Coulomb polarizability of the deuteron and the $E1$ capture mechanism in the $^4He(d,\gamma)^6Li$ reaction

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The ^4He $(d,\gamma)^6Li$ reaction, which proceeds through the overwhelming $E2$ transition, but with a small admixture of the $E1$ transition, is a possible mechanism for the ^6Li nucleosynthesis in the Big Bang. The $E1$ radiation is, however, strongly suppressed in the radiative deuteron capture reaction in self-conjugate nuclei. Usually, the observed small, but not negligible, $E1$ radiation has been attributed to the spin-dependent electric dipole operator. In this study it is shown how the Coulomb polarization of the deuteron generates electric dipole transition as well in the $^4He(d,\gamma)^6Li$ reaction. Expressly we investigate the $E1$ capture process which proceeds with a small change of the deuteron internal structure in the entrance channel state. The structural change of the deuteron is engendered by the Coulomb field in the system of target and deuteron and such a deviant deuteron state is surveyed by calculating the expectation value of the relative coordinate between the proton and neutron in the deuteron. We then show how the electrically polarized deuteron state releases the usual $E1$ selection rule, and subsequently we calculate the relative intensity of the $E1$ versus $E2$ radiation in the $^4He(d,\gamma)^6Li$ reaction.

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I. INTRODUCTION

There has been recently much interest [1–5] in the ^4He $(d,\gamma)^6Li$ radiative capture reaction at astrophysical low incident deuteron energies. This is because the radiative capture of the deuteron by an α particle is believed to be a possible mechanism for a large amount of the ^6Li nucleosynthesis in the early Universe. Additionally, the study of this reaction allows us to obtain information on the D-state component [6–8] of the system composed of the deuteron and the α particle. An interesting feature of the radiative deuteron capture reaction resides also in the strong exothermic character of the reaction, as we can see from Q values of the reactions (d,γ) for various light nuclear targets. Indeed, the Q values of the radiative deuteron capture reaction are 22.28, 16.69, 15.82, 25.19, 18.68, 10.27, and 20.73 MeV for the targets ^6Li, ^7Li, ^9Be, ^{10}B, ^{11}B, ^{12}C, and ^{14}N, respectively. However, contrary to these, the Q value of the $^4He(d,\gamma)^6Li$ reaction is only 1.47 MeV, which is very low. Recent experimental results [1], obtained for the incident energy down to 1 MeV in the center-of-mass system, show that the differential cross section exhibits a typical characteristic of the $E2$ transition but with a small forward-backward asymmetry. The small asymmetry of the angular distribution has been explained by the interference between the overwhelming $E2$ transition and the minor $E1$ transition. However, because of the extremely small electric dipole moment of the d-α system, the $E1$ transition is very unlikely. In other words, since both the initial and final states have isospin $T=0$ in the direct $^4He(d,\gamma)^6Li$ reaction, the usual spin-independent $E1$ operator has no contribution because of the isospin selection rule. Accordingly, the provenance of the observed small $E1$ radiation has been ascribed to the spin-dependent electric dipole operator. Furthermore, it is also believed that a significant contribution from the $M1$ transition is improbable. Arguments in favor of such hindrance of the $M1$ transition should be reviewed [9] very carefully. In an earlier paper [10], we have investigated the radiative deuteron capture reaction in the giant resonance region of ^{16}O and provided an explanation for the experimental gamma-ray angular distribution [11] which also exhibits an admixture character of electric dipole and quadrupole transitions.

In this paper, we discuss various possible $E1$ transition mechanisms in the direct radiative deuteron capture reactions in self-conjugate nuclei, especially in the $^4He(d,\gamma)^6Li$ reaction. In particular, we investigate the possibility of $E1$ radiation induced by the Coulomb polarization of the deuteron. The $E1$ transition selection rule is actually based on the usual spin-independent part of the $E1$ operator which has been deduced with the help of the long-wavelength approximation for photons. There are, however, several unusual $E1$ transition operators by which the $E1$ transition may yet take place. For example, the dipole moment resulting from a distribution of magnetization associated with the spins of nucleons is the origin of the aforementioned spin-dependent $E1$ transition operator. In addition to this, we may also have $E1$ radiation caused by higher-order terms in the long-wavelength approximation for photons, which have been customarily neglected in the usual spin-independent $E1$ transition operator. Generally contributions of these unusual operators to the $E1$ radiation are appreciably smaller than the contribution of the usual spin-independent charge density and they have therefore been neglected. It has been, however, suggested that the only source of the observed $E1$ radiation in the $^4He(d,\gamma)^6Li$ reaction is the spin-dependent $E1$ operator. However, there exists another possibility for the direct $E1$ capture process which comes about through the structural change of the deuteron internal state engendered by the Coulomb interaction between the incident deuteron and

the target. The deuteron is a particle composed of two loosely bounded nucleons and the Coulomb interaction acts, not between the center of mass of the deuteron and the target, but between protons of the target and the proton of the deuteron. The fact that the center of mass and the center of charge of the deuteron does not coincide has an effect to bring in a spatial asymmetry in the deuteron structure and accordingly the deuteron is electrically polarized. This is the so-called Coulomb polarizability of the deuteron [12–15]. The spatial asymmetry of the deuteron could not be very sizable, but it is still enough to loosen in part the usual selection rule for the direct deuteron $E1$ capture process in self-conjugate nuclei. In other words, the initially vanishing matrix element of the usual spin-independent $E1$ transition operator between the deuteron ground state and the final nucleus becomes nonvanishing when the electrically polarized deuteron state is introduced. The Coulomb polarization vector or the spatial asymmetry may be defined as the expectation value of the relative coordinate vector between the proton and the neutron in the deuteron which undergoes the Coulomb interaction. Such an average value certainly vanishes in the absence of the Coulomb field.

In this work, we estimate the $E1$ and $E2$ transition probabilities within the framework of a naive cluster model as well as a simple shell model for the ^{6}Li nucleus. The use of the cluster model, such as the $\alpha+d$ model for ^{6}Li, has been widely used [5] for the analysis of the ^{4}He$(d,\gamma)^6$Li reaction.

In Sec. II, we first give a short description of the usual $E1$ and $E2$ transition operators and show how the $E1$ transition is suppressed in self-conjugate nuclei. We then estimate the contributions coming from the unusual $E1$ transition operators by computing the relative intensity of the $E1$ radiation versus the predominant $E2$ radiation. In Sec. III, we discuss the influence of the Coulomb interaction between the target and the incident deuteron on the deuteron internal structure and this is realized by calculating the Coulomb polarization of the deuteron. In Sec. IV, we discuss the electrically polarized deuteron internal wave function as well as the resulting nonvanishing $E1$ transition matrix element for the radiative deuteron capture process. Finally, we give a summary and conclusion for general features of the $d\gamma$ reaction mechanism in self-conjugate nuclei, in particular, in the ^{4}He$(d,\gamma)^6$Li reaction.

II. RELATIVE INTENSITY OF UNUSUAL $E1$ VERSUS PREDOMINANT $E2$ RADIATION

Since the usual electric multipole transition operators are quite well known, we first describe briefly the parts of transition operators which are needed for discussion of their suppressive character in the radiative deuteron capture process in self-conjugate nuclei.

The effective charges in the electromagnetic transition operators in nuclei arise when the center-of-mass motion is extracted from the relative motion between nucleons or, equivalently, when the recoil correction is made. The effective $E1$ transition operators [16] are then expressed as

$$Q_{1m}=\tfrac{1}{2}C_\gamma\left[\frac{N}{A}\sum_{i=1}^{A}(1+\tau_{iz})-\frac{Z}{A}\sum_{i=1}^{A}(1-\tau_{iz})\right]r_i Y^*_{1m}(\hat{\mathbf{r}}_i)$$
$$=\tfrac{1}{2}C_\gamma\sum_{i=1}^{A}\left[\frac{N-Z}{A}+\tau_{iz}\right]r_i Y^*_{1m}(\hat{\mathbf{r}}_i) \tag{2.1}$$

for the $E1$ transition and

$$Q_{2m}=\tfrac{1}{2}e(-k_\gamma^2\sqrt{\pi/15})\times\sum_{i=1}^{A}\left[\left[1-\frac{2}{A}+\frac{2Z}{A^2}\right]+\left[1-\frac{2}{A}\right]\tau_{iz}\right]r_i^2 Y^*_{2m}(\hat{\mathbf{r}}_i) \tag{2.2}$$

for the $E2$ transition, where $C_\gamma=-iek_\gamma\sqrt{4\pi/3}$, k_γ is the photon wave number, and τ_{iz} is the third component of the isospin operator of the ith nucleon. As we see from (2.2), the effective charges for the $E2$ transition may not be of much significance.

For a pair of nucleons which are actually involved in the deuteron capture reaction, the $E1$ and $E2$ transition operators can be expressed in terms of the center-of-mass coordinate, $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$, and the relative coordinate, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. For the proton and neutron pair we get

$$Q_{1m}=C_\gamma\left[\frac{N-Z}{A}RY^*_{1m}(\hat{\mathbf{R}})+\tfrac{1}{2}rY^*_{1m}(\hat{\mathbf{r}})\right], \tag{2.3}$$

for the $E1$ transition and

$$Q_{2m}=-ek_\gamma^2\sqrt{\pi/15}\left\{\left[1-\frac{2N}{A^2}\right][R^2Y^*_{2m}(\hat{\mathbf{R}})+\tfrac{1}{4}r^2Y^*_{2m}(\hat{\mathbf{r}})]\right.$$
$$\left.+2\sqrt{10\pi/3}\left[1-\frac{2}{A}\right]Rr\sum_{m'}\langle 1m'1m-m'|2m\rangle Y^*_{1m'}(\hat{\mathbf{R}})Y^*_{1m-m'}(\hat{\mathbf{r}})\right\} \tag{2.4}$$

for the $E2$ transition. It is observed that we may equally formulate the transition operators in terms of the separation coordinate between the deuteron and the core, $\rho=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)-\mathbf{r}_{\text{core}}$. For example, when the origin of the coordinate system is not the center of mass of the α particle, the $E2$ transition operator with the separation coordinate vector in the d-α cluster is multiplied [5] by a numerical factor, $\frac{2}{3}$. Actually the second terms in (2.3) and (2.4) correspond to the $\Delta T=1$ transitions in isospin language. The fact that the first term in (2.3) vanishes identically for self-conjugate nuclei

and that the second term in (2.4) contains only odd order radial operators illustrates clearly which transitions are allowed and which transitions are not. When the two-particle states constructed out of the final nucleus are either in S or D states, such as is the case in the cluster model of ^{6}Li, we see that the $E1$ transition in self-conjugate nuclei is strictly forbidden, since the matrix element of the operator rY^*_{1m} between the deuteron ground state and the final two-particle S or D states vanishes. Accordingly, only the $E2$ transition is allowed and it becomes overwhelming. To argue further the necessity of investigating the Coulomb polarizability of the deuteron in relation to the experimental differential cross section [1], it will be necessary to estimate first the order of magnitude of the contribution of the unusual $E1$ transition operators and this may be realized following the arguments put forward in Refs. [17–21].

Actually the relative intensity of the habitually neglected higher-order terms in the usual $E1$ transition operator in the long-wavelength approximation for photons may be estimated in the single-particle picture by taking into account the higher-order expansion terms of the spherical Bessel function in the vector potential. Using the relation

$$j_L(k_\gamma r)\approx\frac{(k_\gamma r)^L}{(2L+1)!!}-\frac{(k_\gamma r)^{L+2}}{2(2L+3)!!}\ , \tag{2.5}$$

where j_L is the spherical Bessel function, the vector potential for the electric multipole transition may be written as

$$\mathbf{A}(e)=\sqrt{2\pi}P\sum_{LM}\frac{(-i)^{L+1}}{\sqrt{L(L+1)}}D^L_{MP}(\mathbf{k}_\gamma)\frac{1}{k_\gamma r}\left\{\left[\frac{(L+1)(2L+1)}{(2L+1)!!}(k_\gamma r)^L-\frac{(L+1)(L+3)}{2(2L+3)!!}(k_\gamma r)^{L+2}\right]\right.$$
$$\left.\times\sqrt{L}\,\mathbf{T}^M_{L,L-1}-\left[\frac{1}{(2L+3)!!}(k_\gamma r)^{L+2}\right]\sqrt{L+1}\mathbf{T}^M_{L,L+1}\right\}, \tag{2.6}$$

where $\mathbf{T}^M_L=\sum\langle 1-\mu\lambda M+\mu|LM\rangle Y_{\lambda M+\mu}(\hat{\mathbf{r}})\xi_{-\mu}$ with $\xi_{-\mu}$ being the spherical basis vector. The first term with $(k_\gamma r)^L$ is the usual operator which we employ commonly for the calculation of electric 2^L pole transitions. The ratios of two other terms to the first term lead, respectively, to

$$\left|\frac{L+3}{(2L+1)2(2L+3)}(k_\gamma R_0)^2\right|^2_{L=1}=(\tfrac{2}{15})^2(k_\gamma R_0)^4 \tag{2.7}$$

and

$$\left|\frac{L+1}{L(2L+3)}(k_\gamma R_0)^2\right|^2_{L=1}=(\tfrac{2}{5})^2(k_\gamma R_0)^4\ , \tag{2.8}$$

where R_0 is of the order of magnitude of the nuclear radius. These ratios give rise to the order of magnitude varying from 10^{-6} to 10^{-5} for $E_\gamma=1-10$ MeV. In view of this, one can safely neglect the contributions of the higher-order terms in the long-wavelength approximation as far as the $E1$ transition is concerned. A similar estimation has been made in Ref. [20].

The spin-dependent $E1$ transition operator is generally given by

$$Q'_{1m}=-\frac{ik_\gamma}{2}\frac{e\hbar}{2Mc}\sum_{j=1}^{A}\mu_j\nabla[r_jY^*_{1m}(\hat{\mathbf{r}}_j)](\mathbf{r}_j\times\boldsymbol{\sigma}_j)\ , \tag{2.9}$$

where μ is the magnetic moment of the particle in Bohr magnetons and σ is the Pauli spin operator. Using the transition operator (2.9), a very crude estimation of the ratio between the spin-dependent $E1$ radiation and the predominant $E2$ radiation can be made in the single-particle transition picture. Following the argument given in Ref. [17], this ratio may be put into the form

$$\left|\frac{\sqrt{4\pi/3}(1/2)k_\gamma\frac{\hbar}{2Mc}R_0[(1/R_0)\mu_p\sigma_pR_0+(1/R_0)\mu_n\sigma_nR_0]}{\sqrt{4\pi/15}k_\gamma R_0^2}\right|^2=\left|\frac{\sqrt{5}(\hbar k_\gamma/4Mc)(\mu_p\sigma_p+\mu_n\sigma_n)}{k_\gamma R_0}\right|^2. \tag{2.10}$$

In deriving (2.10) we have replaced the operator ∇ by R_0^{-1}. When we further remark that $\mathbf{L}/(L+1)$ is of the same order of magnitude as $\mu\sigma$ and that the operator $\mathbf{L}$ gives rise to a multiplication by a number of the order L, we see that the ratio (2.10) amounts to be of the order of 10^{-2} or less for $E_\gamma=1-10$ MeV. However, this estimation represents merely the order of magnitude and may not be very reliable due to several crude approximations we made. Actually the intrinsic magnetic moments μ of the nucleons are somewhat larger than unity and therefore $\mu\sigma$ is probably a few times larger than $\mathbf{L}/(L+1)$. The resulting underestimation may be counterbalanced with the overestimation arising from the consideration on the same footing of the magnetic moments of proton and neutron.

In order to obtain an estimate of the actual value of the

ratio, we have to assume a specific nuclear model. For the pair of proton-neutron, the spin-dependent $E1$ operators (2.9) can be transformed into the form

$$Q'_{1m}=ik_\gamma\sqrt{3/4\pi}\frac{e\hbar}{2Mc}\tfrac{1}{2}(\mu_p+\mu_n)\times[2\mathbf{R}\times\mathbf{S}+\tfrac{1}{2}\mathbf{r}\times(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)]\xi^*_m\ , \tag{2.11}$$

where ξ_m is the aforementioned spherical basis vector and $\mathbf{S}=\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$ is the total spin operator of the pair. Since the expectation value of the operator $\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2$ between two triplet states vanishes, the only operator to be considered is the total spin part. For practical calculations, use will be made of the identity

$$(\mathbf{R}\times\mathbf{S})\xi^*_m=i\sqrt{8\pi/3}\sum_\nu\langle 1m-\nu 1\nu|1m\rangle RY_{1\nu}(\hat{\mathbf{R}})S_{m-\nu}\ , \tag{2.12}$$

where $S_{m-\nu}$ is the $m-\nu$ component of the total spin operator.

We now write the entrance channel state as

$$|J_iM_i\rangle=\sum_{l_dm_dJM_J}4\pi i^{l_d}\langle l_dm_d1\sigma_d|JM_J\rangle\times Y^*_{l_dm_d}(\hat{\mathbf{k}}_d)\Phi(l_dJM_J)\Psi_i(\alpha)\ , \tag{2.13}$$

where $\mathbf{k}_d$ is the wave-number vector of the incident deuteron,

$$\Psi(l_dJM_J)=\sum_{m_d\sigma_d}\langle l_dm_d1\sigma_d|JM_J\rangle\times f_{l_d}(k_dR)Y_{l_dm_d}(\hat{\mathbf{R}})\phi_d(r)\bar{\chi}(1\sigma_d)\ , \tag{2.14}$$

and Ψ_i denotes the α-particle wave function. In (2.14), $\bar{\chi}$ and ϕ_d stand for the deuteron spin and internal wave functions, respectively. The radial function in the entrance channel f_{l_d} may take the form

$$f_{l_d}(\rho)=\exp i(\eta_{l_d}+\delta_{l_d})[F_{l_d}(\rho)\cos\delta_{l_d}+G_{l_d}(\rho)\sin\delta_{l_d}]/\rho\ , \tag{2.15}$$

where $\rho=k_dR$, η is the Coulomb phase, δ is the phase shift related to the optical potential, and F and G are the regular and irregular Coulomb functions, respectively. As in Ref. [1], we may employ the distorted wave generated by the McIntyre-Haeberli potential [22] for the incoming continuous state. Similarly, the final state may be written as

$$|J_fM_f\rangle=\sum_{M_LM_S}C\langle L_fM_LS_fM_S|J_fM_f\rangle\Phi_f(L_fM_L;\mathbf{R},\mathbf{r})\bar{\chi}(S_fM_S)\Psi_f(\text{core})\ , \tag{2.16}$$

where Φ_f stands for the spatial part of the two-particle state out of the core denoted by Ψ_f and $\bar{\chi}$ is the two-particle spin function. The explicit forms of C and Φ depend on the specific model for the final nucleus. For the d-α cluster model, the coordinate $\mathbf{R}$ may represent the separation coordinate between the α particle and the deuteron. For a simple shell model in which the core consists of $(0s)^4$ particles, the function Φ represents the Moshinsky-Brody [24] transformed harmonic-oscillator function.

Having calculated all transition matrix elements between the initial and final states defined by (2.13) and (2.16), the relative intensity of the Q'_{1m} transition probability versus the Q_{2m} transition probability may now be put into the form

$$\frac{[(\hbar/2Mc)(1/2)(\mu_p+\mu_n)]^2\left[\sum_{i=0}^{1}a_i\int_0^\infty u_i(R)f_1(k_dR)R^3dR\right]^2}{\left[\sum_{i=0}^{1}b_i\int_0^\infty u_i(R)f_2(k_dR)R^4dR\right]^2}\ , \tag{2.17}$$

where the coefficients a_i and b_i stand for various numerical factors related to specific nuclear models, including the radial integrals over dr. For example, for a simple cluster model without the D-state component, $a_1=b_1=0$ and u_0 represents the S state of the d-α cluster. For the harmonic-oscillator two-particle states, u_i denotes the radial function $\mathcal{R}_{nl}(R)$ with different combinations of n and l.

For the simple cluster model [23], the radial function u_0 is given by

$$u_0(R)=R^2[\exp(-c_1R^2)+c_2\exp(-c_3R^2)]\ , \tag{2.18}$$

where $c_1=0.18$ fm^{-2}, $c_2=0.25$, and $c_3=0.065$ fm^{-2}. When use is made of this function, the relative intensity is shown to be at most of the order of 10^{-2} at the deuteron incident energy, $E_{\text{c.m.}}=2.1$ MeV, which is slightly smaller than the corresponding value extracted from Ref. [1]. However, because of the different nuclear model used and also of the normalization constants involved in Ref. [1], it seemed less suitable to make a straight comparison of two estimations. In this approximation and in what follows we have assumed a structureless α particle.

When we use the harmonic-oscillator wave function for the two-particle state out of the core, the result is about four times larger than the previous estimation, but the order of magnitude remains unchanged.

III. COULOMB POLARIZABILITY OF DEUTERON

The usual definition [12–15] of the Coulomb polarizability of deuteron is the ratio of the polarization energy, which is actually the change in the binding energy for the ground state, to an adiabatically applied uniform electric field $\mathcal{E}$,

$$\bar{\alpha}=-2\frac{W_p}{\mathcal{E}^2} . \tag{3.1}$$

Here W_p is the energy corresponding to the second-order Stark effect arising from the perturbation $-\frac{1}{2}ez\mathcal{E}$, where z is the third component of the relative distance $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$ along the direction of the electric field $\mathcal{E}$. However, the Coulomb field in the target-deuteron system is not static and thus the adiabatic application of the Coulomb field does not correspond to the physical reality. The inconvenience of such an adiabatic application of the Coulomb field will be seen more clearly in the following discussion.

The Hamiltonian for a deuteron in the field of a target charge Ze may be written as

$$H=-\frac{\hbar^2}{4M}\Delta_R-\frac{\hbar^2}{M}\Delta_r+V_{np}(\mathbf{r})+\frac{Ze^2}{|\mathbf{R}-\frac{1}{2}r|} , \tag{3.2}$$

where Δ_R and Δ_r are the operators with respect to the coordinates $\mathbf{R}$ and $\mathbf{r}$, respectively, $\mathbf{R}$ being the center-of-mass coordinate of the deuteron.

The Schrödinger equation which describes the deuteron scattering is then given by

$$(E-H_0)\Psi(\mathbf{R},\mathbf{r})=H'\Psi(\mathbf{R},\mathbf{r}) , \tag{3.3}$$

where

$$H'=Ze^2\left[\frac{1}{|\mathbf{R}-\frac{1}{2}\mathbf{r}|}-\frac{1}{R}\right] , \tag{3.4}$$

with $H_0=H-H'$. In Eq. (3.3) E is the sum of the incident kinetic energy of the deuteron and the deuteron binding energy. We now assume that the total wave function $\Psi(\mathbf{R},\mathbf{r})$ is composed of two parts, $\Psi_0(\mathbf{R},\mathbf{r})$ and $\chi(\mathbf{R},\mathbf{r})$, in such a way that $\Psi_0(\mathbf{R},\mathbf{r})$ is the product of the Coulomb scattering function of the deuteron $\Phi(\mathbf{R})$ and the unperturbed deuteron internal wave function $\phi_d(\mathbf{r})$. The polarization vector or the polarization of the deuteron is then defined by the expectation value of the relative coordinate $\mathbf{r}$,

$$\begin{aligned}\mathbf{P}&=\langle\Psi(\mathbf{R},\mathbf{r})|\mathbf{r}|\Psi(\mathbf{R},\mathbf{r})\rangle\\&=2\,\mathrm{Re}\langle\chi(\mathbf{R},\mathbf{r})|\mathbf{r}|\Phi(\mathbf{R})\phi_d(\mathbf{r})\rangle .\end{aligned} \tag{3.5}$$

Since H' is assumed to be small, we may replace, as a first approximation, the function $\Psi(\mathbf{R},\mathbf{r})$ on the right-hand side of Eq. (3.3) by $\Psi_0(\mathbf{R},\mathbf{r})$ and we get

$$(E-H_0)\chi(\mathbf{R},\mathbf{r})=H'\Psi_0(\mathbf{R},\mathbf{r}) . \tag{3.6}$$

Neglecting the two-nucleon interaction potential in Eq. (3.6) in accordance with the introduction of an explicit form of the deuteron ground-state wave function into Ψ_0, Eq. (3.6) can be written in a tractable form. In fact, the omission of the potential $V_{np}(\mathbf{r})$ is based on the assumption that the critical values of r lie outside the range of the potential. In the studies [12–15] of the Coulomb polarizability of the deuteron, it has been argued that, when the Coulomb polarizability $\bar{\alpha}$ of (3.1) is expressed explicitly in terms of two-body-potential-dependent and -independent parts, the contribution of the term with $V_{np}(\mathbf{r})$ to the polarizability is small since the short-range two-body interaction contributes mainly for S states but the matrix elements with the z-coordinate operator in (3.1) vanish for states of even parity. In this context we expect that the overlook of such an interaction potential $V_{np}(\mathbf{r})$ may not have much influence on the evaluation of the Coulomb polarization of the deuteron. We get

$$\left[E+\frac{\hbar^2}{4M}\Delta_R-\frac{\hbar^2}{M}q^2-\frac{Ze^2}{R}\right]\chi(\mathbf{R},\mathbf{q})=F(\mathbf{R},\mathbf{q}) , \tag{3.7}$$

where $\chi(\mathbf{R},\mathbf{q})$ and $F(\mathbf{R},\mathbf{q})$ are the Fourier-transformed functions which take explicitly the forms

$$\chi(\mathbf{R},\mathbf{q})=\frac{1}{(2\pi)^{3/2}}\int\chi(\mathbf{R},\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r} \tag{3.8}$$

and

$$F(\mathbf{R},\mathbf{q})=\frac{1}{(2\pi)^{3/2}}\int H'\phi_d(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r} . \tag{3.9}$$

Upon performing again the Fourier transformation of H', the function $F(\mathbf{R},\mathbf{q})$ is transformed into

$$F(\mathbf{R},\mathbf{q})=\frac{4}{(2\pi)^{5/2}}Ze^2N_dF'(\mathbf{R},\mathbf{q}) \tag{3.10}$$

with

$$F'(\mathbf{R},\mathbf{q})=\Phi(\mathbf{R})\int\frac{1}{p^2}\left[\frac{1}{\alpha^2+(\frac{1}{2}\mathbf{p}-\mathbf{q})^2}-\frac{1}{\alpha^2+q^2}\right]\times e^{-i\mathbf{p}\cdot\mathbf{R}}d\mathbf{p} . \tag{3.11}$$

In deriving Eq. (3.10), we have used the exponential form of the deuteron internal wave function, $\phi_d(\mathbf{r})=N_d\exp(-\alpha r)/r$ with $N_d=\sqrt{\alpha/2\pi}$.

At this stage it is worthwhile to discuss the inconvenience of the adiabatic application of the Coulomb field to the calculation of the polarization. When a deuteron is placed in a uniform Coulomb field, the peturbation Hamiltonian H' takes the form given below Eq. (3.1). In this case, the Coulomb polarization, Eq. (3.5), may be simply written as

$$\mathbf{P}=2\langle\phi_1(\mathbf{r})|\mathbf{r}|\phi_d(\mathbf{r})\rangle , \tag{3.12}$$

where

$$\phi_1(\mathbf{r})=-\frac{M}{\hbar^2}\int G(\mathbf{r},\mathbf{r}')H'\phi_d(\mathbf{r})d\mathbf{r}' . \tag{3.13}$$

The Green's function $G(\mathbf{r},\mathbf{r}')$ satisfies the equation

$$(-\Delta+\alpha^2)G(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}') . \tag{3.14}$$

Equation (3.13) can be solved by using the Green's function (3.14) and the polarization is then shown to be

$$\mathbf{P}=\frac{1}{16B_d}\frac{Ze^2}{R}\frac{\mathbf{R}}{\alpha^2R^2}\ , \tag{3.15}$$

where $B_d=\hbar^2\alpha^2/M$. This polarization is in a very concise and simple form. However, the shortcoming of this expression resides in the fact that the polarization of the deuteron goes to infinity when R tends to zero, which is quite inconceivable. In fact, the validity of the result (3.15) and therefore the applicability of the perturbation $H'=-ez\mathcal{E}/2$ is limited to the region where $R>r$. This is understood by expanding the dynamical perturbation Hamiltonian H', Eq. (3.4), as

$$H'=Ze^2\sum_{\lambda=0}^{\infty}\frac{1}{R}\left[\frac{r}{2R}\right]^{\lambda}P_\lambda(\cos\Theta_{\mathbf{Rr}})\quad\text{for } R>r\ , \tag{3.16}$$

where $\Theta_{\mathbf{Rr}}$ is the angle between $\mathbf{R}$ and $\mathbf{r}$. We see that the term with $\lambda=1$ corresponds precisely to the uniform electric field with which we have calculated the polarization of the deuteron, (3.15).

Using the Fourier-transformed functions we have derived so far, the general formula for the Coulomb polarization of the deuteron can now be expressed by

$$\mathbf{P}=-\frac{2i}{\pi^3}N_dZe^2\Phi^*(\mathbf{R})\int\frac{\mathbf{q}\chi'(\mathbf{R},\mathbf{q})}{(\alpha^2+q^2)^2}d\mathbf{q}\ , \tag{3.17}$$

where the function $\chi'(R,q)$ satisfies the equation

$$\left[E+\frac{\hbar^2}{4M}\Delta_R-\frac{\hbar^2}{M}q^2-\frac{Ze^2}{R}\right]\chi'(\mathbf{R},\mathbf{q})=F'(\mathbf{R},\mathbf{q})\ . \tag{3.18}$$

Putting

$$\chi'(\mathbf{R},\mathbf{q})=\Phi(\mathbf{R})\Xi(\mathbf{R},\mathbf{q})\ , \tag{3.19}$$

the polarization is finally shown to be

$$\mathbf{P}=-i\frac{MZe^2\alpha}{\hbar^2\pi^4}|\Phi(\mathbf{R})|^2\int\frac{\mathbf{q}\Xi(\mathbf{R},\mathbf{q})}{(\alpha^2+q^2)^2}d\mathbf{q}\ , \tag{3.20}$$

where the function $\Xi(\mathbf{R},\mathbf{q})$ is now the solution to the equation

$$\frac{1}{4}\left[\Delta_R+\frac{2\nabla_R\Phi(\mathbf{R})}{\Phi(\mathbf{R})}\nabla_R\right]\Xi(\mathbf{R},\mathbf{q})-(\alpha^2+q^2)\Xi(\mathbf{R},\mathbf{q})=\int\frac{\mathbf{p}}{p^2[\alpha^2+(\mathbf{p}/2-\mathbf{q})^2]}e^{-i\mathbf{p}\cdot\mathbf{R}}d\mathbf{p}\ . \tag{3.21}$$

The calculation of the polarization is then reduced practically to the evaluation of the function $\Xi(\mathbf{R},\mathbf{q})$ satisfied by Eq. (3.21). As a very crude approximation we neglect the terms in the square brackets on the left-hand side of Eq. (3.21). In this case the function $\Xi(\mathbf{R},\mathbf{q})$ takes a simple integral form which can be analytically integrated and the polarization takes the form

$$\frac{\mathbf{P}}{R_d}=\frac{1}{16}\left[\frac{Ze^2/R}{B_d}\right]\frac{R_d\mathbf{R}}{R^2}|\Phi(\mathbf{R})|^2\times\left[1-\exp\left[-\frac{4R}{R_d}\right]\left[1+\frac{4R}{R_d}+\frac{8R^2}{R_d^2}\right]\right]\ , \tag{3.22}$$

where R_d is the deuteron radius. For the region of small R where the Coulomb force acts strongly, the polarization becomes simply

$$\mathbf{P}=\left[\frac{2}{3}\frac{Ze^2/R}{B_d}\mathbf{R}\right]|\Phi(\mathbf{R})|^2\ , \tag{3.23}$$

which is finite. This result is to be compared to the previous estimation, Eq. (3.15), obtained from the adiabatic application of the electric field. The numerical variation of the quantity $\mathbf{P}/R_d$ in units of 10^{-2} as a function of R can be shown to be 1.21, 0.72, and 0.39 for distances $R=0.5R_d$, $R=R_d$, and $R=1.5R_d$, respectively. For a practical application to specific nuclei, these values are to be multiplied by the factor $Z|\Phi(\mathbf{R})|^2$. The imperfection of this result is that, because of the omission of the first two terms in Eq. (3.21), it is independent of the incident deuteron wave number k_d. A more accurate evaluation of the Coulomb polarization which takes into account the deuteron incident energy can be carried out by putting

$$\frac{\nabla_R\Phi(\mathbf{R})}{\Phi(\mathbf{R})}=i\mathbf{k}_d\ . \tag{3.24}$$

Actually this approximation implies that the ratio of the gradient of $\Phi(\mathbf{R})$ to $\Phi(\mathbf{R})$ may not be very much different from the corresponding ratio obtained from the plane-wave approximation, though the scattering wave function $\Phi(\mathbf{R})$ is not the plane wave. The polarization is then expressed as

$$\mathbf{P}=-i\frac{MZe^2\alpha}{\hbar^2\pi^4}|\Phi(\mathbf{R})|^2\int\int\frac{\mathbf{q}e^{-i\mathbf{p}\cdot\mathbf{R}}}{p^2(\alpha^2+q^2)^2[\alpha^2+(\mathbf{p}/2-\mathbf{q})^2][\alpha^2+q^2+\frac{1}{4}(p^2-2\mathbf{p}\cdot\mathbf{k}_d)]}d\mathbf{q}\,d\mathbf{p}\ . \tag{3.25}$$

Upon performing the integration over $\mathbf{q}$, we get

$$\mathbf{P}=i\frac{MZe^2}{4\hbar^2\pi^2}|\Phi(\mathbf{R})|^2\int\frac{\mathbf{p}}{p^4}g(\mathbf{p},\mathbf{k}_d)e^{-i\mathbf{p}\cdot\mathbf{R}}d\mathbf{p}\ , \tag{3.26}$$

where

$$g(\mathbf{p},\mathbf{k}_d)=\left[\frac{1}{\alpha^2+p^2/16}-\frac{4}{(\alpha+\beta)^2}\right]+\frac{2\mathbf{p}\cdot\mathbf{k}_d}{p^2-2\mathbf{p}\cdot\mathbf{k}_d}\left\{\frac{1}{\alpha^2+p^2/16}-\frac{64\alpha}{p(p^2-2\mathbf{p}\cdot\mathbf{k}_d)}\left[\tan^{-1}\frac{2(\alpha+\beta)}{p}-\tan^{-1}\frac{4\alpha}{p}\right]\right\}\ , \tag{3.27}$$

for $\beta^2=\frac{1}{4}(p^2-2\mathbf{p}\cdot\mathbf{k}_d)+\alpha^2>0$ and

$$g(\mathbf{p},\mathbf{k}_d)=\left[\frac{1}{\alpha^2+p^2/16}-\frac{4}{(\alpha-i\beta_0)^2}\right]+\frac{2\mathbf{p}\cdot\mathbf{k}_d}{p^2-2\mathbf{p}\cdot\mathbf{k}_d}\left\{\frac{1}{\alpha^2+p^2/16}-\frac{64\alpha}{p(p^2-2\mathbf{p}\cdot\mathbf{k}_d)}\left[-\tan^{-1}\frac{4\alpha}{p}-\frac{1}{2}i\ln\frac{p/2+\beta_0+i\alpha}{p/2-\beta_0-i\alpha}\right]\right\}, \tag{3.28}$$

for $\beta^2<0$ with $\beta_0^2=-\beta^2$. The integral over $d\mathbf{p}$ is somewhat cumbersome but can be evaluated numerically. As a first step, let us assume $k_d=0$, i.e., the deuteron wave is assumed to be standing. The polarization then becomes

$$\mathbf{P}=\frac{32MZe^2}{\pi\hbar^2}\frac{\mathbf{R}}{R}R^2|\Phi(\mathbf{R})|^2(2I_2-3I_1)\ . \tag{3.29}$$

The explicit forms of the two integrals I_1 and I_2 are given in Appendix A. The variation of the quantity $\mathbf{P}/R$ in units of 10^{-2} is now seen to be 2.03, 0.83, and 0.41 for $R=0.5R_d$, $R=R_d$, and $R=1.5R_d$, respectively. For a practical application to a specific nucleus, these values are also multiplied by the factor $Z|\Phi(\mathbf{R})|^2$. It is remarked that these values of the polarization are very close to the previous results obtained from expression (3.22), except for the region where R is small. Incidentally, the two approximations are similar due to the fact that both calculations are implicitly independent of the deuteron wave number. According to expressions (3.15), (3.22), and (3.29), the polarization $\mathbf{P}$ is directed to the radial vector $\mathbf{R}$, as it ought to be.

Generally, k_d is not equal to zero and β^2 is either positive or negative. By putting $\mathbf{p}\cdot\mathbf{k}_d=pk_dx$ and $\mathbf{p}R=\mathbf{y}$, we see that the inequalities for β^2 may be written as

$$\frac{y^2+a^2/4}{\gamma ay}\gtrless x\ , \tag{3.30}$$

where $a=4\alpha R$ and $\gamma=k_d/2\alpha$. By taking into account these inequalities, the general form of the polarization of the deuteron is now given by

$$\mathbf{P}=\frac{16MZe^2}{\pi\hbar^2}R\,\mathbf{R}|\Phi(\mathbf{R})|^2I_\lambda\ , \tag{3.31}$$

where the functions I_λ are various double integrals over dx and dy. When $k_d=2\alpha$, the integral $I_{\lambda=3}$ takes the form

$$I_3=\int_{-1}^{+1}dx\int_0^\infty dy\,x\sin(xy)f_1(x,y)\ , \tag{3.32}$$

where the function f_1 is composed of several elementary functions and shown in Appendix A.

For $\beta^2>0$, the general expression of the integral $I_{\lambda=4}$ may be written as

$$I_{\lambda=4}=\left[\int_0^{b_-}dy\int_{-I}^{+1}dx+\int_{b_-}^{b_+}dy\int_{-1}^{c(y)}dx+\int_{b_+}^\infty dy\int_{-1}^{+1}dx\right][x\sin(xy)f_2(x,y)]\ , \tag{3.33}$$

where $b_\pm=(a/2)(\gamma\pm\sqrt{\gamma^2-1})$ and $c(y)=(y^2+a^2/4)/\gamma ay$. The explicit forms of the function f_2 as well as functions f_3 and f_4 in Eq. (3.34) are given in Appendix A.

For $\beta^2<0$, the integral becomes

$$I_{\gamma=5}=\int_{b_-}^{b_+}dy\int_{c(y)}^{+1}dx[x\sin(xy)f_3(x,y)-x\cos(xy)f_4(x,y)]\ . \tag{3.34}$$

The explicit forms of the functions $f_i(x,y)$ are rather cumbersome but they are composed of elementary functions only. Table I displays the numerical results of the deuteron polarization in units of 10^{-2}. For actual values of the polarization, these values are also to be multiplied by the factor $Z|\Phi(\mathbf{R})|^2$, as was remarked before. It is observed that the polarization is not very sensitive to the change of the wave number k_d. In the case of $\beta^2<0$, the integral over $d\mathbf{p}$ in (3.26) can be carried out only under particular conditions and this is because the inequality leads to very restricted and unusual angles between $\mathbf{p}$ and $\mathbf{k}_d$. The convergence of the integral is accordingly suffered and the resulting polarization at a small region of R becomes rather inapposite, though it is still finite. In what follows we show only the result for $\beta^2>0$. Generally the polarization decreases when R increases and it decreases also for increasing k_d, as was expected.

The square modulus of the function $\Phi(\mathbf{R})$ may be calculated by using, for example, the Coulomb scattering function expressed in parabolic coordinates,

$$\Phi(\mathbf{R})=\exp(-\tfrac{1}{2}\eta\pi)\Gamma(1+i\eta)\exp(ik_d\bar{Z})F(-i\eta,1,iK_d\zeta)\ , \tag{3.35}$$

where F is the confluent hypergeometric function with $\eta=Ze^2/\hbar v_d$, $\zeta=R-\bar{Z}$, and $v_d=\hbar^2k_d^2/4M$, $\bar{Z}$ being the third component of $\mathbf{R}$. The square modulus of the Coulomb function (3.35) can be evaluated by computing directly the confluent hypergeometric function in terms of complex variables. At the origin, the square modulus becomes

TABLE I. The ratio $\mathbf{P}/R_d$ in units of 10^{-2}. For a particular nucleus these values are multiplied by the factor $Z|\Phi(\mathbf{R})|^2$.

E_d (MeV)	$R=0.5R_d$	$R=R_d$	$R=1.5R_d$	$R=2R_d$
2.2	0.81	0.62	0.37	0.21
5	0.59	0.42	0.35	0.27
9	0.41	0.34	0.27	0.22
14	0.31	0.27	0.22	0.21
20	0.26	0.23	0.18	0.15

$$|\Phi(0)|^2=\frac{2\pi\eta}{\exp(2\pi\eta-1)} . \tag{3.36}$$

In fact, the function $\Phi(\mathbf{R})$ may be assumed to be not very much different from the plane wave $\exp(i\mathbf{k}_d\cdot\mathbf{R})$ for small values of η and therefore we may put $|\Phi(\mathbf{R})|^2\approx 1$ for high incident energies. We have already discussed the usefulness of the plane-wave approximation for the function $\Phi(\mathbf{R})$ in relation to the reduction of the very complicated equation (3.21). When η is large enough, the square modulus of function (3.35) becomes very small in the vicinity of the target nucleus. Instead of (3.35), we may equally use the Coulomb wave function expressed in spherical coordinates. Nevertheless, because of the infinite summation over the partial waves involved in this case, use of such a Coulomb wave function may be less convenient than form (3.35). The square modulus of the scattering function (3.35) can also be expressed in terms of the real regular Coulomb function (see Appendix B). We have

$$|\Phi(\mathbf{R})|^2=[F_0(\eta,\rho)]^2+\left[\left[\eta+\frac{1}{\rho}\right]F_0(\eta,\rho)-\sqrt{1+\eta^2}F_1(\eta,\rho)\right]^2 , \tag{3.37}$$

where $\rho=\frac{1}{2}k_d\zeta$ and $F_l(\eta,\rho)$ is the regular Coulomb function. Since actual values of F_0 and F_1 are very easily available, we can carry out readily the numerical value of (3.37). Table II exhibits the polarization of the deuteron in the ^{4}He$(d,\gamma)^6$Li reaction, calculated with the help of the result shown in Table I. The comment made previously on the general feature of the Coulomb polarization in relation to Table I holds also for the result in Table II.

TABLE II. The ratio $\mathbf{P}/R_d$ in units of 10^{-2} in the system $d+^4$He. Here $\zeta=R-\bar{Z}$ and B_d is the deuteron binding energy [see Eq. (3.35)].

	$R=0.5R_d$		$R=R_d$		$R=1.5R_d$		$R=2R_d$	
	$\zeta=0$	$\zeta=R$	$\zeta=0$	$\zeta=R$	$\zeta=0$	$\zeta=R$	$\zeta=0$	$\zeta=R$
$E_d=B_d$	0.97	1.28	0.75	1.20	0.44	0.78	0.35	0.44
$E_d=4B_d$	0.64	0.81	0.53	0.70	0.42	0.52	0.35	0.44
$E_d=9B_d$	0.45	0.54	0.38	0.44	0.31	0.36	0.25	0.30

IV. COULOMB POLARIZATION AND THE ^{4}He$(d,\gamma)^6$Li REACTION

Since the deuteron scattering wave function $\Psi(\mathbf{R},\mathbf{r})$ is assumed to be expressed as

$$\Psi(\mathbf{R},\mathbf{r})=\Phi(\mathbf{R})\phi_d(r)+\chi(\mathbf{R},\mathbf{r}) , \tag{4.1}$$

the matrix element of the $E1$ transition operator (2.3) between the initial and final states for the radiative deuteron capture process may be represented in short by

$$\langle\Phi_f(L_fM_f;\mathbf{R},\mathbf{r})|\tfrac{1}{2}rY^*_{1M}(\hat{\mathbf{r}})|\chi(\mathbf{R},\mathbf{r})\rangle . \tag{4.2}$$

The explicit form of the function $\chi(\mathbf{R},\mathbf{r})$ follows from Eqs. (3.8) and (3.19). We get

$$\chi(\mathbf{R},\mathbf{r})=\frac{N_dZe^2}{4\pi^4}\Phi(\mathbf{R})\int\Xi(\mathbf{R},\mathbf{q})\exp(i\mathbf{q}\cdot\mathbf{r})d\mathbf{q} , \tag{4.3}$$

where the function $\Xi(\mathbf{R},\mathbf{q})$ is the solution to Eq. (3.21). Explicitly $\Xi(\mathbf{R},\mathbf{q})$ takes the form

$$\Xi(\mathbf{R},\mathbf{q})=-\frac{M}{\hbar^2}\int d\mathbf{p}\frac{e^{-i\mathbf{p}\cdot\mathbf{R}}}{p^2[\alpha^2+q^2+\frac{1}{4}(p^2-2\mathbf{p}\cdot\mathbf{k}_d)][\alpha^2+(\mathbf{p}/2-\mathbf{q})^2]} . \tag{4.4}$$

To understand how the Coulomb polarization of the deuteron releases the initially forbidden $E1$ transition, we take an illustrative example by assuming the polarized deuteron wave function in the form

$$\chi(\mathbf{R},\mathbf{r})=\Phi(\mathbf{R})[\phi_d(r)+\phi_1(\mathbf{r})] , \tag{4.5}$$

where $\phi_d(r)$ denotes the unperturbed ground state of the deuteron and $\phi_1(\mathbf{r})$ is the electrically polarized deuteron internal wave function. When use is made of the adiabatic perturbation Hamiltonian $H'=-\frac{1}{2}ez\mathcal{E}$ with $\mathcal{E}=(Ze/R^3)\mathbf{R}$, the perturbed function $\phi_1(\mathbf{r})$ can be obtained by solving Eq. (3.13). We get

$$\phi_1(\mathbf{r})=N_d\frac{\alpha Ze^2}{8B_dR}\frac{\mathbf{R}\cdot\mathbf{r}}{R^2}\exp(-\alpha r) , \tag{4.6}$$

where B_d is the deuteron binding energy. Owing to the presence of the factor $\mathbf{R}\cdot\mathbf{r}$ in (4.6), we see that the matrix element (4.2) now leads to a nonvanishing value, which implies that the usual selection rule for the electric dipole capture process is released as far as the polarized deuteron state $\phi_1(\mathbf{r})$ is concerned. We understand in this manner how the $E1$ transition in self-conjugate nuclei proceeds through the Coulomb polarization of the deuteron. Since the present spatial asymmetry disappears in the absence of the Coulomb interaction, the Coulomb polarization of the deuteron has to be regarded as a dynamical quantity.

The relative intensity of the $E1$ transition induced by the deuteron polarization versus the predominant $E2$ transition in the ^{4}He$(d,\gamma)^6$Li reaction can be estimated as before by calculating explicitly the matrix element (4.2) with the help of the polarized wave function (4.6). The calculation with the harmonic-oscillator two-particle wave function for the final state yields 0.12 for the relative intensity at deuteron incident energy $E_{\text{c.m.}}=2.1$ MeV. The use of the simple cluster model wave function in place of the harmonic-oscillator function does not make any substantial modification of the relative intensity. This rather large value of relative intensity is undoubtedly overestimated due to the use of the adiabatic perturbation Hamiltonian. As we have seen previously, the adiabatic application of a uniform electric field leads to the irrelevant polarization at small values of R and there-

fore the radial integral over dR without cutoff in the matrix element (4.2) may turn out to be fallacious.

The actual relative intensity can be estimated by introducing the reliable wave function $\chi(\mathbf{R},\mathbf{r})$ which is in the double integral form and which can be integrated only numerically. The method of calculation of the matrix element (4.2) becomes somewhat similar to the calculation of the polarization, since the $E1$ transition operator can be transformed into the relative coordinate vector $\mathbf{r}$ via the identity $rY^*_{1M}(\hat{\mathbf{r}})=\sqrt{3/4\pi}\xi^*_M\mathbf{r}$, where ξ_M is the spherical basis vector.

Upon performing all numerical computations involved, we get finally 0.05 for the relative intensity of the $E1$ versus $E2$ radiation at the same incident energy as before. This value is about half of the previous result obtained in the framework of the simple adiabatic polarization model. The use of the simple cluster model leads to a comparable conclusion. The present intensity is of the same order of magnitude as the ratio of the total $E1$ to $E2$ capture cross section, shown in Ref. [1]. It should be, however, observed that the present calculation is actually concerned with the relative intensity obtained in a slightly different nuclear model having no adjustable normalization parameters, and therefore a simple and direct comparison of the present result with the analysis in Ref. [1] has to be made with much caution.

V. SUMMARY AND CONCLUSION

In the present work we have shown how the Coulomb polarization of the deuteron induces the electric dipole radiation in the direct deuteron capture process in self-conjugate nuclei. Instead of analyzing systematically the experimental capture cross section of the ^{4}He$(d,\gamma)^6$Li reaction, we have rather devoted ourselves to the inquiry about the origin of the observed small but not negligible $E1$ radiation.

The expectation value of the relative coordinate vector of the proton-neutron pair in the entrance channel state, which is actually the definition of the Coulomb polarization, is contiguously connected with the transition matrix element of the usual spin-independent $E1$ operator of the pair and thus the initially vanishing matrix element of this operator gives rise to a specific contribution to the $E1$ radiation. A marked feature of the present study is that the small but non-negligible $E1$ transition probability originating from the Coulomb polarization of the deuteron is comparable with the corresponding transition probability estimated with the help of the dipole moments associated with the spins of nucleons. In other words, the relative intensity of the $E1$ versus $E2$ radiation, calculated by means of the polarized deuteron wave function but using the usual $E1$ transition operator, has been shown to be of the same order of magnitude as the corresponding quantity, estimated by employing the unusual spin-dependent $E1$ operator and the unpolarized deuteron internal wave function.

The usual calculation of the Coulomb polarization depends somewhat on the accuracy of the solution to Eq. (3.21), which is rather unwieldy. Since the approximation (3.24) is believed to be satisfactorily reliable as is the resulting solution to Eq. (3.21), the present result of the expectation value of the relative coordinate may not be very much different from the actual value. A more exact calculation of the Coulomb polarization and the subsequent relative intensity of the $E1$ versus $E2$ radiation may consist in inclusion of all possible nuclear interactions in the Hamiltonian (3.2). However, in view of the complex numerical procedure already involved in the present work, such an elaborated calculation will be beyond the present scope of feasibility.

APPENDIX A

The integrals I_1 and I_2 in expression (3.28) for the Coulomb polarization have the forms

$$I_1=\int_0^\infty \frac{\sin z}{z^3}\left[\frac{1}{z^2+a^2}-\frac{4}{(a+\sqrt{a^2+4z^2})^2}\right]dz\ , \tag{A1}$$

$$I_2=\int_0^\infty \frac{\sin z}{z^3}\left[\frac{a}{(z^2+a^2)^2}-\frac{4}{(a+\sqrt{a^2+4z^2})^2\sqrt{a^2+4z^2}}\right]dz\ , \tag{A2}$$

where $z=pR$ and $a=4\alpha R$.

The functions f_λ in the integrals (3.31)–(3.33) explicitly take the forms

$$f_2(x,y)=\frac{1}{(y^2+a^2)(y-\gamma ax)}+\frac{1}{y^3(y-\gamma ax)^2}\left\{\gamma a^2x\tan^{-1}\frac{y[a/2-\sqrt{a^2/4+y(y-\gamma ax)}]}{y^2+a[a/2+\sqrt{a^2/4+y(y-\gamma ax)}]}-\left[\frac{a}{2}-\sqrt{a^2/4+y(y-\gamma ax)}\right]^2\right\}, \tag{A3}$$

$$f_1(x,y)=f_2(x,y)\quad\text{with }\gamma=1\ , \tag{A4}$$

$$f_3(x,y)=\frac{1}{(y^2+a^2)(y-\gamma ax)}+\frac{1}{y^3(y-\gamma ax)^2}\left\{\gamma a^2x\left[\tfrac{1}{2}\tan^{-1}\frac{a}{\gamma ax-2y}+\tan^{-1}\frac{a}{y}\right]-\left[\frac{a^2}{2}+y(y-\gamma ax)\right]\right\}, \tag{A5}$$

$$f_4(x,y)=\frac{\gamma a^2 x}{4y^3(y-\gamma a x)^2}\ln\frac{[y+\sqrt{y(\gamma a x-y)-a^2/4}]^2+a^2/4}{[y-\sqrt{y(\gamma a x-y)-a^2/4}]^2+a^2/4}-\frac{a}{y^3(y-\gamma a x)^2}\sqrt{y(\gamma a x-y)-a^2/4}\ . \tag{A6}$$

APPENDIX B

Using the relation between the regular Coulomb function and the confluent hypergeometric function

$$F_l(\eta,\rho)=C_l(\eta)\rho^{l+1}\exp(-i\rho)F(l+1-i\eta,2l+2,2i\rho)\ , \tag{B1}$$

where

$$C_l(\eta)=2^l\exp(-\tfrac{1}{2}\pi\eta)\frac{|\Gamma(l+1+i\eta)|}{(2l+1)!}\ , \tag{B2}$$

we get the identity

$$\mathrm{Im}[\exp(-i\rho)F(-i\eta,1,2i\rho)]=-\frac{F_0(\eta,\rho)}{C_0(\eta)}\ . \tag{B3}$$

On the other hand, successive use of the recursion relation

$$\exp(-i\rho)F(-i\eta,1,2i\rho)=\exp(i\rho)F(i\eta,1,-2i\rho)-2i\rho\exp(i\rho)F(1+i\eta,2,-2i\rho)\ , \tag{B4}$$

leads to a relation

$$\begin{aligned}F(-i\eta,1,2i\rho)=&\frac{2}{1+i\eta}(1+2\eta\rho)F(1-i\eta,2,2i\rho)\\&-\tfrac{2}{3}(1-i\eta)\rho^2F(2-i\eta,4,2i\rho)\\&-\frac{1-i\eta}{1+i\eta}\exp(2i\rho)F(i\eta,1,-2i\rho)\ .\end{aligned} \tag{B5}$$

In obtaining (B3) and (B4), use is made of Kummer's identity [25]

$$F(-i\eta,1,2i\rho)=\exp(2i\rho)F(1+i\eta,1,-2i\rho)\ . \tag{B6}$$

In view of (B5), we get

$$\mathrm{Re}[\exp(-i\rho)F(-i\eta,1,2i\rho)]=(1+\eta\rho)\frac{F_0(\eta,\rho)}{\rho C_0(\eta)}-(1+\eta^2)\frac{F_1(\eta,\rho)}{3C_1(\eta)}\ . \tag{B7}$$

Using (B3) and (B7), we finally obtain the square modulus of the function $\Phi(\mathbf{R})$ in terms of the regular Coulomb functions. We have

$$|\Phi(\mathbf{R})|^2=[F_0(\eta,\rho)]^2+\left[\left(\eta+\frac{1}{\rho}\right)F_0(\eta,\rho)-\sqrt{1+\eta^2}F_1(\eta,\rho)\right]^2\ . \tag{B8}$$

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