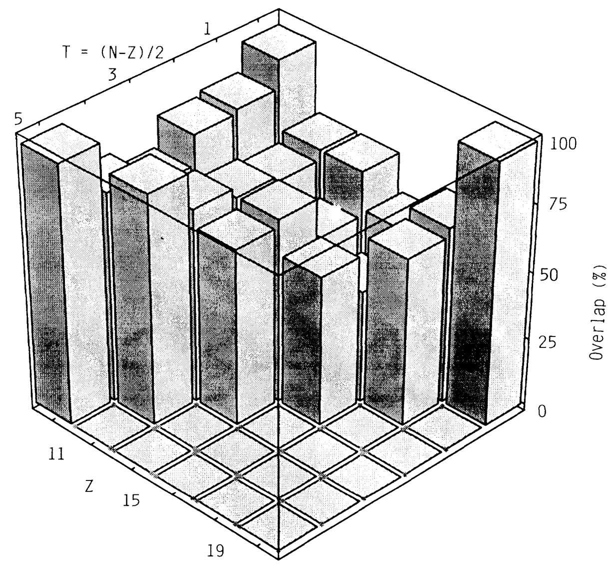

FIG. 2. Barchart representation of the overlaps between the lowest $J^\pi = 1^+$ or 0^+ states of all odd-odd nuclei and the eigenstates of the SU(4) Casimir operator corresponding to its lowest eigenvalue and compatible with the given A, T, and J. (The nuclei where 0^+ is used instead of 1^+ can be seen in Table II.)

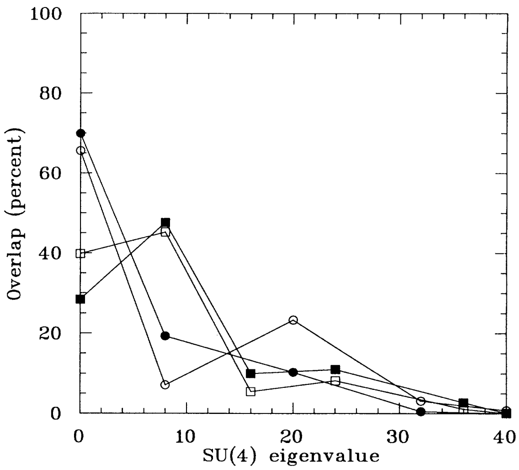

FIG. 3. Complete decomposition in terms of the SU(4) representations of the 0^+ ground states of ^{24}Ne (empty squares) and ^{32}Si (full squares), and of the lowest 1^+ states in ^{22}Na (full circles) and ^{34}Cl (empty circles). On the abscissa axis are the SU(4) Casimir operator eigenvalues, relative to the lowest one for each nucleus.

symmetry violation. French and Parikh [14] have shown that the overlaps with the lowest SU(4) set of states increase dramatically if one makes all single-particle states degenerate. We have come to a very similar conclusion. With standard single-particle energies the overlaps with the lowest Young tableau are 51.2% and 39.8%, while for degenerate single particle (s.p.) states they become 89.1% and 90.6%.

Next, we would like to test the role of the residual interaction. By reducing *all* matrix elements of the interaction by a factor of 10, and keeping the s.p. energies as they were, we arrive at states that have no resemblance to SU(4). The lowest tableau gives only 6.3% and 11.0% for the two cases. Thus, inclusion of the residual interaction "restores" to some extent the SU(4) symmetry that was initially totally broken by the spin-orbit splitting.

One can look at these tendencies also from a different perspective. We could consider the Hamiltonian with degenerate s.p. states but including the two-body interaction as a starting point, and the spin-orbit force as a symmetry breaking perturbation. Since we saw above that the Wildenthal interaction is to a large extent SU(4) symmetry conserving, the zeroth-order states would then have good SU(4) symmetry. The one-body spin-orbit force can shift only one block in the Young tableaux [17], and therefore typically connects tableaux with close eigenvalues. Thus, the pattern exhibited in Fig. 3 can be understood.

When comparing the even-even and odd-odd nuclei we concluded that the pairing interaction is effective in breaking SU(4) symmetry. Pairing is not a realistic approximation for the two-body interaction in the *sd* shell, but becomes increasingly more effective in heavier nuclei. Since we use the *sd* shell primarily as a testing ground, we replaced the Wildenthal interaction with the charge-independent pairing interaction that acts between $J = 0$, $T = 1$ states, that is $\langle j_1 j_1; J,T|V|j_2 j_2; J,T\rangle = -\chi\sqrt{2j_1+1)(2j_2+1)}\delta_{J0}\delta_{T1}$. [The pairing interaction, even when treated in the isospin conserving way, clearly violates SU(4) symmetry, since the Casimir operator, Eq. (1), contains interaction terms in *both* the $T = 0$ and $T = 1$ channels, with a fixed relative strength.]

The coupling constant $\chi = 0.390\,28$ MeV was chosen by fitting the $T = 1$ two-body matrix elements of the Wildenthal *sd*-shell interaction to a pairing plus a density-density interaction composed of monopole, quadrupole, and hexadecapole components. For the two cases considered above the first overlaps become 10.1% and 16.8% showing that pairing indeed effectively destroys SU(4) symmetry. We then repeated the pairing interaction diagonalization with degenerate s.p. energies. As expected, the overlaps with the lowest tableau grew, to 48.9% and 86.1%. The symmetry, however, has not been restored, and a closer look shows that the eigenstates of pairing are very different from eigenstates of the realistic interaction in their decomposition in terms of the SU(4) representations. Whereas we observed an obvious correlation between the overlap and the Casimir operator eigenvalue for a realistic interaction, the pairing situation is different. Even for the case of degenerate orbits, the admixed SU(4) representations are far removed from the lowest Young tableau.

The results in Tables I and II were obtained with the Wildenthal interaction. How different are the overlaps

TABLE III. SU(4) overlaps of the lowest $J^\pi, T = 0^+, 2$ state of ^{24}Ne and $J^\pi, T = 1^+, 0$ state of ^{22}Na for different interactions.

^{24}Ne				
C[SU(4)]	Wildenthal	Bonn pot.	Kuo-Brown bare	Kuo-Brown renorm.
0.0	39.8	50.9	55.8	61.1
8.0	45.2	39.7	38.1	33.2
16.0	5.5	3.8	2.7	2.1
24.0	8.2	5.0	2.9	3.3
36.0	1.1	0.5	0.4	0.3
>36.0	0.22	0.05	0.02	0.03
^{22}Na				
C[(SU4)]	Wildenthal	Bonn pot.	Kuo-Brown bare	Kuo-Brown renorm.
0.0	69.9	76.6	83.3	82.4
8.0	19.3	17.2	12.7	13.2
20.0	10.3	5.9	3.8	4.2
32.0	0.5	0.2	0.1	0.2
>32.0	0.04	0.02	0.02	0.02

with other popular effective interactions? To answer this question we calculated the complete decomposition of the $J^\pi, T = 0^+, 2$ ground state of ^{24}Ne and of the lowest $J^\pi, T = 1^+, 0$ state of ^{22}Na for three other widely used effective *sd*-shell interactions (Bonn potential based *G*-matrix [23], bare Kuo-Brown, and renormalized Kuo-Brown [24]). The results are shown in Table III. One can see that the Wildenthal interaction has the smallest overlap, while the renormalized Kuo-Brown interaction has the largest overlap. We believe that this is a general result. At the same time, the pattern of SU(4) breaking, and the larger overlap in odd-odd nuclei in comparison to the even-even ones, is a common feature of all interactions. One should keep in mind, however, that the fitted Wildenthal interaction describes the spectra of *sd*-shell nuclei much better than the other ones. For example, in the two considered nuclei, Wildenthal interaction describes quite well the splitting between the first and second 0^+ and 1^+ states while the other three interactions underestimate it considerably.

IV. *fp*-SHELL RESULTS

Unlike the *sd* shell, the *fp* shell contains too many particles and cannot be treated by shell-model codes in a full generality. Thus, it is impossible to extend the same treatment, described previously, of SU(4) symmetry breaking to heavier *fp*-shell nuclei. However, one can see in Fig. 1 that the most important quantities, the overlaps of the ground state with the lowest Young tableaux, are qualitatively similar for the oxygen isotopes (purely neutron states) and in the other *sd*-shell nuclei. It appears that the amount of the overlap depends primarily on the number of nucleons in the shell, and to a lesser degree on their isospin.

Guided by these considerations, we have evaluated the overlaps between several low-lying $J = 0^+$ states of even-even Ca isotopes and the eigenstates of the SU(4) Casimir operator. Both calculations were performed for the full *fp* shell without restrictions, using the interaction proposed by Richter *et al.* [25]. The results are displayed in Fig. 4.

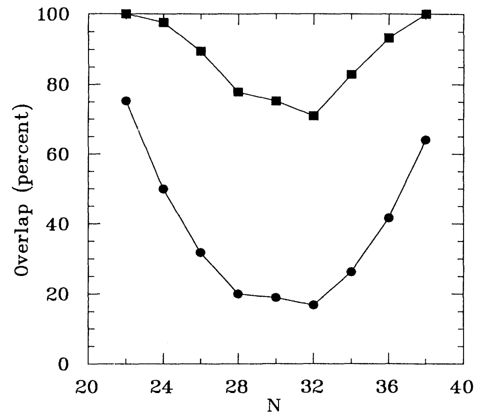

FIG. 4. Overlaps between the $J = 0^+$ ground states of the Ca isotopes and the eigenstates of the SU(4) Casimir operator corresponding to its lowest eigenvalue (circles) and to the sum of the lowest and next eigenvalues (squares).

The overlaps with the lowest SU(4) Young tableaux are even smaller for the Ca isotopes than for the analogous oxygen isotopes. In the middle of the *fp* shell, the ground states contain only about 20% of the lowest SU(4) irreducible representation. However, our previous finding that the lowest and the next-to-lowest Young tableaux account for most of the ground state wave function remains true, as one can also see in Fig. 4.

For $^{42-48}$Ca one can test to what extent the spin-orbit splitting is responsible for the SU(4) breaking. When the diagonalization is repeated with the realistic interaction, but forcing all neutrons to be in the $f_{7/2}$ subshell, one obtains overlaps with the lowest Young tableau of 57.1, 31.8, 18.6, and 12.1% for neutron numbers 2, 4, 6, and 8, respectively. By comparing with the overlaps in Fig. 4, we see that the finite separation between the $f_{7/2}$ and the rest of the *fp* shell "restores" the symmetry somewhat, as expected, but the restoration is insufficient. [Let us note that the overlaps for the $(f_{7/2})^n$ configurations quoted above are interaction independent.]

V. CONCLUSION

In conclusion, we have evaluated the overlaps between the wave functions that describe the ground states (and a few low-lying states) of all even *A* *sd*-shell and some *fp*-shell nuclei and the eigenstates of the SU(4) Casimir operator. By adding the squares of such overlaps for all degenerate states belonging to the same irreducible representation (Young tableau) of SU(4) we can make a quantitative statement about the validity of the SU(4) symmetry in these nuclei. We find that SU(4) symmetry is badly broken for all nuclei with more than a few nucleons (or holes) outside closed shells. We identify the spin-orbit splitting as the main SU(4) symmetry breaking mechanism, in agreement with previous results. In addition, the pairing interaction also has symmetry breaking effects.

Diagonalization in the full shell-model space "restores" SU(4) symmetry to some extent. However, this effect is insufficient to make SU(4) a practical starting point in the development of a truncation scheme (or of a perturbation theory expansion) aimed at reducing the huge dimensions inherent in the shell-model approach to heavier nuclei. On the other hand, our calculations show that the overlaps with the SU(4) eigenstates belonging to the lowest possible Casimir operator eigenvalues are significantly larger than one would expect on purely statistical grounds. We also find that the decomposition of the nuclear wave functions in terms of SU(4) eigenstates has characteristic regularities, i.e., states belonging to higher SU(4) eigenvalues have smaller overlaps. These tendencies are consequences of two opposing trends. The two-body interaction is largely SU(4) symmetry conserving, namely, it contains the space exchange Majorana force as a major component. At the same time, the symmetry breaking is caused mostly by the one-body spin-orbit force which has very simple selection rules in SU(4).

The features of nuclear spectra, mentioned in the Introduction, which are often considered as indicators of the SU(4) symmetry, are general consequences of the spin-isospin structure of the nucleon-nucleon interaction. The existence of the giant Gamow-Teller state, for example, is caused by the repulsive $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ particle-hole force; by itself the existence of the giant Gamow-Teller state, and even its closeness to the isobar analog state, is not a guarantee of the SU(4) symmetry.

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[1] E. P. Wigner, Phys. Rev. **51**, 106 (1937); **51**, 947 (1937); **56**, 519 (1939).
[2] P. Franzini and L. A. Radicati, Phys. Lett. **6**, 322 (1963).
[3] Yu. V. Gaponov, N. B. Shulgina, and D. M. Vladimirov, Nucl. Phys. **A391**, 93 (1982).
[4] J. I. Fujita and D. Ikeda, Nucl. Phys. **67**, 145 (1965).
[5] J. Bernabeu, B. Desplanques, J. Navarro, and S. Noguerra, Z. Phys. C **46**, 323 (1990).
[6] P. Vogel, M. Ericson, and J. D. Vergardos, Phys. Lett. B **212**, 259 (1988).
[7] K. Muto, Phys. Lett. B **277** , 13 (1992).
[8] D. C. Zheng, L. Zamick, and N. Auerbach, Phys. Rev. C **40**, 936 (1989); Ann. Phys. **197**, 343 (1990).
[9] M. Hino, K. Muto, and T. Oda, Phys. Rev. C **37**, 1328 (1988).
[10] G. Mairle, K. T. Knopfle, and M. Seeger, Nucl. Phys. **A490**, 371 (1988).
[11] P. T. Nang, Nucl. Phys. **A185**, 413 (1972).
[12] N. C. Mukhopodhyay and F. Cannata, Phys. Lett. **51B**, 225 (1974).
[13] B. H. Wildenthal, in *Progress in Particles and Nuclear Physics*, edited by D. H. Wilkinson (Pergamon, Oxford, 1984), Vol. **11**, p. 5.
[14] J. B. French and J. C. Parikh, Phys. Lett. **35B**, 1 (1971).
[15] K. T. Hecht, Annu. Rev. Nucl. Sci. **23**, 123 (1973).
[16] W. D. M. Rae, A. Etchegoyen, and B. A. Brown, OXBASH, the Oxford-Buenos Aires-MSU Shell-Model Code, Michigan State University Cyclotron Laboratory Report No. 524, 1988.
[17] C. M. Vincent, Nucl. Phys. **A106**, 199 (1967).
[18] Z. Pluhař, Nucl. Phys. **A167**, 33 (1971).
[19] J. P. Draayer, Nucl. Phys. **A216**, 457 (1973).
[20] W. G. McKay and J. Patera, *Tables of Dimensions, Indices, and Branching Rules for Representations of Simple Lie Algebras* (Marcel Dekker, New York, 1981).
[21] J. B. McGrory, Phys. Lett. **47B**, 481 (1973).
[22] K. T. Hecht and S. C. Pang, J. Math. Phys. **10**, 1571 (1969).
[23] Supplied by T. T. S. Kuo to the OXBASH program package (1988).
[24] T. T. S. Kuo, Nucl. Phys. **A103**, 71 (1967).
[25] W. A. Richter *et al.*, Nucl. Phys. **A523**, 325 (1991).

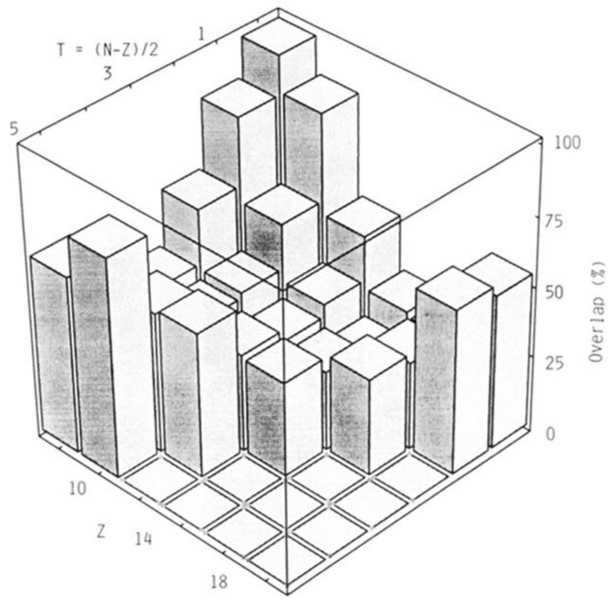

FIG. 1. Barchart representation of the overlaps between the $J^\pi = 0^+$ ground states of all even-even nuclei and the eigenstates of the SU(4) Casimir operator corresponding to its lowest eigenvalue and compatible with the given A, T, and J.

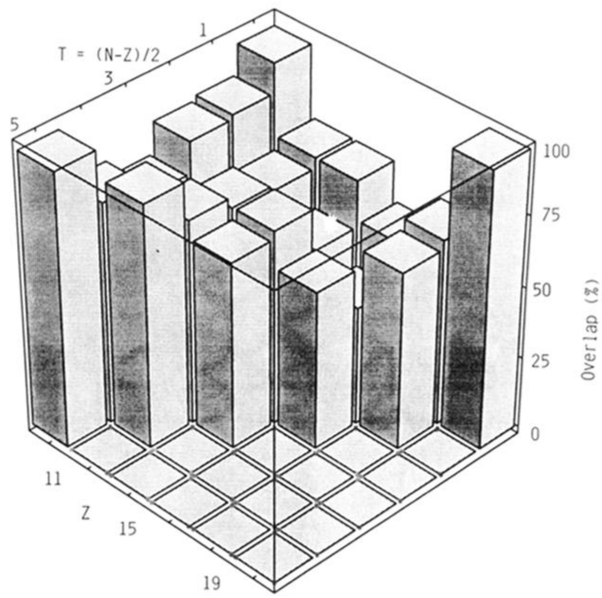

FIG. 2. Barchart representation of the overlaps between the lowest $J^{\pi} = 1^{+}$ or 0^{+} states of all odd-odd nuclei and the eigenstates of the SU(4) Casimir operator corresponding to its lowest eigenvalue and compatible with the given A, T, and J. (The nuclei where 0^{+} is used instead of 1^{+} can be seen in Table II.)