Nuclear density of states for moving fused compound systems

M. Rajasekaran and R. Premanand
Department of Nuclear Physics, University of Madras, Guindy Campus, Madras 600 025, India
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The single-particle distribution function of moving fused compounds, with velocities of the order of a few tenths of the velocity of light, are calculated using the discrete spectrum of single-particle states of deformed nuclei. The polarization of single-particle momenta in the direction of flight is clearly brought out. Results are presented for ^{40}Ca, ^{64}Ni, ^{71}As, and ^{90}Zr. For small temperatures when the internal excitations are small, the nuclear level density varies as $E(E^2-M^2)^{1/2}$ where E is the total energy of the moving nuclei, and, as excitation increases, the internal degrees of freedom dominate and the level density $\rho\sim e^S$, where S is the entropy of the system, varies exponentially. The level densities thus calculated are used to evaluate light charged-particle emission from hot fused compounds in motion.

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Recent experiments in heavy-ion collisions in which fused compounds are moving with nearly a few tenths of the velocity of light yield interesting data on the nuclear level density based on the evaporated particle spectra. In Ref. [1], the spectra of the emitted particle are fitted by assuming a moving source (or sources) and a Maxwellian distribution function. In view of these developments it becomes necessary to theoretically investigate the phase space available for moving hot nuclei.

The statistical theory employed here is the same as the one used in our earlier calculations [2], except for the modification necessitated due to the translational motion of the system, where we have included only deformation and rotational degrees of freedom in the superfluid nuclei [3]. The partition function in the present calculation has two parts:

$$Z=Z_{\rm int}Z_{\rm trans}\ , \tag{1}$$

where

$$Z_{\rm int}=\prod_i\left[1+\exp(\mu-\epsilon_i)\beta\right]\ ,$$
$$Z_{\rm trans}=\exp(-E_k\beta)\ , \tag{2}$$

where μ is the chemical potential, β $(=1/T)$ is the inverse of the temperature, and

$$E_k=\mathbf{v}\cdot\sum_i\mathbf{p}_i\ , \tag{3}$$

where $\mathbf{v}$ is the velocity of the moving frame and $\mathbf{p}_i$ are the single-particle momenta of the nucleons.

The grand partition function is given by

$$Z=\sum_i\exp[-\beta(E_i+E_k)+\alpha N_i]\ , \tag{4}$$

where E_i is the internal total energy

$$E_i=\sum_j n_j\epsilon_j\ , \tag{5}$$

with N_i the number of fermions given by

$$N_i=\sum_j n_j\ . \tag{6}$$

The Lagrangian multipliers α and β conserve the average particle number and energy of the system. In view of Eq. (3), another Lagrangian multiplier v for total momentum conservation has to be introduced and it could be easily recognized as the collective translational velocity of the nucleus [4]. With these modifications, the statistical theory is made suitable to study hot nuclei in motion.

A moving fused compound with internal degrees of freedom cannot be treated as a rigid body in motion. The nucleonic motion with Fermi energy $E_F\sim 40$ MeV and $k_F\sim 1.36$ fm^{-1} inside the nucleus at rest should have net momenta $\mathbf{P}=\mathbf{0}$. The total momentum is composed of an equal number of nucleons moving with $+p_i$ and $-p_i$, so that

$$\sum_i n_i^{\pm}p_i^{\pm}=0\ , \tag{7}$$

where $\pm$ indicate the direction of single-particle momenta, i.e., parallel or antiparallel to the direction of flight of the nucleus and $n_i^{\pm}$ are the corresponding occupation probabilities.

In the moving frame these occupational probabilities may be written after applying the necessary transformation for single-particle energy ϵ as [5]

$$n_i=\{1+\exp[(E\epsilon_i-\mathbf{p}_i\cdot\mathbf{P})/M-\mu]\beta\}^{-1}\ , \tag{8}$$

where $\mathbf{p}_i$ is the single-particle momentum of the ith nucleon, E and $\mathbf{P}$ are the total energy and momentum of the nucleus, M is the rest mass of the nucleus of mass number A, and μ is the chemical potential fixed by number conservation. The term $\mathbf{p}_i\cdot\mathbf{P}$ is the term due to translational momentum $\mathbf{P}$. If $\mathbf{p}_i=\mathbf{p}_{zi}$ and $\mathbf{P}$ is along the z direction, the occupational probability is given by

$$n_i=\{1+\exp[(E\epsilon_i-\mathbf{p}_{zi}\cdot\mathbf{P})/M-\mu]\beta\}^{-1}\ . \tag{9}$$

The single-particle distribution function in the Fermi gas model is given by [5]

$$g(p_z)=\int\frac{2V_m/(2\pi)^3}{\hbar^3}d^2p_\perp \times\{1+\exp[(E\epsilon-\mathbf{p}_z\cdot\mathbf{P})/M-\mu]/T\}^{-1}\ , \tag{10}$$

where V_m is the volume in the moving frame.

In the limit $E \sim P$ or $E/P \sim 1$, i.e., in the infinite momentum frame,

$$g(x)=\frac{2V/(2\pi)^2}{\hbar^3}M^2Tx\ \ln\{1+\exp[(\mu-Mx/2)/T]\}\ , \tag{11}$$

where we have assumed

$$p_z=xP\ . \tag{12}$$

The parameter x which varies from 0 to 1 is the momentum fraction of the nucleons. The number conservation equation,

$$\int g(x)dx = A\ , \tag{13}$$

fixes μ in Eq. (11). When $T\rightarrow 0$,

$$g(x)=\frac{M^2x}{\hbar^3}(\mu-Mx/2)[2V/(2\pi)^2]\ . \tag{14}$$

Hence

$$\mu=(3\pi^2\hbar^3A/V)^{1/3}\ . \tag{15}$$

The nucleon distribution function is shown in Fig. 1 in the limit $E \sim P$ (infinite momentum frame) as a function of x, the momentum fraction of the nucleons. This corresponds to the totally aligned motion of nucleons in the direction of flight such that momentum p_z of the nucleons is given by $p_z=xP$, where P is the total momentum of the nucleus.

In general, we can treat the relative velocity v (in units of c) of the moving frame as a Lagrangian multiplier and write [6]

$$n_i^m(\epsilon_i)=(1+\exp\{\beta[\gamma(\epsilon_i-\mathbf{v}\cdot\mathbf{p}_i)-\mu]\})^{-1}\ , \tag{16}$$

where $\gamma=1/(1-v^2)^{1/2}$.

For simplicity let us use a single-particle spectrum generated by a deformed oscillator potential of the Nilsson type [7]. We may then write [8]

$$\langle p_z^2\rangle^{1/2}=[(\nu+\tfrac{1}{2})m\omega\hbar]^{1/2}\ , \tag{17}$$

where ν is the oscillator quantum number corresponding to the z direction and takes values $0,1,\ldots N$, N is the total oscillator quantum number:

$$N=\nu+\lambda\ , \tag{18}$$

λ being the oscillator quantum number corresponding to the perpendicular direction.

If the relative velocity $\mathbf{v}$ is along the z direction then

$$\mathbf{v}\cdot\mathbf{p}_i=\pm v\langle p_z\rangle\ . \tag{19}$$

The total energy of nuclear system is given by

$$E=\gamma\left[\sum_i n_i^{\pm}\epsilon_i+v\sum_i n_i^{\pm}p_i^{\pm}\right]\ , \tag{20}$$

and the total momentum is given by

$$P=\sum_i n_i^{\pm}p_i^{\pm}\ . \tag{21}$$

The second term in the energy equation is the contribution of translational motion of the whole nucleus.

Since the total number of particles A is fixed,

$$A=\sum_i n_i^{\pm}\ . \tag{22}$$

The above equation determines μ for a given v and T.

The statistical nuclear density of states in the stationary frame, i.e., $v=0$, which has been investigated in Refs.

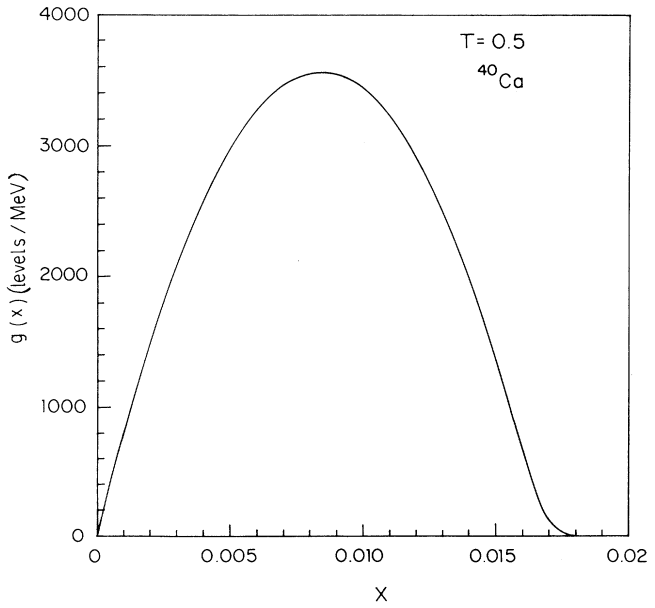

FIG. 1. The distribution function $g(x)$ vs the scaling parameter x for ^{40}Ca at $T=0.5$ MeV.

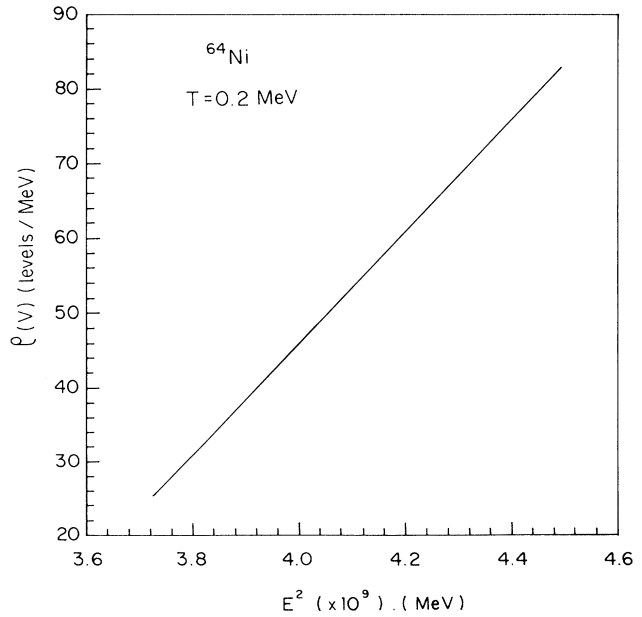

FIG. 2. Nuclear level densities in a moving frame as a function of E^2 for ^{64}Ni at $T=0.2$ MeV.

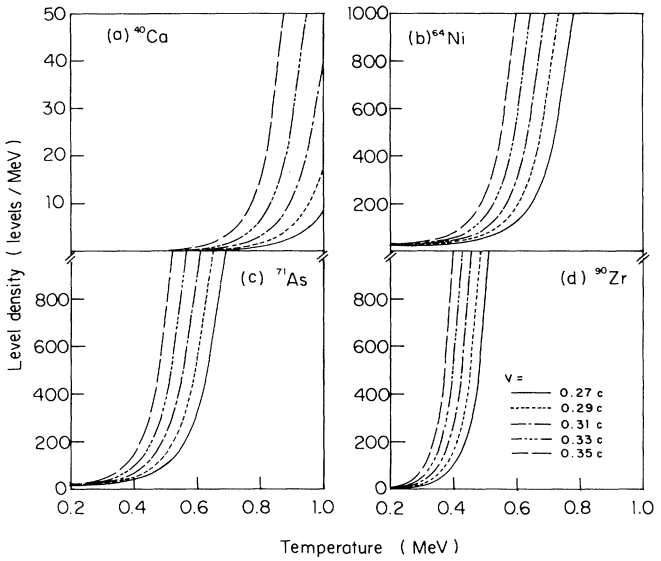

FIG. 3. Plot of the nuclear level densities for (a) ^{40}Ca, (b) ^{64}Ni, (c) ^{71}As, and (d) ^{90}Zr in a moving frame vs temperature for various relative velocities corresponding to the deformation parameter $\delta=0.0$.

[3,9] may be written as

$$\rho_I=\beta\exp(S_R)/S_{\max}\ , \tag{23}$$

where S_R, the entropy in the rest frame, is given by

$$S_R=-\sum_i [n_i^R \ln n_i^R+(1-n_i^R)\ln(1-n_i^R)] \tag{24}$$

with

$$n_i^R(\epsilon_i)=\{1+\exp[\beta(\epsilon_i-\mu)]\}^{-1}\ . \tag{25}$$

The subscript and superscript R denote rest-frame values. The normalization factor $S_{\max}$ depends upon the dimensionality of phase space which is the number of eigenstates used [9]. The subscript I of ρ denotes the internal density of states of the nuclear system at a temperature T, due to the rearrangement of particles in the eigenstates of the excited nucleus.

When the nucleus is moving with kinetic energy $E_{\rm kin}$, then the density of states due to translational motion

$$\rho_E=4\pi(E^2-m^2)dp/dE$$
$$\propto E(E^2-m^2)^{1/2}\ . \tag{26}$$

The total density of states is given by

$$\rho=\rho_I\rho_E\ . \tag{27}$$

This can be calculated directly using the thermodynamic quantities in the moving frame. The entropy in the moving frame,

$$S_m=-\sum_i [n_i^m \ln n_i^m+(1-n_i^m)\ln(1-n_i^m)]\ , \tag{28}$$

includes both translational entropy and the entropy due to the finite temperature of the system. The corresponding level density is

$$\rho_m(v)=\beta\exp[S_m(v,T)]/S_{\max}\ . \tag{29}$$

The occupational probabilities n_i^m are given by Eq. (16).

The nuclear level density as already stated according to Eq. (27) is made of two parts, the internal nuclear level density ρ_I due to the temperature T and the translational contribution proportional to E^2. At low temperatures when the internal contribution ρ_I is very small, the total nuclear level density ρ is linearly proportional to E^2. This

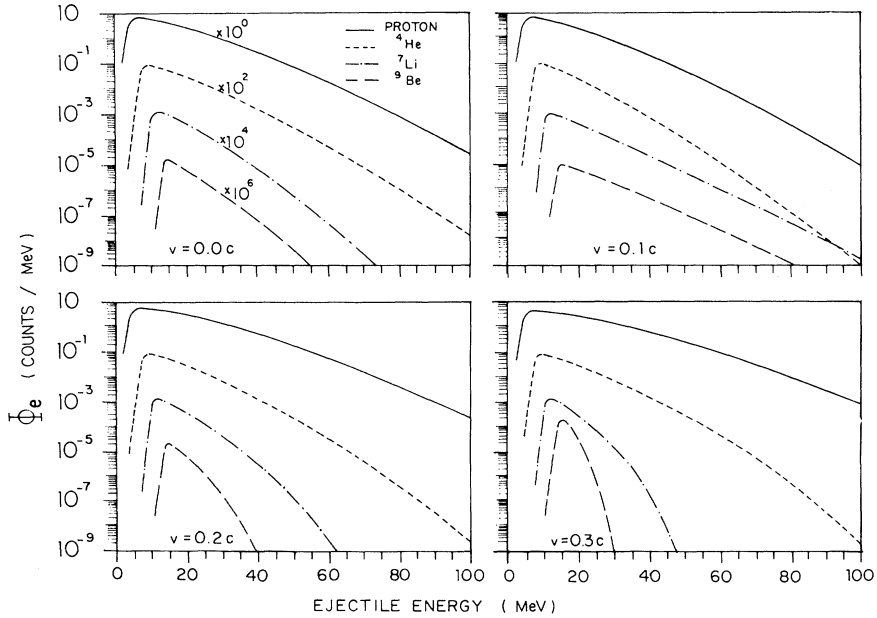

FIG. 4. (a) The emission probabilities are plotted against the ejectile energies for the various ejectiles: (i) proton (ii) ^{4}He, (iii) ^{7}Li, and (iv) ^{9}Be, for $v=0$. Each graph is normalized to 100. (b) Same as (a) for $v=0.1c$. (c) Same as (a) for $v=0.2c$. (d) Same as (a) for $v=0.3c$.

is shown in Fig. 2. At high temperatures, the internal contribution ρ_I, which follows the relation $\rho_I \propto \exp(2aT)$, increases exponentially with T and therefore the net nuclear level density also shows exponential behavior. This is shown in Fig. 3.

In Ref. [1], the emission probabilities are calculated by fitting the data using a Maxwell-Boltzmann distribution in the moving frames of the emitting systems. Here we calculate the emission probability using quantal Fermi-Dirac statistics, to calculate the phase space available for the light charged-particle emission.

The emission probability is written as [10]

$$\Phi_e = C\rho_m(U)E_e T_l \ , \tag{30}$$

where ρ_m, the level density [Eq. (29)], is a function of the effective excitation energy U and the average velocity of the fused compound system [6]. $U = E^* - S_e - E_e$, where $E^* = E(T) - E(0)$, S_e is the ejectile separation energy, and E_e is the ejectile energy. T_l is the transmission probability. For neutron emission $T_l = 1$ and for charged particles $T_l < 1$ [11].

In Fig. 4, the emission probabilities in the reaction $^{15}N + ^{56}Fe \rightarrow ^{71}As$ are plotted against the ejectile energies for different light charged particles. All the plots are normalized to 100. The peaks are shifted towards higher energies as the ejectile mass increases. This shows that at lower energies the proton emission is more probable.

In conclusion, we state that a method of evaluating nuclear level densities in heavy-ion reactions, when the fused compound is highly excited and moves with velocities greater than the average nucleonic velocities, is presented. Its application is illustrated in the reaction $^{56}Fe(^{15}N,x)B$. The effect of translational motion on nuclear level density at a given temperature is illustrated in the Fermi gas model as well as in the discrete single-particle model. An extension of this method to rotating nuclei in motion is underway.

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