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Particle-hole state densities with good isospin

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Earlier results for two-component particle-hole state densities with good isospin have been rederived, removing a number of earlier approximations. The new results are shown to be consistent with the corresponding isospin-mixed state densities and are more suitable for light nuclei. Approximate results appropriate for reaction calculations are presented.

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I. INTRODUCTION

In the study of preequilibrium nuclear reactions in the continuum, the extent to which isospin is a conserved quantum number is still an open question. Most reaction calculations have been performed assuming (at least implicitly) that isospin is fully mixed, but a few preliminary studies [1,2] indicate that there may be a significant amount of isospin conservation, at least in the early stages of nuclear equilibration. Evidence from compound nucleus reactions (Ref. [3] and references therein) supports this possibility.

More recently, Ryckbosch *et al.* [4] have studied this problem more directly by following the mixing of $T=T_z+1$ strength into the more numerous $T=T_z$ states during the preequilibrum phase of a reaction. Using the unified exciton model and fairly rudimentary state densities, they found that for the two systems considered, most preequilibrium particle emission should occur prior to isospin mixing. Thus the evidence is mounting that isospin can be a significant quantum number in preequilibrium reactions.

In order to include isospin in preequilibrium reaction calculations, it is necessary to have particle-hole state densities for specific isospin values. These are typically evaluated assuming that there is a one-to-one correspondence between states of the same isospin, T, in a set of isobaric nuclei. Letting $\omega(Y,E,T,T_z)$ denote the density of states specified by E, T, T_z, and Y (all other relevant quantum numbers), the one-to-one correspondence implies

$$\omega(Y,E,T,T)=\omega(Y,E+E(T,T-1),T,T-1)$$
$$=\omega(Y,E+E(T,T-2),T,T-2)=\cdots \; . \tag{1}$$

(The symbols used here are defined in Table I.)

An earlier paper [5] showed that Eq. (1), while frequently useful, is not fulfilled for two-component particle-hole state densities where $Y=p_\pi,h_\pi,p_\nu,h_\nu$. It fails for these states because the isospin-flip transitions connecting different members of an isospin multiplet can convert a neutron degree of freedom into a proton degree of freedom or vice versa. Thus the "quantum numbers" Y are not preserved in these transitions, while Eq. (1) inherently assumes that they are. The validity of classifying states according to p_π,h_π,p_ν,h_ν in an isospin-dependent formalism is discussed in the next section.

While not exact, Eq. (1) is often a good approximation [5]. As a result, it was used to derive a simple formula [5] for the isospin-dependent state density in terms of the more usual isospin-mixed state densities:

$$\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)=\omega(p_\pi,h_\pi,p_\nu,h_\nu,E-E(T,T_z),T)$$
$$-\left[\frac{p_\pi+h_\nu}{2T+p_\pi+h_\nu}+\frac{2T}{2T+p_\pi+h_\nu}\,\frac{p_\pi h_\nu(n-1)(n-2)}{g_\pi g_\nu[E-E(T+1,T_z)-A(p_\pi,h_\pi,p_\nu,h_\nu)]^2}\right]$$
$$\times\,\omega(p_\pi,h_\pi,p_\nu,h_\nu,E-E(T+1,T_z),T+1)\; . \tag{2}$$

[The factors $(n-1)(n-2)$ were inadvertently omitted in the publication of that work but included in the calculations.]

Since the T-mixed state densities on the right-hand side of Eq. (2) do not explicitly depend on T_z, this label may be dropped. The common formula for them when the single particle states are taken to be equally spaced in energy is [6]

$$\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)$$
$$=\frac{(g_\pi)^{n_\pi}(g_\nu)^{n_\nu}[E-A(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!}\; . \tag{3}$$

While Eq. (2) should be adequate for some preequilibri-

TABLE I. Identification of symbols.

Symbol(s)	Identification
E	excitation energy
T	total isospin quantum number
T_z	z component of the isospin $=(N-Z)/2$
$E(T,T_z)$	excitation energy of the lowest isospin T state in a nucleus with T_z
$p_\pi,h_\pi,n_\pi=p_\pi+h_\pi$	numbers of proton particle and hole degrees of freedom and their sum
$p_\nu,h_\nu,n_\nu=p_\nu+h_\nu$	same for neutron degrees of freedom
$n=n_0+n_\nu$	total number of excitons
g_π,g_ν	proton and neutron single particle state densities in the equispacing model
$A(p_\pi,h_\pi,p_\nu,h_\nu)$	Pauli principle correction function
$\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)$	isospin-dependent state density
$\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z)\equiv$ $\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)$	isospin-mixed state density

um calculations, it has difficulties of ambiguity and inaccuracy and is of uncertain consistency with T-mixed state densities.

A. The ambiguity question

First, the exact form of the factor inside the large parentheses in Eq. (2) depends on how often and at what stage in the derivation various approximations are introduced. For instance, small changes in the derivation could change the factor $2T$ in the numerator into $2T-2$ or make the denominator in the first term $2T+p_\pi+h_\nu+1$. Clearly, these ambiguities are not significant so long as $2T\gg 2$, but this will not always be the case, as discussed below.

B. The accuracy question

The accuracy of Eq. (2) can suffer when the approximations made in its derivation break down. These approximations are the following: (1) The single particle state densities g_π and g_ν are assumed to be approximately equal. (2) All of the Pauli correction functions connected with the derivation of a single T-conserved state density are taken to be approximately equal. Thus $A(p_\pi,h_\pi,p_\nu,h_\nu)$ is used to replace quantities like $A(p_\pi-1,h_\pi,p_\nu+1,h_\nu)$, $A(p_\pi,h_\pi+1,p_\nu,h_\nu-1)$, and $A(p_\pi-1,h_\pi,p_\nu,h_\nu-1)$. (3) The quantity $2T+p_\pi+h_\nu$ is taken to be much larger than unity. (4) The quantity $2T+p_\pi+h_\nu$ is also much larger than the typical numbers of passive particles and holes. (Passive particles and holes are those that occupy fixed locations and are not degrees of freedom. They are discussed in Refs. [7] and [8].)

Relative to approximation 1, the quantities g_π and g_ν are typically taken to be proportional to the Z and N of the nucleus, respectively, and differences between them were shown [5] not to be of practical significance to the calculated state densities.

The validity of approximation 2 depends largely on the size of the Pauli correction functions relative to the excitation energy from which they are subtracted. For the simple, few-exciton states important in preequilibrium reaction calculations, the A's are generally small relative to the excitation energy except near threshold for a given term.

The third and fourth approximations both essentially require that $2T+p_\pi+h_\nu\gg 1$. In preequilibrium reaction calculations, the most important states are those with small (or even zero) values of p_π and h_ν, so the requirement becomes that $2T\gg 1$. Since the quantity T tends to be close to $|T_z|=|N-Z|/2$, the approximations fail totally for simple states in nuclei with a small neutron (or proton) excess. It was this observation that led to the present study.

C. The consistency question

Even for those nuclei in which all of the approximations are valid and the ambiguities are unimportant, there is a third concern: Are the results of Eq. (2) consistent with the usual isospin-mixed state densities? Do they lead to the same total number of states? This question was not explicitly addressed in Ref. [5], but comparisons made there suggest that consistency may not exist.

D. Aims of the present work

In order to resolve the above questions of ambiguity, accuracy, and consistency, I have decided to embark on a rederivation of the particle-hole state densities with good isospin quantum number. The rederivation involved (1) removing the earlier approximations, (2) demonstrating consistency with the usual T-mixed state densities, and (3) looking for simpler approximate expressions that are adequate for preequilibrium reaction calculations. The present paper describes this process, and the results it produced.

II. THE NEW DERIVATIONS

The derivation here follows the general outlines of Ref. [5]. First the particle-hole state densities for a given isospin value, T, in isobaric nuclei with different values of T_z are related to one another. Then if the state densities in one of these nuclei can be expressed in terms of T-mixed state densities, the state densities in the other nuclei can as well.

The derivation is done assuming that all of the nuclei involved have an excess of neutrons. The extension to the case of a proton excess is discussed at the end of the paper.

The state densities derived here and in Ref. [5] are designed for use in the two-component exciton model, and their function is to estimate available phase space in the evaluation of particle emission rates and internal transition rates. The exciton model, as well as other simple preequilibrium reaction models, is designed to trace the energy equilibration process in a nuclear reaction by studying the mixing of strength from the simple, few-particle–few-hole doorway states into the more complex configurations characteristic of an equilibrated compound nucleus. It is never assumed that the classes of configurations are classes of eigenstates. Indeed, they cannot be eigenstates, or there would be no residual interactions and no equilibration.

With regard to isospin, it is certainly true that any specific p_π,h_π,p_ν,h_ν configuration will not generally have good T (unless only one value is energetically possible) and conversely a physical eigenstate of good T will have components with various different p_π,h_π,p_ν,h_ν labels. Nevertheless, in the context of statistical reactions in the continuum, and specifically statistical preequilibrium reactions, it is useful to know the effective number of states (or cumulative strength) with given T that looks like p_π,h_π,p_ν,h_ν. This is what is represented by the state densities $\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)$.

A. Types of isospin flips and their consequences

The states in an isospin multiplet are related to each other by isospin-flip interactions. Describing the types and relative importance of the interactions encompasses much of the physics involved in the derivations.

Reference [5] identified the three different types of isospin flips pictured in Fig. 1. In each case, a neutron in the nucleus with the larger neutron excess is converted into a proton in the corresponding single particle state to form a state in the isobaric nucleus. While all of these isospin flips leave the quantum numbers p and h unchanged, the three types lead to different combinations of p_π,h_π,p_ν,h_ν.

It should be noted that the three analog configurations in Fig. 1 (and others like them) are degenerate in energy and should mix with one another. This is a consequence of the assumed charge independence of the nuclear force which makes an isospin formalism reasonable. Given this mixing, it may be asked whether it makes sense to label or classify states by the quantities p_π,h_π,p_ν,h_ν. The answer depends on the purpose of the labeling. If the purpose is to describe eigenstates of the system, then the answer is probably "no." On the other hand, if the purpose is to identify phase space or reaction strength accessible in a given experimental situation, the proposed scheme should be quite useful.

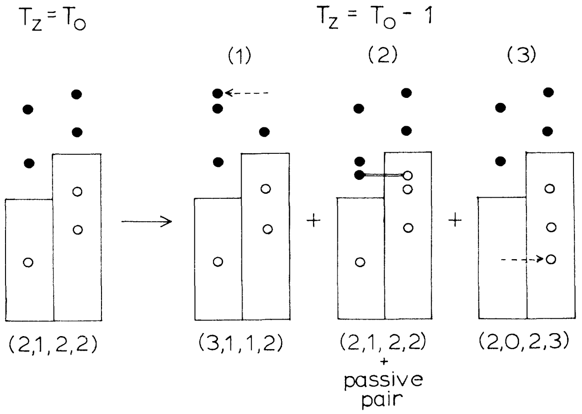

FIG. 1. Schematic representation of the components in the $T_z = T_0 - 1$ isobaric analog of a $(p_\pi,h_\pi,p_\nu,h_\nu)=(2,1,2,2)$, $T=T_0$ configuration in a $T_z=T_0$ nucleus. The blocks indicate single particle states occupied in the ground state of this nucleus, with energy increasing in the vertical direction. The excited particles are denoted as solid circles above the blocks and the excited holes as open circles inside the blocks. (Taken from Ref. [5].)

Thus the mere fact of mixing between the analog configurations shown in Fig. 1 does not automatically render the evaluation of state densities using p_π,h_π,p_ν,h_ν meaningless for reaction calculations (with or without the specification of the isospin quantum numbers) any more than any other configuration mixing. The main question is whether the mixing is adequately accounted for in the calculations. In the two-component exciton model $\Delta n = 0$ mixing (of which this is an example) is generally considered along with the $\Delta n = +2$ transitions which are assumed to dominate early in the equilibrium. The adequacy of this treatment is outside the scope of this paper. In addition, it is important in developing T-dependent state densities to be careful that all states are accounted for and none are double counted.

B. Isospin-flip weighting factors

Referring back to Fig. 1, the simplest weighting factors for the three types of isospin flips given in terms of the quantum numbers in the nucleus with the larger neutron excess would seem to be p_ν, $2T_z$, and h_π, respectively, but these estimates need refinement.

First, the relative weights for all members of an isospin multiplet are most naturally evaluated in the analog configuration where T is the ground state isospin. The weights in the T,T configuration represent "reservoirs" of candidates for the three types of isospin flips. In the configuration with T,T_z, the weights are reduced by one unit for each isospin flip of that type already used in getting to the T_z analog component being considered. Thus the weights in a configuration specified by T,T_z will depend on how the $T-T_z$ isospin constraints are distributed among the three types of isospin flips. Second, the weights need to be corrected for Pauli blocking.

Beginning with the type-2 isospin flips, we note that the base number of candidates in the T,T analog of the T,T_z configuration is $2T$. This estimate is modified for the presence of passive particles and holes by replacing T with an effective value T_e. In reaction calculations, the passives are derived from the target configuration so that

$$T_e = T - T_z + T_{z,\mathrm{targ}} \; . \tag{4}$$

(Previously [5], the effect of passives was put in explicitly, but was later largely removed as approximations were made.)

The number $2T_e$ is reduced by blocking from proton particle or neutron hole degrees of freedom that occupy single particle states in the neutron excess region [i.e., that have excitation energies between zero and $2T_e/g_a$, where $g_a = (g_\pi + g_\nu)/2$]. The fraction of the excitons in this excitation range should be similar to the result for T-mixed state densities at the effective energy $E - E(T,T_z)$. The result is

$$f_2(n,E,T,T_z) = 1 - \left[\frac{E - E(T,T_z) - A(p_\pi,h_\pi,p_\nu,h_\nu) - 2T_e/g_a}{E - E(T,T_z) - A(p_\pi,h_\pi,p_\nu,h_\nu)} \right]^{n-1} \tag{5}$$

if the numerator of the fraction is greater than zero. Otherwise, f_2 is unity. Here E is assumed to have been measured in the analog nucleus specified by T,T_z so that $E - E(T,T_z)$ is the corresponding excitation energy in the T,T analog configuration. Thus values of f_2 are (and indeed must be) independent of the isobaric nucleus in which E and T_z are evaluated.

This blocking fraction is weighted by the number of proton particles and neutron holes *in the T,T configuration* and a correction for double blocking is applied to give a reservoir of candidates for type-2 isospin flips that is essentially

$$B_2(n,p_\pi,h_\nu,E,T,T_z) = 2T_e - (p_\pi + h_\nu) f_2(n,E,T,T_z) + \frac{p_\pi h_\nu}{2T_e} [f_2(n,E,T,T_z)]^2 \; . \tag{6}$$

Moving on to consider type-1 isospin flips, the base number will be the number of neutron particle degrees of freedom in the T,T analog configuration. These will be blocked by the Pauli exclusion principle any time the T,T analog has a proton particle and a neutron particle in corresponding single particle states. A similar consideration pertains to proton and neutron hole states for type-3 isospin flips.

The fraction of the type-1 candidates that is blocked is found by noting that the blocked states effectively have one less proton particle degree of freedom than the others since once the neutron particle state is chosen, the proton particle state is specified. Because of the similarity of type-1 and type-3 isospin flips, it is useful to define the quantity

$$f_{13}(n,E,T,T_z) = \frac{n-1}{2g_a} \frac{[E - E(T,T_z) - A(p_\pi,h_\pi,p_\nu,h_\nu) - 2T_e/g_a]^{n-2}}{[E - E(T,T_z) - A(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}} \; . \tag{7}$$

In terms of f_{13} and the numbers of excitons *in the T,T configuration,* the number of candidates for type-1 and type-3 isospin flips are

$$B_1(n,p_\pi,p_\nu,E,T,T_z) = p_\nu[1 - p_\pi f_{13}(n,E,T,T_z)] \; , \tag{8a}$$

$$B_3(n,h_\nu,h_\pi,E,T,T_z) = h_\pi[1 - h_\nu f_{13}(n,E,T,T_z)] \; . \tag{8b}$$

In each T,T_z isobar component configuration, each of the three weights will be reduced by the number of isospin flips of that type which were used in converting the T,T configuration into that T,T_z isobar component. The numbers used are denoted by i, j, and k for types 1, 2, and 3, respectively, where $T - T_z = i + j + k$.

To express the three weights in terms of the exciton quantum numbers in the T,T_z configuration being considered, we note that the particle-hole numbers in the two configurations are related to one another as

T,T_z	T,T
p_π	$p_\pi - i$
h_π	$h_\pi + k$
p_ν	$p_\nu + i$
h_ν	$h_\nu - k$

Thus the total number of isospin flips still possible in the T,T_z analog component is

$$\begin{aligned} C(p_\pi - i, h_\pi + k, p_\nu + i, h_\nu - k, E, T, T_z) - i - j - k \\ = [B_1(n,p_\pi - i, p_\nu + i, E, T, T_z) - i] + [B_2(n,p_\pi - i, h_\nu - k, E, T, T_z) - j] + [B_3(n, h_\nu - k, h_\pi + k, E, T, T_z) - k] \; , \end{aligned} \tag{9}$$

where each pair of brackets gives the weight for one type of isospin flip. (In the earlier study [5], the derivations were performed without any consideration of the blocking of type-1 and type-3 flips, and type-2 isospin flips were considered only in the $f_2 = 0$ and $f_2 = 1$ limits.)

C. Relation between state densities in an isobaric multiplet

Beginning with the above weighting factors, we now relate the isospin-conserved state densities from different isobaric nuclei. For light particle reactions, states with isospins up to $T=|T_z|+2$ are needed.

Recalling that the weights are given in terms of the quantum numbers of the nucleus with the greater neutron excess (or T_z value), the result for neighboring isobars is

$$\begin{aligned}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)= & \frac{B_1(p_\pi-1-i,p_\nu+1+i,E,T,T_z)-i}{C(p_\pi-1-i,h_\pi+k,p_\nu+1+i,h_\nu-k,E,T,T_z)-i-j-k}\\ & \times\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_e(T,T_z),T,T_z+1)\\ & +\frac{B_2(p_\pi-i',h_\nu-k',E,T,T_z)-j'}{C(p_\pi-i',h_\pi+k',p_\nu+i',h_\nu-k',E,T,T_z)-i'-j'-k'}\\ & \times\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_e(T,T_z),T,T_z+1)\\ & +\frac{B_3(h_\nu-1-k'',h_\pi+1+k'',E,T,T_z)-k''}{C(p_\pi-i'',h_\pi+1+k'',p_\nu+i'',h_\nu-1-k'',E,T,T_z)-i''-j''-k''}\\ & \times\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_e(T,T_z),T,T_z+1)\end{aligned} \tag{10}$$

where the i's, j's, and k's refer to the isospin flips used in going from the T,T to the T,T_z+1 isobar, and $E_e(T,T_z)$ is

$$E_e(T,T_z)=E-E(T,T_z)+E(T,T_z+1)\ . \tag{11}$$

In the B's and C's, the argument set $E_e(T,T_z),T,T_z+1$ is equivalent to E,T,T_z, and the argument n has been dropped from the notation. Note that for each term in the right-hand side of Eq. (10) different states will have different combinations of the i's, j's, and k's, and a weighted sum over these combinations is really needed. The result is only explicitly defined for $T=T_z+1$, the case with no constraints on the right-hand side. Thus we begin with this case and work up to higher $T-T_z$, each time expressing the $\omega(Y,E,T,T_z)$ in terms of the various $\omega(Y',E-E(T,T_z),T,T)$.

Many of the effective excitation energies occurring in the T-mixed state densities are simplified by evaluating the symmetry energies from the corresponding terms in the semiempirical mass equation [9]. This gives symmetry energies that are proportional to $T^2-(T_z)^2$.

For the explicit case, the effective excitation energy becomes $E_e(T_z+1,T_z)=E-E(T_z+1,T_z)$, and it is convenient to define

$$E_m=E-E(T_z+m,T_z)\ . \tag{12}$$

The desired state density relationship is thus

$$\begin{aligned}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z+1,T_z)= & X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1,T_z+1,T_z+1)\\ & +X_2(p_\pi,h_\pi,p_\nu,h_\nu,E_1)\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_1,T_z+1,T_z+1)\\ & +X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1,T_z+1,T_z+1)\ ,\end{aligned} \tag{13}$$

with the weighting factors

$$X_1(p_\pi,h_\pi,p_\nu,h_\nu,E_1)=\frac{B_1(p_\pi,p_\nu,E_1,T_z+1,T_z+1)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_1,T_z+1,T_z+1)}\ , \tag{14a}$$

$$X_2(p_\pi,h_\pi,p_\nu,h_\nu,E_1)=\frac{B_2(p_\pi,h_\nu,E_1,T_z+1,T_z+1)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_1,T_z+1,T_z+1)}\ , \tag{14b}$$

$$X_3(p_\pi,h_\pi,p_\nu,h_\nu,E_1)=\frac{B_3(h_\nu,h_\pi,E_1,T_z+1,T_z+1)}{C(p_\pi,h_\pi,p_\nu,h_\pi,E_1,T_z+1,T_z+1)}\ . \tag{14c}$$

Here $2T_e$ in the B's becomes $2(T_{ze}+1)$ and T_{ze} is typically the T_z of the target. In the X's, the isospin labels are omitted since E_i is always associated with T_z+i,T_z+i. [In the earlier work, approximations were made at this stage in order to recover the results of Eq. (1).]

The next step is to express the $\omega(Y,E,T_z+2,T_z)$ in terms of the $\omega(Y',E_2,T_z+2,T_z+2)$. Applying Eq. (10) twice and combining identical terms yields

$$\begin{aligned}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z+2,T_z) = {} & X_{11}(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)\omega(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2,T_z+2,T_z+2) \\ & +2X_{12}(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2,T_z+2,T_z+2) \\ & +2X_{13}(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\omega(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2,T_z+2,T_z+2) \\ & +X_{22}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_2,T_z+2,T_z+2) \\ & +2X_{23}(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2,T_z+2,T_z+2) \\ & +X_{33}(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)\omega(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2,T_z+2,T_z+2) \ . \end{aligned} \tag{15}$$

Here the X factors are given by

$$X_{11}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)=\frac{B_1(p_\pi,p_\nu,E_2)-1}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)-1}\,\frac{B_1(p_\pi,p_\nu,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)}\ , \tag{16a}$$

$$X_{12}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)=\frac{B_1(p_\pi,p_\nu,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)-1}\,\frac{B_2(p_\pi,h_\nu,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)}\ , \tag{16b}$$

$$X_{13}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)=\frac{B_1(p_\pi,p_\nu,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)-1}\,\frac{B_3(h_\nu,h_\pi,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)}\ , \tag{16c}$$

$$X_{22}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)=\frac{B_2(p_\pi,h_\nu,E_2)-1}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)-1}\,\frac{B_2(p_\pi,h_\nu,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)}\ , \tag{16d}$$

$$X_{23}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)=\frac{B_2(p_\pi,h_\nu,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)-1}\,\frac{B_3(h_\nu,h_\pi,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)}\ , \tag{16e}$$

$$X_{33}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)=\frac{B_3(h_\nu,h_\pi,E_2)-1}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)-1}\,\frac{B_3(h_\nu,h_\pi,E_2)}{C(p_\pi,h_\pi,p_\nu,h_\nu,E_2)}\ , \tag{16f}$$

where, for simplicity, the isospin designations T_z+2,T_z+2 have been dropped from the B's and C's as they were from the X's.

The result for $\omega(Y,E,T_z+3,T_z)$ in terms of the $\omega(Y',E_3,T_z+3,T_z+3)$ takes three applications of Eq. (10) and is analogous to Eq. (15) but with many more terms. The X factors are analogous to Eq. (16) but with three subscripts and three factors each. (The X's are all independent of the order of the subscripts.)

D. Evaluation of isospin-conserved state densities

To express the isospin-conserved state densities in terms of the more common isospin-mixed state densities, we start, as in Ref. [5], at low excitation energies where only states with the ground state isospin $T=T_z$ are possible and work up to successively higher energy domains where additional isospin values become allowed. The results from lower domains are used to evaluate the analog state densities in each new domain. Then the $T=T_z$ state densities are obtained by subtracting the $T>T_z$ contributions from the full T-mixed state density, taking account of the constrained particle-hole pair produced in type-2 isospin flips.

The resulting expressions for the T-conserved state densities in terms of the T-mixed state densities are found to become increasingly more complicated for each energy domain. On the other hand, the second and higher order correction terms show significant cancellation in the weights of individual T-mixed state densities. Thus these higher order terms have been neglected, leading to results applicable in all energy domains. These results are

$$\begin{aligned}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z,T_z) = {} & \omega(p_\pi,h_\pi,p_\nu,h_\nu,E)-X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1) \\ & -X_2(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_1)\omega(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_1) \\ & -X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\ , \end{aligned} \tag{17}$$

$$
\begin{aligned}
&\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z+1,T_z)\\
&\quad=X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)\\
&\qquad+X_2(p_\pi,h_\pi,p_\nu,h_\nu,E_1)\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_1)+X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\\
&\qquad-X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)[X_1(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)\omega(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)\\
&\qquad\qquad+X_2(p_\pi-2,h_\pi,p_\nu+1,h_\nu-1,E_2)\omega(p_\pi-2,h_\pi,p_\nu+1,h_\nu-1,E_2)\\
&\qquad\qquad+X_3(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\omega(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)]\\
&\qquad-X_2(p_\pi,h_\pi,p_\nu,h_\nu,E_1)[X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\\
&\qquad\qquad+X_2(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_2)\omega(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_2)\\
&\qquad\qquad+X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)]\\
&\qquad-X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)[X_1(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\omega(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\\
&\qquad\qquad+X_2(p_\pi-1,h_\pi+1,p_\nu,h_\nu-2,E_2)\omega(p_\pi-1,h_\pi+1,p_\nu,h_\nu-2,E_2)\\
&\qquad\qquad+X_3(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)\omega(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)]
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
&\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z+2,T_z)\\
&\quad=X_{11}(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)\omega(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)+2X_{12}(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\\
&\qquad+2X_{13}(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\omega(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\\
&\qquad+X_{22}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_2)\\
&\qquad+2X_{23}(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)\\
&\qquad+X_{33}(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)\omega(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)\\
&\qquad-X_{11}(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)[X_1(p_\pi-3,h_\pi,p_\nu+3,h_\nu,E_3)\omega(p_\pi-3,h_\pi,p_\nu+3,h_\nu,E_3)\\
&\qquad\qquad+X_2(p_\pi-3,h_\pi,p_\nu+2,h_\nu-1,E_3)\omega(p_\pi-3,h_\pi,p_\nu+2,h_\nu-1,E_3)\\
&\qquad\qquad+X_3(p_\pi-2,h_\pi+1,p_\nu+2,h_\nu-1)\omega(p_\pi-2,h_\pi+1,p_\nu+2,h_\nu-1,E_3)]\\
&\qquad-2X_{12}(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)[X_1(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_3)\omega(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_3)\\
&\qquad\qquad+X_2(p_\pi-2,h_\pi,p_\nu+1,h_\nu-1,E_3)\omega(p_\pi-2,h_\pi,p_\nu+1,h_\nu-1,E_3)\\
&\qquad\qquad+X_3(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_3)\omega(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_3)]\\
&\qquad-2X_{13}(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_3)\\
&\qquad\quad\times[X_1(p_\pi-2,h_\pi+1,p_\nu+2,h_\nu-1,E_3)\omega(p_\pi-2,h_\pi+1,p_\nu+2,h_\nu-1,E_3)\\
&\qquad\qquad+X_2(p_\pi-2,h_\pi+1,p_\nu+1,h_\nu-2,E_3)\omega(p_\pi-2,h_\pi+1,p_\nu+1,h_\nu-2,E_3)\\
&\qquad\qquad+X_3(p_\pi-1,h_\pi+2,p_\nu+1,h_\nu-2,E_3)\omega(p_\pi-1,h_\pi+2,p_\nu+1,h_\nu-2,E_3)]\\
&\qquad-X_{22}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)[X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_3)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_3)\\
&\qquad\qquad+X_2(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_3)\omega(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_3)\\
&\qquad\qquad+X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_3)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_3)]\\
&\qquad-2X_{23}(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)[X_1(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_3)\omega(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_3)\\
&\qquad\qquad+X_2(p_\pi-1,h_\pi+1,p_\nu,h_\nu-2,E_3)\omega(p_\pi-1,h_\pi+1,p_\nu,h_\nu-2,E_3)\\
&\qquad\qquad+X_3(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_3)\omega(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_3)]\\
&\qquad-X_{33}(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)[X_1(p_\pi-1,h_\pi+2,p_\nu+1,h_\nu-2,E_3)\omega(p_\pi-1,h_\pi+2,p_\nu+1,h_\nu-2,E_3)\\
&\qquad\qquad+X_2(p_\pi-1,h_\pi+2,p_\nu,h_\nu-3,E_3)\omega(p_\pi-1,h_\pi+2,p_\nu,h_\nu-3,E_3)\\
&\qquad\qquad+X_3(p_\pi,h_\pi+3,p_\nu,h_\nu-3,E_3)\omega(p_\pi,h_\pi+3,p_\nu h_\nu-3,E_3)]\ .
\end{aligned}
\tag{19}
$$

Equations (17)–(19) are much more complicated than Eq. (2), but contain almost none of its approximations and none of its ambiguities.

E. Evaluating the state densities

In order to use the above results, two quantities must be defined: the T-mixed state densities and the symmetry energies, $E(T,T_z)$. While there is no required form for the isospin-mixed state densities, the most common choice, and the one that has been used here, is the traditional equispacing model formula given by Eq. (3) with $g_\pi = Z/(13\text{ MeV})$, $g_\nu = N/(13\text{ MeV})$, and the Pauli correction function

$$A(p_\pi,h_\pi,p_\nu,h_\nu) = \frac{(q_\pi)^2}{g_\pi} + \frac{(q_\nu)^2}{g_\nu} - \frac{p_\pi(p_\pi+1)+h_\pi(h_\pi+1)}{4g_\pi} - \frac{p_\nu(p_\nu+1)+h_\nu(h_\nu+1)}{4g_\nu} \times \frac{(p_\pi-1)^2+(h_\pi-1)^2}{g_\pi G_\pi(p_\pi,h_\pi,p_\nu,h_\nu,E)} + \frac{(p_\nu-1)^2+(h_\nu-1)^2}{g_\nu G_\nu(p_\pi,h_\pi,p_\nu,h_\nu,E)} . \tag{20}$$

Here, the quantity q_π is the *total* number of proton particle-hole pairs, typically given by

$$q_\pi = \max(p_\pi,h_\pi) \tag{21}$$

with a similar relation for q_ν. The first two terms in A represent the threshold energy for the configuration,

$$E_{\text{th}}(p_\pi,h_\pi,p_\nu,h_\nu) = \frac{(q_\pi)^2}{g_\pi} + \frac{(q_\nu)^2}{g_\nu} . \tag{22}$$

The first four terms in A are all that is normally used. They are energy independent and yield accurate state densities except near the threshold energy. Improved accuracy near threshold is accomplished [8] by including the last two, energy-dependent terms. Here G_π is

$$G_\pi(p_\pi,h_\pi,p_\nu,h_\nu,E) = \frac{12q_\pi}{q_\pi+q_\nu} + \frac{4g_\pi[E - E_{\text{th}}(p_\pi,h_\pi,p_\nu,h_\nu)]}{q_\pi} \tag{23}$$

and G_ν has a similar form. (The G's are slightly different from those given in Ref. [8].)

In this work, the symmetry energies have been taken from the volume symmetry term of the Myers and Swiatecki semiempirical mass equation [9]. Thus we have

$$E(T,T_z) = \frac{112\text{ MeV}}{A}[T^2-(T_z)^2] . \tag{24}$$

Obviously, the surface symmetry term could also be included, or the symmetry energies for $T = T_z + 1$ could be evaluated from (p,n) reaction Q values as proposed by Anderson *et al.* [10].

F. Approximations in the present results

While the present results avoid the major approximations of the earlier study, they are still not "exact." It has already been stated that they contain only first order correction terms. In addition, several approximations have been made in implementing the blocking factors f_2 and f_{13} given by Eqs. (5) and (7). They are the following: (1) The blocking fractions are evaluated using the energy-independent Pauli corrections even when the full energy-dependent Eq. (20) is used in the calculations of the T-mixed state densities. (2) The blocking fractions use the Pauli correction from the main configuration even for the analog states with different configurations. None of these approximations is expected to have a significant impact on the results.

III. CONSISTENCY CONDITIONS

Given the present expressions, we now consider their consistency with T-mixed particle-hole state densities, total Fermi-gas state densities, and Fermi-gas state densities with good isospin. In a real sense, consistency between the present results and the isospin-mixed state densities was built into the derivations because the T_z,T_z state density for a given class of states was found by subtracting the $T > T_z$ contributions from the total (isospin-mixed) state density. This intrinsic consistency also extends to the isospin-mixed Fermi-gas state densities which have been shown [6] to be consistent with the summed isospin mixed particle-hole state densities. The main consistency conditions are therefore inherently met.

It is appealing, however, to try to verify this intrinsic consistency through numerical checks such as the one on the total number of states:

$$\sum_p \sum_{p_\pi} \sum_T \omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z) = \sum_p \sum_{p_\pi} \omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z) , \tag{25}$$

where p_π-h_π and p_ν-h_ν are held constant in the sums because particles and holes are created and annihilated in pairs. However, this and similar conditions turn out not to be valid for two principle reasons: (i) the presence in many states with $T > T_z$ of constrained proton particle-neutron hole pairs, as shown in Fig. 1, and (ii) the influence of passive particles and holes on the sums of particle-hole state densities. Thus, we examine the role of passive particles and holes in summed particle-hole state densities and then look at the validity of possible consistency conditions.

A. Passive particle and holes in consistency checks

We begin by reviewing some definitions. A "particle" is a filled single particle state above the Fermi level, and a "hole" is an empty single particle state below the Fermi level. Thus the *total* number of proton particles must equal the *total* number of proton holes, and similarly for neutrons. Most particles and holes are "active." They represent degrees of freedom because they carry excitation energy that can be given up in subsequent interactions. A few may be constrained to specific single particle states adjacent to the Fermi level (or elsewhere if T is conserved). They have no permutable excitation energy and are termed "passive" [7,8]. (For instance, a passive hole is produced when a nucleon is incident on a target and the Fermi level moves up to its new position, leaving an empty single particle state below it.)

We next consider the effect of the passives on sums of isospin-mixed particle-hole state densities. The key point here is that the zero-passive state density $\omega(p_\pi,h_\pi=p_\pi,p_\nu,h_\nu=p_\nu,E)$ counts all possible configurations of the excitons (active particles and holes) which have the specified excitation energy. This state density thus counts, as special cases, the states in which some of the particles and holes occupy states adjacent to the Fermi level. That is, the no-passive state densities *contain* the ones which have passives. From this it should be clear that the total state density of the system is given by

$$\omega(E)=\sum_{p_\pi}\sum_{p_\nu}\omega(p_\pi,p_\pi,p_\nu,p_\nu,E)\ . \tag{26}$$

As a corollary, if state densities containing passives are summed holding p_π-h_π and p_ν-h_ν constant, the result will always be less than $\omega(E)$.

Looked at functionally, the role of the passive particles and holes is to tie up excitation energy, leaving less of it to be permuted among the excitons. This implies fewer permutations and thus a lower state density than for no-passive configurations with the same E and n.

B. Validity of consistency checks

We now consider the consistency condition given in Eq. (25). For this comparison to be valid, the same states must be summed in both cases; i.e., the same number and type of passives should be present in both sums. The number of passives in the T-mixed state densities on the right-hand side is unambiguous and is determined by p_π-h_π and p_ν-h_ν. On the right-hand side, however, the isospin constraints mean that a single isospin-dependent state density has components which are given by (or in a very real sense "look like," "sum like") a variety of T-mixed state densities with different numbers of passive particles and holes. As a result, the condition expressed in Eq. (25) should probably not be rigorously fulfilled even for completely consistent state densities. On the other hand, it is still useful.

First, in the $p_\pi=h_\pi$ and $p_\nu=h_\nu$ limit the right-hand side of Eq. (25) gives the total particle-hole state density of the nucleus. Thus, for any p_π,h_π,p_ν,h_ν, the left-hand side should always be less than or equal to this limit, and a valid condition is

$$\sum_{p}\sum_{p_\pi}\sum_{T}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)\le\sum_{p}\sum_{p_\pi}\omega(p_\pi,p_\pi,p_\nu,p_\nu,E,T_z)\ . \tag{27}$$

In addition, Eq. (25) should be approximately fulfilled. The reason is that when T-conserved state densities for a given configuration are expressed in terms of T-mixed state densities, the main terms from states with isospin $T+1$ will largely cancel the correction terms from states with isospin T. To the extent that this cancellation is complete, all that is left in the sum over T is the main term for $T=T_z$ which is identical to the state density inside the sums on the right side of the equation. Thus, *to the extent that the cancellation is complete,* Eq. (25) will be fulfilled. Additional conditions that might also be approximately fulfilled are

$$\sum_{p_\pi}\sum_{T}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)=\sum_{p_\pi}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z)\ , \tag{28}$$

$$\sum_{T}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)=\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z)\ . \tag{29}$$

Since there will not be an exact cancellation of the other terms, conditions that minimize the size of these terms will lead to better fulfillment of the conditions. Thus both large symmetry energies and high exciton numbers are favorable while high excitation energies make the terms more comparable and fulfillment worse.

C. Comparisons of summed particle-hole state densities

The comparisons suggested by Eqs. (25), (28), and (29) have been made for both ^{58}Ni and ^{208}Pb. Comparisons are limited to excitation energies where T_z+2 is the maximum isospin allowed. Values of 5, 10, 20, and 30 MeV for ^{58}Ni and 20, 40, and 60 MeV for ^{208}Pb have been used. The difference in the energy scales is due to the difference in the symmetry energies. At the lowest energy in each case, only the ground state isospin is allowed so that all of the conditions are exactly fulfilled. The situation at the other energies is discussed below.

It is found that the conditions expressed in Eqs. (28) and (29) are satisfied to very nearly the same extent whenever p and E are the same, so it is simplest to look at the combined results of Eq. (28). These are shown in Fig. 2 as the ratio of the summed T-dependent state density to the summed T-mixed state density. As expected, the ratio is closer to unity at the lower excitation energies for a given nucleus. Similarly, at comparable excitation energies ^{208}Pb shows better fulfillment of the condition than ^{58}Ni. In all cases the ratio starts above one and decreases as the exciton number, n, increases. The ratio generally levels off very close to unity, except for the (p,n) reaction producing ^{58}Ni at 30 MeV. Here the ratio again begins to decrease rapidly. [This unusual behavior is due to a

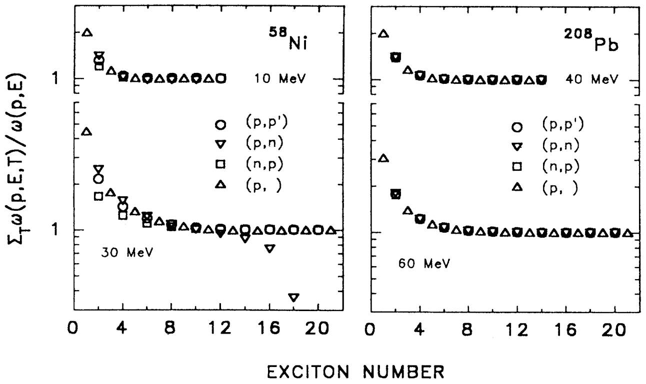

FIG. 2. Ratio of the isospin-dependent state densities summed over T and p_π to the corresponding isospin-mixed state densities summed over p_π. The results are plotted as a function of exciton number for the indicated excitation energies.

peculiarity of the reaction system. Because of the role of the passive particles and holes and because $E(T_z+1,T_z)$ is only 5.7 MeV, the correction terms in the $T=T_z$ state densities at high n actually have *more* permutable excitation energy than the main term.]

With regard to the condition of Eq. (25), the results for ^{58}Ni are shown in Table II, again as the ratio between the T-conserved and T-mixed state densities. In general the T-conserved results are slightly greater than the corresponding T-mixed ones, but the differences are only of the order of 1% or less. For (p,n), the ratio is less than one because of the effect noted above, and the difference is 6% or less. For ^{208}Pb, the full space was not explored, but sums up through $n=24$ indicate a ratio of essentially unity in all cases. Thus, Eq. (25) is quite well fulfilled.

This leaves the limit of Eq. (26) to consider. The reaction yielding $p_\pi=h_\pi$ and $p_\nu=h_\nu$ is inelastic scattering for which Table II shows that the T-conserved state density exceeds the limit but only at the level of about 1%. For the other reactions, the summed T-conserved state densities are all significantly smaller than the full state density.

D. Consistency checks with Fermi-gas state densities

Williams [6] has already demonstrated that the total T-mixed particle-hole state density of Eq. (3) with $p_\pi=h_\pi$ and $p_\nu=h_\nu$ agrees with the total two-component Fermi-gas state density

$$\omega(E)=\frac{1}{12}\left[\frac{\pi^2}{a}\right]E^{-5/3}\exp[2(aE)^{1/2}]\ , \tag{30}$$

particularly at higher excitation energies. Here the level density parameter is $a=\pi^2(g_\pi+g_\nu)/6$. Sample calculations indicate that for ^{58}Ni, the level of agreement is about 15% at 5 MeV and improves to about 4% at 30 MeV.

Thus, since the total T-conserved state densities derived here are consistent with the corresponding T-mixed state densities, they are also consistent with the Fermi-gas results, but their distribution in isospin may be different. Typically, the isospin-conserved Fermi-gas state densities are given by (e.g., Ref. [11])

$$\omega(E,T,T_z)=\omega(E-E(T,T_z))-\omega(E-E(T+1,T_z))\ , \tag{31}$$

although sometimes the correction term is ignored. The results of Eq. (31) are compared with the present results summed over p and p_π in Table III. Also included in this table are the inelastic scattering results from a simple expression suggested by Jensen [11]

$$\begin{aligned}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)&\\ =\omega(p_\pi,h_\pi,p_\nu,h_\nu,E-E(T,T_z))&\\ -\omega(p_\pi,h_\pi,p_\nu,h_\nu,E-E(T+1,T_z))&\ ,\end{aligned} \tag{32}$$

which gives the same isospin distribution as the Fermi-gas formula.

Table III shows that for the present results, the distribution of the states in isospin depends on the passive particles and holes that are present. While inelastic scattering (with no passives in the main $T=T_z$ term) yields $T=T_z$ state densities that are close to the Fermi-gas result, it is the proton incident composite nucleus that best reproduces the T dependence of the Fermi-gas state densities. By contrast, the isospin dependence from Eq. (32) shows little variation with the passives present.

The differences in the isospin distributions in the Fermi-gas and the present particle-hole descriptions do not necessarily mean that either one is incorrect. Much of the difference could be related to the role of passive particles and holes and the fact that p_π-h_π and p_ν-h_ν are held constant in the sums. (As was pointed out [12], they are not always constant in physical reactions, and the effect of passives at the Fermi level should wash out at equilibrium.) This explanation is supported by observing

TABLE II. Ratios of summed ^{58}Ni state densities.

E	$\sum_{p,p_\pi,T}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)\Big/\sum_{p,p_\pi}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z)$			
(MeV)	^{58}Ni(N,N')	^{57}Co$+p$	^{58}Co(p,n)	^{58}Cu(n,p)
5	1.000	1.000	1.000	1.000
10	1.003	1.001	0.980	1.002
20	1.009	1.003	0.957	1.006
30	1.012	1.004	0.943	1.009

TABLE III. Isospin dependence of summed ^{58}Ni state densities at 30 MeV.

	$\sum_{p,p_\pi}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)$				Jensen	Fermi
T	^{58}Ni(N,N')	^{57}Co+p	^{58}Co(p,n)	^{58}Cu(n,p)	(N,N')	gas
1	9.41×10^9	4.18×10^9	1.49×10^9	2.08×10^9	8.90×10^9	9.22×10^9
2	2.34×10^8	2.96×10^8	4.91×10^8	0.33×10^8	6.31×10^8	6.61×10^8
3	0.84×10^6	1.20×10^6	2.15×10^6	0.09×10^6	3.29×10^6	3.55×10^6
Sum	9.64×10^9	4.48×10^9	1.98×10^9	2.12×10^9	9.53×10^9	9.99×10^9

that the summed particle-hole state densities for a given T are never larger than the corresponding Fermi-gas result (except for the 2% effect for $T=T_z$ in inelastic scattering).

IV. CALCULATED STATE DENSITIES

Having demonstrated that the present results are consistent with the usual isospin-mixed equations, we move on to examine their behavior. Again, calculations have been performed for both ^{58}Ni and ^{208}Pb, but with a given configuration being followed as a function of excitation energy.

As in Ref. [5], the new results are compared with the Jensen formula of Eq. (32) and with its leading term. The Jensen result is useful here mainly as a point of comparison. Figures 3 and 4 give the comparisons for the initial configurations which dominate the behavior of preequilibrium reactions. They correspond to Figs. 2 and 3 in Ref. [5] but with the addition of the configurations from (n,p) reactions.

For ^{58}Ni, the $T=T_z=1$ results are qualitatively the same as in Ref. [5]. Below the threshold for the $T=2$ states, all of the results agree, while above the threshold they diverge, with the present results falling between the Jensen state densities and their leading term. For the higher isospins, the present results are shifted at all energies either up or down from this trend, depending on the number and type of passives present. This shift was not seen in the earlier study. In all cases, the energy dependence of the present state densities is closer to that of the leading term in Eq. (32) than to that of the full expression. For ^{208}Pb, the trends are the same but all of the effects are very much smaller.

Figure 5 shows results for eight exciton configurations and corresponds to Fig. 4 in Ref. [5]. Only the ^{58}Ni results are shown because for ^{208}Pb the various curves are indistinguishable. The increased exciton number here means that the corrections terms are less important. On the other hand, the upward or downward shifts of the

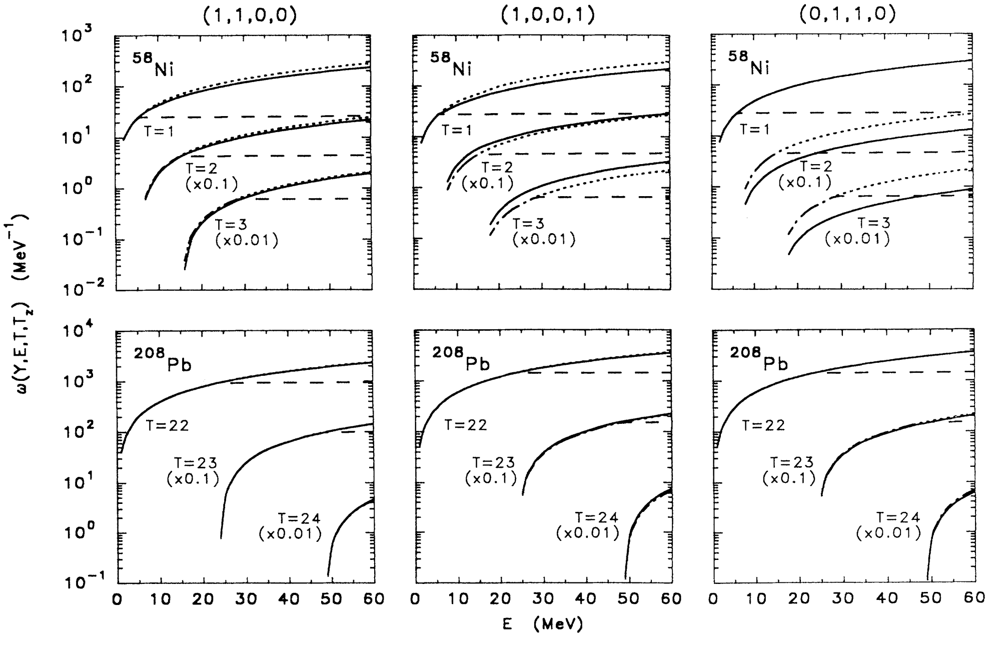

FIG. 3. Isospin-dependent state densities as a function of excitation energy for two exciton states. The solid curves give the present results, the long dashed curves are from the Jensen results [11], and the short dashed curves are from the leading term of the Jensen result (used in Ref. [4]).

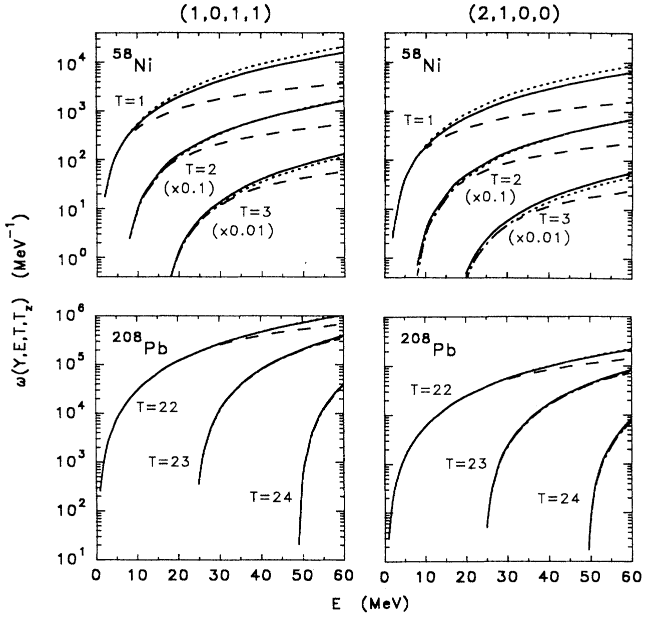

FIG. 4. Isospin-dependent state densities as a function of excitation energy for three exciton states. Curves are as in Fig. 3.

present results for $T > T_z$ relative to the Jensen state densities are still present.

These comparisons suggest that the present state densities represent a real improvement over the results of earlier work [5,11], especially for $T > T_z$. We now search for simpler expressions that will be adequate for routine preequilibrium reaction calculations.

V. APPROXIMATE EXPRESSIONS FOR REACTION CALCULATIONS

The improvements over Ref. [5] are the result of four factors: (1) not neglecting differences in the Pauli correction functions in order to combine the various main terms and/or correction terms in a single state density expression; (2) including realistic blocking fractions for all types of isospin flip; (3) treating passives by using the T_z of the target nucleus in estimating the weights for type-2 isospin flips; (4) not assuming that type-2 isospin flips are predominant. Of these factors, the first two are candidates for possible relaxation. Analogs of Eqs. (17)–(19) have thus been derived in which factors (1), (2), or both (1) and (2) are relaxed.

A. Uniformly applied approximations

Results from the expressions with the simplifications applied to all terms show that for ^{208}Pb, any of these approximations are adequate while for ^{58}Ni, none are. In particular, for ^{58}Ni we note that the upward or downward shifts of the current results for $T > T_z$ relative to the simple or Jensen results are due primarily to three factors: (i) the weighting factors for the leading terms, (ii) the differences in the Pauli correction functions, and (iii) the blocking corrections. Of these, the first is totally dominant for the very simple states in Fig. 3. As the exciton number increases, however, the other two factors become rapidly more important, especially at lower excitation energies. Thus it appears that neither factor (1) nor factor (2) can be totally neglected.

Neglecting only the blocking of type-1 and type-3 iso-

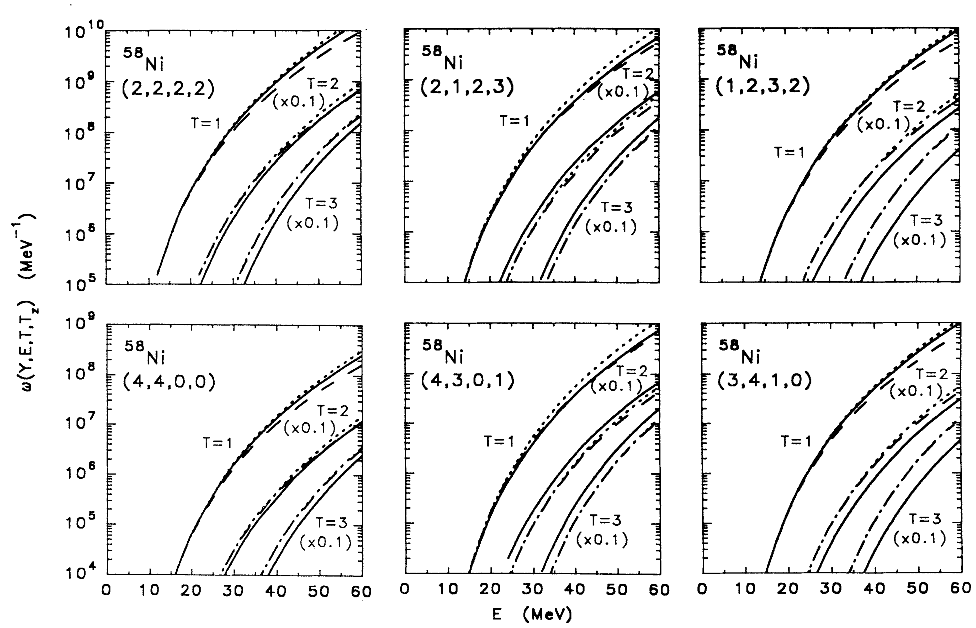

FIG. 5. Isospin-dependent state densities as a function of excitation energy for eight exciton configurations. Curves are as in Fig. 3.

spin flips introduces errors of no more than about 10% into ^{58}Ni state densities with $p \leq 4$ but does not substantially reduce the computational effort required.

B. Additional approximations in the correction terms

Starting with the results with no type-1 and -3 blocking corrections, we next try additional approximations in the corrections terms only.

Relaxing factor (1) by ignoring differences in the Pauli correction functions simplifies calculations because the various correction terms can all be combined and written in terms of $\omega(p_\pi,h_\pi,p_\nu,h_\nu,E-E(T+1,T_z))$. Neglecting blocking of type-2 isospin flips as well was also tried and found to make only a very small difference in the results so these latter expressions were tested on the ^{58}Ni state densities.

For the simple configurations which dominate preequilibrium reactions ($n=2$ in the residual nuclei and $n=3$ in the composite nucleus), the new results agree with the full results to within about 3%. The agreement gets worse as the excition number increases, especially for $T=T_z$ and for the (p,n) configurations. At $n=8$, the errors can be as large as 20%. Still, these results should be adequate for reaction calculations.

C. Approximate expressions adopted

The approximate expressions for the isospin-conserved state densities adopted for reaction calculations thus neglect type-1 and -3 blocking corrections in all terms and neglect blocking of type-2 isospin flips and differences between Pauli corrections in the correction terms.

Neglecting the type-1 and -3 blocking causes the weighting factors for the three types of isospin flips that occur in the X coefficients of Eqs. (14) and (16) to change as follows:

$$B_1(p_\pi,p_\nu,E,T,T_z)\rightarrow p_\nu \ , \tag{33a}$$

$$B_2(p_\pi,h_\nu,E,T,T_z)\rightarrow \text{unchanged, but drop subscript 2} \ , \tag{33b}$$

$$B_3(h_\nu,h_\pi,E,T,T_z)\rightarrow h_\pi \ , \tag{33c}$$

$$C(p_\pi,h_\pi,p_\nu,h_\nu,E,T,T_z)=p_\nu+h_\pi+B(p_\pi,h_\nu,E,T,T_z) \ . \tag{33d}$$

For the correction terms, $B_2\rightarrow 2T_e$, and additional factors are introduced when the correction terms are combined. To accommodate this, new Y coefficients are defined as

$$Y_1(p_\pi,h_\pi,p_\nu,h_\nu,E_i)=\frac{p_\pi+1}{p_\nu+h_\pi+2(T_{ze}+i)}\frac{g_\nu}{g_\pi} \ , \tag{34a}$$

$$Y_2(p_\pi,h_\pi,p_\nu,h_\nu,E_i)=\frac{2(T_{ze}+i)}{p_\nu+h_\pi+2(T_{ze}+i)}\frac{(p_\pi+1)(h_\nu+1)(n-1)(n-2)}{g_\pi g_\nu[E-A(\text{main})]^2} \ , \tag{34b}$$

$$Y_3(p_\pi,h_\pi,p_\nu,h_\nu,E_i)=\frac{h_\nu+1}{p_\nu+h_\pi+2(T_{ze}+i)}\frac{g_\pi}{g_\nu} \ . \tag{34c}$$

In terms of the Y's and the modified X's, the appropriate T-dependent state densities adopted are

$$\begin{aligned}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z,T_z)={}&\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)-[\,Y_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)+Y_2(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_1)\\&+Y_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)]\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_1) \ ,\end{aligned} \tag{35}$$

$$\begin{aligned}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z+1,T_z)={}&X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)\\&+X_2(p_\pi,h_\pi,p_\nu,h_\nu,E_1)\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_1)\\&+X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\\&-\left\{X_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_1)\frac{p_\pi}{p_\nu+1}\frac{g_\nu}{g_\pi}\right.\\&\times[\,Y_1(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)+Y_2(p_\pi-2,h_\pi,p_\nu+1,h_\nu-1,E_2)\\&+Y_3(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)]\end{aligned}$$

$$
\begin{aligned}
&+X_2(p_\pi,h_\pi,p_\nu,h_\nu,E_1)[\,Y_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\\
&\qquad\qquad +Y_2(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_2)\\
&\qquad\qquad +Y_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)]\\
&+X_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_1)\frac{h_\nu}{h_\pi+1}\frac{g_\pi}{g_\nu}\\
&\quad\times[\,Y_1(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)+Y_2(p_\pi-1,h_\pi+1,p_\nu,h_\nu-2,E_2)\\
&\qquad +Y_3(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)]\Bigg\}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_2)
\end{aligned}
\tag{36}
$$

and

$$
\begin{aligned}
\omega(p_\pi,h_\pi,p_\nu,h_\nu,E,T_z+2,T_z)={}&X_{11}(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)\omega(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)\\
&+2X_{12}(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\omega(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\\
&+2X_{13}(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\omega(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\\
&+X_{22}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_2)\\
&+2X_{23}(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)\omega(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)\\
&+X_{33}(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)\omega(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)\\
&-\Bigg\{X_{11}(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_2)\frac{p_\pi}{p_\nu+1}\frac{p_\pi-1}{p_\nu+2}\left[\frac{g_\nu}{g_\pi}\right]^2\\
&\qquad\times[\,Y_1(p_\pi-3,h_\pi,p_\nu+3,h_\nu,E_3)\\
&\qquad\quad +Y_2(p_\pi-3,h_\pi,p_\nu+2,h_\nu-1,E_3)+Y_3(p_\pi-2,h_\pi+1,p_\nu+2,h_\nu-1,E_3)]\\
&\quad +2X_{12}(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_2)\frac{p_\pi}{p_\nu+1}\frac{g_\nu}{g_\pi}\\
&\qquad\times[\,Y_1(p_\pi-2,h_\pi,p_\nu+2,h_\nu,E_3)\\
&\qquad\quad +Y_2(p_\pi-2,h_\pi,p_\nu+1,h_\nu-1,E_3)+Y_3(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_3)]\\
&\quad -2X_{13}(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_2)\frac{p_\pi}{p_\nu+1}\frac{h_\nu}{h_\pi+1}\\
&\qquad\times[\,Y_1(p_\pi-2,h_\pi+1,p_\nu+2,h_\nu-1,E_3)\\
&\qquad\quad +Y_2(p_\pi-2,h_\pi+1,p_\nu+1,h_\nu-2,E_3)+Y_3(p_\pi-1,h_\pi+2,p_\nu+1,h_\nu-2,E_3)]\\
&\quad +X_{22}(p_\pi,h_\pi,p_\nu,h_\nu,E_2)\\
&\qquad\times[\,Y_1(p_\pi-1,h_\pi,p_\nu+1,h_\nu,E_3)\\
&\qquad\quad +Y_2(p_\pi-1,h_\pi,p_\nu,h_\nu-1,E_3)+Y_3(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_3)]\\
&\quad +2X_{23}(p_\pi,h_\pi+1,p_\nu,h_\nu-1,E_2)\frac{h_\nu}{h_\pi+1}\frac{g_\pi}{g_\nu}\\
&\qquad\times[\,Y_1(p_\pi-1,h_\pi+1,p_\nu+1,h_\nu-1,E_3)\\
&\qquad\quad +Y_2(p_\pi-1,h_\pi+1,p_\nu,h_\nu-2,E_3)+Y_3(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_3)]\\
&\quad +X_{33}(p_\pi,h_\pi+2,p_\nu,h_\nu-2,E_2)\frac{h_\nu}{h_\pi+1}\frac{h_\nu-1}{h_\pi+2}\left[\frac{g_\pi}{g_\nu}\right]^2\\
&\qquad\times[\,Y_1(p_\pi-1,h_\pi+2,p_\nu+1,h_\nu-2,E_3)+Y_2(p_\pi-1,h_\pi+2,p_\nu,h_\nu-3,E_3)\\
&\qquad\quad +Y_3(p_\pi,h_\pi+3,p_\nu,h_\nu-3,E_3)]\Bigg\}\omega(p_\pi,h_\pi,p_\nu,h_\nu,E_3)\;.
\end{aligned}
\tag{37}
$$

These results do not look substantially simpler than the full results, but the X's and Y's are simpler to evaluate and the total number of T-mixed state densities that must be calculated is significantly reduced.

VI. SYSTEMS WITH A PROTON EXCESS

All of the work described so far has assumed that the nuclei involved have a neutron excess. For nuclei with a proton excess, the changes needed in the equations are the following: (i) Replace T_z with $|T_z|$ everywhere that it is used in a calculation, and similarly for T_{ze}. (ii) Reverse the roles of proton and neutron degrees of freedom. This means reversing the subscripts ν and π everywhere that they occur (and reordering the resulting "quantum numbers" when they appear in designations of state densities). With these changes the existing results are applicable.

VII. SUMMARY AND CONCLUSIONS

A new derivation of two-component particle-hole state densities with good isospin was carried out eliminating all of the major approximations made in the previous work [5]. When second order and higher correction terms are ignored, the resulting state density expressions are given by Eqs. (17)–(19).

The new isospin-dependent state density results are consistent with the usual expressions for the isospin-mixed and total state densities. They suggest a different distribution in T of the total state density than what is typically assumed in the Fermi-gas model, but this difference is likely related to the role of passive particles and holes. It does not necessarily mean an inconsistency.

The current isospin-dependent state densities are different both from the Jensen result [11] (as seen in Figs. 3–5) and from the earlier results of Ref. [5] (which did not show the sizable shifts currently seen for the $T > T_z$ state densities). The differences are significant for ^{58}Ni which has a very small neutron excess, but much smaller for ^{208}Pb.

In a search for simpler approximate results for use in reaction calculations, it was concluded that the blocking of type-1 and -3 isospin flips could be neglected for all terms and that both the blocking of type-2 isospin flips and the differences between the Pauli correction functions of the different analog configurations could be neglected in the correction terms. The working expressions are given by Eqs. (35)–(37), where the X factors of Eqs. (14) and (16) incorporate the simplifications of Eq. (33), and the Y factors are given by Eq. (34). These results are suitable for implementation in two-component exciton model codes and for studying the importance of isospin conservation in preequilibrium nuclear reactions.

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