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Intermittent behavior of nuclear multifragments

P. L. Jain, G. Singh, and A. Mukhopadhyay
High Energy Experimental Laboratory, Department of Physics, State University of New York at Buffalo, Buffalo, New York 14260
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The observed intermittent behavior in nuclear fragment distributions for ^{238}U at 0.96A GeV, ^{84}Kr at 1.52A GeV, and ^{131}Xe at 1.22A GeV may be due to the mixing of fragments produced under different initial conditions. Intermittent effects are significantly reduced by selecting a sample under the same initial conditions. Except the cumulants of order two, all other moments are consistent with zero within their statistical errors.

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Recently the concept of intermittency [1] corresponding to the existence of large nonstatistical fluctuations, which possess self-similarity at all scales, has been quite popular in particle and nuclear collisions [2]. In this technique the data are analyzed in terms of normalized scale factorial moments (SFM) as a function of decreasing rapidity bins. This procedure allows us to test the statistical significance of the observed density fluctuations in order to find whether they are simply statistical, or they have dynamical origin leading to an intermittency pattern in multiparticle production. In the former case the SFM's are predicted to saturate with decreasing bin width, whereas in the case of intermittency the moments continue to increase according to a power law with decreasing bin width, down to the experimental resolution.

It was recently shown that nuclear fragmentation [3,4] processes, in which several smaller fragments each more massive than α particles are produced in a decay of a single highly excited nuclear system, show signs of intermittent behavior. It was further shown that the percolation model [5], when applied to multifragmentation, also gives similar intermittency behavior [3,4].

In hadronic and nuclear collisions, it has been found that the observed increase in factorial moments is due to the short-range correlations [6] and therefore the introduction of a power law behavior may not be essential in such collisions. In hadronic as well as in nuclear collisions when ordinary factorial moments are decomposed into cumulant moments, the former are seen to be dominated by the cumulants of order two [7] and all other moments are consistent with zero within their statistical uncertainties. This may be a reasonable explanation for the presence of intermittency in hadronic and nuclear collisions; one is thus interested to search for the possible trivial origins of the intermittent behavior of fragment yields in nuclear multifragmentation process and this is the subject of the present paper.

We have recently shown that the charge distribution in nuclear fragments for nonfissile events of ^{238}U at 0.96A GeV [4], of ^{84}Kr at 1.52A GeV [8], and of ^{131}Xe at 1.22A GeV [9] in nuclear emulsion is fitted with a power law. The method of SFM was used to study fluctuations in the nuclear fragmentation and intermittent behavior was observed in all three projectiles. The charges of the projectile fragments were estimated by a combination of different methods, such as—measurements of gap density, grain density, relative track width, etc. and δ-ray counting. For charges $Z \leq 6$ the resolution of these estimations was 1e and for $Z > 6$ it was 2e (for details of charge measurement see Refs. [8] and [10]). In these experiments, we collected the data event by event basis with fragmentations corresponding to many different impact parameters [4,8–10]. The impact parameter that characterizes the initial state is not a directly measurable quantity, but proton multiplicity in the final state is strongly correlated with the impact parameter. For the hardness of the collision, we used the release of $Z=1$ projectile fragment N_p. By assuming the geometrical prescription of Ref. [11], in Figs. 1(a) and 1(b) are shown for ^{238}U [4,10] and ^{84}Kr [8] beams a monotonic relationship between the proton multiplicity and impact parameter through $[b(N_p)/b_{\max}]^2 = \int_{N_p}^{\infty} dp_{N_p}$, where dp_{N_p} is the normalized probability distribution for the measured proton multiplicity and $b_{\max}$ is the maximum impact parameter for proton detection. The results shown for ^{238}U and ^{84}Kr beams in Fig. 1 are associated to a wide range of excitation energies of the fragmenting transient system. Hence they give rise to large fluctuations of the initial conditions for the nuclear disassembly reaction. For the intermittency calculation [4,8], we used practically all events and when taken together they correspond to

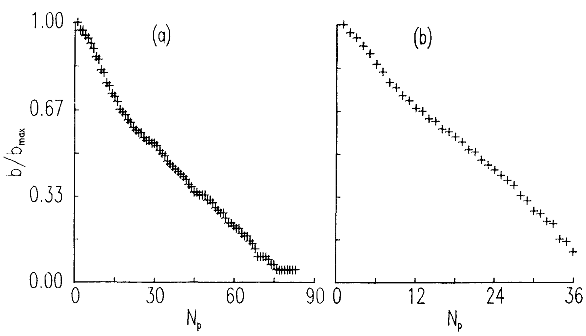

FIG. 1. Relation between impact parameter $b/b_{\max}$ and proton multiplicity (N_p) as obtained from geometrical prescription of Ref. [12]: (a) ^{238}U at 0.96A GeV and (b) ^{84} Kr at 1.52A GeV.

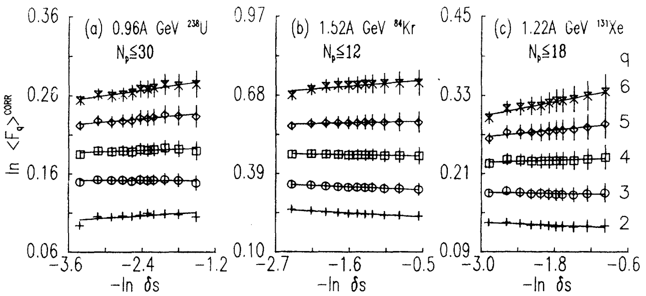

FIG. 2. Variation of $\ln\langle F_q\rangle^{\rm corr}$ as a function of $-\ln\delta s$ for the events with (a) $N_p \le 30$ for ^{238}U, (b) $N_p \le 12$ for ^{84}Kr, and (c) $N_p \le 18$ for ^{131}Xe. Values of q are depicted in Fig. 3(c) for the ^{131}Xe data and are also true for the ^{238}U and ^{84}Kr data samples. The solid lines in these figures are the least-squares fits to the data points.

different excitation energies and this may have caused [12] the presence of intermittency in multifragmentation process. Now if we look at Figs. 1(a) and 1(b) it appears quite resonable to distinguish between "central," "mid-central," and "peripheral" collisions. But we may caution here that the relative scale between impact parameter and proton multiplicity must not be overinterpreted, since considerable fluctuations of proton multiplicity must be expected even for collisions of a well defined impact parameter.

In order to separate the events produced possibly under the same excitation conditions, let us consider only peripheral events. We used only those events which have $N_p \le 30$ for ^{238}U $(Z=92)$, $N_p \le 12$ $[\approx(36/92)\times 30]$ for ^{84}Kr $(Z=36)$, and $N_p \le 18$ $[\approx(54/92)\times 30]$ for ^{131}Xe $(Z=54)$ beams. We plot the variations of $\ln\langle F_q\rangle^{\rm corr}$ as a function of $-\ln\delta s$ [4,9] for ^{238}U, ^{84}Kr, and ^{131}Xe beams, respectively, where SFM F_q of order q are calculated by

$$\langle F_q\rangle = \frac{1}{\langle N_f\rangle^q}\left\langle \frac{1}{M}\sum_{m=1}^{M} n_m(n_m-1)\cdots(n_m-q+1)\right\rangle$$

where

$$\langle N_f\rangle = \left\langle \frac{1}{M}\sum_{m=1}^{M} n_m\right\rangle \tag{1}$$

for events with mean fragment multiplicity $\langle N_f\rangle$ in the fragment charge interval Δs, which is divided into M bins of size $\delta s = \Delta s/M$. The number of fragments in the mth bin $(m=1,2,3,\ldots,M)$ is n_m. For nonflat fragment multiplicity distributions varying within a finite bin of width δs introduces an extra M-dependent correction factor R_q [13], which is given by

$$R_q = \frac{1}{M}\sum_{m=1}^{M} M^q\langle n_m\rangle^q/\langle N_f\rangle^q \ . \tag{2}$$

The values of the slopes α_q of fitted straight lines for different orders of q for ^{238}U, ^{84}Kr, and ^{131}Xe as shown in Fig. 2(a), 2(b), and 2(c), respectively, up to $q=6$ are: $\alpha_2=0.0046\pm0.0019$, -0.0131 ± 0.0009, -0.0038 ± 0.0008; $\alpha_3=0.0004\pm0.0009$, -0.0093 ± 0.0007, -0.0017 ± 0.0009; $\alpha_4=0.0026\pm0.0010$, -0.0037 ± 0.0006, 0.0026 ± 0.0010; $\alpha_5=0.0067\pm0.011$, 0.0056 ± 0.0017, 0.0088 ± 0.0012; $\alpha_6=0.0111\pm0.0011$, 0.0202 ± 0.0040, 0.0181 ± 0.0014. These (α_q) values are

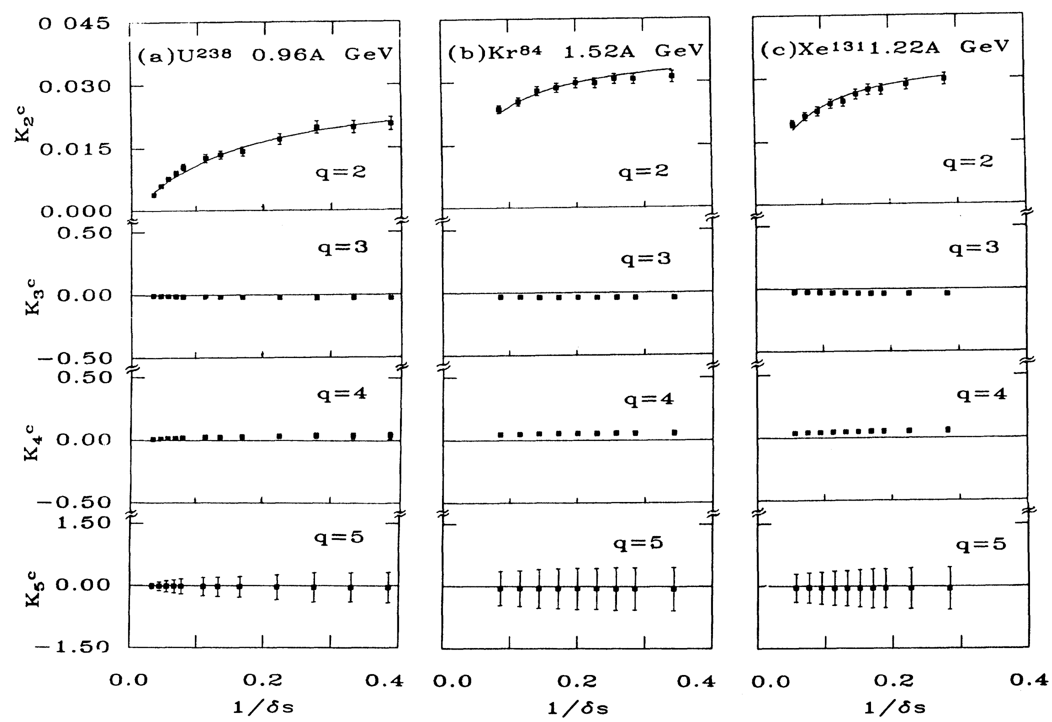

FIG. 3. Variation of cumulant moments K_q, $q=2$, 3, 4, and 5 with $1/\delta s$ for (a) ^{238}U at $0.96A$ GeV, (b) ^{84}Kr at $1.52A$ GeV, and (c) ^{131}Xe at $1.22A$ GeV obtained from factorial moments. The solid curve in all parts represents the best fitted values of K_2 according to Eq. (5) with experimental data. $K_q=0$ line is shown by solid lines.

much smaller than for the whole sample [4,8]. We notice that some values of the slopes are even negative and the pattern resembling intermittency practically disappears in almost all the beams excepting for $q=5$ and 6, and that may be due to mixing of some of the events at higher excitation energy (midcentral events) with peripheral events. Now if we select only central events with $N_p \geq 35$ for ^{238}U and ^{131}Xe and $N_p \geq 25$ for ^{84}Kr, the slopes of the fitted straight lines up to $q=6$ are: $\alpha_2=0.0025\pm0.0001$, 0.0009 ± 0.0001, 0.0012 ± 0.0001; $\alpha_3=0.0039\pm0.0002$, 0.0014 ± 0.0001, 0.0019 ± 0.0002; $\alpha_4=0.0052\pm0.0002$, 0.0019 ± 0.0001, 0.0025 ± 0.0003; $\alpha_5=0.0065\pm0.0003$, 0.0025 ± 0.0001, 0.0031 ± 0.0003; $\alpha_6=0.0079\pm0.0004$, 0.0030 ± 0.0002, 0.0039 ± 0.0005. These values just like peripheral events are also much smaller in comparison with the values of the whole sample [4,8] for their respective beams. There is still a slight mixture of the sample in each beam with different excitation energies as indicated earlier. It shows that N_p cuts do not absolutely eliminate central events from peripheral events and even a few percentage of mismatched events in the sample would not take away completely the intermittent signal.

If intermittency is due to two-fragment correlation in nuclear fragments then one may apply the technique of cumulant moment where the largest parts of the factorial moments of all orders come from the two-fragment correlation. The true correlation of any specific order is revealed only when contributions from lower orders are subtracted out of it. Thus to obtain the true contributions of higher order correlations ($q>2$), it is advisable to express the factorial moments [7] in terms of factorial cumulants,

$$F_2=K_2+1, F_3=K_3+3K_2+1 ,$$
$$F_4=K_4+4K_3+3\overline{(K_2)^2}+6K_2+1 , \quad (3)$$
$$F_5=K_5+5K_4+10K_3+10\overline{K_3K_2}+15\overline{(K_2)^2}+10K_2+1 .$$

Here,

$$K_q(\delta y)=\frac{1}{M(\delta y)^q}\sum_m \int_{\Omega_m}\prod_i dy_i \frac{C_q(y_1,\ldots,y_q)}{(\bar{\rho}_m)^q} \quad (4)$$

is the cumulant of qth order, and

$$C_2(y_1,y_2)=\rho_2(y_1,y_2)-\rho_1(y_1)\rho_1(y_2) ,$$
$$C_3(y_1,y_2,y_3)=\rho_3(y_1,y_2,y_3)-\rho_1(y_1)\rho_2(y_2,y_3)$$
$$-\rho_1(y_2)\rho_2(y_3,y_1)-\rho_1(y_3)\rho_2(y_1,y_2)$$
$$+2\rho_1(y_1)\rho_1(y_2)\rho_1(y_3) ,$$

etc. are the two and three particle correlation functions. Here, $\rho_1(y_1)$, $\rho_2(y_2,y_2)$, and $\rho_3(y_1,y_2,y_3)$ are one, two, and three particle densities, respectively. In the above expressions of F_4 and F_5, the bar averaging is done in the following way: $\overline{K_2K_3}=1/M\sum_{m=1}^{M}K_2K_3$, and so on.

Cumulants are obtained from the corrected factorial moments $F_q^C(\delta s)=F_q(\delta s)/R_q(\delta s)$. The relation between $F_q^C(\delta s)$ and the cumulants $K_q^C(\delta s)$ is not exactly governed by the relation like Eq. (3). Such equations are only approximately valid in this case.

The cumulant moments thus obtained will be vanishing in the absence of any true dynamical correlations. We have thus computed the values of cumulant moments of lowest orders (i.e., $q=2,3,4,5$) on the nuclear fragment data for ^{238}U, ^{84}Kr, and ^{131}Xe beams and they are shown in Figs. 3(a), 3(b), and 3(c), respectively. Each event of the data sets used for calculating the K_q moments has at least 25 projectile fragments for ^{238}U, and at least 10 fragments for ^{84}Kr as well as ^{131}Xe (as are used in Refs. [4],[8], and [9], respectively). In each beam K_2 values of second order cumulants are significantly different from zero and with decreasing δs, these values undergo a saturation following an initial rise. The small variations from zero in the values of cumulant moments for $q=3$ and $q=4$ may be due to the approximation involved in calculating the cumulant moments using Eq. (4). For $q=5$, the values of K_q are always zero for all the beams used here.

We have also parametrized the experimental data in terms of the algebraic expression

$$K_2=2\gamma\xi^2[(\delta s/\xi)-1+e^{-\delta s/\xi}]/\delta s^2 . \quad (5)$$

Here γ is a parameter related to the c.m. energy of the system, and according to the one-dimensional statistical

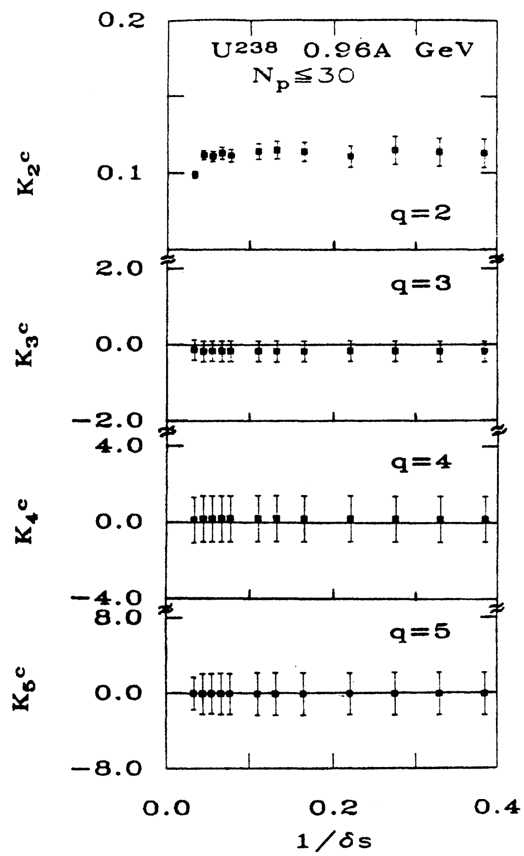

FIG. 4. Variation of cumulant moments, K_q, $q=2$, 3, 4, and 5 with $1/\delta s$ for ^{238}U at 0.96A GeV only for events with $N_p \leq 30$, obtained from factorial moments. $K_q=0$ line is shown by solid lines for $q=3$, 4, and 5.

model of Ref. [8], ξ indicates how far the system is from the critical point. In all cases the fitting process yield reasonably low χ^2 values. The values of the parameter γ, the strength of the correlation length ξ and χ^2/d.o.f. are for beams: ^{238}U: $\gamma=0.029$, $\xi=2.58$, χ^2/d.o.f. $=1.26$; ^{84}Kr: $\gamma=0.038$, $\xi=6.47$, χ^2/d.o.f. $=0.71$; ^{131}Xe: $\gamma=0.036$, $\xi=6.83$, χ^2/d.o.f. $=0.66$. These values are significantly different than those obtained from a similar analysis done for shower particles in heavy ion collisions [7]. When the same analysis is performed for events with specific excitation energies which correspond either to peripheral or central collisions, i.e., with a cut on the proton number, we found that the general observation of nonzero cumulants of order 2 ($K_2>0$), and vanishing cumulants of higher order ($K_q=0$ for $q=3,4,5$), holds true for all three data sets. As an example, the schematic representation of the variation of K_q moments with $1/\delta s$ is shown in Fig. 4 only for the ^{238}U induced reactions having $N_p \leq 30$. Comparatively larger statistical errors in this diagram are due to partitioning of the sample size into a smaller subset.

We conclude that the observation of intermittency in nuclear fragmenting events is due to the presence of events produced with different impact parameters and consequently with different excitation energies. In multifragmentation process, there is two-fragment correlation and correlations higher than two do not exist beyond the statistical uncertainties. The strength of two-fragment correlation length as seen from the values of the fitted parameter ξ is different than in hadronic matter.

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