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^{6}He beta decay to the $\alpha + d$ channel in a three-body model

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^{6}He beta decay to $\alpha + d$ continuum is studied in a three-body $\alpha + N + N$ approach. The branching ratio for beta-delayed deuteron emission is found to be $(3-4) \times 10^{-5}$ for three-body ^{6}He and ^{6}Li wave functions reproducing the ft value for ^{6}He with great accuracy. The results are sensitive to the neutron halo structure of the ^{6}He nucleus.

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The development of relativistic heavy ion physics and of modern experiments with radioactive nuclear beams opens new possibilities to investigate the structure of neutron drip line nuclei. Up to now, various types of nuclear reactions are the only way to obtain reliable information on their structure. Some of them (^{6}He, ^{11}Li, ^{14}Be, ^{17}B) have very specific structures such as the neutron halo, revealed in the enormously large cross sections, the narrow momentum distributions, and the large Coulomb dissociation cross sections [1]. From these properties the problem of the halo structure arises, particularly the neutron correlations in the halo. Note that ^{6}He has many common features with ^{11}Li from both experimental and theoretical points of view (see, e.g., [2]).

Numerous calculations of $A = 6$ nuclei [3–6] have shown the three particle $\alpha + N + N$ structure of the ground and low lying states. In the papers [5, 6] strong correlations between valence neutrons (configurations of "cigar" and "dineutron" type) in the ^{6}He ground state wave function (WF) were found. Such correlations may reveal themselves in high energy ^{6}He fragmentation reactions [7]. Another experiment that could reveal the correlations is the ^{6}He β decay into the ^{6}Li continuum. It is known that the ^{6}He half-life time is characterized by the fact that the Gamow-Teller transition into the ^{6}Li ground state has the largest matrix element $B_{\rm GT}$ among the nuclei. This matrix element depends mainly on the $S = 0$ and $S = 1$ component ratio in the ^{6}He and ^{6}Li ground state WF's while the corresponding radial WF's in these nuclei are very similar and their overlap is of order ~ 0.95 [8].

Recently, a unique experiment concerning β decay of the ^{6}He nucleus to the $\alpha + d + e^- + \bar{\nu}$ channel was carried out [9]. The experimental branching ratio was estimated to be $B = (2.8 \pm 0.5) \times 10^{-6}$. From a theoretical point of view, there are at least two reasons why the ^{6}He β decay into the ^{6}Li two-body continuum has a very low branching compared to the β decay to the ^{6}Li ground state in spite of the fact that in both cases the Gamow-Teller transition dominates. First of all, it is connected with the orthogonality of the $\alpha + d$ channel WF to that of the ground state (^{6}Li). Qualitative considerations point out the peripheral character of the reaction since the ^{6}He and ^{6}Li ground state WF's differ mainly in the asymptotic region due to different binding energies. (In the inner region the difference of the WF's related to its magnitude is really negligible). Second, there is an additional difference between the $\alpha + d$ channel and ^{6}He in the peripheral region. In this region for the $\alpha + d$ channel only deuteron correlation exists whereas in the ^{6}He ground state both "dineutron" and "cigar" types of correlations [5, 6] are very important. Therefore the overlap integral for the Gamow-Teller transition to the continuum will select only "dineutron" configuration. This means that the overlap integral would contain information about the neutron halo structure of the ^{6}He nucleus. This provides an opportunity to extract information on valence neutron correlations in ^{6}He from the $^6\text{He} \to \alpha + d + e^- + \bar{\nu}$ process. In [10] the β-delayed deuteron emission of ^{6}He was calculated in a two-body potential model assuming that both valence neutrons in ^{6}He form a "dineutron." The calculated branching ratio is 2.0×10^{-4}. This result is considerably larger than the experimental value. Therefore it is necessary to provide a more detailed study of the ^{6}He β decay to the $\alpha + d$ continuum in a framework of a three-body model where the neutron halo structure of ^{6}He would reveal itself in a more transparent way.

Below, we present a theoretical calculation of this process based on a three-body $\alpha + N + N$ model. In this model we apply three-body ^{6}He and ^{6}Li ground state WF's obtained using the hyperspherical harmonics method (HH) [6, 11]. It is convenient to introduce translation invariant Jacobi coordinates $\mathbf{x} = \sqrt{\frac{1}{2}}\mathbf{r}_{12}$, $\mathbf{y} = \sqrt{\frac{4}{3}}\,\mathbf{r}_{(12)3}$, where $\mathbf{r}_{12}$ is the distance between the two valence nucleons and similarly $\mathbf{r}_{(12)3}$ is the distance between c.m. of the valence nucleons and the α particle.

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In the HH method, the WF's of three particle relative motion (in LS representation) are

$$\Psi^{T}_{JM}(\mathbf{x},\mathbf{y}) = \rho^{-\frac{5}{2}} \sum \chi^{LS}_{Kl_x l_y}(\rho) \left\{ \mathcal{Y}^{l_x l_y}_{KL}(\Omega_5) \otimes X_S \right\}_{JM} X_{TM_T} \quad (1)$$

where the *hyperradius* $\rho = (x^2 + y^2)^{1/2}$ is a collective translationally, rotationally, and permutationally invariant variable. The HH basis functions $\mathcal{Y}^{l_x l_y}_{KL}(\Omega_5)$ are functions of the five angles $\Omega_5 = (\theta, \hat{x}, \hat{y})$ with $\theta = \arctan(x/y)$ and $X_S(1,2)$ is the coupled two-nucleon spin function and likewise for isospin. The extra quantum number $K = l_x + l_y + 2n$ (n=0,1,2,...) is called the *hypermoment.* The explicit form for $\mathcal{Y}^{l_x l_y}_{KL}(\Omega_5)$ is given in [6, 11].

Inserting the WF (1) in the three-body Schrödinger equation with binary interaction potentials describing the dynamics of the binary subsystems, the corresponding ^{6}He and ^{6}Li ground state WF's have been obtained [6, 11]. These WF's were tested [11] on a large set of experimental data and, in particular, the *ft* value for the ^{6}He β decay to the ^{6}Li ground state was reproduced with an accuracy of 2–3%. In the following we will use the same WF's. To simplify the calculations we will use only terms with K=0,2 from the WF expansion (1) because only these terms give significant contributions to the ^{6}He and ^{6}Li ground state norm values (about 96%) and their overlap integrals. The spatial part of the ^{6}He WF with quantum numbers $L = S = 0$, $l_x = l_y = 0$ (the sum of K=0,2) is shown in Fig. 1. It is easy to see from Fig. 1 that this WF contains two types of strong correlations between valence neutrons. One of them looks like a "dineutron" (two neutrons are close to each other and their c.m. is far from the α particle). Another one looks like a "cigar" (the valence neutrons are on opposite sides of the α particle). The origin of these correlations is the so-called Pauli focusing quantum effect [6], caused by approximate dominance of a single K=2 component with the aforementioned quantum numbers in the framework of the HH method.

The $\alpha + d$ channel WF was chosen in the product form $\Psi_{\alpha+d} = \phi_d(\mathbf{x})F(k\mathbf{y})$ where ϕ_d is an internal deuteron WF and $F(k\mathbf{y})$ is the $\alpha + d$ scattering WF with corresponding asymptotic behavior. [For simplicity we omitted the α-particle WF from this formula as well as from (1), and also the spin and isospin terms.] Taking into account the selection rules for the allowed Gamow-Teller transition we can rewrite the $\alpha + d$ channel WF as $\Psi_{\alpha+d} = \phi_d(x)F_{l=0}(ky)$ where ϕ_d contains only s-wave components and $F_{l=0}(ky)$ is a pure s-wave $\alpha + d$ scattering WF.

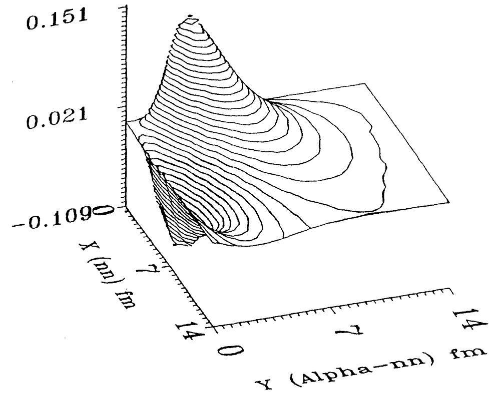

FIG. 1. The spatial part of the ^{6}He wave function (1) with quantum numbers $L = S = 0$, $l_x = l_y = 0$.

To study the sensitivity of the results to the size of the deuteron and the form of the deuteron WF we choose ϕ_d in analytical form with the correct asymptotic behavior at the origin and at large distances. The rms matter radius of the deuteron was varied between 1.6 and 2.4 fm and the norm value of ϕ_d was fixed at $\sim$ 95% in correspondence with the "real" weight of the s-wave component in the deuteron WF.

For the allowed β transition, the final state WF must be orthogonalized (it is not an eigenfunction of the three-body Hamiltonian) to the ^{6}Li ground state WF. Taking into account that the Gamow-Teller transition operator $\hat{O} = \boldsymbol{\sigma}\tau_-$ does not contain spatial coordinates, it is easy to show that instead of orthogonalization of the exit channel WF we can use the spatial part of the ^{6}He WF with S=0 orthogonalized to the spatial part of the ^{6}Li WF with S=1. This orthogonalized ^{6}He WF - $\Psi^{^6\mathrm{He}}_{\mathrm{ort}}$ has the form

$$\Psi^{^6\mathrm{He}}_{\mathrm{ort}} = \sum_{K=0,2} \left(|\mathrm{He}, K\rangle - |\mathrm{Li}, K\rangle \frac{\sum_{K'=0,2}\langle \mathrm{Li}, K' | \mathrm{He}, K'\rangle}{\sum_{K''=0,2}\langle \mathrm{Li}, K'' | \mathrm{Li}, K''\rangle} \right) , \quad (2)$$

where the brackets $|\mathrm{He}, K\rangle$ and $|\mathrm{Li}, K\rangle$ on the right side of (2) mean only the coordinate parts of the ^{6}He and ^{6}Li WF's (1).

The orthogonalized ^{6}He WF $\Psi^{^6\mathrm{He}}_{\mathrm{ort}}$ (2) is shown in Fig. 2. Comparing Figs. 1 and 2, it is easy to see that the orthogonalization is very essential and changes the initial ^{6}He WF drastically. First of all, the WF becomes smaller by the order of magnitude and, second, the "dineutron" correlation, giving the maximum overlap with the $\alpha + d$ channel, is pushed out for large distances 8–12 fm (it is connected with a lower binding energy of ^{6}He as compared to ^{6}Li) and is essentially broader than in the nonorthogonalized WF.

Finally for fixed $\Psi^{^6\mathrm{He}}_{\mathrm{ort}}$ and $\Psi_{\alpha+d}$ WF's, the branching ratio for the β decay to the $(\alpha + d)$ channel per energy unit (in the no-recoil approximation) can be written

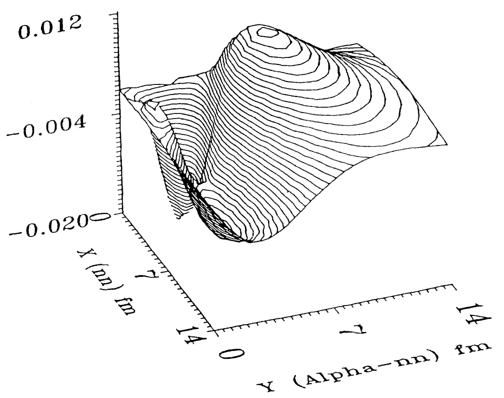

FIG. 2. The spatial part of the orthogonalized ^{6}He wave function (2) with quantum numbers $L = S = 0$, $l_x = l_y = 0$.

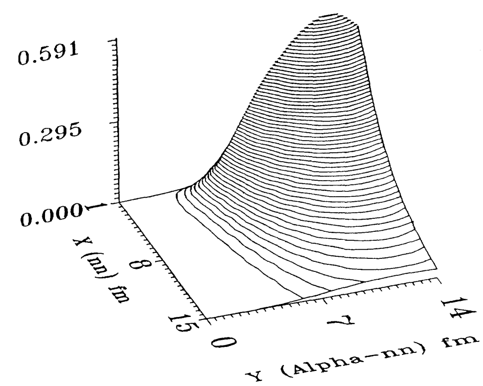

FIG. 3. The spatial part of the $\alpha + d$ channel WF with quantum numbers L=0, $l_x = l_y = 0$ for the energy E_d=0.6 MeV and pure repulsive $\alpha + N$ potential from Table I.

$$\frac{dB}{dE} = \frac{dW(^{6}\mathrm{He} \to \alpha + d + e^- + \bar{\nu})}{W(^{6}\mathrm{He} \to ^{6}\mathrm{Li}_{\mathrm{g.s.}})dE} = \frac{1}{2\pi^2 v}\frac{B_{\mathrm{GT}}(E)}{B_{\mathrm{GT0}}}\frac{f(Q-E)}{f_0}, \quad (3)$$

$$B_{\mathrm{GT}}(E) = 6\left|\int \phi_d(\mathbf{x})F(k\mathbf{y}) \cdot \Psi_{\mathrm{ort}}^{^{6}\mathrm{He}}(\mathbf{x},\mathbf{y})d\mathbf{x}\,d\mathbf{y}\right|^2, \quad (4)$$

where k and v are the wave number and the relative velocity in the $\alpha+d$ channel at c.m. energy E, $Q = 2.543$ MeV is the threshold energy with respect to the $\alpha + d$ channel, $f(Q-E)$ is the space volume integral, and f_0 the space volume integral for the β decay to the ^{6}Li ground state. We choose $B_{\mathrm{GT0}} = 4.84$ from [11] where this value was calculated with the same WF's (1).

The $\alpha + d$ relative motion WF's from (4) were calculated for Woods-Saxon potentials with parameters $V_0 = -78.0$ MeV, $R = 1.85$ fm, a=0.71 fm [12] (attractive potential with forbidden states) and V_0=60.0 MeV, R=3.7 fm, a=0.71 fm (pure repulsive potential, fitted to the experimental $\alpha + d$ s-wave phases). Note that the pure repulsive $\alpha + d$ s-wave potential is more consistent with our three-body WF's because these WF's also have been obtained for the $\alpha + N$ potentials containing the s-wave Pauli repulsive cores. The $\alpha + d$ channel WF for the energy E_d=0.6 MeV and pure repulsive $\alpha+N$ potential is shown in Fig. 3. It is important to stress that for both types of the potential, the relative motion WF's are strongly enhanced at the surface region (at $\alpha+d$ distances ~10–25 fm) and suppressed in the interior as shown in Fig. 3 (see also [10]). This phenomenon is mainly connected with small relative energies in the $\alpha + d$ channel and the Pauli principle for the α particle and the deuteron. Comparing Figs. 2 and 3 we may expect that mainly "dineutron" configuration from Fig. 2 will contribute to the overlap integral (4).

Results of the branching ratio calculations for the ^{6}He β decay to the ($\alpha+d$) channel are shown in Table I where we also include in the first row the result for the $\alpha+d$ relative motion WF's in the plane wave approximation, and in the second row the result for the $\alpha+d$ relative motion WF's obtained with pure soft Coulomb repulsion between the α particle and deuteron (taken as for homogeneously charged sphere R=1.7 fm). In the third and fourth rows are shown the results for Woods-Saxon attractive and repulsive $\alpha + d$ interactions. The columns correspond to the deuteron WF's giving different rms deuteron radii. The upper value in each row is the total branching ratio and the lower one is the branching ratio for deuterons with energy more than 370 keV. The new experimental branching ratio [13] is $B(E_d >370\ \mathrm{keV})=(8 \pm 1) \times 10^{-6}$.

The calculated deuteron spectrum (for the rms R_d=2.1 fm and pure repulsive $\alpha+d$ interaction) together with the experimental data [13] are shown in Fig. 4.

The proximity of the results shown in Table I for principally different $\alpha + d$ potentials means that the inner structure of the WF's is not as important for the problem as their asymptotic behavior. Monotonous and weak growth of the branching ratio with deuteron rms radius shows that mainly dineutron configuration in ^{6}He makes a contribution in the overlap integral (4). It means that the effective size of the dineutron configuration in the orthogonalized ^{6}He WF (2) is larger than the size of the real deuteron. It should be stressed here that the main effect, which is responsible for the stability of the results (Table I) within the order of magnitude despite a quite different choice of the exit channel WF's, is the effect of the orthogonalization (2).

Figure 4 (and also Table I) shows that for all the

TABLE I. Branching ratios in 10^{-5} units.

$\alpha + d$ channel	$\langle R_d^2\rangle^{1/2}$, fm		
	1.64	2.1	2.44
Plane		23.7	
wave		1.6	
Soft		11.5	
Coulomb		3.6	
$V_0 = -78$ MeV, R=1.85 fm	6.40	8.45	10.4
(attractive)	2.56	4.06	4.14
V_0=60 MeV, R=3.71 fm	4.79	7.07	8.38
(repulsive)	1.81	3.05	3.15

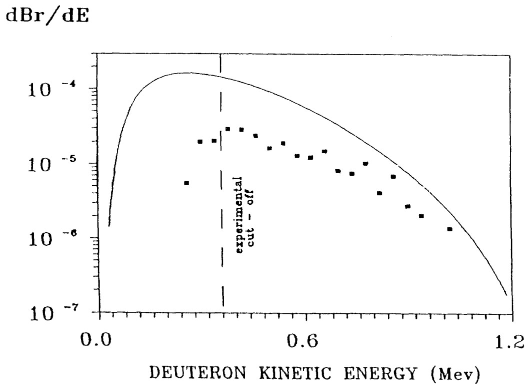

FIG. 4. Branching ratio dB/dE (3) energy distribution (in MeV^{-1} units). The solid line corresponds to the results of the calculations with repulsive $\alpha + d$ potential (fourth row from Table I). The experimental points are taken from [13].

considered cases, the maximum of the calculated energy distributions is situated below the experimental cutoff threshold. It means that in recent experiments only the tail of the distribution has been measured. So it is possible to expect that the registration effectiveness near the threshold is essential and this fact might change the experimental data to some extent.

Our results overestimate the experimental branching ratio [13], but nevertheless they are approximately 3–4 times smaller (and much closer to the experimental data) than the result of the calculations [10] in the framework of the two-body model.

There are few reasons for such a discrepancy with the results [10]. First of all, the neutron correlations in ^{6}He lead to two distinct spatial configurations having approximately equal weights—"cigar" and "dineutron" (see Fig. 1). The "cigar" type term of the three-body ^{6}He WF almost does not contribute to the overlap integral (4). But if we want to make a calculation with some model "dineutron" WF, we have to reduce its value by a factor $2^{1/2}$. It means that B would decrease by a factor of 2 in comparison with pure dineutron WF [10]. Second, the overlap integral between deuteron WF and dineutron configuration from (2) is less than 0.9 (as it was assumed in [10]). It is seen from Table I that B as a function of deuteron radius R_d achieves its maximum for R_d values larger than the real rms deuteron radius. The third factor, which decreases the B value in comparison with the two-body model, is the correct three-body asymptotic of the ^{6}He WF which decreases more rapidly than for the two-body model. This fact was demonstrated in the two-body model [10] by the essential increasing of the ^{6}He binding energy as compared with the experimental one.

The three-body approach described above is also not completely self-consistent because the final state WF is not an exact eigenfunction of the three-body Hamiltonian. In reality the deuteron is a very soft nucleus easily deformed near the α particle. This effect should be seen in a strict three-body calculation of the $\alpha + d$ continuum WF's. However, it is well known from the resonating group method [14] that the deuteron polarization is important only at distances comparable with the α-particle radius. So we may believe that strict three-body calculations will change only the internal part of the $\alpha + d$ channel WF, as the B value depends mainly on the external part of this WF. Nevertheless, strict three-body calculations with the continuum binary channel would be very useful.

Concluding, the ^{6}He beta decay to $\alpha + d$ continuum was studied in the three-body $\alpha + N + N$ approach. The branching ratio for beta-delayed deuteron emission is found to be $(3\text{–}4)\times 10^{-5}$ for three-body ^{6}He and ^{6}Li wave functions reproducing the ft value for ^{6}He with great accuracy. The results are sensitive to the neutron halo structure of the ^{6}He nucleus. So this reaction provides a very interesting possibility to extract information about the valence neutron correlations in the ^{6}He nucleus.

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