QCD sum rules for nucleons in nuclear matter II

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A correlator of QCD interpolating fields for the nucleon is evaluated in nuclear matter by applying an operator product expansion. All density dependence is incorporated into the composite operator matrix elements (condensates) taken in the nuclear matter ground state. The in-medium condensates up to dimension five are estimated to first order in the nucleon density. The key phenomenological inputs used in determining the quark and gluon condensates up to dimension four are the nucleon σ term, the strangeness content of the nucleon, and quark and gluon distribution functions deduced from deep-inelastic scattering experiments. Dimension-five quark and quark-gluon condensates, which give small contributions to the correlator, are estimated by a variety of techniques. Contributions from dimension-six four-quark condensates are also included in a factorized form.

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I. INTRODUCTION

A major goal of modern nuclear theory is to relate nuclear physics phenomenology to quantum chromodynamics (QCD), the underlying theory of the strong interaction. One promising approach applies QCD sum-rule methods [1] at finite density. In a recent series of papers [2–4], sum rules for nucleons in nuclear matter have been developed, with an emphasis on testing the predictions of relativistic nuclear phenomenology for nucleon self-energies. In this paper we continue the development.

The QCD sum-rule approach was introduced by Shifman, Vainshtein, and Zakharov [1] to predict features of low-energy hadronic physics in the absence of a full solution to QCD at large distances. The goal of the sum rules is to calculate resonance properties (e.g., masses) in terms of QCD Lagrangian parameters and vacuum matrix elements of composite quark and gluon operators (condensates), which are used to parametrize nonperturbative physics. Numerous applications of these sum rules in recent years have been phenomenologically successful and have provided insight into the nature of hadrons [5–9].

QCD sum rules for nucleons in nuclear matter focus on a correlation function of interpolating fields, built from quark fields, that have the quantum numbers of a nucleon [see Eqs. (3.1) and (3.4)]. The analytic properties of this correlator as a function of energy can be manifested by a Lehmann representation; the nucleon self-energies characterize the singularities associated with a quasinucleon excitation. By introducing a simple ansatz for the spectral density, guided by constraints from covariance and from nuclear phenomenology, one obtains a representation of the correlator throughout the complex energy plane. On the other hand, one can apply an operator product expansion (OPE) to calculate the correlator in regions of the energy plane far away from the quasinucleon singularities. By using these two different representations in appropriately weighted integrals, we obtain QCD sum rules that relate the nucleon spectral properties (such as the quasinucleon self-energies) to QCD Lagrangian parameters and finite-density condensates [4].

A truncated version of the finite-density nucleon sum rules indicates that large and canceling Lorentz scalar and vector self-energies arise naturally in finite-density QCD due to changes in the quark condensate and the quark density [2,4]. Such self-energies are consistent with those predicted by relativistic nuclear phenomenology [10–13]. In Ref. [4], these simple sum rules were extended by considering the contributions of higher-energy states and higher-order terms in the OPE. In this paper we expand on the discussion of the OPE given in Refs. [2,4] and give details on the calculation of the nucleon correlator at finite density. We adopt simplified rules for this calculation, based on the usual treatment of vacuum sum rules, in which all nonperturbative physics and density dependence resides in the in-medium condensates, which have coefficient functions constructed in leading-order perturbation theory. We estimate a number of condensates not considered in Ref. [3]. Some of these results have already been used in Ref. [4]. Additional numerical details of the sum rules will be reported in a future work [14].

At zero density, the Lorentz invariance of the vacuum implies that only matrix elements of scalar operators are nonvanishing. At finite density, the ground state is no longer Lorentz invariant; however, matrix elements in this state do have well-defined Lorentz transformation properties. In medium there is an additional four-vector u^μ, the nuclear matter four-velocity, that must be transformed when comparing observations in different reference frames and must be included when building tensors or identifying invariant functions. As a result, expectation values of local operators with any integer spin can be nonzero in the medium. Thus the medium implies not only density dependence for condensates present in the vacuum sum rules, but a large number of new terms in the sum rules that are proportional to new condensates. For the sum rules to be practical, we must find phenomenological estimates for the condensates that make the largest numerical contributions. We consider the contributions from dimension-four gluon condensates and from quark and quark-gluon condensates up to dimension five; in addition, we include contributions from the dimension-six four-quark condensates that arise at tree level.

The in-medium condensates up to dimension five are estimated to first order in the nucleon density ρ_N. We approximate the ground-state expectation value of an operator $\hat{O}$ as $\langle \hat{O} \rangle_{\rho_N} = \langle \hat{O} \rangle_{\text{vac}} + \langle \hat{O} \rangle_N \rho_N$, where $\langle \hat{O} \rangle_{\text{vac}}$ is the vacuum expectation value of the operator and $\langle \hat{O} \rangle_N$ is the expectation value in a nucleon state (at rest). Contributions of higher order in the nucleon density due to the Fermi motion of the nucleons and interactions between nucleons are neglected, so the first-order density expansion is only accurate at sufficiently low densities. In the case of the quark condensate $\langle \bar{q}q \rangle_{\rho_N}$, model calculations suggest that such higher-order contributions are less than 20% the size of the linear contribution at nuclear matter saturation density [3]. However, the models considered in Ref. [3] are not sophisticated enough to allow a definitive conclusion. We assume in the present work that the first-order density expansion gives a reasonable estimate of the in-medium condensates up to saturation density. Such a treatment should be suitable for estimating the sizes of the scalar and vector nucleon self-energies; however, it is not sufficient for an analysis of nuclear matter saturation, which depends on the interplay of the scalar and vector self-energies as a function of density.

The dimension-three quark condensate $\langle \bar{q}q \rangle_{\rho_N}$ is related to the nucleon σ term, and $\langle q^\dagger q \rangle_{\rho_N}$ is exactly proportional to the nucleon density. The dimension-four quark condensate $\langle \bar{q}iD_0 q \rangle_{\rho_N}$ is also exactly proportional to the nucleon density; $\langle q^\dagger iD_0 q \rangle_{\rho_N}$ depends on quark and antiquark distribution functions measured in deep-inelastic scattering [15]. The two dimension-four gluon condensates, proportional to $\langle \mathbf{E}^2 - \mathbf{B}^2 \rangle_{\rho_N}$ and $\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle_{\rho_N}$, are estimated in terms of the σ term, the strangeness content of the nucleon, and the gluon distribution function in the nucleon. Dimension-five quark and quark-gluon condensates, $\langle \bar{q}iD_0 iD_0 q \rangle_{\rho_N}$, $\langle q^\dagger iD_0 iD_0 q \rangle_{\rho_N}$, $\langle g_s \bar{q}\sigma\cdot\mathcal{G} q \rangle_{\rho_N}$, and $\langle g_s q^\dagger \sigma\cdot\mathcal{G} q \rangle_{\rho_N}$, are estimated in terms of quantities such as quark and antiquark distribution functions and the coefficient of the leading power correction to the Gross–Llewellyn Smith sum rule.

Following standard practice in vacuum sum rules, we estimate the dimension-six four-quark condensates using the factorization, or ground-state saturation, approximation. Thus the in-medium four-quark condensates are estimated in terms of $\langle \bar{q}q \rangle^2_{\rho_N}$, $\langle \bar{q}q \rangle_{\rho_N} \langle q^\dagger q \rangle_{\rho_N}$, and $\langle q^\dagger q \rangle^2_{\rho_N}$. It is not obvious that such a treatment of the four-quark condensates is justified at finite density; a sum-rule analysis of nucleon self-energies suggests that the factorization approximation may strongly overestimate the density dependence of the four-quark condensates [4].

There have been several recent papers that discuss similar or related topics. Drukarev and Levin [16,17] have studied QCD sum rules for nucleons in nuclear matter using an OPE and dispersion relations that differ from those considered here and in Ref. [4]. They have focused on the properties of nuclear matter, such as the saturation curve. We note that an accurate treatment of nuclear matter saturation requires detailed knowledge of the density dependence of the condensates, particularly the in-medium quark condensate $\langle \bar{q}q \rangle_{\rho_N}$. In Refs. [16,17], $\langle \bar{q}q \rangle_{\rho_N}$ is estimated using a simple nonrelativistic model of nuclear matter in which uncorrelated nucleons interact via single massless pion exchange. It is not obvious that this treatment of $\langle \bar{q}q \rangle_{\rho_N}$ is sufficiently realistic to allow a meaningful description of saturation. In Refs. [18,19] nucleon sum rules were used to estimate the density dependence of the neutron-proton mass difference, which might account for the Nolen-Schiffer anomaly. Sum rules for vector mesons in nuclear matter were discussed in Ref. [15], which includes estimates of various condensates that are also relevant for nucleon sum rules.

The outline of this paper is as follows. In Sec. II we review the QCD vacuum condensates and establish notation. An OPE for the nucleon correlator in nuclear matter is developed in Sec. III, and the in-medium condensates that appear are estimated in Sec. IV. Section V is a summary. Some additional results and details are given in the Appendixes.

II. QCD VACUUM CONDENSATES

In this section we review the basic QCD field operators and vacuum condensates in order to establish our notation and conventions. We start with the fields for the up, down, and strange quarks, $u_{a\alpha}$, $d_{a\alpha}$, and $s_{a\alpha}$, where we use $a,b,c,\ldots=1-3$ for quark color indices and $\alpha,\beta,\gamma,\ldots=1-4$ for Dirac indices. In this paper we are mainly concerned with up and down quark degrees of freedom. Due to isospin symmetry, we often do not distinguish between these two quark flavors; we use $q_{a\alpha}$ to refer to either an up or down quark field.

The gluon field is denoted A^A_λ, where we use $A,B,C,\ldots=1-8$ for gluon color indices and $\lambda,\mu,\nu,\ldots=0-3$ for Lorentz indices. A matrix form of the gluon field is obtained by multiplying the color components of the gluon field by the generators of the SU(3) Lie algebra:

$$\mathcal{A}^\mu_{ab} \equiv A^{A\mu} t^A_{ab} \;, \tag{2.1}$$

where $t^A \equiv \lambda^A/2$ are the SU(3) generators in the fundamental representation (λ^A are the Gell-Mann matrices [20]). They satisfy the following relations:

$$[t^A,t^B]=if^{ABC}t^C,\quad \mathrm{tr}(t^A)=0,\quad \mathrm{tr}(t^At^B)=\tfrac{1}{2}\delta^{AB}\ , \qquad (2.2)$$

where f^{ABC} are totally antisymmetric structure constants [20] and tr denotes a trace over quark color indices.

It is useful to introduce the gluon field tensor, which can be defined as

$$\mathcal{G}_{\mu\nu}\equiv G^A_{\mu\nu}t^A\equiv D_\mu\mathcal{A}_\nu-D_\nu\mathcal{A}_\mu\ , \qquad (2.3)$$

where $D_\mu\equiv\partial_\mu-ig_s\mathcal{A}_\mu$ is the covariant derivative (g_s is the quark-gluon coupling constant). Useful identities that follow from Eqs. (2.2) and (2.3) are

$$\mathcal{G}_{\mu\nu}=\frac{i}{g_s}[D_\mu,D_\nu] \qquad (2.4)$$

and

$$G^A_{\mu\nu}=\partial_\mu A^A_\nu-\partial_\nu A^A_\mu+g_sf^{ABC}A^B_\mu A^C_\nu\ . \qquad (2.5)$$

We also define the dual of the gluon field tensor,

$$\tilde{G}^{A\kappa\lambda}\equiv\tfrac{1}{2}\epsilon^{\kappa\lambda\mu\nu}G^A_{\mu\nu}\ . \qquad (2.6)$$

The quark fields are coupled to the gluon field by replacing ordinary derivatives with covariant derivatives in the free quark Lagrangians. Thus the quark fields q and $\bar{q}$ (with current quark mass m_q) satisfy the following equations of motion:

$$(i\not{D}-m_q)q=0,\quad \bar{q}(i\overleftarrow{\not{D}}+m_q)=0\ , \qquad (2.7)$$

where $\overleftarrow{D}_\mu\equiv\overleftarrow{\partial}_\mu+ig_s\mathcal{A}_\mu$. These equations of motion will be used in the subsequent discussion to simplify operator matrix elements. In nucleon sum rules, the contributions from finite up and down current quark masses are numerically small; therefore, we neglect these contributions.

We now consider vacuum condensates, which are nonvanishing expectation values of composite operators built from the fundamental quark and gluon fields. The Lorentz invariance of the vacuum state $|\mathrm{vac}\rangle$ dictates that only spin-0 operators can have nonvanishing vacuum expectation values; the lowest-dimensional vacuum condensates are

$$\begin{aligned}&\langle\bar{q}q\rangle_{\mathrm{vac}},\quad d=3\ ,\\ &\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}},\quad d=4\ ,\\ &\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\mathrm{vac}},\quad d=5\ ,\\ &\langle\bar{q}\Gamma_1q\bar{q}\Gamma_2q\rangle_{\mathrm{vac}},\quad d=6\ ,\\ &\langle g_s^3fG^3\rangle_{\mathrm{vac}},\quad d=6\ ,\end{aligned} \qquad (2.8)$$

where d denotes the mass dimension of the condensate. We take $\sigma_{\mu\nu}\equiv i[\gamma_\mu,\gamma_\nu]/2$; we have also introduced the notation $\langle\hat{O}\rangle_{\mathrm{vac}}\equiv\langle\mathrm{vac}|\hat{O}|\mathrm{vac}\rangle$, $G^2\equiv G^A_{\mu\nu}G^{A\mu\nu}$, $\sigma\cdot\mathcal{G}\equiv\sigma_{\mu\nu}\mathcal{G}^{\mu\nu}$, and $fG^3\equiv f^{ABC}G^{A\mu}_\lambda G^{B\nu}_\mu G^{C\lambda}_\nu$. Other condensates, such as $\langle\bar{q}D^2q\rangle_{\mathrm{vac}}$, can be related to those listed in Eq. (2.8) by using the field equations.

We first consider the quark condensate $\langle\bar{q}q\rangle_{\mathrm{vac}}$. Due to isospin symmetry one has

$$\langle\bar{u}u\rangle_{\mathrm{vac}}\simeq\langle\bar{d}d\rangle_{\mathrm{vac}}\equiv\langle\bar{q}q\rangle_{\mathrm{vac}}\ . \qquad (2.9)$$

The numerical value of $\langle\bar{q}q\rangle_{\mathrm{vac}}$ can be determined from the Gell-Mann–Oakes–Renner relation,

$$(m_u+m_d)\langle\bar{q}q\rangle_{\mathrm{vac}}=-m_\pi^2f_\pi^2[1+O(m_\pi^2)]\ , \qquad (2.10)$$

where m_π and f_π are the pion mass and pion decay constant, and m_u and m_d are the up and down current quark masses. Both sides of Eq. (2.10) are renormalization-group invariant [21]; therefore, given the current quark masses at a particular renormalization scale, one can determine the quark condensate at that same scale. We take $m_\pi=138$ MeV and $f_\pi=93$ MeV; using the standard values of the light quark masses one obtains $m_u+m_d=14\pm4$ MeV at a renormalization scale of 1 GeV [22]. Thus one has

$$\langle\bar{q}q\rangle_{\mathrm{vac}}\simeq-(0.225\pm0.025\ \mathrm{GeV})^3 \qquad (2.11)$$

at a renormalization scale of 1 GeV [22].

The gluon condensate was first estimated from an analysis of leptonic decays of ρ^0 and ϕ^0 mesons [23] and from a sum-rule analysis of the charmonium spectrum [1]. Its numerical value is taken to be [24]

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}}\simeq(0.33\pm0.04\ \mathrm{GeV})^4\ . \qquad (2.12)$$

(Also see Ref. [24] for a discussion of lattice QCD extractions of the gluon condensate.) Note that the product $(\alpha_s/\pi)G^2$ is approximately renormalization-group invariant; violations of renormalization-group invariance are of higher order in α_s [21].

The quark-gluon condensate $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\mathrm{vac}}$ is expressed in terms of the quark condensate $\langle\bar{q}q\rangle_{\mathrm{vac}}$:

$$\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\mathrm{vac}}=2\langle\bar{q}D^2q\rangle_{\mathrm{vac}}\equiv2\lambda_q^2\langle\bar{q}q\rangle_{\mathrm{vac}}\ , \qquad (2.13)$$

where we have used Eqs. (2.4) and (2.7) to obtain the first equality. Thus λ_q^2 parametrizes the average vacuum gluon field strength and the average virtuality (momentum squared) of the quarks in the QCD vacuum. The standard QCD sum-rule estimate of this quantity is $\lambda_q^2=0.4\pm0.1$ GeV^2 [6,25]. Somewhat larger values have been obtained from a lattice calculation ($\lambda_q^2=0.55\pm0.05$ GeV^2) [26] and a QCD sum-rule analysis of the pion form factor using nonlocal quark and gluon condensates ($\lambda_q^2=0.7\pm0.1$ GeV^2) [27]. A much larger value for λ_q^2 is obtained in a sum-rule analysis of the pion wave function using nonlocal condensates [28]. The value suggested by this analysis is $\lambda_q^2\simeq1.2$ GeV^2, which agrees with the value obtained with an instanton liquid model [29].

In QCD sum-rule applications, higher-dimensional condensates are usually approximated in terms of $\langle\bar{q}q\rangle_{\mathrm{vac}}$ and $\langle(\alpha_s/\pi)G^2\rangle_{\mathrm{vac}}$. For example, the four-quark condensates are frequently estimated in terms of $\langle\bar{q}q\rangle^2_{\mathrm{vac}}$ by using the factorization, or vacuum-saturation, approximation. This approximation corresponds to inserting a complete set of intermediate states in the middle of the four-quark matrix element, but retaining only the dominant vacuum intermediate state. An analogous approximation is commonly used in many-body physics [30]. The factorization approximation has been justified in large-N_c QCD [31]; in QCD with $N_c=3$, it has been argued that the contribution to four-quark condensates from single-pion intermediate states (the lowest excitations of the vacuum) is small compared to that of the vacuum intermediate state [1]. Four-quark condensates in vacuum are thus estimated as

$$\langle \bar{q}\Gamma_1 q\bar{q}\Gamma_2 q\rangle_{\text{vac}} = \tfrac{1}{16}\langle\bar{q}q\rangle^2_{\text{vac}}[\text{Tr}(\Gamma_1)\text{Tr}(\Gamma_2) - \tfrac{1}{3}\text{Tr}(\Gamma_1\Gamma_2)] , \quad (2.14)$$

$$\langle \bar{q}\Gamma_1\lambda^A q\bar{q}\Gamma_2\lambda^A q\rangle_{\text{vac}} = -\tfrac{1}{9}\langle\bar{q}q\rangle^2_{\text{vac}}\text{Tr}(\Gamma_1\Gamma_2) , \quad (2.15)$$

where Γ_1 and Γ_2 are Dirac matrices, and Tr denotes a trace over Dirac indices. A more detailed discussion of the factorization approximation and the estimation of four-quark condensates at finite density, including those of mixed flavor (not shown here), is in Appendix A. Note that phenomenological studies [32,33] and instanton liquid models [24] suggest strong deviations from the factorized results in some cases.

The value of the three-gluon condensate has been estimated in terms of the two-gluon condensate using the dilute instanton gas approximation [1]:

$$\langle g_s^3 fG^3\rangle_{\text{vac}} \simeq \frac{48\pi^2}{5}\rho_c^{-2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\text{vac}} , \quad (2.16)$$

where the instanton size cutoff is $\rho_c \sim (200\ \text{MeV})^{-1}$. This instanton-based estimate agrees well with the phenomenological estimate $\langle g_s^3 fG^3\rangle_{\text{vac}} \simeq 0.06\ \text{GeV}^6$ [32].

Thus there are a small number of condensates up to dimension six that could contribute to nucleon sum rules in the vacuum. To generalize the sum rules to finite density, the density dependence of these condensates must be estimated. In addition, there are a number of new condensates that vanish in the vacuum, but are nonzero in nuclear matter.

III. NUCLEON CORRELATOR

In this section we review some basic features of the operator product expansion (OPE), which is then used to calculate the nucleon correlation function in nuclear matter. We work to leading order in perturbation theory; leading-logarithmic corrections can be included through anomalous-dimension factors. Contributions proportional to the up and down current quark masses are neglected, since they give numerically small contributions. In the OPE for the nucleon correlator, we consider pure gluon condensates up to dimension four and quark and quark-gluon condensates up to dimension five. At dimension six, we consider only the four-quark condensates, which give numerically important contributions to nucleon sum rules in free space [5] and in nuclear matter [4]. All other dimension-six and higher-dimensional condensates are neglected, since their contributions are expected to be small.

We calculate the nucleon correlator, which is the Fourier transform of a time-ordered correlation function of the nucleon interpolating field η (constructed from quark fields) evaluated in the ground state of nuclear matter $|\Psi_0\rangle$:

$$\Pi(q) \equiv i\int d^4x\, e^{iq\cdot x}\langle\Psi_0|T[\eta(x)\bar{\eta}(0)]|\Psi_0\rangle . \quad (3.1)$$

The nuclear matter ground state is characterized by the rest-frame nucleon density ρ_N and the four-velocity u^μ. We assume that the ground state is invariant under parity and time reversal.

We consider nucleon interpolating fields that contain no derivatives and couple to spin $\frac{1}{2}$ only. There are two linearly independent fields with these characteristics. One way to construct these independent fields is to consider the proton to consist of an up quark coupled to an up-down diquark, where the diquark is constructed so as to have vanishing spin and isospin. The remaining structure of the fields is dictated by the constraint that the proton interpolating field behave like a Dirac spinor under parity. Such an interpolating field will then have the same quantum numbers as the proton. Thus the two independent interpolating fields are taken to be

$$\eta_1(x) = \epsilon_{abc}[u_a^T(x)C\gamma_5 d_b(x)]u_c(x) , \quad (3.2)$$

$$\eta_2(x) = \epsilon_{abc}[u_a^T(x)Cd_b(x)]\gamma_5 u_c(x) , \quad (3.3)$$

where T denotes a transpose in Dirac space and C is the charge conjugation matrix [20]. The analogous fields for the neutron follow by interchanging the up and down quark fields.

In lattice QCD calculations, the interpolating field η_1 is usually used since this field has a nonrelativistic limit (η_2 vanishes in the nonrelativistic limit) and tends to minimize statistical uncertainties. The choice of nucleon interpolating field in QCD sum-rule applications is based on different criteria: The goals are to minimize the coupling of the interpolating field to the nucleon intermediate state relative to other (continuum) states, while minimizing the contributions of higher-order terms in the OPE [5]. (For a detailed discussion, see Ref. [34].) These goals cannot be simultaneously realized; however, the optimal choice of interpolating field seems to be the one made by Ioffe [5],

$$\eta(x) = \epsilon_{abc}[u_a^T(x)C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x) , \quad (3.4)$$

which is proportional to $\eta_1 - \eta_2$. We use this interpolating field in subsequent calculations.

Lorentz covariance, parity, and time reversal imply that $\Pi(q)$ has the form [2,4]

$$\Pi(q) \equiv \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \Pi_u(q^2, q\cdot u)\not{u} . \quad (3.5)$$

There are three distinct structures, scalar, $\not{q}$, and $\not{u}$, that multiply three invariants functions of the two scalars q^2 and $q\cdot u$. The invariant functions can be projected out by taking traces of the correlator:

$$\Pi_s(q^2, q\cdot u) = \tfrac{1}{4}\text{Tr}[\Pi(q)] , \quad (3.6)$$

$$\Pi_q(q^2, q\cdot u) = \frac{1}{q^2 - (q\cdot u)^2} \times\left[\frac{1}{4}\text{Tr}[\not{q}\Pi(q)] - \frac{q\cdot u}{4}\text{Tr}[\not{u}\Pi(q)]\right] , \quad (3.7)$$

$$\Pi_u(q^2,q\cdot u)=\frac{1}{q^2-(q\cdot u)^2}\times\left[\frac{q^2}{4}\mathrm{Tr}[\not u\,\Pi(q)]-\frac{q\cdot u}{4}\mathrm{Tr}[\not q\,\Pi(q)]\right] . \quad (3.8)$$

These projections require $q^2-(q\cdot u)^2\neq 0$, i.e., $q^\mu \neq \mathrm{const}\times u^\mu$. (This implies $\mathbf{q}\neq 0$ in the nuclear matter rest frame.) If $q^2-(q\cdot u)^2$ vanishes, then there are only two independent invariant functions.

We use the operator product expansion to provide a QCD expansion of the finite-density correlator. The OPE is a useful tool for extracting phenomenological information from renormalizable quantum field theories. The central idea is that the time-ordered product of two local (elementary or composite) operators at short distances can be expanded in terms of a complete set of regular local operators $\hat{O}_n(0)$ [35,36]:

$$T[A(x)B(0)] \overset{x\to 0}{=} \sum_n C_n^{AB}(x)\hat{O}_n(0) . \quad (3.9)$$

The c-number coefficients C_n^{AB} of the expansion are called Wilson coefficients. In this expansion the singularities at short distances are factored out from the regular operators, and the terms in the expansion are organized in decreasing order of singularity. The OPE has only been proven in perturbation theory; the validity of the OPE in the presence of nonperturbative effects is not obvious.

There has been a series of papers discussing the nature of the OPE beyond perturbation theory [37,38]. It was shown that to unambiguously apply the OPE, one must define the coefficient functions and composite operators by introducing an auxiliary parameter, the normalization point μ:

$$T[A(x)B(0)] \overset{x\to 0}{=} \sum_n C_n^{AB}(x,\mu)\hat{O}_n(0,\mu) . \quad (3.10)$$

Both the coefficient functions and the operators contain, in general, nonperturbative as well as perturbative contributions. However, in practical applications of the OPE (in particular, in the QCD sum-rule formalism) one usually applies a simplified version. Namely, the Wilson coefficients are evaluated in perturbation theory, while the nonperturbative effects are contained entirely in the vacuum expectation values of composite operators, which are assumed to contain no perturbative contributions.

This simplified version of the OPE is justified in part by the phenomenological success of QCD sum rules. In Ref. [37], the reason behind this success is attributed to the following: There seems to be a range in μ in which μ is large enough with respect to Λ_{QCD} (the QCD scale parameter) so that nonperturbative corrections to the coefficients are small and can be neglected, but small enough so that the values of the condensates are quite insensitive to variations in μ. In other words, there seems to be a "window" in QCD, where the simplified version of the OPE applies. Perturbative corrections to the operators can be taken into account in the leading-logarithmic approximation through anomalous-dimension factors [1].

In this paper we adopt the simplified version of the OPE to expand the time-ordered product in Eq. (3.1) at short distances. In the sum-rule applications of Ref. [4], the correlator is studied in the limit that q_0 becomes large and imaginary, while $|\mathbf{q}|$ remains fixed (in the nuclear matter rest frame). This limit takes $q^2\to -\infty$ with $|q^2/q\cdot u|\to\infty$, which satisfies the conditions discussed in Ref. [36] for a short-distance expansion.[1] We also apply Borel transforms in q_0^2 (or, equivalently, in q^2), which implies that only terms in the expansion that are discontinuous across the real q_0 axis contribute to the sum rules (for example, polynomials are eliminated).

At finite density, the OPE for the invariant functions of the nucleon correlator takes the general form

$$\Pi_i(q^2,q\cdot u)=\sum_n C_n^i(q^2,q\cdot u)\langle \hat{O}_n\rangle_{\rho_N} , \quad (3.11)$$

where we have introduced the notation $\langle \hat{O}_n\rangle_{\rho_N}\equiv\langle\Psi_0|\hat{O}_n|\Psi_0\rangle$. The $C_n^i(q^2,q\cdot u)$ $(i=\{s,q,u\})$ are the Wilson coefficients, which depend on QCD Lagrangian parameters such as the quark masses and the strong coupling constant. The Wilson coefficients in the OPE only depend on q^μ, and the ground-state expectation values of the operators are proportional to tensors constructed from the nuclear matter four-velocity u^μ, the metric $g^{\mu\nu}$, and the antisymmetric tensor $\epsilon^{\kappa\lambda\mu\nu}$. In Eq. (3.11) we incorporate the contraction of q^μ (from the OPE) and u^μ (from the ground-state expectation values of the operators) into the definition of the Wilson coefficients $C_n^i(q^2,q\cdot u)$. Thus the dependence on $q\cdot u$ is solely in the form of polynomial factors. We have suppressed the dependence on the normalization point μ.

The $\hat{O}_n$ are local composite operators constructed from quark and gluon fields; examples of such operators are $\bar{q}q$ and $(\alpha_s/\pi)G^2$. The ground-state expectation values of these operators are the in-medium condensates. The operators are defined so that the density dependence of the correlator resides solely in the in-medium condensates. Thus the only substantial difference from the vacuum calculations is that more composite operator matrix elements are nonzero in nuclear matter. The operators $\hat{O}_n$ are ordered by mass dimension and the $C_n^i(q^2,q\cdot u)$ for higher-dimensional operators fall off by corresponding powers of $Q^2\equiv -q^2$. Therefore, for sufficiently large Q^2, the operators of lowest dimension dominate, and the OPE can be truncated after a small number of lower-dimensional operators.

Since the nucleon interpolating field is a color singlet, the nucleon correlator is gauge invariant; therefore, one can evaluate the correlator in any desired gauge. For convenience, we use the fixed-point gauge, which was introduced for use in electrodynamics in Refs. [39,40], and reintroduced for use in QCD in Ref. [41]. Simple rules

[1]Note that in Ref. [17] the finite-density correlator is studied using kinematics analogous to that of deep inelastic scattering (i.e., $q^2/q\cdot u$ is fixed and finite). This ensures that only the light cone is probed, but does not imply a short-distance expansion. Furthermore, the identification of quasinucleon intermediate states is obscured.

have been formulated for the calculation of correlation functions using the fixed-point gauge and background-field techniques [42–44,9]. The fixed-point gauge condition is

$$x_\mu \mathcal{A}^\mu(x)=0 \ . \tag{3.12}$$

In this gauge the gluon field $\mathcal{A}_\mu$ can be expressed directly in terms of the gluon field tensor $\mathcal{G}_{\mu\nu}$ [41–45,9],

$$\mathcal{A}_\nu(x)=\int_0^1 d\alpha\, \alpha x^\mu \mathcal{G}_{\mu\nu}(\alpha x) = \tfrac{1}{2}x^\mu \mathcal{G}_{\mu\nu}(0)+\tfrac{1}{3}x^\lambda x^\mu (D_\lambda \mathcal{G}_{\mu\nu})_{x=0}+\cdots \ , \tag{3.13}$$

which allows one to obtain manifestly gauge-invariant results in a relatively simple way.

In the background-field method, the presence of nonperturbative quark and gluon condensates is parametrized by Grassmann background quark fields, $\chi^q_{a\alpha}$ and $\bar{\chi}^q_{a\alpha}$, and a classical background gluon field $F^A_{\mu\nu}$. It is most convenient to first calculate the correlator in coordinate space and then transform to momentum space. The coordinate-space quark propagator for massless quarks in the presence of the background fields takes the following form in the fixed-point gauge [9]:

$$\begin{aligned} S^q_{ab,\alpha\beta}(x,0) &\equiv \langle T[q_{a\alpha}(x)\bar{q}_{b\beta}(0)]\rangle_{\rho_N} \\ &= \frac{i}{2\pi^2}\delta_{ab}\frac{1}{(x^2)^2}[\not{x}]_{\alpha\beta}+\chi^q_{a\alpha}(x)\bar{\chi}^q_{b\beta}(0) \\ &\quad -\frac{ig_s}{32\pi^2}F^A_{\mu\nu}(0)t^A_{ab}\frac{1}{x^2}[\not{x}\sigma^{\mu\nu}+\sigma^{\mu\nu}\not{x}]_{\alpha\beta}+\cdots \ , \end{aligned} \tag{3.14}$$

where the first term is the free quark propagator, and the second and third terms are the contributions due to the background quark and gluon fields, respectively.[2] The gluonic contribution to Eq. (3.14) comes from a single gluon insertion, retaining only the leading term in the short-distance expansion of the gluon field [see Eq. (3.13)]. Contributions from derivatives of the gluon field tensor, which are less singular at $x \to 0$, and additional gluon insertions are not included. These refinements are not needed given the level of truncation in the OPE considered here.

The calculation of the nucleon correlator using background-field techniques is similar to ordinary perturbation theory. The time-ordered product in Eq. (3.1) is evaluated using Wick's theorem, retaining only those contributions in which the quark fields are fully contracted. However, the background quark propagator [Eq. (3.14)], rather than the free quark propagator, is used for each contraction. Working to leading order in perturbation theory, we obtain

$$\Pi(q)=-2i\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x\, e^{iq\cdot x}\mathrm{Tr}[S^{uT}_{aa'}(x,0)C\gamma_\mu S^u_{bb'}(x,0)\gamma_\nu C]\gamma_5\gamma^\mu S^d_{cc'}(x,0)\gamma^\nu\gamma_5 \ . \tag{3.15}$$

We perform the integration over x using the formulas [44,9]

$$\int \frac{d^4x}{x^2}e^{iq\cdot x}=-\frac{4\pi^2 i}{q^2} \ , \tag{3.16}$$

$$\int \frac{d^4x}{(x^2)^n}e^{iq\cdot x}=\frac{i(-1)^n 2^{4-2n}\pi^2}{\Gamma(n-1)\Gamma(n)}(q^2)^{n-2}\ln(-q^2)+P_{n-2}(q^2) \quad (n\ge 2) \ , \tag{3.17}$$

and their derivatives with respect to q^μ. $P_m(q^2)$ is a polynomial in q^2 of degree m with divergent coefficients. The precise forms of the polynomials are not important, since they do not contribute to the Borel transformed sum rules.

The leading perturbative contribution to the nucleon correlator is obtained by using only the free part of the propagator in Eq. (3.15). Higher-order contributions to the correlator are conveniently calculated using the background-field method. Products of Grassmann background quark fields and classical background gluon fields obtained in Eq. (3.15) correspond to ground-state matrix elements of the corresponding quark and gluon operators:

$$\begin{aligned} \chi^q_{a\alpha}(x)\bar{\chi}^q_{b\beta}(0) &= \langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N} \ , \\ F^A_{\kappa\lambda}F^B_{\mu\nu} &= \langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_{\rho_N} \ , \\ \chi^q_{a\alpha}\bar{\chi}^q_{b\beta}F^A_{\mu\nu} &= \langle q_{a\alpha}\bar{q}_{b\beta}G^A_{\mu\nu}\rangle_{\rho_N} \ , \\ \chi^q_{a\alpha}\bar{\chi}^q_{b\beta}\chi^q_{c\gamma}\bar{\chi}^q_{d\delta} &= \langle q_{a\alpha}\bar{q}_{b\beta}q_{c\gamma}\bar{q}_{d\delta}\rangle_{\rho_N} \ , \end{aligned} \tag{3.18}$$

where the fields are evaluated at $x=0$ unless otherwise noted. In Eq. (3.18) we have only shown those matrix elements that are needed in order to carry out the OPE to the level we are considering. Thus we evaluate the fields in the higher-dimensional operators at the same point, since nonlocalities would only introduce condensates that are higher in dimension than those we wish to consider. The operators on the right-hand side of Eq. (3.18) are implicitly normal ordered with respect to the perturbative vacuum at zero density; we can write these matrix elements in terms of scalar local condensates by projecting

[2]Note that some calculations are simpler if one writes the gluonic contribution to the propagator as

$$\frac{ig_s}{8\pi^2}\tilde{F}^A_{\mu\nu}(0)t^A_{ab}\frac{1}{x^2}x^\mu[\gamma^\nu\gamma_5]_{\alpha\beta}+\cdots \ .$$

However, to keep our discussion succinct, we only explicitly consider the form used in Eq. (3.14).

out the Dirac, Lorentz, and color structure and performing a short-distance expansion if necessary. We discuss this procedure in detail; it is through this discussion that we introduce the relevant condensates for our sum-rule calculations.

The Dirac and color structure of the matrix element $\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N}$ can be projected out to obtain

$$\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N} = -\frac{\delta_{ab}}{12}[\langle\bar{q}(0)q(x)\rangle_{\rho_N}\delta_{\alpha\beta}+\langle\bar{q}(0)\gamma_\lambda q(x)\rangle_{\rho_N}\gamma^\lambda_{\alpha\beta}] , \tag{3.19}$$

since nuclear matter is colorless and the ground state is (assumed to be) invariant under parity and time reversal. [Other matrix elements of the form $\langle\bar{q}(0)\Gamma q(x)\rangle_{\rho_N}$ do not contribute due to parity and/or time reversal.] We evaluate Eq. (3.19) at short distances by expanding the quark field $q(x)$ in a Taylor series:

$$q(x)=q(0)+x^\mu(\partial_\mu q)_{x=0}+\tfrac{1}{2}x^\mu x^\nu(\partial_\mu\partial_\nu q)_{x=0}+\cdots . \tag{3.20}$$

However, since the nucleon correlator is gauge invariant, the ordinary derivatives in Eq. (3.20) must ultimately become covariant derivatives. In standard calculations, gluon fields in higher-order terms of the OPE combine with the ordinary derivatives in lower-order terms to form covariant derivatives.

The situation is much more straightforward in the fixed-point gauge; the ordinary derivatives can be replaced with covariant derivatives immediately. We follow the discussion of Ref. [44]. Using the fixed-point gauge condition in Eq. (3.12), and expanding the gluon field, one obtains

$$x^\nu\mathcal{A}_\nu(0)+x^\mu x^\nu(\partial_\mu\mathcal{A}_\nu)_{x=0} +\tfrac{1}{2}x^\lambda x^\mu x^\nu(\partial_\lambda\partial_\mu\mathcal{A}_\nu)_{x=0}+\cdots=0 . \tag{3.21}$$

Since x is arbitrary, the individual terms of Eq. (3.21) must vanish; using this fact, one can readily show

$$\begin{aligned} x^\mu(D_\mu q)_{x=0}&=x^\mu(\partial_\mu q)_{x=0} ,\\ x^\mu x^\nu(D_\mu D_\nu q)_{x=0}&=x^\mu x^\nu(\partial_\mu\partial_\nu q)_{x=0} , \end{aligned} \tag{3.22}$$

and so on. Combining this result with Eq. (3.20) one derives the following covariant Taylor expansion:

$$q(x)=q(0)+x^\mu(D_\mu q)_{x=0}+\tfrac{1}{2}x^\mu x^\nu(D_\mu D_\nu q)_{x=0}+\cdots . \tag{3.23}$$

An analogous expansion of the gluon field tensor at short distances [used in Eq. (3.13)] is proved to all orders using mathematical induction in Ref. [45]. Therefore, we obtain

$$\begin{aligned}\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N}=-\frac{\delta_{ab}}{12}[&(\langle\bar{q}q\rangle_{\rho_N}+x^\mu\langle\bar{q}D_\mu q\rangle_{\rho_N}+\tfrac{1}{2}x^\mu x^\nu\langle\bar{q}D_\mu D_\nu q\rangle_{\rho_N}+\cdots)\delta_{\alpha\beta}\\ &+(\langle\bar{q}\gamma_\lambda q\rangle_{\rho_N}+x^\mu\langle\bar{q}\gamma_\lambda D_\mu q\rangle_{\rho_N}+\tfrac{1}{2}x^\mu x^\nu\langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N}+\cdots)\gamma^\lambda_{\alpha\beta}] ,\end{aligned} \tag{3.24}$$

where all fields and field derivatives in the condensates are evaluated at $x=0$.

It is useful to note one particular calculational convenience. Since the Dirac matrices involved in calculating the Wilson coefficient of $\langle\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\rangle_{\rho_N}[\langle\bar{q}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}q\rangle_{\rho_N}]$ are the same as those involved in calculating the Wilson coefficient of $\langle\bar{q}q\rangle_{\rho_N}[\langle\bar{q}\gamma_\mu q\rangle_{\rho_N}]$, we conclude that the coordinate-space coefficients are related as follows:

$$C_{\bar{q}D_{\mu_1}\cdots D_{\mu_n}q}(x)=\frac{1}{n!}x^{\mu_1}\cdots x^{\mu_n}C_{\bar{q}q}(x) , \tag{3.25}$$

$$C_{\bar{q}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}q}(x)=\frac{1}{n!}x^{\mu_1}\cdots x^{\mu_n}C_{\bar{q}\gamma_\mu q}(x) . \tag{3.26}$$

This implies that the momentum-space Wilson coefficients are related by

$$C_{\bar{q}D_{\mu_1}\cdots D_{\mu_n}q}(q)=\frac{(-i)^n}{n!}\left[\frac{\partial}{\partial q_{\mu_1}}\cdots\frac{\partial}{\partial q_{\mu_n}}\right]C_{\bar{q}q}(q) , \tag{3.27}$$

$$C_{\bar{q}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}q}(q)=\frac{(-i)^n}{n!}\left[\frac{\partial}{\partial q_{\mu_1}}\cdots\frac{\partial}{\partial q_{\mu_n}}\right]C_{\bar{q}\gamma_\mu q}(q) . \tag{3.28}$$

We now proceed to evaluate the condensates appearing in Eq. (3.24) in terms of expectation values of scalar operators multiplied by quantities that contain the Lorentz structure of the original condensates. In vacuum, these condensates can only be expressed in terms of the metric tensor $g^{\mu\nu}$ and the antisymmetric tensor $\epsilon^{\kappa\lambda\mu\nu}$; thus condensates with an odd number of uncontracted Lorentz indices must vanish in the vacuum. In-medium condensates, however, can also be expressed in terms of the nuclear matter four-velocity u^μ, which leads to new condensates and new Lorentz structures. The general procedure for evaluating the condensates in Eq. (3.24) is to write each as a sum of all possible Lorentz structures with unknown coefficients. These coefficients, which will turn out to be expectation values of scalar operators, can then be determined by taking appropriate traces over the Lorentz indices.

The condensates involving vector operators in Eq. (3.24), $\langle\bar{q}\gamma_\mu q\rangle_{\rho_N}$ and $\langle\bar{q}D_\mu q\rangle_{\rho_N}$, must be proportional to u_μ:

$$\langle \bar{q}\gamma_\mu q\rangle_{\rho_N} = \langle \bar{q}\not{u} q\rangle_{\rho_N} u_\mu \,, \tag{3.29}$$

$$\langle \bar{q}D_\mu q\rangle_{\rho_N} = \langle \bar{q}u\cdot Dq\rangle_{\rho_N} u_\mu \,. \tag{3.30}$$

The condensates $\langle \bar{q}\gamma_\mu D_\nu q\rangle_{\rho_N}$ and $\langle \bar{q}D_\mu D_\nu q\rangle_{\rho_N}$ can be decomposed into terms proportional to $u_\mu u_\nu$ and $g_{\mu\nu}$:

$$\begin{aligned}\langle \bar{q}\gamma_\mu D_\nu q\rangle_{\rho_N} &= \tfrac{4}{3}\langle \bar{q}\not{u}u\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \tfrac{1}{4}g_{\mu\nu}) - \tfrac{1}{3}\langle \bar{q}\not{D}q\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}) \\ &= \tfrac{4}{3}\langle \bar{q}\not{u}u\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \tfrac{1}{4}g_{\mu\nu}) \,,\end{aligned} \tag{3.31}$$

$$\begin{aligned}\langle \bar{q}D_\mu D_\nu q\rangle_{\rho_N} &= \tfrac{4}{3}\langle \bar{q}u\cdot Du\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \tfrac{1}{4}g_{\mu\nu}) - \tfrac{1}{3}\langle \bar{q}D^2 q\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}) \\ &= \tfrac{4}{3}\langle \bar{q}u\cdot Du\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \tfrac{1}{4}g_{\mu\nu}) - \tfrac{1}{6}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}) \,.\end{aligned} \tag{3.32}$$

The second equality in Eq. (3.31) follows from Eq. (2.7); the second equality in Eq. (3.32) follows from Eq. (2.13). (The current quark mass is neglected in both cases.) The condensate $\langle \bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N}$ can be decomposed into terms proportional to $u_\lambda u_\mu u_\nu$, $u_\lambda g_{\mu\nu}$, $u_\mu g_{\lambda\nu}$, and $u_\nu g_{\lambda\mu}$:

$$\begin{aligned}\langle \bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} = {} & 2\langle \bar{q}\not{u}u\cdot Du\cdot Dq\rangle_{\rho_N}[u_\lambda u_\mu u_\nu - \tfrac{1}{6}(u_\lambda g_{\mu\nu} + u_\mu g_{\lambda\nu} + u_\nu g_{\lambda\mu})] - \tfrac{1}{3}\langle \bar{q}\not{u}D^2 q\rangle_{\rho_N}(u_\lambda u_\mu u_\nu - u_\lambda g_{\mu\nu}) \\ & - \tfrac{1}{3}\langle \bar{q}u\cdot D\not{D}q\rangle_{\rho_N}(u_\lambda u_\mu u_\nu - u_\mu g_{\lambda\nu}) - \tfrac{1}{3}\langle \bar{q}\not{D}u\cdot Dq\rangle_{\rho_N}(u_\lambda u_\mu u_\nu - u_\nu g_{\lambda\mu}) \,.\end{aligned} \tag{3.33}$$

(A potential contribution proportional to $u^\kappa \epsilon_{\kappa\lambda\mu\nu}$ vanishes due to parity and time-reversal invariance.) The condensate in the fourth term of Eq. (3.33) can be reexpressed as $\langle \bar{q}\not{D}u\cdot Dq\rangle_{\rho_N} = -\langle \bar{q}\overleftarrow{\not{D}}u\cdot Dq\rangle_{\rho_N}$, which follows from translation invariance (see Appendix B). Thus the condensates in the third and fourth terms of Eq. (3.33) are proportional to the current quark mass by Eq. (2.7), and we neglect these terms. Therefore, one obtains

$$\langle \bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} = 2\langle \bar{q}\not{u}u\cdot Du\cdot Dq\rangle_{\rho_N}[u_\lambda u_\mu u_\nu - \tfrac{1}{6}(u_\lambda g_{\mu\nu} + u_\mu g_{\lambda\nu} + u_\nu g_{\lambda\mu})] - \tfrac{1}{6}\langle g_s\bar{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}(u_\lambda u_\mu u_\nu - u_\lambda g_{\mu\nu}) \,, \tag{3.34}$$

where the second term follows from the relation $\langle \bar{q}\not{u}D^2 q\rangle_{\rho_N} = \frac{1}{2}\langle g_s\bar{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$. Thus the expansion of $\langle q_{a\alpha}(0)\bar{q}_{b\beta}(x)\rangle_{\rho_N}$ up to dimension five includes quark condensates and quark-gluon condensates.

Another source of quark-gluon condensates is from contributions of the form $\chi^q_{a\alpha}\bar{\chi}^q_{b\beta}F^A_{\mu\nu}$ in Eq. (3.15). The corresponding matrix element can be decomposed as

$$\begin{aligned}\langle g_s q_{a\alpha}\bar{q}_{b\beta}G^A_{\mu\nu}\rangle_{\rho_N} = -\frac{t^A_{ab}}{96}\{ & \langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}[\sigma_{\mu\nu} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}]_{\alpha\beta} + \langle g_s\bar{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}[\sigma_{\mu\nu}\not{u} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)]_{\alpha\beta} \\ & - 4\langle \bar{q}u\cdot Du\cdot Dq\rangle_{\rho_N}[\sigma_{\mu\nu} + 2i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}]_{\alpha\beta}\} \,,\end{aligned} \tag{3.35}$$

which is obtained by projecting out the color, Dirac, and Lorentz structures by taking appropriate traces. The appearance of the condensate $\langle \bar{q}u\cdot Du\cdot Dq\rangle_{\rho_N}$ can be understood from the identity $\langle \bar{q}D_\mu D_\nu q\rangle_{\rho_N} = -\frac{1}{2}\langle ig_s\bar{q}\gamma_\mu\gamma^\lambda\mathcal{G}_{\lambda\nu}q\rangle_{\rho_N}$, which we prove in Sec. IV. Many of the "condensates" encountered in the derivation of Eq. (3.35) vanish due to the assumed parity and time-reversal invariance of the nuclear matter ground state.

We include contributions to the nucleon correlator from all dimension-four gluon condensates, which arise from factors of $F^A_{\kappa\lambda}F^B_{\mu\nu}$ in Eq. (3.15). Note that this factor comes from single gluon insertions in two of the quark propagators. In principle, one could also consider the case in which two gluon lines emanate from the same quark propagator; however, we neglect these contributions, since they vanish in the massless quark limit [42]. The matrix element $\langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_{\rho_N}$ can be written as

$$\begin{aligned}\left\langle \frac{\alpha_s}{\pi}G^A_{\kappa\lambda}G^B_{\mu\nu}\right\rangle_{\rho_N} = \frac{\delta^{AB}}{96}\Bigg\{ & \left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_{\rho_N}(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu}) \\ & -2\left\langle \frac{\alpha_s}{\pi}[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\right\rangle_{\rho_N}[g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu} \\ & -2(g_{\kappa\mu}u_\lambda u_\nu - g_{\kappa\nu}u_\lambda u_\mu - g_{\lambda\mu}u_\kappa u_\nu + g_{\lambda\nu}u_\kappa u_\mu)]\Bigg\} \,,\end{aligned} \tag{3.36}$$

where $(u\cdot G)^2 \equiv (u^\lambda G^A_{\lambda\nu})(u_\mu G^{A\mu\nu})$ [$\tilde{G}^A_{\mu\nu}$ is defined in Eq. (2.6)]. In the nuclear matter rest frame, the two in-medium gluon condensates $\langle (\alpha_s/\pi) G^2 \rangle_{\rho_N}$ and $\langle (\alpha_s/\pi)[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\rangle_{\rho_N}$ are proportional to $\langle \mathbf{E}^2 - \mathbf{B}^2 \rangle_{\rho_N}$ and $\langle \mathbf{E}^2 + \mathbf{B}^2\rangle_{\rho_N}$, respectively, where $\mathbf{E}^A$ and $\mathbf{B}^A$ are the color-electric and color-magnetic fields ($\mathbf{E}^2 \equiv \mathbf{E}^A\cdot\mathbf{E}^A$ and $\mathbf{B}^2 \equiv \mathbf{B}^A\cdot\mathbf{B}^A$).

Contributions to the nucleon correlator from higher-dimensional pure gluon condensates are expected to be small compared to those from quark condensates of equal dimension due to small numerical factors in the gluon condensate Wilson coefficients; therefore, these condensates can be neglected. One possible exception is the dimension-six three-gluon condensate $\langle g_s^3 f G^3\rangle_{\rho_N}$, since it has a nonvanishing vacuum value. It is a simple matter to compare the contribution of this three-gluon condensate to that of the two-gluon condensate $\langle g_s^2 G^2\rangle_{\rho_N}$, since they are diagrammatically similar. Two-gluon contributions to Eq. (3.15) are those in which two of the three background-field quark propagators are replaced with the gluonic part of the propagator, while the third is replaced with the free part of the propagator. The three-gluon contribution simply uses the gluonic part of the propagator for each. Thus one expects the ratio R of the three-gluon condensate contribution to the correlator to that of the two-gluon condensate to be given roughly by

$$R \sim \frac{\langle g_s^3 f G^3\rangle_{\rho_N}}{M^2 \langle g_s^2 G^2\rangle_{\rho_N}} , \tag{3.37}$$

where M is a relevant mass scale for the nucleon correlator. Taking $M\sim 1$ GeV and neglecting the (small) density dependence of the condensates, we obtain $R\sim 10\%$. The contribution of the two-gluon condensate to the nucleon sum rules is small, and the contribution from the three-gluon condensate is evidently smaller still; therefore, we neglect contributions from three-gluon condensates along with all other higher-dimensional pure gluon condensates.

At dimension six we only consider the four-quark condensates. The leading-order four-quark condensate contributions to the nucleon correlator arise at tree level; thus they do not carry the suppression factors associated with loops. Such contributions appear as terms proportional to $\chi^u_{a\alpha}\bar{\chi}^u_{b\beta}\chi^u_{c\gamma}\bar{\chi}^u_{d\delta}$ and $\chi^u_{a\alpha}\bar{\chi}^u_{b\beta}\chi^d_{c\gamma}\bar{\chi}^d_{d\delta}$ in the evaluation of Eq. (3.15) . The Lorentz, Dirac, and color structure of these matrix elements can be projected out in a manner similar to that discussed above; one then obtains the four-quark condensates in an unfactorized form. Here we consider the four-quark condensates only in a factorized form in which the four-quark matrix elements are approximated as follows [30]:

$$\langle u_{a\alpha}\bar{u}_{b\beta}u_{c\gamma}\bar{u}_{d\delta}\rangle_{\rho_N} \simeq \langle u_{a\alpha}\bar{u}_{b\beta}\rangle_{\rho_N}\langle u_{c\gamma}\bar{u}_{d\delta}\rangle_{\rho_N} - \langle u_{a\alpha}\bar{u}_{d\delta}\rangle_{\rho_N}\langle u_{c\gamma}\bar{u}_{b\beta}\rangle_{\rho_N} , \tag{3.38}$$

$$\langle u_{a\alpha}\bar{u}_{b\beta}d_{c\gamma}\bar{d}_{d\delta}\rangle_{\rho_N} \simeq \langle u_{a\alpha}\bar{u}_{b\beta}\rangle_{\rho_N}\langle d_{c\gamma}\bar{d}_{d\delta}\rangle_{\rho_N} . \tag{3.39}$$

The two-quark matrix elements in Eqs. (3.38) and (3.39) are then simplified using Eq. (3.19); thus the four-quark condensates are approximated in terms of $\langle\bar{q}q\rangle^2_{\rho_N}$, $\langle\bar{q}q\rangle_{\rho_N}\langle\bar{q}\not{u}q\rangle_{\rho_N}$, and $\langle\bar{q}\not{u}q\rangle^2_{\rho_N}$ with no new parameters. Additional details of the factorization (or ground-state saturation) approximation are discussed in Appendix A.

The three invariant structures of the nucleon correlator can be projected out by using Eqs. (3.6)–(3.8). For convenience we separate the correlators into pieces that are even and odd in $q\cdot u$:

$$\Pi_s(q^2,q\cdot u) \equiv \Pi_s^E(q^2,(q\cdot u)^2) + q\cdot u\,\Pi_s^O(q^2,(q\cdot u)^2) , \tag{3.40}$$

$$\Pi_q(q^2,q\cdot u) \equiv \Pi_q^E(q^2,(q\cdot u)^2) + q\cdot u\,\Pi_q^O(q^2,(q\cdot u)^2) , \tag{3.41}$$

$$\Pi_u(q^2,q\cdot u) \equiv \Pi_u^E(q^2,(q\cdot u)^2) + q\cdot u\,\Pi_u^O(q^2,(q\cdot u)^2) . \tag{3.42}$$

The results of our calculations are

$$\Pi_s^E(q^2,(q\cdot u)^2) = \frac{1}{4\pi^2} q^2 \ln(-q^2)\langle\bar{q}q\rangle_{\rho_N} + \frac{4}{3\pi^2}\frac{(q\cdot u)^2}{q^2}[\langle\bar{q}(iu\cdot D)(iu\cdot D)q\rangle_{\rho_N} + \tfrac{1}{8}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}] , \tag{3.43}$$

$$\Pi_s^O(q^2,(q\cdot u)^2) = -\frac{1}{2\pi^2}\ln(-q^2)\langle\bar{q}(iu\cdot D)q\rangle_{\rho_N} - \frac{4}{3q^2}\langle\bar{q}q\rangle_{\rho_N}\langle\bar{q}\not{u}q\rangle_{\rho_N} , \tag{3.44}$$

$$\begin{aligned}\Pi_q^E(q^2,(q\cdot u)^2) = {} & -\frac{1}{64\pi^4}(q^2)^2\ln(-q^2) + \frac{1}{9\pi^2}\left[5\ln(-q^2) - \frac{8(q\cdot u)^2}{q^2}\right]\langle\bar{q}\not{u}(iu\cdot D)q\rangle_{\rho_N} - \frac{1}{32\pi^2}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rho_N} \\ & - \frac{1}{144\pi^2}\left[\ln(-q^2) - \frac{4(q\cdot u)^2}{q^2}\right]\left\langle\frac{\alpha_s}{\pi}[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\right\rangle_{\rho_N} - \frac{2}{3q^2}\langle\bar{q}q\rangle^2_{\rho_N} - \frac{4}{3q^2}\langle\bar{q}\not{u}q\rangle_{\rho_N} ,\end{aligned} \tag{3.45}$$

$$\begin{aligned}\Pi_q^O(q^2,(q\cdot u)^2) = {} & \frac{1}{3\pi^2}\ln(-q^2)\langle\bar{q}\not{u}q\rangle_{\rho_N} + \frac{1}{18\pi^2 q^2}\langle g_s\bar{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N} \\ & - \frac{2}{3\pi^2 q^2}\left[1 + \frac{2(q\cdot u)^2}{q^2}\right][\langle\bar{q}\not{u}(iu\cdot D)(iu\cdot D)q\rangle_{\rho_N} + \tfrac{1}{12}\langle g_s\bar{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}] ,\end{aligned} \tag{3.46}$$

$$\begin{aligned}\Pi_u^E(q^2,(q\cdot u)^2) = {} & \frac{2}{3\pi^2}q^2\ln(-q^2)\langle\bar{q}\not{u}q\rangle_{\rho_N} - \frac{1}{12\pi^2}\ln(-q^2)\langle g_s\bar{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N} \\ & + \frac{4}{\pi^2}\frac{(q\cdot u)^2}{q^2}[\langle\bar{q}\not{u}(iu\cdot D)(iu\cdot D)q\rangle_{\rho_N} + \tfrac{1}{12}\langle g_s\bar{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}] ,\end{aligned} \tag{3.47}$$

$$\Pi_u^O(q^2,(q\cdot u)^2)=-\frac{20}{9\pi^2}\ln(-q^2)\langle\bar{q}\not{u}(iu\cdot D)q\rangle_{\rho_N}+\frac{1}{36\pi^2}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}[(u\cdot G)^2+(u\cdot\tilde{G})^2]\right\rangle_{\rho_N}-\frac{8}{3q^2}\langle\bar{q}\not{u}q\rangle_{\rho_N}^2 . \tag{3.48}$$

Here we have assumed isoscalar nuclear matter and take $\langle\bar{u}\hat{O}u\rangle_{\rho_N}\simeq\langle\bar{d}\hat{O}d\rangle_{\rho_N}\equiv\langle\bar{q}\hat{O}q\rangle_{\rho_N}$, with $\hat{O}$ a combination of Dirac matrices, gluon field tensors, and covariant derivatives. All polynomials in q^2 in Eqs. (3.43)–(3.48), which vanish under the Borel transform, have been omitted.

IV. IN-MEDIUM CONDENSATES

To calculate the nucleon correlator at finite density we need to know the condensates in nuclear matter. In this section we estimate the various condensates appearing in the invariant functions. We work in the rest frame, where $u^\mu\rightarrow u'^\mu\equiv(1,\mathbf{0})$, and expand the in-medium condensates in terms of the rest-frame nucleon density. To first order in the nucleon density we have

$$\langle\hat{O}\rangle_{\rho_N}=\langle\hat{O}\rangle_{\rm vac}+\langle\hat{O}\rangle_N\rho_N+\cdots , \tag{4.1}$$

where $\cdots$ denotes correction terms that are of higher order in the nucleon density. Note that this expansion is *not* a Taylor series expansion in ρ_N. The spin-averaged nucleon matrix element is

$$\langle\hat{O}\rangle_N=\int d^3x(\langle\tilde{N}|\hat{O}(\mathbf{x})|\tilde{N}\rangle-\langle{\rm vac}|\hat{O}(\mathbf{x})|{\rm vac}\rangle) , \tag{4.2}$$

where $|\tilde{N}\rangle$ is the state vector for a nucleon at rest normalized to unity ($\langle\tilde{N}|\tilde{N}\rangle=1$). Using more conventional notation, the nucleon matrix element is given by

$$\langle\hat{O}\rangle_N=\langle N|\hat{O}|N\rangle , \tag{4.3}$$

where $|N\rangle$ is once again the state vector for a nucleon at rest. In this case, the connected matrix element is implied, which is equivalent to making a vacuum subtraction as in Eq. (4.2), and the nucleon plane-wave states are normalized as follows:

$$\langle N(p)|N(p')\rangle=\frac{\omega_p}{M_N}(2\pi)^2\delta^3(\mathbf{p}-\mathbf{p}') , \tag{4.4}$$

where $\omega_p=p_0=\sqrt{\mathbf{p}^2+M_N^2}$.

For a general operator $\hat{O}$ there is not a systematic way to study contributions to $\langle\hat{O}\rangle_{\rho_N}$ that are of higher order in ρ_N. In the case of $\langle\bar{q}q\rangle_{\rho_N}$, however, higher-order corrections can be systematically studied with an application of the Hellmann-Feynman theorem, although the corrections are necessarily model dependent. Estimates of $\langle\bar{q}q\rangle_{\rho_N}$ in Ref. [3] show that the linear approximation is reasonably good (higher-order corrections $\sim$20% of the linear term) up to nuclear matter saturation density, although the models considered in this work are not sophisticated enough to allow a definitive conclusion. In this paper, we assume that the first-order approximation of the condensates is good up to nuclear matter saturation density.

The in-medium quark condensates that appear in the nucleon correlator [Eqs. (3.43)–(3.48)] can be written in the rest frame as $\langle\bar{q}q\rangle_{\rho_N}$, $\langle q^\dagger q\rangle_{\rho_N}$, $\langle\bar{q}iD_0q\rangle_{\rho_N}$, $\langle q^\dagger iD_0q\rangle_{\rho_N}$, $\langle\bar{q}iD_0iD_0q\rangle_{\rho_N}$, and $\langle q^\dagger iD_0iD_0q\rangle_{\rho_N}$. The correlator also depends on the quark-gluon condensates $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$ and $\langle g_sq^\dagger\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$. In the rest frame, it is convenient to take the two independent gluon condensates to be $\langle(\alpha_s/\pi)(\mathbf{E}^2-\mathbf{B}^2)\rangle_{\rho_N}$ and $\langle(\alpha_s/\pi)(\mathbf{E}^2+\mathbf{B}^2)\rangle_{\rho_N}$, where $\mathbf{E}^A$ and $\mathbf{B}^A$ are the color-electric and color-magnetic fields. In terms of the original forms of these condensates one has

$$\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2-\mathbf{B}^2)\right\rangle_{\rho_N}=-\frac{1}{2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rho_N} , \tag{4.5}$$

$$\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2+\mathbf{B}^2)\right\rangle_{\rho_N}=-\left\langle\frac{\alpha_s}{\pi}[(u'\cdot G)^2+(u'\cdot\tilde{G})^2]\right\rangle_{\rho_N} . \tag{4.6}$$

We now proceed to estimate the in-medium condensates.

A. Dimension-three condensates

The most important condensates in finite-density QCD sum rules for the nucleon are the dimension-three quark condensates $\langle\bar{q}q\rangle_{\rho_N}$ and $\langle q^\dagger q\rangle_{\rho_N}$. These condensates alone contribute to the leading-order sum-rule results for the nucleon self-energies [2,4]. The in-medium quark condensate $\langle\bar{q}q\rangle_{\rho_N}$ can be expanded in terms of the nucleon density as

$$\langle\bar{q}q\rangle_{\rho_N}=\langle\bar{q}q\rangle_{\rm vac}+\langle\bar{q}q\rangle_N\rho_N+\cdots , \tag{4.7}$$

where we have used Eq. (4.1). This condensate has been discussed extensively in Refs. [3,16,17]; therefore, we simply quote the result here. The nucleon matrix element $\langle\bar{q}q\rangle_N$ is related to the nucleon σ term $\sigma_N\equiv(m_u+m_d)\langle\bar{q}q\rangle_N$, where m_u and m_d are the up and down current quark masses. Therefore, one obtains

$$\langle\bar{q}q\rangle_N=\frac{\sigma_N}{m_u+m_d} . \tag{4.8}$$

Note that σ_N is renormalization-group invariant; therefore, as with $\langle\bar{q}q\rangle_{\rm vac}$, the normalization scale of $\langle\bar{q}q\rangle_N$ is determined by that of m_q. The most recent estimate of the σ term is $\sigma_N\simeq45\pm10$ MeV [46]; thus the quark condensate is 30–45 % smaller than its vacuum value at nuclear matter saturation density.

The other dimension-three quark condensate is $\langle q^\dagger q\rangle_{\rho_N}$. Since the baryon current is conserved, $\langle q^\dagger q\rangle_{\rho_N}$ is proportional to the nucleon density:

$$\langle q^\dagger q\rangle_{\rho_N}=\tfrac{3}{2}\rho_N , \tag{4.9}$$

where the factor $\frac{3}{2}$ corresponds to three colors (i.e., three valence quarks) divided by two flavors. This is an exact result.

B. Dimension-four condensates

We now estimate the two dimension-four quark condensates $\langle \bar{q}iD_0q\rangle_{\rho_N}$ and $\langle q^\dagger iD_0q\rangle_{\rho_N}$ and the two dimension-four gluon condensates $\langle(\alpha_s/\pi)(\mathbf{E}^2-\mathbf{B}^2)\rangle_{\rho_N}$ and $\langle(\alpha_s/\pi)(\mathbf{E}^2+\mathbf{B}^2)\rangle_{\rho_N}$. We start by considering the quark condensate $\langle \bar{q}iD_0q\rangle_{\rho_N}$, which can be evaluated exactly. Using the identity

$$D_\mu \equiv \tfrac{1}{2}(\gamma_\mu \not{D}+\not{D}\gamma_\mu) , \tag{4.10}$$

one obtains

$$\langle \bar{q}iD_0q\rangle_{\rho_N}=\tfrac{1}{2}(\langle \bar{q}\gamma_0 i\not{D}q\rangle_{\rho_N}+\langle \bar{q}i\not{D}\gamma_0 q\rangle_{\rho_N}) . \tag{4.11}$$

Translation invariance of the nuclear matter ground state implies $\langle \bar{q}i\not{D}\gamma_0 q\rangle_{\rho_N}=-\langle \bar{q}i\overleftarrow{\not{D}}\gamma_0 q\rangle_{\rho_N}$ (see Appendix B); using the quark field equations [Eq. (2.7)], one then finds

$$\langle \bar{q}iD_0q\rangle_{\rho_N}=m_q\langle q^\dagger q\rangle_{\rho_N}=\tfrac{3}{2}m_q\rho_N , \tag{4.12}$$

where we have also used Eq. (4.9). This is an exact result; we neglect this condensate since it is proportional to the current quark mass.

The remaining dimension-four condensates are expanded to first order in the nucleon density using Eq. (4.1). In order to implement this expansion one must first determine the vacuum values of these condensates. For example, the vacuum value of the quark condensate $\langle q^\dagger iD_0q\rangle_{\rho_N}$ is given by

$$\begin{aligned}\langle q^\dagger iD_0q\rangle_{\text{vac}}&=u'_\mu u'_\nu\langle \bar{q}\gamma^\mu iD^\nu q\rangle_{\text{vac}}\\ &=\frac{m_q}{4}\langle \bar{q}q\rangle_{\text{vac}}\simeq 0 ,\end{aligned} \tag{4.13}$$

where $u'_\mu\equiv(1,0)$. We have used the fact that $\langle \bar{q}\gamma^\mu iD^\nu q\rangle_{\text{vac}}$ can only be proportional to $g^{\mu\nu}$. The vacuum values of the other condensates are determined by similar considerations. Thus the remaining dimension-four condensates are expanded as follows:

$$\langle q^\dagger iD_0q\rangle_{\rho_N}=\langle q^\dagger iD_0q\rangle_N\rho_N+\cdots , \tag{4.14}$$

$$\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2-\mathbf{B}^2)\right\rangle_{\rho_N}=-\frac{1}{2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\text{vac}}+\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2-\mathbf{B}^2)\right\rangle_N\rho_N+\cdots , \tag{4.15}$$

$$\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2+\mathbf{B}^2)\right\rangle_{\rho_N}=\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2+\mathbf{B}^2)\right\rangle_N\rho_N+\cdots . \tag{4.16}$$

The QCD trace anomaly is used to estimate $\langle(\alpha_s/\pi)(\mathbf{E}^2-\mathbf{B}^2)\rangle_N$. The details are discussed in Refs. [3,16,17]; therefore, we simply quote the result here:

$$\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2-\mathbf{B}^2)\right\rangle_N=\frac{4}{9}(M_N-\sigma_N-S_N) , \tag{4.17}$$

where M_N is the nucleon mass, $\sigma_N\equiv(m_u+m_d)\langle\bar{q}q\rangle_N$ is the nucleon σ term, and $S_N\equiv m_s\langle\bar{s}s\rangle_N$ is the strangeness contribution to the nucleon mass. The strangeness content of the nucleon is commonly parametrized by the dimensionless quantity $y\equiv\langle\bar{s}s\rangle_N/\langle\bar{q}q\rangle_N$. Calculations that analyze the mass spectrum of the baryon octet in the context of SU(3) flavor symmetry indicate that the strangeness content of the nucleon is related to the σ term in the following manner [46]:

$$\sigma_N=\frac{\sigma_N^0}{1-y} , \tag{4.18}$$

where σ_N^0 is the σ term in the limit of vanishing strangeness content. Therefore, S_N can be parametrized as

$$S_N=\left[\frac{m_s}{m_u+m_d}\right](\sigma_N-\sigma_N^0) . \tag{4.19}$$

We take $\sigma_N\simeq 45\pm 10$ MeV, $\sigma_N^0\simeq 35\pm 5$ MeV (from second-order perturbation theory in m_s-m_q), and $m_s/(m_u+m_d)\simeq 13$ [46]; thus we have the following estimate for the nucleon matrix element in Eq. (4.17):

$$\left\langle\frac{\alpha_s}{\pi}(\mathbf{E}^2-\mathbf{B}^2)\right\rangle_N\simeq 0.325\pm 0.075\ \text{GeV} . \tag{4.20}$$

At nuclear matter saturation density, $\langle(\alpha_s/\pi)(\mathbf{E}^2-\mathbf{B}^2)\rangle_{\rho_N}$ is 5–10 % smaller than its vacuum value.

The matrix elements $\langle q^\dagger iD_0q\rangle_N$ and $\langle(\alpha_s/\pi)(\mathbf{E}^2+\mathbf{B}^2)\rangle_N$ can be related to moments of parton distribution functions measured in deep-inelastic scattering experiments [15–17]. We consider the twist-two operators $\bar{q}\{\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}\}q$ and $\text{tr}\{\mathcal{G}_{\lambda\mu_1}D_{\mu_2}\cdots D_{\mu_{n-1}}\mathcal{G}_{\mu_n}{}^{\lambda}\}$, where $\{\cdots\}$ makes the quantity enclosed symmetric and traceless in its Lorentz indices. (Recall that twist = dimension − spin.) These operators give the leading contributions to the nucleon structure functions in the Bjorken limit [36,47,48]. Nucleon matrix elements of these operators can be defined as follows:

$$\langle N(p)|\bar{q}\{\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}\}q|N(p)\rangle\equiv\frac{(-i)^{n-1}}{2M_N}A_n^q(\mu^2)\{p_{\mu_1}\cdots p_{\mu_n}\} , \tag{4.21}$$

$$\langle N(p)|\text{tr}\{\mathcal{G}_{\lambda\mu_1}D_{\mu_2}\cdots D_{\mu_{n-1}}\mathcal{G}_{\mu_n}{}^{\lambda}\}|N(p)\rangle\equiv\frac{(-i)^{n-2}}{2M_N}A_n^g(\mu^2)\{p_{\mu_1}\cdots p_{\mu_n}\} , \tag{4.22}$$

where $A_n^q(\mu^2)$ and $A_n^g(\mu^2)$ are reduced matrix elements, μ is the renormalization scale, and the nucleon plane-wave states are normalized as in Eq. (4.4). The reduced matrix elements can be expressed as moments of parton distribution functions [48–50]:

$$A_n^q(\mu^2)=2\int_0^1 dx\,x^{n-1}[q(x,\mu^2)+(-1)^n\bar{q}(x,\mu^2)] , \tag{4.23}$$

$$A_n^g(\mu^2)=\frac{1+(-1)^n}{2}\int_0^1 dx\,x^{n-1}g(x,\mu^2) , \tag{4.24}$$

where $q(x,\mu^2)$ and $\bar{q}(x,\mu^2)$ are the scale-dependent distribution functions for quarks and antiquarks (of flavor q) in the nucleon, and $g(x,\mu^2)$ is the gluon distribution function in the nucleon.[3] Given a generic parton distribution function $p(x,\mu^2)$, the product $p(x,\mu^2)dx$ gives the number of partons of type p with a fraction of the total nucleon momentum between x and $x+dx$ at the scale μ^2 [36,47,48].

For the purpose of evaluating in-medium condensates to first order in ρ_N, one can consider nuclear matter to consist of uniformly moving noninteracting nucleons; thus one takes $p_\mu = M_N u_\mu$. To evaluate $\langle q^\dagger iD_0 q\rangle_N$ and $\langle(\alpha_s/\pi)(\mathbf{E}^2+\mathbf{B}^2)\rangle_N$, we consider Eqs. (4.21) and (4.22) for the $n=2$ case; evaluating the trace over color indices in Eq. (4.22) we obtain

$$\langle \bar{q}\{\gamma_\mu D_\nu\}q\rangle_N = -\frac{i}{2} M_N A_2^q(\mu^2)(u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}) , \quad (4.25)$$

$$\langle \{G^A_{\lambda\mu} G^{A\lambda}_\nu\}\rangle_N = M_N A_2^g(\mu^2)(u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}) , \quad (4.26)$$

where we now work in the rest frame. Alternatively, by projecting out the Lorentz structure of the matrix elements in the standard manner we get

$$\langle \bar{q}\{\gamma_\mu D_\nu\}q\rangle_N = \tfrac{4}{3}\langle q^\dagger D_0 q\rangle_N (u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}) , \quad (4.27)$$

$$\begin{aligned}\langle \{G^A_{\lambda\mu} G^{A\lambda}_\nu\}\rangle_N &= -\tfrac{4}{3}[\langle (u'\cdot G)^2\rangle_N - \tfrac{1}{4}\langle G^2\rangle_N](u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}) \\ &= \tfrac{2}{3}\langle \mathbf{E}^2+\mathbf{B}^2\rangle_N (u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}) , \quad (4.28)\end{aligned}$$

where Eq. (4.27) follows from Eq. (3.31). Matching Eq. (4.25) [Eq. (4.26)] with Eq. (4.27) [Eq. (4.28)] one finds

$$\langle q^\dagger iD_0 q\rangle_N = \tfrac{3}{8} M_N A_2^q(\mu^2) , \quad (4.29)$$

$$\left\langle \frac{\alpha_s}{\pi}(\mathbf{E}^2+\mathbf{B}^2)\right\rangle_N = \frac{3}{2\pi} M_N \alpha_s(\mu^2) A_2^g(\mu^2) . \quad (4.30)$$

In order to obtain numerical values for these matrix elements we need to know the parton distribution functions.

From a theoretical point of view, the parton distributions are unknown functions; they can only be deduced from experimental data. However, the evolution with μ^2 of the parton distribution functions is governed by the QCD renormalization-group equations [36,47,48]. As a result, if one knows the parton distributions at some particular value of μ^2, then one can, in principle, calculate the parton distributions at all values of μ^2 where perturbative evolution is valid.

In practice, boundary conditions are imposed on the parton distribution functions at an appropriate lower scale $\mu_0^2 \simeq 0.2$ GeV^2 [51,52]; these boundary conditions together with the measured distribution functions at $\mu^2 \simeq 10$ GeV^2 uniquely determine the various parton distribution functions at all values of μ^2 greater than μ_0^2. Since the relevant scale in nucleon sum rules is ~ 1 GeV, we evaluate $q(x,\mu^2)$, $\bar{q}(x,\mu^2)$, and $g(x,\mu^2)$ at $\mu^2 = 1$ GeV^2. We use the leading-order (LO) scheme of Refs. [51,52]. In this scheme, all the higher-order terms in the QCD renormalization-group equations are neglected, and the parton distribution functions are parametrized as functions of x and μ^2. Using the LO parametrization of parton distributions given in Ref. [51] we get $A_2^q(1$ GeV$^2)\simeq 0.47$ and $A_2^g(1$ GeV$^2)\simeq 0.50$. Using the parametrizations in Ref. [52], we obtain $A_2^q(1$ GeV$^2)\simeq 0.55$ and $A_2^g(1$ GeV$^2)\simeq 0.43$. We evaluate $\alpha_s(\mu^2)$ at $\mu^2 = 1$ GeV^2 using the values of $\Lambda_{\rm QCD}$ used for the LO parametrization in Refs. [51,52], which yield $\alpha_s(1$ GeV$^2)\simeq 0.50$ and $\alpha_s(1$ GeV$^2)\simeq 0.48$, respectively. Thus we have the following estimates for the nucleon matrix elements:

$$\langle q^\dagger iD_0 q\rangle_N \simeq 0.18 \pm 0.01 \text{ GeV} , \quad (4.31)$$

$$\left\langle \frac{\alpha_s}{\pi}(\mathbf{E}^2+\mathbf{B}^2)\right\rangle_N \simeq 0.10 \pm 0.01 \text{ GeV} . \quad (4.32)$$

C. Dimension-five condensates

The condensates of interest here are the two in-medium quark condensates $\langle \bar{q} iD_0 iD_0 q\rangle_{\rho_N}$ and $\langle q^\dagger iD_0 iD_0 q\rangle_{\rho_N}$ and the two quark-gluon condensates $\langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_{\rho_N} = -2\langle \bar{q} iD_\mu iD^\mu q\rangle_{\rho_N}$ and $\langle g_s q^\dagger \sigma\cdot\mathcal{G} q\rangle_{\rho_N} = -2\langle q^\dagger iD_\mu iD^\mu q\rangle_{\rho_N}$. Following Eq. (4.1), we parametrize the density dependence of these condensates in the rest frame as

$$\langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_{\rho_n} = \langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_{\rm vac} + \langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_N \rho_N + \cdots , \quad (4.33)$$

$$\langle g_s q^\dagger \sigma\cdot\mathcal{G} q\rangle_{\rho_N} = \langle g_s q^\dagger \sigma\cdot\mathcal{G} q\rangle_N \rho_N + \cdots , \quad (4.34)$$

$$\begin{aligned}\langle \bar{q} iD_0 iD_0 q\rangle_{\rho_N} &+ \tfrac{1}{8}\langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_{\rho_N} \\ &= (\langle \bar{q} iD_0 iD_0 q\rangle_N + \tfrac{1}{8}\langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_N)\rho_N + \cdots , \quad (4.35)\end{aligned}$$

$$\begin{aligned}\langle q^\dagger iD_0 iD_0 q\rangle_{\rho_N} &+ \tfrac{1}{12}\langle g_s q^\dagger \sigma\cdot\mathcal{G} q\rangle_{\rho_N} \\ &= (\langle q^\dagger iD_0 iD_0 q\rangle_N + \tfrac{1}{12}\langle g_s q^\dagger \sigma\cdot\mathcal{G} q\rangle_N)\rho_N + \cdots . \quad (4.36)\end{aligned}$$

In Eqs. (4.35) and (4.36) we consider the quark condensates with quark-gluon contributions added since these combinations occur naturally.

We estimate $\langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_{\rho_N}$ by analogy to the treatment of this condensate in free space [see Eq. (2.13)]:

$$\begin{aligned}\langle g_s \bar{q}\sigma\cdot\mathcal{G} q\rangle_{\rho_N} &= 2\langle \bar{q} D^2 q\rangle_{\rho_N} \\ &\equiv 2\lambda_q^{*2}(\rho_N)\langle \bar{q}q\rangle_{\rho_N} . \quad (4.37)\end{aligned}$$

[3] In the literature, the nucleon matrix elements in Eqs. (4.21) and (4.22) are actually taken to be *proton* matrix elements, and $q(x,\mu^2)$, $\bar{q}(x,\mu^2)$, and $g(x,\mu^2)$ are the parton distribution functions in the proton. We account for this difference in convention by taking $A_n^q(\mu^2) = \frac{1}{2}[A_n^u(\mu^2) + A_n^d(\mu^2)]$, with $A_n^u(\mu^2)$ and $A_n^d(\mu^2)$ defined as in Eq. (4.23), but with up and down quark and antiquark distribution functions.

For an initial estimate we assume $\lambda_q^{*2}(\rho_N)\sim\lambda_q^2$; thus we obtain

$$\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N\sim\frac{2\lambda_q^2\sigma_N}{m_u+m_d}\simeq 3\pm 1\ \text{GeV}^2\ , \tag{4.38}$$

where we have used $\lambda_q^2\simeq 0.4\pm 0.1$ GeV^2 [6,25], $\sigma_N\simeq 45\pm 10$ MeV [46], and $m_u+m_d\simeq 14\pm 4$ MeV [22]. We will assess the sensitivity our nucleon sum-rule results to a conservative range of values for $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N$ in Ref. [14].

The nucleon matrix element $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N$ has been estimated previously [53,54], as it is proportional to the leading power correction to the Gross–Llewellyn Smith sum rule.[4] In Ref. [53] this matrix element is estimated by performing a rough fit to experimental data for the Gross–Llewellyn Smith sum rule; the result is

$$\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N\sim\tfrac{1}{2}\ \text{GeV}^2 \tag{4.39}$$

(assuming $\Lambda_{\text{QCD}}\simeq 100$ MeV). However, in Ref. [54] an analysis based on QCD sum rules gives $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N\simeq -0.33$ GeV^2, and this result is shown to agree reasonably well with other estimates based on vector dominance ($\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N\simeq -0.22\pm 0.04$ GeV^2) and a nonrelativistic quark model ($\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N\simeq -0.2$ GeV^2). Thus the sign of $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N$ is uncertain, although it seems reasonable to assume that the magnitude of this matrix element is less than 1 GeV^2. Numerical studies of nucleon sum rules using this range of values will be presented in Ref. [14].

In Appendix C we estimate $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N$ and $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N$ using a bag model. We obtain $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N\simeq 0.62$ GeV^2 and $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N\simeq 0.66$ GeV^2, although these results cannot be considered to be reliable. The quark and gluon fields of the bag model are not the same operators as the quark and gluon fields of QCD; in particular, the quarks and gluons in the bag model do not satisfy the QCD field equations: The quark field equation $i\not{D}q=0$ is violated on the bag surface, and the gluon fields are treated as eight independent Abelian fields (nonlinear gluon self-interactions are neglected).

We now consider the dimension-five quark condensates with their respective quark-gluon contributions [see Eqs. (4.35) and (4.36)], which can be estimated in terms of twist-two and twist-three distribution functions. In principle, $\langle\bar{q}iD_0iD_0q\rangle_N+\frac{1}{8}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N$ can be related to a moment of the twist-three distribution function $e(x,\mu^2)$. Nucleon matrix elements of the twist-three operator $\bar{q}\{D_{\mu_1}\cdots D_{\mu_n}\}q$ are defined as follows [55]:

$$\langle N(p)|\bar{q}\{D_{\mu_1}\cdots D_{\mu_n}\}q|N(p)\rangle \equiv(-i)^n e_n(\mu^2)\{p_{\mu_1}\cdots p_{\mu_n}\}\ , \tag{4.40}$$

where the nucleon plane-wave states are normalized as in Eq. (4.4) and

$$e_n(\mu^2)\equiv\int dx\, x^n e(x,\mu^2)\ . \tag{4.41}$$

For the $n=2$ case in the rest frame we have

$$\langle\bar{q}\{D_\mu D_\nu\}q\rangle_N=-M_N^2 e_2(\mu^2)(u'_\mu u'_\nu-\tfrac{1}{4}g_{\mu\nu})\ . \tag{4.42}$$

Alternatively, one can project out the Lorentz structure of the operator matrix element in the standard manner to obtain

$$\langle\bar{q}\{D_\mu D_\nu\}q\rangle_N=\tfrac{4}{3}(\langle\bar{q}D_0D_0q\rangle_N-\tfrac{1}{8}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N)\times(u'_\mu u'_\nu-\tfrac{1}{4}g_{\mu\nu})\ , \tag{4.43}$$

which follows from Eq. (3.32). By matching Eqs. (4.42) and (4.43) we find

$$\langle\bar{q}iD_0iD_0q\rangle_N+\tfrac{1}{8}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N=\tfrac{3}{4}M_N^2e_2(\mu^2)\ . \tag{4.44}$$

There is no known practical way to measure $e(x,\mu^2)$ [55]; however, we can obtain a rough estimate of $e_2(\mu^2)$ by using a qualitative feature of the bag model. We proceed by reexpressing $\langle\bar{q}iD_0iD_0q\rangle_N$ in terms of a quark-gluon matrix element. Using Eqs. (4.10) and (2.4) one can show

$$\langle\bar{q}D_\mu D_\nu q\rangle_N=-\tfrac{1}{2}(\langle ig_s\bar{q}\gamma_\mu\gamma^\lambda\mathcal{G}_{\lambda\nu}q\rangle_N-\langle\bar{q}\gamma_\mu D_\nu\not{D}q\rangle_N-\langle\bar{q}\not{D}\gamma_\mu D_\nu q\rangle_N)\ . \tag{4.45}$$

Translation invariance enables us to write the last term as $\langle\bar{q}\not{D}\gamma_\mu D_\nu q\rangle_N=-\langle\bar{q}\overleftarrow{\not{D}}\gamma_\mu D_\nu q\rangle_N$ (see Appendix B); using the quark field equations [Eq. (2.7)] one therefore obtains

$$\langle\bar{q}iD_0iD_0q\rangle_N=\tfrac{1}{2}\langle ig_s q^\dagger\gamma^\lambda\mathcal{G}_{\lambda 0}q\rangle_N+m_q\langle q^\dagger iD_0q\rangle_N\ . \tag{4.46}$$

We estimate the quark-gluon matrix element on the right-hand side of Eq. (4.46) using a bag model. (We neglect the term proportional to the current quark mass.) In a bag model, the nucleon is considered to consist of three quarks moving freely in the perturbative vacuum inside the bag, while the region outside the bag is the physical nonperturbative vacuum. Thus one obtains

$$\langle\bar{q}iD_0iD_0q\rangle_N=-\frac{ig_s}{2}\int_{\text{bag}}d^3x\,\langle\tilde{N}|q^\dagger\boldsymbol{\gamma}\cdot\mathbf{E}^At^Aq|\tilde{N}\rangle\ , \tag{4.47}$$

where the nucleon states are normalized such that $\langle\tilde{N}|\tilde{N}\rangle=1$. Since the color-electric field vanishes inside the bag (see Appendix C), the bag model predicts $\langle\bar{q}iD_0iD_0q\rangle_N=0$. Therefore, one has

$$\langle\bar{q}iD_0iD_0q\rangle_N+\tfrac{1}{8}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N\sim\tfrac{1}{8}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N\ , \tag{4.48}$$

which can be considered to be no more than a rough estimate. Using the estimate of $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N$ in Eq. (4.38) we obtain

$$\langle\bar{q}iD_0iD_0q\rangle_N+\tfrac{1}{8}\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N\sim\frac{\lambda_q^2\sigma_N}{4(m_u+m_d)}\simeq 0.3\pm 0.1\ \text{GeV}^2\ . \tag{4.49}$$

[4]These studies estimate the reduced nucleon matrix element of the twist-four operator $g_s\bar{q}\tilde{\mathcal{G}}^{\mu\nu}\gamma_\nu\gamma_5q$; due to time-reversal and translation invariance one finds $\langle g_sq^\dagger\sigma\cdot\mathcal{G}q\rangle_N=-2u'_\mu\langle g_s\bar{q}\tilde{\mathcal{G}}^{\mu\nu}\gamma_\nu\gamma_5q\rangle_N$.

Using the bag-model estimate of $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N$ from Appendix C instead yields a result that is a factor of 4 smaller than the one in Eq. (4.49).

The nucleon matrix element $\langle q^\dagger iD_0 iD_0 q\rangle_N + \frac{1}{12}\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N$ can be evaluated in terms of quark and antiquark distribution functions. We consider Eq. (4.21) for the $n=3$ case:

$$\langle\bar{q}\{\gamma_\lambda D_\mu D_\nu\}q\rangle_N = -\tfrac{1}{2}M_N^2 A_3^q(\mu^2)[u'_\lambda u'_\mu u'_\nu - \tfrac{1}{6}(u'_\lambda g_{\mu\nu}+u'_\mu g_{\lambda\nu}+u'_\nu g_{\lambda\mu})] , \tag{4.50}$$

where we work in the rest frame. Alternatively, by projecting out the Lorentz structure of the matrix element in the standard manner, one obtains

$$\langle\bar{q}\{\gamma_\lambda D_\mu D_\nu\}q\rangle_N = 2(\langle q^\dagger D_0 D_0 q\rangle_N - \tfrac{1}{12}\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N) \times[u'_\lambda u'_\mu u'_\nu - \tfrac{1}{6}(u'_\lambda g_{\mu\nu}+u'_\mu g_{\lambda\nu}+u'_\nu g_{\lambda\mu})] , \tag{4.51}$$

which follows from Eq. (3.34). A comparison of Eq. (4.50) with Eq. (4.51) then leads to the following result:

$$\langle q^\dagger iD_0 iD_0 q\rangle_N + \tfrac{1}{12}\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N = \tfrac{1}{4}M_N^2 A_3^q(\mu^2) . \tag{4.52}$$

As with the dimension-four condensates, we take $\mu^2=1$ GeV^2. Using the LO parametrization of parton distributions given in Refs. [51,52], we obtain $A_3^q(1\ \text{GeV}^2)\simeq 0.14$. Thus we find

$$\langle q^\dagger iD_0 iD_0 q\rangle_N + \tfrac{1}{12}\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N \simeq 0.031\ \text{GeV}^2 . \tag{4.53}$$

With the exception of $\langle q^\dagger iD_0 iD_0 q\rangle_{\rho_N} + \frac{1}{12}\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$, none of the dimension-five condensates have been determined accurately; however, terms proportional to these condensates make only small contributions to the nucleon sum rules. Thus the sensitivity of our sum-rule results to the precise values of these condensates is small. A numerical analysis of this sensitivity will be given in Ref. [14].

V. SUMMARY

In this paper we studied a time-ordered correlation function of nucleon interpolating fields in nuclear matter [see Eqs. (3.1) and (3.4)]. The Lorentz structure of the correlator at finite density differs from its vacuum form due to the presence of a four-vector, the nuclear matter four-velocity u^μ, that yields new structures. The momentum-space correlator can be decomposed into three distinct invariant functions of q^2 and $q\cdot u$.

In the nuclear matter rest frame, these functions depend on q_0 and $|\mathbf{q}|$. To derive sum rules for nucleon self-energies in nuclear matter, we consider the limit in which q_0 is large and imaginary, while $|\mathbf{q}|$ remains fixed [4]. This limit implies that we probe the correlator at short distances, so we evaluate it using an operator product expansion (OPE) [36]. The Wilson coefficients for the OPE are calculated to leading order in perturbative QCD, and nonperturbative effects are reflected through in-medium condensates, which depend implicitly on the density. We have defined the operators in the OPE such that all density dependence of the correlator resides in the condensates. A large number of condensates that vanish in vacuum are nonzero in nuclear matter.

We have included contributions to the nucleon correlator from quark and quark-gluon condensates up to dimension five and dimension-four gluon condensates; we have also included contributions from dimension-six four-quark condensates. The in-medium condensates up to dimension five have been estimated to first order in the nucleon density, and the four-quark condensates have been estimated by their factorized forms. Motivated by earlier considerations of the lowest-dimensional condensates [3], we assume that deviations from the first-order approximation are numerically small up to the nuclear matter saturation density. This assumption should be investigated further.

The lowest-dimensional condensates are the quark condensates $\langle\bar{q}q\rangle_{\rho_N}$ and $\langle q^\dagger q\rangle_{\rho_N}$. The deviation of $\langle\bar{q}q\rangle_{\rho_N}$ from its vacuum value is related to the nucleon σ term; at nuclear matter saturation density, the in-medium quark condensate is decreased by 30–45 % relative to its vacuum value. The other dimension-three quark condensate $\langle q^\dagger q\rangle_{\rho_N}$ is simply proportional to the nucleon density.

The dimension-four quark condensate $\langle\bar{q}iD_0 q\rangle_{\rho_N}$ is proportional to the current quark mass times the nucleon density (thus we neglect it), and $\langle q^\dagger iD_0 q\rangle_{\rho_N}$ depends on quark and antiquark distribution functions determined by deep-inelastic scattering experiments. In nuclear matter there are two independent dimension-four gluon condensates, $\langle(\alpha_s/\pi)(\mathbf{E}^2-\mathbf{B}^2)\rangle_{\rho_N}$ and $\langle(\alpha_s/\pi)(\mathbf{E}^2+\mathbf{B}^2)\rangle_{\rho_N}$. The trace anomaly is used to obtain a model-independent estimate of $\langle(\alpha_s/\pi)(\mathbf{E}^2-\mathbf{B}^2)\rangle_{\rho_N}$ in terms of the σ term and the strangeness content of the nucleon; at saturation density, this in-medium gluon condensate is about 5–10 % smaller than its vacuum value. The other gluon condensate $\langle(\alpha_s/\pi)(\mathbf{E}^2+\mathbf{B}^2)\rangle_{\rho_N}$ is estimated in terms of the gluon distribution function in the nucleon.

We also considered contributions to the nucleon correlator from the dimension-five quark condensates $\langle\bar{q}iD_0 iD_0 q\rangle_{\rho_N}$ and $\langle q^\dagger iD_0 iD_0 q\rangle_{\rho_N}$ and the quark-gluon condensates $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$ and $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$. These condensates were estimated in terms of such quantities as quark and antiquark distribution functions and the coefficient of the leading power correction to the Gross–Llewellyn Smith sum rule. A simple bag-model estimate of $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$ and $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_{\rho_N}$ is shown in Appendix C.

The dominant higher-dimensional contribution to the nucleon correlator arises from four-quark condensates, since these enter in tree-level diagrams. In vacuum, one usually estimates the four-quark condensate using the factorization, or vacuum-saturation, approximation; all four-quark condensates are proportional to $\langle\bar{q}q\rangle^2_{\text{vac}}$ in this approximation. In this paper we use factorization to estimate the in-medium four-quark condensates in terms of $\langle \bar{q}q \rangle_{\rho_N}^2$, $\langle \bar{q}q \rangle_{\rho_N} \langle q^\dagger q \rangle_{\rho_N}$, and $\langle q^\dagger q \rangle_{\rho_N}^2$. However, in nuclear matter a simple ground-state saturation approximation may not be justified. Studies of nucleon self-energies with QCD sum rules suggest that the factorization approximation may strongly overestimate the density dependence of these condensates [4]. Thus it is important to improve our understanding of these in-medium condensates.

Some of the results of this paper have already been used to estimate nucleon self-energies in nuclear matter [2,4]. A more detailed numerical analysis including all contributions to the correlator discussed here will be presented in a subsequent work [14].

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APPENDIX A

In this appendix we estimate the values of the four-quark condensates by using the ground-state saturation approximation. We wish to estimate the following in-medium four-quark condensates:

$$\begin{aligned} &\langle \bar{q}\Gamma_1 q\bar{q}\Gamma_2 q \rangle_{\rho_N}, \quad \langle \bar{q}\Gamma_1\lambda^A q\bar{q}\Gamma_2\lambda^A q \rangle_{\rho_N} , \\ &\langle \bar{u}\Gamma_1 d\bar{d}\Gamma_2 u \rangle_{\rho_N}, \quad \langle \bar{u}\Gamma_1\lambda^A d\bar{d}\Gamma_2\lambda^A u \rangle_{\rho_N} , \\ &\langle \bar{u}\Gamma_1 u\bar{d}\Gamma_2 d \rangle_{\rho_N}, \quad \langle \bar{u}\Gamma_1\lambda^A u\bar{d}\Gamma_2\lambda^A d \rangle_{\rho_N} , \end{aligned} \tag{A1}$$

where Γ_1 and Γ_2 are Dirac matrices. It is a trivial matter to deduce the vacuum limit (discussed in Sec. II) from these finite-density results.

The essential ingredients of the ground-state saturation approximation are the factorization formulas

$$\begin{aligned} \langle \bar{q}_{a\alpha}q_{b\beta}\bar{q}_{c\gamma}q_{d\delta} \rangle_{\rho_N} \simeq & \langle \bar{q}_{a\alpha}q_{b\beta} \rangle_{\rho_N} \langle \bar{q}_{c\gamma}q_{d\delta} \rangle_{\rho_N} \\ & - \langle \bar{q}_{a\alpha}q_{d\delta} \rangle_{\rho_N} \langle \bar{q}_{c\gamma}q_{b\beta} \rangle_{\rho_N} , \end{aligned} \tag{A2}$$

$$\langle \bar{u}_{a\alpha}u_{b\beta}\bar{d}_{c\gamma}d_{d\delta} \rangle_{\rho_N} \simeq \langle \bar{u}_{a\alpha}u_{b\beta} \rangle_{\rho_N} \langle \bar{d}_{c\gamma}d_{d\delta} \rangle_{\rho_N} . \tag{A3}$$

We motivate these relations by considering a nonrelativistic analogue [30]. For simplicity we neglect interactions. Consider the quantity

$$Q = \langle \Psi_0 | N(\psi_i^\dagger \psi_j \psi_k^\dagger \psi_l) | \Psi_0 \rangle , \tag{A4}$$

where ψ_i is a nonrelativistic fermion field operator (i is a generic index), $|\Psi_0\rangle$ is the ground state of a noninteracting finite-density system, and the normal ordering (denoted here by N) is relative to the zero-density vacuum state. This vacuum state, denoted by $|0\rangle$, is defined by the relation $\psi_i|0\rangle=0$. One can then perform the normal ordering to obtain

$$Q = -\langle \Psi_0 | \psi_i^\dagger \psi_k^\dagger \psi_j \psi_l | \Psi_0 \rangle . \tag{A5}$$

In order to evaluate Q, it is useful to perform a canonical transformation and write the field operator ψ_i in terms of particle destruction operators a_i and hole creation operators $b_i^\dagger$ [30]. Under such a transformation, the state $|\Psi_0\rangle$ is effectively a new vacuum state. One has $\psi_i = a_i + b_i^\dagger$, where $a_i|\Psi_0\rangle=0$ and $b_i|\Psi_0\rangle=0$. Therefore, one obtains

$$\begin{aligned} Q &= -\langle \Psi_0 | (a_i^\dagger + b_i)(a_k^\dagger + b_k)(a_j + b_j^\dagger)(a_l + b_l^\dagger) | \Psi_0 \rangle \\ &= -\langle \Psi_0 | b_i b_k b_j^\dagger b_l^\dagger | \Psi_0 \rangle . \end{aligned} \tag{A6}$$

From the anticommutation relation $\{b_i, b_j^\dagger\} = \delta_{ij}$ one finds

$$\begin{aligned} b_i b_k b_j^\dagger b_l^\dagger = & -\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk} + \delta_{ij} b_l^\dagger b_k - \delta_{il} b_j^\dagger b_k \\ & - \delta_{jk} b_l^\dagger b_i + \delta_{kl} b_j^\dagger b_i + b_j^\dagger b_l^\dagger b_i b_k , \end{aligned} \tag{A7}$$

which implies

$$Q = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk} . \tag{A8}$$

Similarly, one has $\langle \Psi_0 | N(\psi_i^\dagger \psi_j) | \Psi_0 \rangle = \delta_{ij}$; therefore, one obtains the factorized result,

$$\langle \Psi_0 | N(\psi_i^\dagger \psi_j \psi_k^\dagger \psi_l) | \Psi_0 \rangle = \langle \Psi_0 | N(\psi_i^\dagger \psi_j) | \Psi_0 \rangle \langle \Psi_0 | N(\psi_k^\dagger \psi_l) | \Psi_0 \rangle - \langle \Psi_0 | N(\psi_i^\dagger \psi_l) | \Psi_0 \rangle \langle \Psi_0 | N(\psi_k^\dagger \psi_j) | \Psi_0 \rangle . \tag{A9}$$

This relation is analogous to the one in Eq. (A2). Similar, simpler considerations lead to the analogue of Eq. (A3). If interactions are included, the factorized forms of the four-fermion matrix elements correspond to the Hartree-Fock approximation [30].

We now consider the four-quark condensates listed in Eq. (A1). It is useful to perform the analysis for arbitrary N_c, where N_c is the number of quark colors. We use the properties of the generalized Gell-Mann matrices,

$$\mathrm{tr}(\lambda^A)=0, \quad \mathrm{tr}(\lambda^A\lambda^B)=2\delta^{AB} , \tag{A10}$$

where the indices A and B now run from 1 to N_c^2-1. Combined with the relation

$$\langle \bar{q}_{a\alpha}q_{b\beta} \rangle_{\rho_N} = \frac{\delta_{ab}}{4N_c} (\langle \bar{q}q \rangle_{\rho_N} \delta_{\beta\alpha} + \langle \bar{q}\gamma_\mu q \rangle_{\rho_N} \gamma^\mu_{\beta\alpha}) , \tag{A11}$$

which follows from Eq. (3.19), the four-quark condensates appearing in Eq. (A1) can be expressed as

$$\langle \bar{q}\Gamma_1 q\bar{q}\Gamma_2 q\rangle_{\rho_N}=\frac{1}{16}\Bigg\{\langle \bar{q}q\rangle^2_{\rho_N}\left[\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\Gamma_2)-\frac{1}{N_c}\mathrm{Tr}(\Gamma_1\Gamma_2)\right]$$
$$+\langle \bar{q}q\rangle_{\rho_N}\langle \bar{q}\gamma_\mu q\rangle_{\rho_N}\left[\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\gamma^\mu\Gamma_2)-\frac{1}{N_c}\mathrm{Tr}(\Gamma_1\gamma^\mu\Gamma_2)+\mathrm{Tr}(\gamma^\mu\Gamma_1)\mathrm{Tr}(\Gamma_2)-\frac{1}{N_c}\mathrm{Tr}(\gamma^\mu\Gamma_1\Gamma_2)\right]$$
$$+\langle \bar{q}\gamma_\mu q\rangle_{\rho_N}\langle \bar{q}\gamma_\nu q\rangle_{\rho_N}\left[\mathrm{Tr}(\gamma^\mu\Gamma_1)\mathrm{Tr}(\gamma^\nu\Gamma_2)-\frac{1}{N_c}\mathrm{Tr}(\gamma^\mu\Gamma_1\gamma^\nu\Gamma_2)\right]\Bigg\}\ , \tag{A12}$$

$$\langle \bar{q}\Gamma_1\lambda^A q\bar{q}\Gamma_2\lambda^A q\rangle_{\rho_N}=-\left[\frac{N_c^2-1}{8N_c^2}\right]\{\langle \bar{q}q\rangle^2_{\rho_N}\mathrm{Tr}(\Gamma_1\Gamma_2)+\langle \bar{q}q\rangle_{\rho_N}\langle \bar{q}\gamma_\mu q\rangle_{\rho_N}[\mathrm{Tr}(\Gamma_1\gamma^\mu\Gamma_2)+\mathrm{Tr}(\gamma^\mu\Gamma_1\Gamma_2)]$$
$$+\langle \bar{q}\gamma_\mu q\rangle_{\rho_N}\langle \bar{q}\gamma_\nu q\rangle_{\rho_N}\mathrm{Tr}(\gamma^\mu\Gamma_1\gamma^\nu\Gamma_2)\}\ , \tag{A13}$$

$$\langle \bar{u}\Gamma_1 d\bar{d}\Gamma_2 u\rangle_{\rho_N}=-\frac{1}{16N_c}[\langle \bar{u}u\rangle_{\rho_N}\langle \bar{d}d\rangle_{\rho_N}\mathrm{Tr}(\Gamma_1\Gamma_2)+\langle \bar{u}u\rangle_{\rho_N}\langle \bar{d}\gamma_\mu d\rangle_{\rho_N}\mathrm{Tr}(\Gamma_1\gamma^\mu\Gamma_2)$$
$$+\langle \bar{u}\gamma_\mu u\rangle_{\rho_N}\langle \bar{d}d\rangle_{\rho_N}\mathrm{Tr}(\gamma^\mu\Gamma_1\Gamma_2)+\langle \bar{u}\gamma_\mu u\rangle_{\rho_N}\langle \bar{d}\gamma_\nu d\rangle_{\rho_N}\mathrm{Tr}(\gamma^\mu\Gamma_1\gamma^\nu\Gamma_2)]\ , \tag{A14}$$

$$\langle \bar{u}\Gamma_1\lambda^A d\bar{d}\Gamma_2\lambda^A u\rangle_{\rho_N}=-\left[\frac{N_c^2-1}{8N_c^2}\right][\langle \bar{u}u\rangle_{\rho_N}\langle \bar{d}d\rangle_{\rho_N}\mathrm{Tr}(\Gamma_1\Gamma_2)+\langle \bar{u}u\rangle_{\rho_N}\langle \bar{d}\gamma_\mu d\rangle_{\rho_N}\mathrm{Tr}(\Gamma_1\gamma^\mu\Gamma_2)$$
$$+\langle \bar{u}\gamma_\mu u\rangle_{\rho_N}\langle \bar{d}d\rangle_{\rho_N}\mathrm{Tr}(\gamma^\mu\Gamma_1\Gamma_2)+\langle \bar{u}\gamma_\mu u\rangle_{\rho_N}\langle \bar{d}\gamma_\nu d\rangle_{\rho_N}\mathrm{Tr}(\gamma^\mu\Gamma_1\gamma^\nu\Gamma_2)]\ , \tag{A15}$$

$$\langle \bar{u}\Gamma_1 u\bar{d}\Gamma_2 d\rangle_{\rho_N}=\tfrac{1}{16}[\langle \bar{u}u\rangle_{\rho_N}\langle \bar{d}d\rangle_{\rho_N}\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\Gamma_2)+\langle \bar{u}u\rangle_{\rho_N}\langle \bar{d}\gamma_\mu d\rangle_{\rho_N}\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\gamma^\mu\Gamma_2)$$
$$+\langle \bar{u}\gamma_\mu u\rangle_{\rho_N}\langle \bar{d}d\rangle_{\rho_N}\mathrm{Tr}(\gamma^\mu\Gamma_1)\mathrm{Tr}(\Gamma_2)+\langle \bar{u}\gamma_\mu u\rangle_{\rho_N}\langle \bar{d}\gamma_\nu d\rangle_{\rho_N}\mathrm{Tr}(\gamma^\mu\Gamma_1)\mathrm{Tr}(\gamma^\nu\Gamma_2)]\ , \tag{A16}$$

$$\langle \bar{u}\Gamma_1\lambda^A u\bar{d}\Gamma_2\lambda^A d\rangle_{\rho_N}=0\ . \tag{A17}$$

APPENDIX B

In this appendix we prove the following relations:

$$\langle \bar{q}\Gamma D_\mu q\rangle_{\rho_N}=-\langle \bar{q}\Gamma\overleftarrow{D}_\mu q\rangle_{\rho_N}\ , \tag{B1}$$

$$\langle \bar{q}\Gamma D_\mu D_\nu q\rangle_{\rho_N}=-\langle \bar{q}\Gamma\overleftarrow{D}_\mu D_\nu q\rangle_{\rho_N}=\langle \bar{q}\Gamma\overleftarrow{D}_\mu\overleftarrow{D}_\nu q\rangle_{\rho_N}\ , \tag{B2}$$

where $D_\mu=\partial_\mu-ig_s\mathcal{A}_\mu$ and $\overleftarrow{D}_\mu=\overleftarrow{\partial}_\mu+ig_s\mathcal{A}_\mu$. These results, and others with higher numbers of derivatives, follow from the translation invariance of the nuclear matter ground state.

We first prove Eq. (B1). This equality follows from considering the nonlocal matrix element

$$\langle \bar{q}(0)\Gamma q(x)\rangle_{\rho_N}=\langle \bar{q}(-x)\Gamma q(0)\rangle_{\rho_N}\ , \tag{B3}$$

where the right-hand side follows from the translation invariance of nuclear matter. As discussed in Sec. III, the fields can be expanded for small x in the fixed-point gauge using a covariant Taylor series expansion; thus one obtains

$$\langle \bar{q}(0)\Gamma q(x)\rangle_{\rho_N}=\langle \bar{q}\Gamma q\rangle_{\rho_N}+x^\mu\langle \bar{q}\Gamma D_\mu q\rangle_{\rho_N}+\tfrac{1}{2}x^\mu x^\nu\langle \bar{q}\Gamma D_\mu D_\nu q\rangle_{\rho_N}+\cdots\ , \tag{B4}$$

$$\langle \bar{q}(-x)\Gamma q(0)\rangle_{\rho_N}=\langle \bar{q}\Gamma q\rangle_{\rho_N}-x^\mu(\bar{q}\Gamma\overleftarrow{D}_\mu q\rangle_{\rho_N}+\tfrac{1}{2}x^\mu x^\nu\langle \bar{q}\Gamma\overleftarrow{D}_\nu\overleftarrow{D}_\mu q\rangle_{\rho_N}+\cdots\ , \tag{B5}$$

where the fields and derivatives on the right-hand side are evaluated at $x=0$. Equation (B3) and the fact that x is arbitrarily imply that analogous terms in Eqs. (B4) and (B5) are equal. For the terms linear in x^μ, this fact implies that the coefficients are equal; therefore, one obtains the result announced in Eq. (B1).

Equation (B2) is proved in a similar manner by considering the relations

$$\langle \bar{q}(0)\Gamma(D_\nu q)_x\rangle_{\rho_N}=\langle \bar{q}(-x)\Gamma(D_\nu q)_{x=0}\rangle_{\rho_N}\ , \tag{B6}$$

$$\langle(\bar{q}\overleftarrow{D}_\mu)_{x=0}\Gamma q(x)\rangle_{\rho_N}=\langle(\bar{q}\overleftarrow{D}_\mu)_{-x}\Gamma q(0)\rangle_{\rho_N}\ , \tag{B7}$$

which follow from translation invariance. This approach can easily be generalized to give relations involving higher numbers of derivatives. Similar relations also hold for nucleon matrix elements of the above operators.

APPENDIX C

In this appendix we estimate the nucleon matrix elements $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N$ and $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N$ using a bag model. The results that we obtain here should be considered to be rough estimates of these matrix elements at best, since the quark and gluon degrees of freedom in the bag model do not coincide exactly with those of QCD. In addition, the vacuum condensate $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_{\rm vac}$ is nonzero due to the spontaneous breaking of chiral symmetry in the vacuum; thus it is likely that the value of $\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N$ is strongly linked to the physics of partial chiral restoration. The bag model used here is not a chiral model.

In the bag model, the nucleon is considered to consist of three quarks moving freely in the perturbative vacuum inside the bag, while the region outside the bag is the physical nonperturbative vacuum. In terms of the color-electric and color-magnetic fields, $\mathbf{E}^A$ and $\mathbf{B}^A$, the above nucleon matrix elements can be expressed as

$$\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N=ig_s\int_{\rm bag}d^3x\langle\tilde{N}|\bar{q}[2\gamma_0\boldsymbol{\gamma}\cdot\mathbf{E}^A-\boldsymbol{\gamma}\cdot(\boldsymbol{\gamma}\times\mathbf{B}^A)]t^Aq|\tilde{N}\rangle\ ,\tag{C1}$$

$$\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N=ig_s\int_{\rm bag}d^3x\langle\tilde{N}|q^\dagger[2\gamma_0\boldsymbol{\gamma}\cdot\mathbf{E}^A-\boldsymbol{\gamma}\cdot(\boldsymbol{\gamma}\times\mathbf{B}^A)]t^Aq|\tilde{N}\rangle\ ,\tag{C2}$$

where we integrate over the region inside the bag (with $\langle\tilde{N}|\tilde{N}\rangle=1$). The color-electric field contribution to $\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N$ vanishes due to time-reversal and translation invariance. In terms of the quark wave functions, these matrix elements can then be written as

$$\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N=\frac{i}{2}g_s\sum_{i=1}^{3}\int_{\rm bag}d^3x(\psi_i^\dagger[2\boldsymbol{\gamma}\cdot\mathbf{E}^A-\gamma_0\boldsymbol{\gamma}\cdot(\boldsymbol{\gamma}\times\mathbf{B}^A)]t^A\psi_i)_N\ ,\tag{C3}$$

$$\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N=-\frac{i}{2}g_s\sum_{i=1}^{3}\int_{\rm bag}d^3x(\psi_i^\dagger\boldsymbol{\gamma}\cdot(\boldsymbol{\gamma}\times\mathbf{B}^A)t^A\psi_i)_N\ ,\tag{C4}$$

where we use the notation $(\cdots)_N$ as a reminder that the three quarks are in a nucleon. This fact will be used to establish appropriate relations between the spin and color projections of the quarks.

The quarks in the nucleon are assumed to be in the lowest eigenstates of the bag model; thus the wave function of the ith quark is taken to be [56,57]

$$\psi_i(\mathbf{x})=\frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0\left[\dfrac{\omega_0 r}{R}\right]\chi_i\\ i\sigma\cdot\hat{\mathbf{r}}j_1\left[\dfrac{\omega_0 r}{R}\right]\chi_i\end{pmatrix}\ ,\tag{C5}$$

where $\omega_0\simeq2.043$ is the lowest solution of the equation $j_0(\omega)=j_1(\omega)$, R is the bag radius, and χ_i is the spin and color wave function for the ith quark. The wave functions are normalized such that the ith quark appears inside the bag with unit probability:

$$N^2=\frac{\omega_0}{2R^3(\omega_0-1)j_0^2(\omega_0)}\ .\tag{C6}$$

The color fields $\mathbf{E}^A$ and $\mathbf{B}^A$ are generated by the three valence quarks. In calculating these fields, we only consider the one-gluon exchange interaction, which is of lowest order in α_s. At the one-gluon exchange level, non-Abelian gluon self-couplings do not contribute; the gluons act as eight independent Abelian fields. Thus the problem reduces to ordinary electrostatics and magnetostatics with appropriate boundary conditions on the bag surface [56,57]. In the limit in which the quark masses are equal, the total color-electric field $\mathbf{E}^A$ vanishes at every point within the bag; therefore, terms with $\mathbf{E}^A$ do not contribute, and this leads to

$$\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N=-\frac{i}{2}g_s\sum_{i=1}^{3}\int_{\rm bag}d^3x(\psi_i^\dagger\gamma_0\boldsymbol{\gamma}\cdot(\boldsymbol{\gamma}\times\mathbf{B}^A)t^A\psi_i)_N\ ,\tag{C7}$$

$$\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N=-\frac{i}{2}g_s\sum_{i=1}^{3}\int_{\rm bag}d^3x(\psi_i^\dagger\boldsymbol{\gamma}\cdot(\boldsymbol{\gamma}\times\mathbf{B}^A)t^A\psi_i)_N\ .\tag{C8}$$

In the limit of equal quark masses, the color-magnetic field is given by [56]

$$\mathbf{B}^A(\mathbf{x})=\frac{1}{8\pi}\sum_{i=1}^{3}\lambda_i^A\left\{\boldsymbol{\sigma}_i\left[2M(r)+\frac{\mu(R)}{R^3}-\frac{\mu(r)}{r^3}\right]+3\hat{\mathbf{r}}(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}})\frac{\mu(r)}{r^3}\right\}\ ,\tag{C9}$$

where

$$\mu(r)=\int_0^r dr'\mu'(r')\ ,\tag{C10}$$

$$M(r)=\int_r^R dr'\frac{\mu'(r')}{r'^3}\ ,\tag{C11}$$

$$\mu'(r)=-\tfrac{2}{3}g_sN^2r^3j_0\left[\frac{\omega_0 r}{R}\right]j_1\left[\frac{\omega_0 r}{R}\right]\ .\tag{C12}$$

In Eq. (C9) we have also introduced the spin and color projections $\boldsymbol{\sigma}_i\equiv\chi_i^\dagger\boldsymbol{\sigma}\chi_i$ and $\lambda_i^A\equiv\chi_i^\dagger\lambda^A\chi_i$. Note that the color-magnetic moment from the ith quark is $\boldsymbol{\mu}_i^A=\boldsymbol{\sigma}_i(\lambda_i^A/2)\mu(R)$.

Straightforward calculations result in

$$\langle g_s\bar{q}\sigma\cdot\mathcal{G}q\rangle_N=\frac{\alpha_s}{R^2}f_1\sum_{i,j}((\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\lambda_i^A\lambda_j^A))_N\ ,\tag{C13}$$

$$\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N=\frac{\alpha_s}{R^2}f_2\sum_{i,j}((\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\lambda_i^A\lambda_j^A))_N\ .\tag{C14}$$

The estimated quark-gluon condensates thus depend on the strong coupling constant α_s and the bag radius R. f_1 and f_2 are defined by

$$f_1 \equiv \frac{\omega_0^2}{24(\omega_0-1)^2\sin^4\omega_0}\int_0^{\omega_0} dy\, y^2 \left\{ \left[j_0^2(y)+\frac{1}{3}j_1^2(y) \right] \left[\frac{\mathcal{M}(\omega_0)}{\omega_0^3} - j_0^2(\omega_0)+j_0^2(y) \right] - \frac{4}{3}j_1^2(y)\frac{\mathcal{M}(y)}{y^3} \right\}, \tag{C15}$$

$$f_2 \equiv \frac{\omega_0^2}{24(\omega_0-1)^2\sin^4\omega_0}\int_0^{\omega_0} dy\, y^2 \left\{ \left[j_0^2(y)-\frac{1}{3}j_1^2(y) \right] \left[\frac{\mathcal{M}(\omega_0)}{\omega_0^3} - j_0^2(\omega_0)+j_0^2(y) \right] + \frac{4}{3}j_1^2(y)\frac{\mathcal{M}(y)}{y^3} \right\}, \tag{C16}$$

where

$$\mathcal{M}(y) \equiv \frac{y}{2} + \frac{y\cos 2y}{4} - \frac{3\sin 2y}{8}. \tag{C17}$$

The magnitudes of the spin and color projections are well known:

$$\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_i = 3, \quad \lambda_i^A \lambda_i^A = \tfrac{16}{3}, \tag{C18}$$

where there is no sum over i. The products of the spin and color projections of different quarks are constrained by the fact that the three quarks must combine to form a spin-$\frac{1}{2}$ color-singlet nucleon. Thus one obtains [56,57]

$$(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)_N = -1, \quad (\lambda_i^A \lambda_j^A)_N = -\tfrac{8}{3}, \tag{C19}$$

for $i \neq j$.

We choose the coupling constant and bag radius to fit the nucleon root-mean-square charge radius and the N-Δ mass splitting; this implies $\alpha_s \simeq 2.55$ and $R \simeq 1.18$ fm [57]. Evaluating the integrals in Eqs. (C15) and (C16) we obtain

$$\langle g_s \bar{q}\sigma \cdot \mathcal{G} q \rangle_N \simeq 0.62\ \text{GeV}^2, \tag{C20}$$

$$\langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_N \simeq 0.66\ \text{GeV}^2. \tag{C21}$$

[1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979); **B147**, 519 (1979).
[2] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. Lett. **67**, 961 (1991).
[3] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C **45**, 1881 (1992).
[4] R. J. Furnstahl, D. K. Griegel, and T. D. Cohen, Phys. Rev. C **46**, 1507 (1992).
[5] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981).
[6] V. M. Belyaev and B. L. Ioffe, Zh. Eksp. Teor. Fiz. **83**, 876 (1982) [Sov. Phys. JETP **56**, 493 (1982)]; **84**, 1236 (1983) [**57**, 716 (1983)].
[7] B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B232**, 109 (1984).
[8] D. B. Leinweber, Ann. Phys. (N.Y.) **198**, 203 (1990).
[9] For a review, see L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985), and references therein.
[10] S. J. Wallace, Annu. Rev. Nucl. Part. Sci. **37**, 267 (1987).
[11] S. Hama, B. C. Clark, E. D. Cooper, H. S. Sherif, and R. L. Mercer, Phys. Rev. C **41**, 2737 (1990).
[12] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
[13] B. D. Serot, Rep. Prog. Phys. **55**, 1855 (1992).
[14] T. D. Cohen, R. J. Furnstahl, D. K. Griegel, and X. Jin (unpublished).
[15] T. Hatsuda and S. H. Lee, Phys. Rev. C **46**, R34 (1992).
[16] E. G. Drukarev and E. M. Levin, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 307 (1988) [JETP Lett. **48**, 338 (1988)]; Zh. Eksp. Teor. Fiz. **95**, 1178 (1989) [Sov. Phys. JETP **68**, 680 (1989)]; Nucl. Phys. **A511**, 679 (1990); **A516**, 715(E) (1990).
[17] E. G. Drukarev and E. M. Levin, Prog. Part. Nucl. Phys. **27**, 77 (1991).
[18] T. Hatsuda, H. Høgaasen, and M. Prakash, Phys. Rev. Lett. **66**, 2851 (1991).
[19] C. Adami and G. E. Brown, Z. Phys. A **340**, 93 (1991).
[20] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
[21] R. Tarrach, Nucl. Phys. **B196**, 45 (1982).
[22] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
[23] A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 60 (1978) [JETP Lett. **27**, 55 (1978)].
[24] E. V. Shuryak, *The QCD Vacuum, Hadrons, and the Superdense Matter* (World Scientific, Singapore, 1988), and references therein.
[25] A. A. Ovchinnikov and A. A. Pivovarov, Yad. Fiz. **48**, 1135 (1988) [Sov. J. Nucl. Phys. **48**, 721 (1988)].
[26] M. Kremer and G. Schierholz, Phys. Lett. B **194**, 283 (1987).
[27] A. P. Bakulev and A. V. Radyushkin, Phys. Lett. B **271**, 223 (1991).
[28] S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D **45**, 1754 (1992).
[29] E. V. Shuryak, Nucl. Phys. **B328**, 85 (1989).
[30] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).
[31] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. **B237**, 525 (1984).
[32] A. R. Zhitnitskiĭ, Yad. Fiz. **41**, 805 (1985) [Sov. J. Nucl. Phys. **41**, 513 (1985)]; **41**, 1035 (1985) [**41**, 664 (1985)]; **41**, 1331 (1985) [**41**, 846 (1985)].
[33] J. Govaerts, L. J. Reinders, F. de Viron, and J. Weyers, Nucl. Phys. **B283**, 706 (1987).
[34] B. L. Ioffe, Z. Phys. C **18**, 67 (1983).
[35] K. G. Wilson, Phys. Rev. **179**, 1499 (1969).
[36] J. C. Collins, *Renormalization* (Cambridge University Press, New York, 1984).
[37] A. I. Vainshtein, V. I. Zakharov, V. A. Novikov, and M. A. Shifman, Yad. Fiz. **41**, 1063 (1985) [Sov. J. Nucl. Phys. **41**, 683 (1985)], and references therein; V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B249**, 445 (1985), and references therein.
[38] F. David, Nucl. Phys. **B263**, 637 (1986), and references therein.
[39] V. Fock, Phys. Z. Sowjetunion **12**, 404 (1937).
[40] J. Schwinger, *Particles, Sources, and Fields* (Addison-

Wesley, Reading, MA, 1970), Vol. I.
[41] C. Cronström, Phys. Lett. **90B**, 267 (1980).
[42] A. V. Smilga, Yad. Fiz. **35**, 473 (1982) [Sov. J. Nucl. Phys. **35**, 271 (1982)].
[43] W. Hubschmid and S. Mallik, Nucl. Phys. **B207**, 29 (1982).
[44] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Fortschr. Phys. **32**, 585 (1984).
[45] M. Shifman, Nucl. Phys. **B173**, 13 (1980).
[46] J. Gasser, H. Leutwyler, and M. E. Sainio, Phys. Lett. B **253**, 252 (1991), and references therein.
[47] See, for example, F. J. Ynduráin, *Quantum Chromodynamics* (Springer-Verlag, New York, 1983); D. J. Gross, in *Methods in Field Theory,* edited by R. Balian and J. Zinn-Justin (North-Holland, Amsterdam, 1976); or R. L. Jaffe, in *Relativistic Dynamics and Quark-Nuclear Physics,* edited by M. B. Johnson and A. Picklesimer (Wiley, New York, 1986).
[48] A. V. Efremov and A. V. Radyushkin, Riv. Nuovo Cimento **3**(2), 1 (1980), and references therein.
[49] J. C. Collins and D. E. Soper, Nucl. Phys. **B194**, 445 (1982).
[50] G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 27 (1980).
[51] M. Glück, E. Reya, and A. Vogt, Z. Phys. C **48**, 471 (1990).
[52] M. Glück, E. Reya, and A. Vogt, Z. Phys. C **53**, 127 (1992).
[53] E. V. Shuryak and A. I. Vainshtein, Phys. Lett. **105B**, 65 (1981); Nucl. Phys. **B199**, 451 (1982).
[54] V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. **B283**, 723 (1987).
[55] R. L. Jaffe and X. Ji, Nucl. Phys. **B375**, 527 (1992).
[56] T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).
[57] R. K. Bhaduri, *Models of the Nucleon: From Quarks to Soliton* (Addison-Wesley, Redwood City, 1988).