Lipkin-Nogami method at finite temperature in the static-path approximation

N. Dinh Dang,* P. Ring, and R. Rossignoli†

Physik-Department, Technische Universität München, D-8046 Garching, Germany

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An approximate particle number projection at finite temperature using the Lipkin-Nogami method in the context of the static-path approximation is proposed. The numerical evaluation, performed in a simple degenerate 2Ω-level model, shows that the present approach can improve the standard unprojected static-path approximation and is superior to the usual prescription based on classical thermodynamics.

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I. INTRODUCTION

Fluctuations play an especially important role in hot finite nuclear systems. A significant effort has been made during recent years to take into account quantal and statistical fluctuations in the mean-field [1–7]. The use of symmetry-breaking density operators induces quantum fluctuations in related conserved quantities. In the case of the particle number operator the symmetry-violating inner states are the BCS wave functions and the quantal fluctuations are characterized by the fluctuations of the gauge angle. Therefore quantal effects can be taken into account by employing various kinds of projection methods [3,8]. Statistical fluctuations increase as growing temperature and to deal with them, the average quantities such as the average pairing gap, the energy, etc., have been used instead of their most probable value [1,2]. The averaging procedure has been usually performed assuming the isothermal probability distribution for the considered physical quantity from the Landau theory of phase transitions in infinite systems [9]. The integration is taken over the phase volume as in the classical limit. So far only a few approaches considering both quantal and statistical fluctuations in the mean-field of hot nuclei have been performed. Among them we point out Ref. [7], where the Lipkin-Nogami (LN) method [10] is extended to finite temperature so that the quantal effects arising from the particle number fluctuations can be tractable in the finite-temperature BCS formalism (FT-BCS). Although the LN-method is an approximate number projection, it allows one to incorporate properly the exchange terms together with the second-order effects caused by the particle number fluctuations in a very simple manner.

Recently, an effective way to go beyond the finite-temperature Hartree theory, which enables one to include microscopically the fluctuations of the mean-field itself, has been developed within the framework of the static-path approximation (SPA) [11–16] to the path integral representation of the partition function [17]. By expanding the effective action as a power series around the average of temperature-independent paths (the static path), the integration is carried out over all static paths and, therefore, sums over fluctuations of the mean-field. The main disadvantage of the SPA is the omission of the Fock terms [11]. Moreover, the grand canonical partition function, calculated in the SPA, neglects as well quantal effects connected with the particle number fluctuations.

In the present paper we suggest an approach based on the finite temperature LN method (FT-LN method) [7] to incorporate simultaneously both types of fluctuations within the SPA. Based on the advantage of the LN method we believe that we can eliminate the shortcomings of the standard SPA without making the usual SPA calculation procedure more complicated.

In Sec. II the approximated partition function based on the FT-LN method within the SPA framework is constructed. Section III discusses the relation between the averaging procedure in the SPA and the traditional one from the statistical theory of phase transitions. Section IV presents the calculations for the energy, pairing gap, and the level density in the 2Ω level degenerate model, where the comparison with the results within the standard SPA framework [12] and those based on the statistical theory of phase transitions [1,2] is discussed. Some conclusions are drawn in Sec. V.

II. FT-LN METHOD IN THE SPA CONTEXT

A. SPA partition function

As we shall consider the influence of the particle number uncertainty together with statistical fluctuations, it is sufficient to work with the pairing Hamiltonian

$$\hat{H} = \hat{H}_0 - G\hat{P}^+\hat{P}, \tag{1}$$

*On leave from the Department of Theoretical and Computational Physics, INST, Vietnam Atomic Energy Commission, Hanoi, Vietnam. Present address: Istituto Nazionale di Fisica Nucleare, Sezione di Catania, I 95129 Catania Corso Italia 57, Italy.

†On leave from Departamento de Física, Universidad Nacional de La Plata, Argentina.

where $\hat{H}_0$ is the single-particle Hamiltonian determined from the single-particle spectrum $\{E_i^0\}$ of a realistic potential and $\hat{P}$ is the operator annihilating a pair of particles. A generalization including the separable multipole (p-h) and (p-p) residual interactions should be straightforward in a standard way. However, the calculations performed in Refs. [7,18] have shown that accounting the quadrupole vibration could give a sensitive contribution to the coefficient for the increase of the level density only at moderate temperature.

The grand partition function has the form [19]

$$Z(\beta,\lambda) = \mathrm{Tr}\exp[-\beta(\hat{H}-\lambda\hat{N})] \tag{2}$$

and the thermodynamic potential is defined as

$$\Omega(\beta,\lambda) = -\beta^{-1}\ln Z(\beta,\lambda), \tag{3}$$

where $\hat{N}$ is the particle number operator, λ is the chemical potential, and β is the inverse temperature.

Replacing the exponential in Eq. (2) by the Trotter product of L time slices and expressing the exponential on each slice as an integral over the one-body Hamiltonian using the Hubbard-Stratonovich transformation [20], the functional representation of the partition function (2) has been obtained [11]. Performing now the integration over the averages ζ and ζ^* of pairing fields $\zeta(t)$ and $\zeta^*(t)$ over the interval β, one ends up with the SPA to the partition function [12]:

$$Z_{\mathrm{SPA}}(\beta,\lambda) = \frac{1}{\pi G}\beta\int\exp\left[-\beta\frac{\zeta\zeta^*}{G}\right]\mathrm{Tr}\exp[-\beta\hat{h}']d\zeta\, d\zeta^*\ , \tag{4}$$

$$\hat{h}' = \hat{H}_0 - G\hat{P}_0 - \zeta^*\hat{P} - \zeta\hat{P}^+ - \lambda\hat{N}\ , \tag{5}$$

where $\hat{P}_0 = \frac{1}{2}(\hat{N}-\Omega)$ with Ω being half the number of states in the single-particle space.

Parametrizing the pairing fields as $\zeta = \Delta\exp(i\phi)$, one obtains the SPA partition function in a form of an integral over Δ [12]

$$Z_{\mathrm{SPA}}(\beta,\lambda) = \frac{2}{G}\beta\int_0^\infty \exp[-\beta\Omega_{\mathrm{SPA}}(\beta,\lambda,\Delta)]\Delta d\Delta \tag{6}$$

where

$$\begin{aligned}\Omega_{\mathrm{SPA}}(\beta,\lambda,\Delta) &= \langle\hat{H}\rangle - \lambda N - \frac{1}{\beta}S\\ &= \sum_i(E_i^0-\lambda-\epsilon_i) + \frac{\Delta^2}{G}\\ &\quad -\frac{2}{\beta}\sum_i\ln[1+\exp(-\beta\epsilon_i)]\end{aligned} \tag{7}$$

with N being the number of particles, $\epsilon_i = \sqrt{(E_i^0-\lambda)^2+\Delta^2}$ the quasiparticle energy, S the quasiparticle entropy, and

$$\langle\hat{H}\rangle = \sum_i E_i^0\left[1-\frac{E_i^0-\lambda}{\epsilon_i}\tanh(\tfrac{1}{2}\beta\epsilon_i)\right] - \Delta^2/G. \tag{8}$$

Equation (6) just contains *the thermodynamic probability*

$$\mathcal{P}_{\mathrm{SPA}}(\beta,\lambda,\Delta) \equiv \exp[-\beta\Omega_{\mathrm{SPA}}(\beta,\lambda,\Delta)] \tag{9}$$

but it differs from the usual *adiabatic* thermodynamic partition function by the presence of the prefactor $\frac{2}{G}\beta$, which affects the energy evaluation, and the metric $\Delta d\Delta$ instead of $d\Delta$, which has been used so far in the thermodynamic average [1,2]. In the mean time before we return to this question in Secs. III and IV, let us notice that when the pairing field ζ is given by its self-consistent value $\zeta = G\langle\hat{P}\rangle$, the energy $\langle\hat{H}\rangle$ (8) is equivalent to the usual FT-BCS energy (*without* Fock terms), with the pairing gap Δ defined from the FT-BCS equations [19]. The SPA becomes *exact* in the high temperature limit.

With this overview of the SPA we are now going to modify it to approximately eliminate the particle number fluctuations and incorporate the Fock terms.

B. FT-LN partition function based on SPA. Energy and level density

In Ref. [7] by extending the LN method to finite temperature, it has been shown that the thermodynamic potential of type (7), with the gap Δ, calculated self-consistently from the FT-BCS like equations, changes under the influence of particle number fluctuations as

$$\Omega_{\mathrm{LN}} = \Omega_{\mathrm{BCS}} - \lambda_2\Delta N^2 \tag{10}$$

In the RHS of Eq. (10) Ω_{BCS} is the FT-BCS thermodynamic potential, including Fock terms calculated by the LN method (see below), and $\lambda_2\Delta N^2$ are the corrections due to the particle number fluctuations in the FT-LN method with the coefficient λ_2 and the particle number fluctuations $\Delta N^2 \equiv \langle\hat{N}^2\rangle - N^2$ derived explicitly in the FT-BCS in Ref. [7] as

$$\lambda_2 = \frac{\frac{1}{2}\sum_i f(\beta,\epsilon_i)/\epsilon_i^3}{\Delta^2\sum_i f(\beta,\epsilon_i)/\epsilon_i^3 + [\sum_i(E_i-\lambda)f(\beta,\epsilon_i)/\epsilon_i^3]^2 + \frac{1}{2}[\sum_i \mathrm{sech}^2(\frac{1}{2}\beta\epsilon_i)][\sum_j f(\beta,\epsilon_j)/\epsilon_j^3]}, \tag{11}$$

$$\Delta N^2 = \Delta N_{\mathrm{QF}}^2 + \Delta N_{\mathrm{SF}}^2, \tag{12}$$

where ΔN_{QF}^2 and ΔN_{SF}^2 are the quantal and statistical parts of the particle number fluctuations ΔN^2 respectively

$$\Delta N_{\mathrm{QF}}^2 = \Delta^2\sum_i(1-2n_i)/\epsilon_i^2, \tag{13}$$

$$\Delta N_{\mathrm{SF}}^2 = 2\sum_i n_i[(E_i-\lambda)^2(1-n_i)+n_i\Delta^2]/\epsilon_i^2, \tag{14}$$

with

$$f(\beta,\epsilon_i) = \tanh(\tfrac{1}{2}\epsilon_i) - \tfrac{1}{2}\beta\epsilon_i\mathrm{sech}^2(\tfrac{1}{2}\beta\epsilon_i) \tag{15}$$

and n_i being the quasiparticle occupation number

$$n_i = [1 + \exp(\beta\epsilon_i)]^{-1}. \quad (16)$$

The reason for the identification with $\Delta N^2_{\rm QF}$ and $\Delta N^2_{\rm SF}$ has already been discussed in Refs. [4,7] and we will not repeat it here.

The single-particle energies in the LN method are *renormalized* by the exchange terms and by the corrections due to the particle number fluctuations $\sim 4\lambda_2 v_i^2$ as

$$E_i = E_i^0 + (4\lambda_2 - G)v_i^2 + \frac{1}{2}G\frac{E_i-\lambda}{\epsilon_i}n_i\left[1 - \frac{E_i-\lambda}{\epsilon_i}\tanh(\tfrac{1}{2}\beta\epsilon_i)\right], \quad (17)$$

where the thermal average has been done in the FT-HFB approximation using Wick's theorem [8,19] for the ensemble average.

In practice the calculations in the FT-BCS have been usually performed without the last terms in Eq. (17). The numerical evaluations performed in heavy nuclei [6,7] have also shown that this $\sim Gn_i$ part of the exchange energy gives a negligibly small contribution to the excitation energy and the level density. In the model case considered in Sec. IV this part gives *no contribution.* On the other hand, the presence of the terms $\sim 4\lambda_2 v_i^2$ nearly suppresses the self-energy correction due to the part $\sim Gv_i^2$ of the exchange terms, which is also an important fact for the accuracy of the method at high temperatures (see Sec. IV). This merit of the LN method allows one to approximately set $E_i \simeq E_i^0$, as a result of the cancellation of the exchange terms $\sim Gv_i^2$ by the effect due to the particle number fluctuations (the terms $\sim 4\lambda_2 v_i^2$).

Based on this advantage of the LN method, we now *modify* the SPA thermodynamic potential (7) in such a way that both effects from the exchange terms and from the particle number fluctuations are simultaneously taken into account. Namely, we define the *modified* SPA thermodynamic potential $\Omega_{\rm LN(SPA)}(\beta,\lambda,\Delta)$ as in Eq. (9), where the gap Δ plays the role of the integration parameter in the integral of the type (6). We use the subscript LN(SPA) to denote quantities calculated in the SPA including LN corrections.

Thus, we define

$$Z_{\rm LN(SPA)}(\beta,\lambda) = \frac{2}{G}\beta\int_0^\infty \mathcal{P}_{\rm LN(SPA)}(\beta,\lambda,\Delta)\Delta d\Delta \ , \quad (18)$$

where the probability $\mathcal{P}_{\rm LN(SPA)}(\beta,\lambda,\Delta)$ has the form (9) containing the thermodynamic potential $\Omega_{\rm LN(SPA)}$ defined similarly to Eq. (10)

$$\Omega_{\rm LN(SPA)} = \Omega_{\rm SPA} - \lambda_2\Delta N^2 + \langle\hat{H}_e\rangle. \quad (19)$$

In Eq. (19) we use the abbreviation $\langle\hat{H}_e\rangle$ to denote the contribution of the exchange terms (with the LN renormalization) consisting of the part $\sim (4\lambda_2 - G)v_i^2$ and the part $\sim Gn_i$ discussed above.

Equations (18) and (19) are the *central point* of our *postulation,* in which we attempt to encompass the exchange terms together with the effects of second order in the particle number operator $\hat{N}$, charaterized by the particle number fluctuations ΔN^2 (12), in the FT-LN.

The energy E of the system can be deduced from the grand partition function Z as [9,19]

$$E = -\frac{\partial \ln Z}{\partial\beta} + \lambda N. \quad (20)$$

The level density ρ, which is the inverse Laplace transform of the grand partition function [21,22], is evaluated by means of the *saddle-point approximation* for one-component system as

$$\rho(N,E) = Z(\beta,\lambda)\exp[\beta(E-\lambda N)]/2\pi\sqrt{D} \quad (21)$$

at the saddle points for the energy [Eq.(20)] and the particle number N

$$N = \frac{\partial \ln Z}{\beta\partial\lambda} \ . \quad (22)$$

In Eq. (21) D is the 2×2 determinant of the second derivative matrix $\partial^2 Z(x)/\partial x_i\partial x_j$ with $x = (\beta, \beta\lambda)$.

III. THERMODYNAMIC AND SPA AVERAGES

So far fluctuations in nuclear systems at finite temperature have been treated as fluctuations of thermodynamic quantities, which are assumed to behave classically. The calculations then have been considerably simplified by applying the results of the Landau theory of phase transitions [9]. Thus, the mean value of a quantity, related to the order parameter $\mathcal{O}$, is expressed through the functional $\Omega(\mathcal{O})$, which is the thermodynamic potential corresponding to the distribution of this quantity by

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{N}}\int \mathcal{O}\exp[-\beta\Omega(\mathcal{O})]d\mathcal{O} \ , \quad (23)$$

where

$$\mathcal{N} = \int \exp[-\beta\Omega(\mathcal{O})]d\mathcal{O}. \quad (24)$$

Using Eqs. (23) and (24) for calculating the thermodynamic average pairing gap $\langle\Delta\rangle$ with the thermodynamic probability (9), the expression

$$\langle\Delta\rangle = \int \Delta\mathcal{P}_{\rm SPA}(\beta,\lambda,\Delta)d\Delta \Big/ \int \mathcal{P}_{\rm SPA}(\beta,\lambda,\Delta)d\Delta \quad (25)$$

has been employed in the literature for the thermodynamic average gap (cf. Refs. [1,2]) and the average energy has been calculated as a function of this thermodynamic average gap. Consequently, in the FT-LN method one can define the LN average gap $\langle\tilde{\Delta}\rangle$ from Eq. (25), where the probability $\mathcal{P}_{\rm SPA}(\beta,\lambda,\Delta)$ (9) should be replaced by $\mathcal{P}_{\rm LN(SPA)}(\beta,\lambda,\Delta)$ with the thermodynamic potential $\Omega_{\rm LN(SPA)}(\beta,\lambda,\Delta)$ defined by Eq. (19) [7].

The range of applicability of these equations has been discussed thoroughly in Refs. [9,23]. The quantal effects are neglected [9]. Moreover, the averaging (25) is defined in an *ad hoc* macroscopic manner with no direct relation to physical observables. Nevertheless the merit of accounting for the statistical fluctuations in the form (25) in hot finite systems is the qualitative smoothing of the phase transition from superfluid to normal in the region of critical temperature T_c and the fact that the average gap $\langle\Delta\rangle$ does not vanish even at $T \gg T_c$ [1,2].

The SPA, on the contrary, is a solid microscopic approach, which is exact in the limit $T \to \infty$ [11]. However there is a discrepancy between the energy calculated in the standard SPA and the exact one at low temperature [12] due to the missing of Fock terms and of the effect of the second order in the particle number operator in the standard SPA.

The SPA energy, calculated from Eq. (20), can be expressed in a form containing the energy $\langle\hat{H}\rangle$. Making use of Eq. (6), the derivative in the RHS of Eq. (20) leads to

$$E_{\rm SPA} \equiv -\frac{\partial \ln Z_{\rm SPA}}{\partial\beta} + \lambda N = \mathcal{E} - \frac{1}{\beta} \;, \qquad (26)$$

where

$$\mathcal{E} = \int_0^\infty \langle\hat{H}\rangle \mathcal{P}_{\rm SPA}(\beta,\lambda,\Delta)\Delta d\Delta \Big/ \int_0^\infty \mathcal{P}_{\rm SPA}(\beta,\lambda,\Delta)\Delta d\Delta \;. \qquad (27)$$

It is apparent that $\mathcal{E}$ and $E_{\langle\Delta\rangle}$, which is the energy $\langle\hat{H}\rangle$ (8), calculated by replacing Δ by the mean value $\langle\Delta\rangle$ (25) of the pairing gap, do not coincide. However Eqs. (26) and (27) show that fluctuations of the mean field in the SPA also include the statistical fluctuations in the sense of the classical thermodynamic with a *new metric* $\Delta d\Delta$ instead of $d\Delta$.

Applying the FT-LN method in the SPA context, we perform in fact an *approximate particle number projection at finite temperature*, which also includes the effect of Fock terms. Therefore, by treating the particle number fluctuations following Eqs. (18)-(19), we believe that we can improve the standard SPA. In short, both quantal and statistical fluctuations are encompassed in an approach, which still gives the accurate high-temperature limit.

IV. NUMERICAL RESULTS

In order to check the present FT-LN approach in the SPA context, we calculate the energy, pairing gap, and energy level density in an exactly soluble model with a single j shell, which is half filled by $N = \Omega$ particles interacting via a monopole pairing force (the 2Ω degenerate model with $\lambda = -\frac{G}{2}$ and $\epsilon = \Delta$). This model has been employed in Ref. [12] when the SPA was used for the first time to treat fluctuations in finite nuclear systems. The exact result for the energy is given by diagonalizing the monopole pairing Hamiltonian in the seniority scheme [3,8] and the pairing gap is expressed as [3]

$$\Delta(\beta) = \sqrt{-GE(\beta)}. \qquad (28)$$

The SPA partition function $Z_{\rm SPA}$ [Eqs. (6) and (7)] in this model takes the simple form, obtained in Ref. [12], with

$$\Omega_{\rm SPA}(\beta,\lambda,\Delta) = \frac{\Delta^2}{G} - N(\lambda+\Delta) - \frac{2N}{\beta}\ln[1+\exp(-\beta\Delta)] \;. \qquad (29)$$

The Fock terms in this simple model are reduced to $\sim Gv_i^2 = \frac{1}{2}G$ because the part $\sim Gn_i$ vanishes. Therefore, accounting for only the exchange terms in this model leads to a constant shift down of the SPA energy by $\frac{N}{4}G$, which makes incorrect the high-temperature limit.

The LN corrections from Eqs. (10)–(19) have the simple expression in this case

$$\lambda_2 = \frac{\Delta}{N}\coth(\tfrac{1}{2}\beta\Delta) \;, \qquad (30)$$

$$\Delta N^2 = N(1-2n+2n^2), \qquad (31)$$

$$n = [1+\exp(\beta\Delta)]^{-1} \;, \qquad (32)$$

$$\beta\lambda_2\Delta N^2 = \beta\Delta\coth(\beta\Delta). \qquad (33)$$

The determinant in Eq. (21) with these corrections can be calculated based on the expressions, derived in Appendix B of Ref. [7].

The results discussed below have been obtained with the same set of values ($G = 0.1$ MeV, $N = 20$), used in Ref. [12].

The energies $E_{\rm SPA}$, $E_{\rm LN(SPA)}$, $E_{\langle\Delta\rangle}$, and $E_{\langle\tilde{\Delta}\rangle}$ are plotted in comparison with the exact one $E_{\rm exact}$ as functions of temperature in Fig. 1. With the LN corrections, the SPA energy (dotted curve) is significantly improved (dashed curve) and the discrepancy with the exact result (solid curve) is nearly eliminated. We notice that there is still a shortcoming at very low (near zero) temperatures due to the prefactor $\propto \beta^{-1}$ in the definition of the SPA partition function (6), which leads to a negative specific heat [12]. This problem could be solved by recourse to a definition of the energy as a trace of the product of the Hamiltonian and the density operator avoiding the presence of this prefactor as has been done in our previous paper [24] (cf. Ref. [15]), but we will not dwell upon it here. In the region of the critical temperature T_c of phase transition the LN corrections give an overestimation as compared to the SPA energy. However the SPA energies (with and without the LN corrections) are always closer to the exact energy as compared to the thermodynamic average ones $E_{\langle\Delta\rangle}$ (dot-dashed curve) and $E_{\langle\tilde{\Delta}\rangle}$ (double-dot–dashed curve). Although the thermodynamic average energies correspond to positive specific heats at very low temperatures, they are not accurate in the temperature region of phase transition and especially at high temperatures.

The temperature dependence of coefficient λ_2 [Eqs.

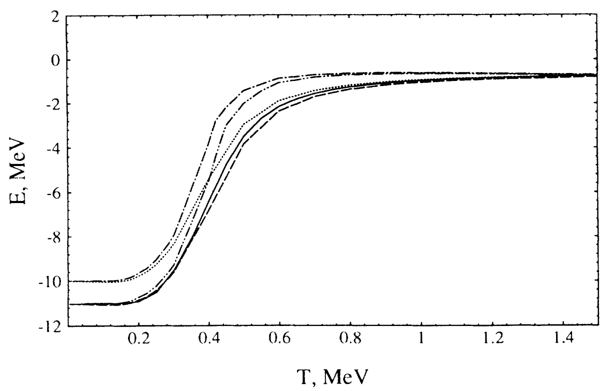

FIG. 1. Energy as a function of the temperature in the degenerate 2Ω-level model. The solid curve corresponds to the exact calculation in the seniority scheme, the dotted curve to the SPA, the dashed curve to the LN method in the SPA context [LN(SPA)], the dot-dashed curve to the thermodynamic average in the FT-BCS ($E_{\langle\Delta\rangle}$) and the double-dot–dashed to the thermodynamic average in the FT-BCS with the LN corrections ($E_{\langle\tilde{\Delta}\rangle}$).

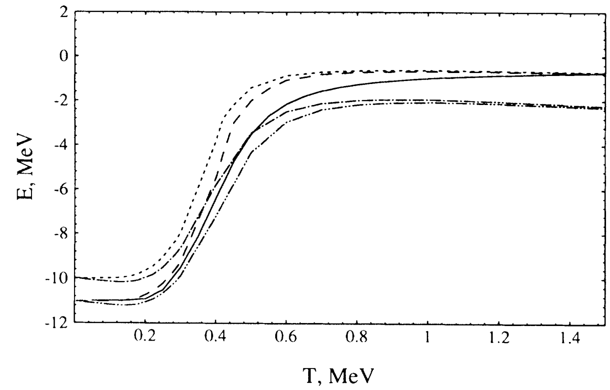

FIG. 3. Thermodynamic average energy $\mathcal{E}$ (27) as a function of the temperature in the degenerate 2Ω-level model. The solid curve corresponds to the exact calculation in the seniority scheme, the dot-dashed curve to $\mathcal{E}$ from Eq. (27), the double-dot–dashed curve to $\mathcal{E}$ with the LN corrections. For a comparison the energies $E_{\langle\Delta\rangle}$ and $E_{\langle\tilde{\Delta}\rangle}$ from Fig. 1 are reproduced, which are denoted here by the dotted and dashed curves respectively.

(11) and (30)] begins at low temperatures with nearly constant value equal to 0.25 (Fig. 2) so as the term $\propto (4\lambda_2 - G)$ in Eq. (17) is completely suppressed. It leads to rather accurate values for energies at low temperatures in both thermodynamic and SPA approaches, when the LN corrections are included. At the same time accounting for only the Fock terms in the SPA in this special case shifts down the SPA energy at zero temperature only from -10 to -10.5 MeV and this shift (by -0.5 MeV) is independent of the temperature. Therefore such simple accounting of the Fock terms in the SPA leads to an incorrect result even in this simple model case. The exchange terms must be included *together* with the LN corrections within the SPA, because they correspond to effects of the same order. The LN corrections practically cancel this incorrect Fock shift in the energy at high temperatures.

The average energy $\mathcal{E}$ (27) is not accurate at finite temperature (Fig. 3). Therefore by comparing the results obtained in the SPA and those from the primitive application of the Landau theory of phase transition to finite nuclear systems [Eqs. (25) and (27)], we have demonstrated that the thermal average in hot finite nuclear systems *cannot* be understood as the average (25) or (27) even in this simple degenerate model. In this sense the *modified* SPA proposed in the present paper is a candidate, which can treat the thermal (quantal and statistical) fluctuations better.

The pairing gap reaches the exact value in the LN method at low temperatures in both thermodynamic and SPA contexts (Fig. 4). At high temperatures only SPA pairing gaps (with or without LN corrections taken into account) are close to the exact result. It is noteworthy to stress again that the LN method is, nevertheless, only an *approximate* number projection, containing up to the second order in the power-series expansion over the particle number operator $\hat{N}$ [10]. The accuracy of the method is determined, strictly speaking, by the assumption that the saddle point in the projected integration is *sharply pronounced.* In the nuclear superfluid model, this means

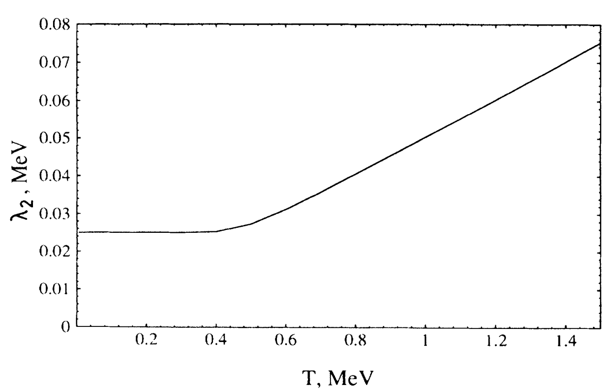

FIG. 2. Coefficient λ_2 as a function of the temperature in the degenerate 2Ω-level model.

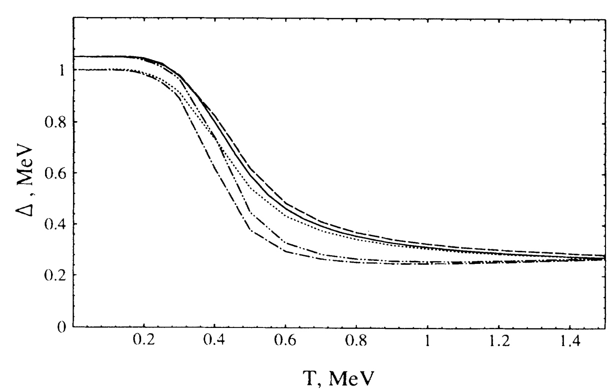

FIG. 4. Pairing gap versus temperature. The various curves correspond to different approximations as in Fig. 1.

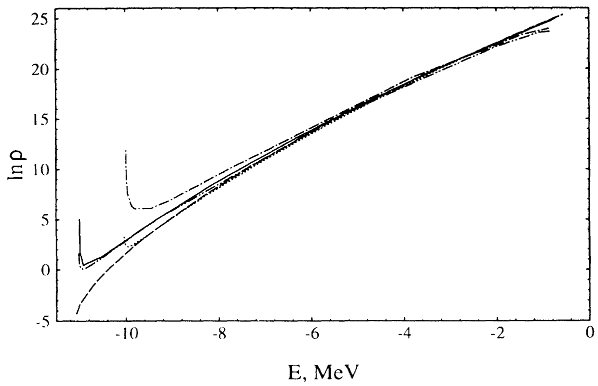

FIG. 5. Logarithm of the level density as a function of the energy in the degenerate 2Ω-level model. See caption to Fig. 1.

that the quantal part $\Delta N^2_{\rm QF}$ (12) of particle number fluctuations is assumed to be large compared with unity [25]. In the temperature region near the critical point T_c and higher, this condition is rather poor (see Refs. [6,7]) and an expansion with higher orders of $\hat{N}$ would be required.

The logarithm of the energy level density calculated within the SPA framework is quite accurate at high temperatures (Fig. 5), while at low temperatures the traditional thermodynamic average including the LN corrections gives a better result (double-dot–dashed curve). However, the LN method in the SPA context allows one to extend the calculations of the level density to lower energies. Some small-amplitude quantal fluctuations could also be, in principle, included as RPA-like contributions to improve the low-energy tail of the level density as has been done in Ref. [14]. To our best knowledge there is no direct connection between the RPA scheme and the LN method. However a study of this kind would be highly desirable.

V. CONCLUSIONS

We have proposed an approach, which encompasses both particle number fluctuations and the statistical fluctuations of the mean field in the context of the SPA. We see the merit of the present approximation in its *simplicity*, which nevertheless allows one to make an *accurate* approximate number projection at finite temperature and at the same time includes automatically the contribution of the exchange terms into the standard SPA. The calculations in the schematic degenerate 2Ω-level model, performed in the present paper, confirms the correctness of our belief. They also show that while at small temperatures the traditional thermodynamic average with the particle number fluctuations taken into account can give good results; it is not more valid at high temperatures, where the SPA is exact.

Because the LN corrections are incorporated in the grand potential, the present approach avoids an integration over the gauge angle as in the exact number projection [8,24]. Therefore it should not be more numerically difficult for calculations in realistic schemes as compared to the standard SPA. The calculations along this way with a Hamiltonian, determined by a realistic single-particle spectrum plus pairing and quadrupole interactions would be highly desirable in comparison with the finite-temperature number projection, proposed in our previous work [24].

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[1] L.G. Moretto, Phys. Lett. **40B**, 1 (1972).
[2] A.L. Goodman, Phys. Rev. C **29**, 1887 (1984).
[3] J.L. Egido, P. Ring, S. Iwasaki, and H.J. Mang, Phys. Lett. **154B**, 1 (1985).
[4] J.L. Egido, Phys. Rev. Lett. **61**, 767 (1988).
[5] R. Rossignoli and A. Plastino, Phys. Rev. C **37**, 314 (1988).
[6] N. Dinh Dang and N. Zuy Thang, J. Phys. G **14**, 1471 (1988).
[7] N. Dinh Dang, Z. Phys. A **335**, 253 (1990).
[8] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
[9] L.D. Landau and E.M. Lifshitz, *Statistical Physics,* Volume 5 of *Course of Theoretical Physics* (Pergamon, 1980).
[10] H.J. Lipkin, Ann. Phys. **31**, 528 (1960); Y. Nogami, Phys. Rev. **134**, B313 (1964); Y. Nogami and I.J. Zucker, Nucl. Phys. **60**, 203 (1964).
[11] P. Arve, G. Bertsch, B. Lauritzen, and G. Puddu, Ann. Phys. (New York) **183**, 309 (1988).
[12] B. Lauritzen, P. Arve, and G. Bertsch, Phys. Rev. Lett. **61**, 2835 (1988).
[13] B. Lauritzen and G. Bertsch, Phys. Rev. C **39**, 2412 (1989).
[14] G. Puddu, P.F. Bortignon, and R.A. Broglia, Phys. Rev. C **42**, 1830 (1991).
[15] B. Lauritzen and J.W. Negele, Phys. Rev. C **44**, 729 (1991).
[16] Y. Alhassid and J. Zingman, Phys. Rev. C **30**, 684 (1984).
[17] A.K. Kerman, S. Levit, and T. Troudet, Ann. Phys. (N.Y.) **148**, 436 (1983); S. Levit and Y. Alhassid, Nucl. Phys. **A413**, 439 (1984).
[18] A.V. Ignatyuk, Yad. Fiz. **21**, 20 (1975) [Sov. J. Nucl. Phys. **21**, 10 (1975)].
[19] D.J. Thouless, *The Quantum Mechanics of Many-Body Systems* (Academic, New York, 1961).
[20] J. Hubbard, Phys. Rev. Lett. **3**, 77 (1959); R.L. Stratonovich, Dokl. Akad. Nauk. SSSR **115**, 1097 (1957) [Sov. Phys. Dokl. **2**, 416 (1958)].
[21] L.G. Moretto, Nucl. Phys. **A182**, 641 (1972).
[22] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. 1.
[23] R.E. Peierls, *Quantum Theory of Solids* (Clarendon, Oxford, 1974).
[24] R. Rossignoli, P. Ring, and N. Dinh Dang, Phys. Lett. B (to be published).
[25] I.N. Mikhailov, Zh. Eksp. Teor. Fiz. **45**, 1102 (1963) [Sov. Phys. JETP **18**, 761 (1964)].