Comment on "Pion-nucleus scattering around the (3,3) resonance"

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We comment on a recent paper by Rahman, Sen Gupta, and Rahman [Phys. Rev. C **41**, 2305 (1990)]. We demonstrate that the theoretical results presented in that paper based on the strong absorption model of Frahn and Venter are at variance with those reported in this Comment. This in turn raises serious doubts regarding their stated conclusions.

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In a recent paper [1], hereafter referred to as RSR, an investigation was undertaken of pion-nucleus scattering around and below the (3,3) resonance for both elastic and inelastic scattering within the framework of the strong absorption model (SAM) of Frahn and Venter [2]. Our earlier communication [3] regarding an error in their basic equation (3) for the scattering amplitude $f(\theta)$ resulted in the authors' subsequent publication of an erratum [4] relative to same. The authors, however, did not indicate whether the correction in any way altered their subsequent results or their stated conclusions. This has led us to reexamine their results. In this Comment we report on a systematic investigation of the RSR paper confined, at present, to the elastic-scattering cases. In particular, we will present the values of nuclear radii and surface thicknesses from our analysis performed within the framework of SAM using RSR's parameters. Also, we give the results of our analytical expressions of the elastic-scattering amplitude derived within the SAM framework starting with the correct equation [3,4] for $f(\theta)$. These expressions are then used to calculate the elastic-scattering cross sections using the RSR parameters for several cases. We will show that our results in each of these areas differ substantially from those presented in RSR.

(1) Nuclear radii and surface thicknesses were calculated. The SAM formalism is characterized by the three free parameters T, Δ, and μ and were obtained by RSR by an analysis of the experimental results examined in their paper. We have therefore taken their quoted values of the three parameters as the basis for our calculations. The radius R and surface thickness d are given by

$$R=\frac{1}{k}[n+(n^2+T^2)^{1/2}]$$

and

$$d=\frac{\Delta}{k}\left[1-\left[\frac{2n}{kR}\right]\right]^{1/2}\left[1-\left[\frac{n}{kR}\right]\right]^{-1}. \qquad (1)$$

Our results are shown in Tables I and II together with those of RSR for comparison. n and k are the Coulomb parameter and wave number, respectively, and are given as follows:

$$n=\pm\frac{Ze^2m}{\hbar^2k},$$

and

TABLE I. SAM calculations for π^+ elastic scattering from nuclei.

Experimental parameters				Theoretical parameters			Calculated values			
Pion energy (MeV)	Nucleus	$k_{\text{c.m.}}$ (fm^{-1})	n	T	Δ	μ	R (fm) a	R (fm) b	d (fm) a	d (fm) b
130	^{28}Si	1.156 82	0.062 13	4.70	0.80	0.448	4.12	4.99	0.69	0.84
180	^{28}Si	1.439 30	0.049 94	5.80	0.95	0.114	4.06	5.21	0.66	0.84
226	^{28}Si	1.688 71	0.042 56	6.40	0.85	0.408	3.82	5.12	0.50	0.67
180	^{40}Ar	1.444 51	0.064 08	6.60	1.00	0.400	4.61	5.93	0.69	0.89
80	^{40}Ca	0.853 95	0.120 43	4.10	0.60	0.504	4.94	5.63	0.70	0.80
130	^{40}Ca	1.160 36	0.088 63	5.50	0.90	0.396	4.82	5.85	0.78	0.94
180	^{40}Ca	1.444 51	0.071 19	6.50	0.90	0.144	4.55	5.85	0.62	0.79
230	^{40}Ca	1.717 20	0.059 89	7.50	0.95	0.418	4.40	5.95	0.55	0.75
130	^{48}Ca	1.161 75	0.088 58	5.70	0.85	0.544	4.98	6.06	0.73	0.89
180	^{48}Ca	1.446 55	0.071 14	6.70	0.85	0.136	4.68	6.02	0.59	0.75
230	^{48}Ca	1.720 00	0.059 83	7.70	1.10	0.572	4.51	6.10	0.64	0.86
80	^{90}Zr	0.856 74	0.240 58	5.20	0.65	0.390	6.36	7.27	0.76	0.86
80	^{208}Pb	0.858 02	0.492 91	6.20	0.65	0.390	7.82	8.99	0.76	0.86

[a]Present calculations.
[b]RSR calculations.

TABLE II. SAM calculations for π^- elastic scattering from nuclei.

Experimental parameters				Theoretical parameters			Calculated values			
Pion energy (MeV)	Nucleus	$k_{c.m.}$ (fm^{-1})	n	T	Δ	μ	R (fm) a	R (fm) b	d (fm) a	d (fm) b
130	^{28}Si	1.156 82	−0.062 13	5.00	0.80	0.448	4.27	5.15	0.69	0.84
180	^{28}Si	1.439 30	−0.049 94	5.90	0.90	0.360	4.06	5.19	0.63	0.80
226	^{28}Si	1.688 71	−0.042 56	6.60	1.00	0.440	3.88	5.19	0.59	0.78
180	^{40}Ar	1.444 51	−0.064 08	6.80	0.80	0.096	4.66	5.96	0.55	0.71
80	^{40}Ca	0.853 95	−0.120 43	4.85	0.50	0.360	5.54	6.27	0.59	0.66
130	^{40}Ca	1.160 36	−0.088 63	5.90	0.85	0.510	5.01	6.04	0.73	0.89
180	^{40}Ca	1.444 51	−0.071 19	6.80	0.95	0.114	4.66	5.95	0.66	0.84
230	^{40}Ca	1.717 20	−0.059 89	7.70	1.05	0.210	4.45	5.98	0.61	0.82
130	^{48}Ca	1.161 75	−0.088 58	6.20	0.70	0.560	5.26	6.36	0.60	0.84
180	^{48}Ca	1.446 55	−0.071 14	7.10	0.95	0.152	4.86	6.21	0.66	0.84
230	^{48}Ca	1.720 00	−0.059 83	8.00	0.90	0.288	4.62	6.21	0.52	0.71
80	^{90}Zr	0.856 74	−0.240 58	6.00	0.50	0.360	6.73	7.61	0.58	0.66
80	^{208}Pb	0.858 02	−0.492 91	7.45	0.50	0.100	8.13	9.21	0.58	0.66

[a]Present calculations.
[b]RSR calculations.

$$k=\frac{m_N}{\hbar}\left[\frac{(E_\pi^2/c^2)-m_\pi^2c^2}{m_\pi^2+m_N^2+2m_N(E_\pi/c^2)}\right]^{1/2},$$

where m_π and m_N are the pion and nuclear rest mass, respectively, and m is their reduced mass. E_π is the total pion energy in the laboratory frame and the + (−) sign is taken for positive (negative) pions. Z is the atomic number of the target nucleus. As can be readily seen from an examination of the tables, RSR's values are consistently higher than ours for both R and d, in many cases by more than 25%.

(2) Analytical expressions for the elastic-scattering amplitude in the format given by RSR were obtained, starting with the correct expression for $f(\theta)$ as noted above, within the framework of the strong absorption model of Frahn and Venter [2]. The assumed form of the nuclear phase shift is

$$\eta_l\exp(-2i\sigma_l)=g(t)+i\mu\frac{dg(t)}{dt}.$$

All symbols used are as defined in Ref. [2]. In the present cases, $1-g(t)$ has the Woods-Saxon form for the nuclear density distribution. For $n\ll 1$, we find that

$$f(\theta)=\frac{T}{k}\left[\frac{\theta}{\sin\theta}\right]^{1/2}F(\Delta\theta)\times\left[i\frac{J_1(T\theta)}{\theta}-\left[\frac{2n}{T\theta^2}-\mu\right]J_0(T\theta)\right], \quad (2)$$

where

$$F(\Delta\theta)=\pi\Delta\theta/\sinh(\pi\Delta\theta).$$

This expression differs from that given by RSR [their Eq. (5)]. It should be noted that the effects of the Coulomb interaction are entirely contained within the n factor: for small scattering angle θ Coulomb effects dominate and our expression becomes

$$f(\theta)\approx-2n/k\theta^2,$$

which leads to the limiting form of the Rutherford cross section at small angles:

$$\sigma_R=4n^2/k^2\theta^4.$$

For $n\gg(2\pi)^{-1}$, two expressions result which are applicable to the respective regions $\theta\le\theta_c$ and $\theta>\theta_c$, where $\theta_c=2\arctan(n/T)$ is the critical angle. For all cases considered in RSR, only the diffraction region $\theta>\theta_c$ is applicable and the result there is

$$f(\theta)=\frac{i}{k}\left[\frac{T}{2\pi\sin\theta}\right]^{1/2}\exp(i\chi)\times\Big\{AF[\Delta(\theta-\theta_c)]\exp[-i(T\theta-\pi/4)]-BF[\Delta(\theta+\theta_c)]\exp[i(T\theta-\pi/4)]\Big\}, \quad (3)$$

with

$$A=G(u)\left[\frac{T}{2\sin\theta_c}\right]^{1/2}-\mu,$$

$$B=(\theta+\theta_c)^{-1}+\mu,$$

and

$$G(u)=\pi^{1/2}\exp[i(u^2+\pi/4)]\mathrm{erfc}[u\exp(i\pi/4)]\left\{1-u\left[\frac{\sin\theta_c}{2T}\right]^{1/2}\left[\frac{1}{\sin\theta_c}+\left[1+i\frac{2}{3}u^2\right]\cot(\theta_c/2)\right]\right\}+\left[\frac{\sin\theta_c}{2T}\right]^{1/2}\left[\frac{1}{\sin\theta_c}+(1+iu^2)\frac{2}{3}\cot(\theta_c/2)\right],$$

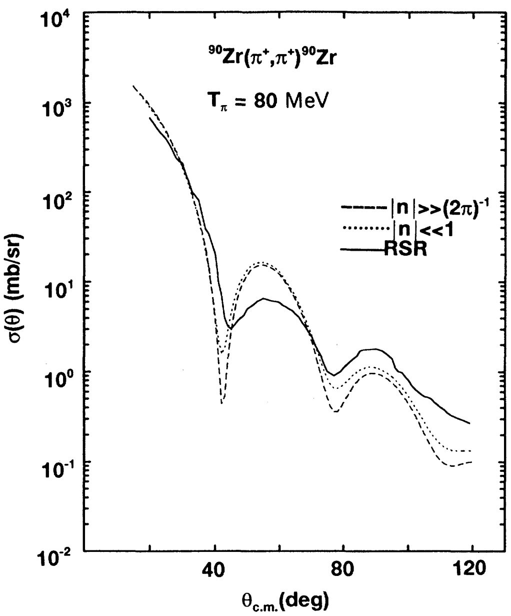

FIG. 1. Comparison of our calculation of the differential scattering cross sections with the RSR theoretical result for 80 MeV positive-pion elastic scattering from ^{90}Zr.

where

$$u=\left[\frac{T}{2\sin\theta_c}\right]^{1/2}(\theta-\theta_c)$$

and

$$\mathrm{erfc}(Z)=\frac{2}{\sqrt{\pi}}\int_Z^\infty \exp(-\tau^2)d\tau\ .$$

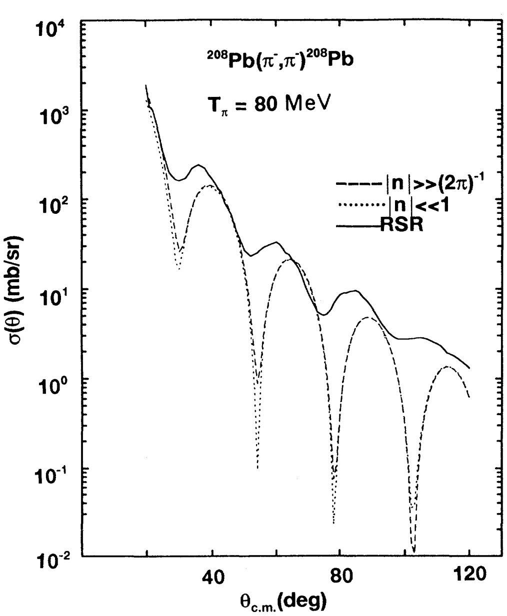

FIG. 2. Comparison of our calculation of the differential scattering cross sections with the RSR theoretical result for 80 MeV negative-pion elastic scattering from ^{208}Pb.

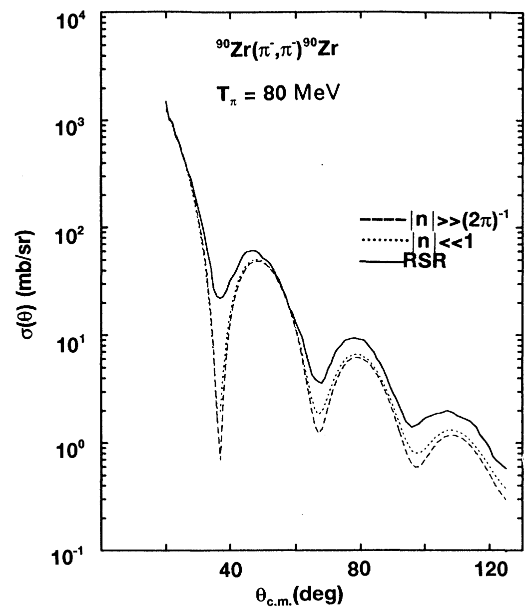

FIG. 3. Comparison of our calculation of the differential scattering cross sections with the RSR theoretical result for 80 MeV negative-pion elastic scattering from ^{90}Zr.

χ is a real number, and it does not figure into the resulting cross section. This expression for $f(\theta)$ is also seen to be different from RSR's Eq. (5); in this case the discrepancy involves the first-order terms in $n^{-1/2}$ corresponding to Frahn and Venter's original expansion [2] for the function $G(u)$. Our expressions are in agreement with those given by Frahn [5] (cf. Eqs. 5.25, 5.26, 6.5, and

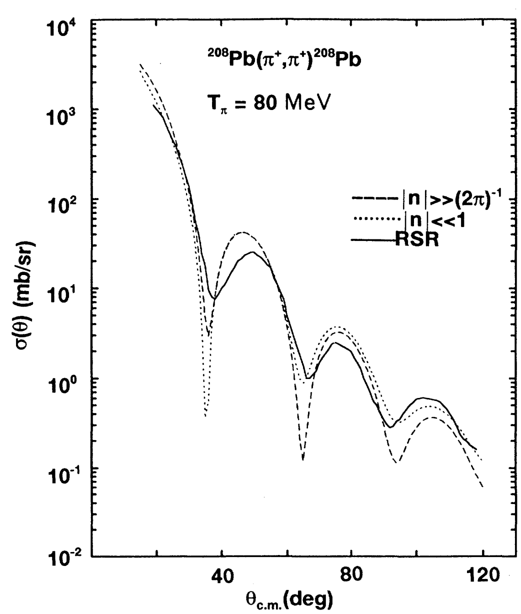

FIG. 4. Comparison of our calculation of the differential scattering cross sections with the RSR theoretical result for 80 MeV positive-pion elastic scattering from ^{208}Pb.

6.8–6.14) applied to the present case.

We have given considerable thought to the sign of the Coulomb parameter n that enters explicitly into Eq. (2) and implicitly into Eq. (3) through the critical angle θ_c for the case of negative pion scattering. Upon a close examination of the underlying mathematical structure of Frahn and Venter's formulation we find that n is implicitly taken as being positive in their derivations of the scattering amplitude. This can be seen, for example, from their use of the standard asymptotic expansion for the Legendre polynomials, valid only for scattering angles in the range $0 \ll \theta \ll \pi$ (cf. Eq. (40) in Venter [2]). The model's underlying mathematical structure seems to require a charge-independent Coulomb parameter n and critical angle θ_c where these appear explicitly in expressions for the scattering amplitude. We have therefore used the absolute value of n in our calculations of the negative pion scattering cases described below. It should be noted, however, that the actual value of the parameter n enters in when considering the kinematics of the scattering problem, i.e., the impact parameter giving the distance of the closest approach in the usual semiclassical geometrical interpretation. This can be seen, for example, in Eq. (1).

(3) Elastic cross sections were calculated for several cases using the above expressions. In all instances the parameters T, Δ, and μ given by RSR were used. Some results of our calculations are shown in Figs. 1–4. Again we have presented the RSR results for comparison. There are significant differences between RSR's results and those of the present investigation. Values at maxima differ by as much as a factor of 2.6 while in some instances values at minima differ by more than a factor of 100. In the case of ^{208}Pb, not only are the absolute values of maxima and minima at variance, but their maxima and minima are slightly shifted relative to ours by 3 to 5 degrees. The ^{90}Zr case with positive-pion scattering also shows a similar shift. Since the values of the Coulomb parameter could reasonably be applicable to either of the above expressions [Eqs. (2) or (3)] we have presented the result for both cases. As can be readily seen, the only large differences occur in the relative depths of the cross sections at minimum. For completeness we also performed calculations of the two negative-pion scattering cases using the actual value of n, and for all four cases using $n = 0$. The results of these calculations also reflect general differences when compared to RSR. It is evident from our calculations that there would be a much poorer fit to the experimental results than is implied by the corresponding cases shown in RSR.

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