Collective effects on transport coefficients of relativistic nuclear matter

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In order to evaluate qualitatively and in order of magnitude the result of some collective effects on dense matter, a specific model is studied: the Walecka's model of nuclear matter. In this model, the meson fields contribute to the various observables in the Hartree approximation only while all correlations give rise to a collision term considered in the relaxation-time approximation. Within these approximations, a transport equation is given and solved in the Chapman-Enskog expansion in first order. The main transport coefficients (thermal conductivity, shear and bulk viscosities) are calculated and compared to the results first obtained by Anderson and Witting.

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I. INTRODUCTION

Transport properties of relativistic dense matter are quite important in many physical or astrophysical situations. Essentially two classes of dense matter have to be dealt with, quark matter or nuclear matter. These states of matter are supposed to appear in heavy-ion collisions, quark stars (quark matter), or neutron stars. In the case of heavy-ion collisions [1] the relaxation of a possible fluid phase is controlled by dissipative effects via transport coefficients (shear and bulk viscosities, thermal conduction, diffusion, etc.) while properties of dense stars [2] (such as the cooling of a neutron star or its possible "glitches" occurring through transport of momentum from its inner part towards its crust) are also concerned with these basic coefficients.

These transport coefficients are generally calculated via the use of relativistic kinetic theory [3] and thereby imply the knowledge of a *collision term* which we briefly discuss below. In such calculations nucleons, quarks, or quasibaryons of either sort propagate freely between two pointlike collisions. This calls for several remarks. First, a collision term must be chosen in conformity with what is known on the dense matter at hand. When dealing with quark matter that is supposed to be created in heavy-ion collisions, it is clear that its states falls in the confinement-deconfinement regime, a phase where presently nothing is really known from either a theoretical point of view or experimentally. As to nuclear matter, the one boson exchange cross section generally used in a Boltzmann-like (or Uhlenbeck-Uehling version [4]), kinetic equation corresponds to the second-order term of a nonconverging expansion of the transition amplitude, assuming also that a correct quantum field theory describing nucleons is known. For these reasons, and also for the sake of simplicity, most authors have preferred the use of a relativistic kinetic theory involving a relaxation-time approximation [5]. Such an approach, somewhat less ambitious than a general one, is, in our opinion, more realistic in the sense that it yields general expressions of the transport coefficients as functions of the temperature, the energy density, etc., whereas all dynamical problems are rejected in a single (or a few [5]) parameter(s), the relaxation times(s), to be evaluated with specific models. In this spirit, most authors have used the Anderson-Witting form [6] of the relativistic kinetic equation [7]

$$p\cdot\partial f(x,p)=-\frac{p\cdot u}{\tau}[f(x,p)-f_{\rm eq}(x,p)]\ , \tag{1.1}$$

where u^μ is the local average four-velocity of the medium, where $f_{\rm eq}(x,p)$ is the *local* equilibrium distribution function, and τ is the relaxation time. Equation (1.1) is next solved at first order in a Chapman-Enskog [3,5,6] expansion and immediately provides the first correction to $f_{\rm eq}$ as

$$f_{(1)}=(\tau p\cdot\partial/p\cdot u)f_{\rm eq}\ . \tag{1.2}$$

Then following standard procedures [3,5–8], transport coefficients are derived.

However (and here is our second remark), when collective effects must be taken into account, such as, e.g., boson condensates or average fields, then Eq. (1.1) has to be modified and, accordingly, so is the case for the transport coefficients of the system. Such a modification occurs in two different ways. First, it appears as a (self-coupled) force term, as we show in Sec. II; next, it affects the scattering amplitudes and hence the relaxation time. However, this last effect is of no importance in the first-order Chapman-Enskog expansion for it appears only in second order.

In this paper, we would like to investigate the influence of such collective effects on the transport properties of relativistic dense matter. To this end, some side effects (such as those ones due to spin [5], isospin, or other internal degrees of freedom) are neglected; moreover, a specific dynamical model must be chosen as to dense matter. In order to get some idea, and also to be more specific, the Walecka model of nuclear matter [9,10] has been chosen, although we are conscious of its problems [10]; the Walecka model is indeed a standard reference as to relativistic nuclear matter. This model is based on the following Lagrangian:

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$$\mathcal{L}=\tfrac{1}{2}\bar{\psi}\gamma\cdot\overleftrightarrow{D}\psi-\bar{\psi}(m-g_s\phi)\psi+\tfrac{1}{2}[(\partial\phi)^2-m_s^2\phi^2]$$
$$-\tfrac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu}+\tfrac{1}{2}m_V^2 V\cdot V\ , \qquad (1.3)$$

where ϕ is a scalar field of mass m_s coupled to the nucleon field ψ via the coupling constant g_s and V^μ is a vector field of mass m_V coupled to ψ through the coupling constant g_V, while $F_{\mu\nu}$ is connected to V_μ through $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$. From this Lagrangian, the main observables are obtained, i.e., the baryonic four-current

$$J^\mu_{0p}=\bar{\psi}\gamma^\mu\psi \qquad (1.4)$$

and the energy-momentum tensor

$$T^{\mu\nu}_{0p}=\frac{i}{2}\bar{\psi}\overleftrightarrow{D}^\nu\gamma^\mu\psi-\frac{1}{2}\eta^{\mu\nu}[(\partial\phi)^2-m_s^2\phi^2]+\partial^\mu\phi\partial^\nu\phi+F^{\mu\rho}F^\nu_\rho$$
$$+m_V^2V^\mu V^\nu-\frac{1}{2}\eta^{\mu\nu}\left[m_V^2V^2-\frac{1}{2}F_{\rho\lambda}F^{\rho\lambda}\right]\ , \qquad (1.5)$$

which are repeatedly used throughout this paper. In Eqs. (1.3) and (1.5) use has been made of the following notation:

$$i\overleftrightarrow{D}^\nu\equiv[i\partial^\nu-g_VV^\nu]\overrightarrow{\ }-\overleftarrow{[}i\partial^\nu+g_VV^\nu]\ . \qquad (1.6)$$

II. TRANSPORT EQUATION

In this section the transport equation is briefly derived and discussed. The basic tool, which recommends itself in a relativistic quantum kinetic theory, is the covariant Wigner function

$$F(x,p)=\frac{1}{(2\pi)^4}\times\int d^4R\,e^{-ip\cdot R}\langle\bar{\psi}(x+\tfrac{1}{2}R)\otimes\psi(x-\tfrac{1}{2}R)\rangle \qquad (2.1)$$

whose properties are studied elsewhere [11,12] (see also Ref. [3]). In Eq. (2.1) the brackets denote a quantum-mechanical statistical average. With $F(x,p)$ the average value of the baryonic four-current $J^\mu_{\rm op}$ and of the fermion part of the energy-momentum tensor $T^{\mu\nu}_{\rm op}$ can be calculated as

$$J^\mu\equiv\langle J^\mu_{\rm op}\rangle={\rm Sp}\int d^4p\ \gamma^\mu F(x,p) \qquad (2.2)$$

and

$$T^{\mu\nu}_{\rm mat}\equiv\langle T^{\mu\nu}_{\rm op(mat)}\rangle={\rm Sp}\int d^4p\ \gamma^\mu p^\nu F(x,p)\ , \qquad (2.3)$$

where Sp indicates a trace operation over the spinor indices involved in γ^μ and F. In fact, one can also define $T^{\mu\nu}_{\rm mat}$ as

$$T^{\mu\nu}_{\rm mat}\equiv\langle(i/2)\bar{\psi}\gamma^\mu\overleftrightarrow{D}^\nu\psi\rangle \qquad (2.4)$$
$$={\rm Sp}\int d^4p\ \gamma^\mu(p^\nu-g_VV^\nu)F(x,p)\ . \qquad (2.5)$$

As a matter of fact, only the knowledge of J^μ and $T^{\mu\nu}_{\rm mat}$ is necessary in the derivation of transport coefficients in a first-order Chapman-Enskog expansion.

A. A kinetic equation for the covariant Wigner function

From the Walecka Lagrangian (1.3), Dirac's equations obeyed by the nucleon field ψ are easily obtained and, after they are used in the definition of the Wigner operator $F_{\rm op}$ [i.e., in Eq. (2.1) where the average value is not taken], one obtains the following equations:

$$[i\gamma\cdot\partial+2(\gamma\cdot p-m)]F_{\rm op}=\frac{2}{(2\pi)^4}\int d^4R\,d^4\xi\exp[-i(p-\xi)\cdot R]\left[g_V\gamma^\mu F_{\rm op}(x,\xi)V_\mu\left[x-\frac{1}{2}R\right]-g_sF_{\rm op}(x,\xi)\phi\left[x-\frac{1}{2}R\right]\right], \qquad (2.6a)$$

$$F_{\rm op}\overleftarrow{[}i\gamma\cdot\partial-2(\gamma\cdot p-m)]$$
$$=-\frac{2}{(2\pi)^4}\int d^4R\,d^4\xi\exp[-i(p-\xi)\cdot R]\left[g_VV_\mu\left[x+\frac{1}{2}R\right]F_{\rm op}(x,\xi)\gamma^\mu-g_s\phi\left[x+\frac{1}{2}R\right]F_{\rm op}(x,\xi)\right]\ , \qquad (2.6b)$$

which are equivalent to Dirac's equations obeyed by the fields ψ and $\bar{\psi}$. Similarly, the equations obeyed by the fields ϕ and V^μ give rise to

$$(\Box+m_s^2)\phi=g_s\,{\rm Sp}\int d^4p\ F_{\rm op}(x,p)\ , \qquad (2.7)$$

$$(\Box+m_V^2)V^\mu=g_V\,{\rm Sp}\int d^4p\ \gamma^\mu F_{\rm op}(x,p)\ . \qquad (2.8)$$

These four equations can be rewritten in a symbolic manner as

$$\mathcal{L}F_{\rm op}=g_V\int\gamma\cdot F_{\rm op}V-g_s\int F_{\rm op}\phi\ , \qquad (2.6a')$$

$$F_{\rm op}\overleftarrow{\mathcal{L}}=-g_V\int V\cdot F_{\rm op}\gamma+g_s\int\phi F_{\rm op}\ , \qquad (2.6b')$$

$$KG\phi=g_s\,{\rm Sp}\int F_{\rm op}\ , \qquad (2.7')$$

$$KGV=g_V\,{\rm Sp}\int\gamma F_{\rm op}\ , \qquad (2.8')$$

so as to obtain a Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy [11,12] by taking average values of the various possible products of operators as, e.g., $\langle F_{\rm op}\rangle$, $\langle F_{\rm op}\phi\rangle$, $\langle F_{\rm op}F_{\rm op}\rangle$, $\langle F_{\rm op}\phi\phi\rangle$, etc. For instance, taking the average value of Eq. (2.6a′) connects $\langle F_{\rm op}\rangle$ to $\langle F_{\rm op}\phi\rangle$ and $\langle F_{\rm op}V\rangle$. Multiplying (2.6a′) by ϕ from the right and taking the average values yields a connection of $\langle F_{\rm op}\phi\rangle$ and $\langle F_{\rm op}\cdot\phi\cdot\phi\rangle$ and also with $\langle F_{\rm op}\cdot\phi\cdot V\rangle$, etc.

In order to make the collective effects and the correlation (i.e., the effects of collisions) apparent in the lowest-order equations of the hierarchy [i.e., in the above system (2.6)–(2.8) where the average value is taken] let us write the second-order moments as (cluster decomposition)

$$\begin{aligned}\langle F_{\rm op}\cdot\phi\rangle &= \langle F_{\rm op}\rangle\langle\phi\rangle + \mathcal{C}(F_{\rm op},\phi)\ ,\\ \langle F_{\rm op}\cdot V\rangle &= \langle F_{\rm op}\rangle\langle V\rangle + \mathcal{C}(F_{\rm op},V)\ ,\end{aligned} \tag{2.9}$$

where the first terms on the right-hand sides are nothing but the collective terms we are looking for, while the second terms represent correlations. Here we do not need other such decompositions. In ordinary kinetic theory the correlation terms lead, via a variety of assumptions, to a specific collision term. Next, it is argued that this collision term can be approximated by a relaxation-time expression. However, it was shown elsewhere [5] that, in the absence of polarization perturbation, such expressions very generally give rise to results identical with the Anderson-Witting ones [6]. Therefore, a simple relaxation term is chosen here, whose properties are studied in Refs. [5,13]; it reads

$$-i(\gamma\cdot u/\tau)(F-F_{\rm eq})\ , \tag{2.10a}$$

$$-i(F-F_{\rm eq})(\gamma\cdot u/\tau)\ , \tag{2.10b}$$

where Eqs. (2.10a) and (2.10b) are the relaxation terms that corresponds to Eqs. (2.6a) and (2.6b), where the cluster expansion (2.9) has been introduced after a statistical quantum average value has been taken on both sides. In Eq. (2.10) u^μ is the average four-velocity of the medium, τ is the relaxation time to be calculated elsewhere or estimated roughly as, e.g., $\tau=(n\sigma\langle v\rangle)^{-1}$, n being a number density, σ a cross section, and $\langle v\rangle$ an average of the (relative) three-velocity between two nucleons. Finally, $F_{\rm eq}$ is the equilibrium Wigner function, which is briefly studied below.

Finally, the relativistic quantum kinetic equation we are looking for reads

$$\mathcal{L}F = g_V\int\gamma\cdot F\cdot\langle V\rangle - g_s\int F\cdot\langle\phi\rangle - (i\gamma\cdot u/\tau)(F-F_{\rm eq})\ , \tag{2.11}$$

$$f\mathcal{L} = -g_V\int\langle V\rangle\cdot F\cdot\gamma + g_s\int\langle\phi\rangle\cdot F - i(F-F_{\rm eq})(\gamma\cdot u/\tau)\ ,$$

to which the equations for the average fields $\langle V\rangle$ and $\langle\phi\rangle$ must be added,

$$\begin{aligned}KG\langle\phi\rangle &= g_s\,\mathrm{Sp}\int F\ ,\\ KG\langle V\rangle &= g_V\,\mathrm{Sp}\int F\gamma\ .\end{aligned} \tag{2.12}$$

The system (2.11) and (2.12) constitutes the relativistic quantum analog of the usual Boltzmann-Vlasov equations of ordinary plasma physics, for the Walecka model of nuclear matter.

B. A simpler kinetic system

At this point two remarks appear to be in order. First, it should be noticed that the sixteen components of the covariant Wigner function F are not at all needed. Indeed, the system (2.12) makes use of the following quantities only:

$$f(x,p)\equiv\tfrac{1}{4}\,\mathrm{Sp}F(x,p)\ , \tag{2.13}$$

$$f^\mu(x,p)\equiv\tfrac{1}{4}\,\mathrm{Sp}\gamma^\mu F(x,p)\ , \tag{2.14}$$

and, moreover, these quantities are also the ones which appear in the calculation of the main macroscopic objects, i.e., J^μ and $T^{\mu\nu}$ [see Eqs. (2.2), (2.3), (2.7), (2.8), and (1.5)]. Hence, it would be highly desirable to get a kinetic equation which would involve f and f^μ only. Technically, this can be achieved when one takes a second remark into account. One must indeed realize that the use of a Chapman-Enskog method (or any other one) for the calculation of transport coefficients does imply a weak gradient assumption for the average fields $\langle\phi\rangle$ and $\langle V\rangle$. This means that, in actual practice, one can take

$$\langle\phi(x\pm\tfrac{1}{2}R)\rangle \sim \langle\phi(x)\rangle \pm \tfrac{1}{2}R\cdot\partial\phi(x) + O((\partial\langle\phi\rangle)^2) \tag{2.15}$$

and a similar expression for $\langle V(x\pm\frac{1}{2}R)\rangle$. This expansion is discussed further in Sec. VI.

Introduced into Eq. (2.6a), Eq. (2.15) yields

$$[i\gamma\cdot\partial + 2(\gamma\cdot\tilde{p}-\tilde{m})]F - i\partial\cdot(\gamma\cdot\tilde{p}-\tilde{m})\cdot\nabla F = -i(\gamma\cdot u/\tau)(F-F_{\rm eq}) \tag{2.16}$$

and a similar equation to Eq. (2.6b). In Eq. (2.16) the following definitions have been used:

$$\tilde{m}\equiv m - g_s\langle\phi(x)\rangle\ , \tag{2.17}$$

$$\tilde{p}^\mu \equiv p^\mu - g_V\langle V^\mu(x)\rangle\ , \tag{2.18}$$

$$\nabla^\lambda \equiv \frac{\partial}{\partial p_\lambda}\ . \tag{2.19}$$

Let us now successively take the trace of Eq. (2.16) (and of the similar equation which has not explicitly been written) with I, γ^μ, and $\sigma^{\alpha\beta}$; let us also make the sum and the difference of the resulting equations; one then gets

$$\partial_\mu f^\mu - \partial_\lambda\tilde{p}_\mu\cdot\nabla^\lambda f^\mu + \partial_\lambda\tilde{m}\cdot\nabla^\lambda f = -(u_\mu/\tau)(f^\mu - f^\mu_{\rm eq})\ , \tag{2.20}$$

$$\tilde{p}_\mu f^\mu = \tilde{m}f\ , \tag{2.21}$$

$$\partial_\mu f - \partial_\lambda\tilde{p}_\mu\cdot\nabla^\lambda f + \partial_\lambda\tilde{m}\cdot\nabla^\lambda f + 4\tilde{p}_\alpha f^{\alpha\mu} = -(u_\mu/\tau)(f-f_{\rm eq})\ , \tag{2.22}$$

$$\partial^{[\alpha}f^{\beta]} - \partial_\lambda\tilde{p}^{[\alpha}\cdot\nabla^\lambda f^{\beta]} + 2\tilde{p}_\mu\varepsilon^{\mu\alpha\beta\sigma}f_{5\sigma} - 4\tilde{m}f^{\alpha\beta} = -u^{[\alpha}[(f^{\beta]} - f^{\beta]}_{\rm eq})/\tau]\ . \tag{2.23}$$

In these last equations use has been made of the usual decomposition of F on the Dirac algebra

$$F = fI + f_\mu\gamma^\mu + if_{\mu\nu}\sigma^{\mu\nu} + if_5\gamma^5 + f_{5\mu}\gamma^5\gamma^\mu\ .$$

Multiplying now Eq. (2.22) by $\tilde{p}_\mu$ and using Eq. (2.21), one is led to

$$\tilde{p}\cdot\partial f - \partial_\mu\tilde{m}\cdot f^\mu - \tfrac{1}{2}[\partial_\lambda(\tilde{p}^2-\tilde{m}^2)]\cdot\nabla^\lambda f = -(u\cdot\tilde{p}/\tau)(f-f_{\rm eq})\ . \tag{2.24}$$

Similar manipulations involving Eq. (2.23) multiplied by $\tilde{p}_\alpha$, and the use of Eq. (2.21), provide

$$\tilde{p}\cdot\partial f^\beta-\partial^\beta\tilde{m}\cdot f-\tfrac{1}{2}[\partial_\lambda(\tilde{p}^2-\tilde{m}^2)]\nabla^\lambda f^\beta+\partial^{[\beta}\tilde{p}^{\alpha]}f_\alpha = -(u\cdot\tilde{p}/\tau)(f^\beta-f^\beta_{\rm eq}) \; . \quad (2.25)$$

Finally, the system (2.24) and (2.25) involves only f and f^μ and is somewhat simpler than the initial system (2.6). In fact, a consistent Chapman-Enskog expansion of the system (2.6) would have led to the same final results, although in a much more involved way.

III. CHAPMAN-ENSKOG EXPANSION AND THE TRANSPORT COEFFICIENTS

In this section, explicit expressions for the main transport coefficients are derived from the Chapman-Enskog expansion of Eqs. (2.12), (2.24), and (2.25). Such an expansion, in its simplest form [5], depends on one dimensionless expansion parameter (see, however, Ref. [5] and the remarks below) ε given by

$$\varepsilon\equiv\tau/L \; , \quad (3.1)$$

where L is a macroscopic (hydrodynamic) length on which various gradients vary in an appreciable way. It can be written as

$$F=F_{\rm eq}+\tau F_{(1)}+\cdots \; , \quad (3.2a)$$

$$\langle\phi\rangle=\phi_{\rm eq}+\tau\phi_{(1)}+\cdots \; , \quad (3.2b)$$

$$\langle V^\mu\rangle=V^\mu_{\rm eq}+\tau V^\mu_{(1)}+\cdots \; , \quad (3.2c)$$

where the subscript eq denotes equilibrium values, taken as the zeroth order in the expansion. Owing to the fact that only f and f^μ are to be effectively considered, instead of Eq. (3.2a), it is preferable to use

$$f=f_{\rm eq}+\tau f_{(1)}+\cdots \; , \qquad f^\mu=f^\mu_{\rm eq}+\tau f^\mu_{(1)}+\cdots \; , \quad (3.3)$$

so that our first task is to determine the equilibrium quantities $f_{\rm eq}$, $f^\mu_{\rm eq}$, $\phi_{\rm eq}$, and $V^\mu_{\rm eq}$. By assuming as usual that the latter are x dependent through the macroscopic parameters (temperature, density, and average four-velocity) involved therein, the Chapman-Enskog expansion provides the off-equilibrium part $J^\mu_{(1)}$ of the four-current and $T^{\mu\nu}_{(1)}$ of the energy-momentum tensor and, ultimately, the transport coefficients.

A. Thermal equilibrium

The thermodynamic equilibrium properties of nuclear matter, assumed to obey to the Walecka model in Hartree approximation, have been thoroughly studied [9,10,14] and hence only the main results are recalled and adapted to the covariant Wigner function used throughout this paper.

In the Hartree approximation, the equilibrium properties of nuclear matter are essentially controlled by the Fermi-Dirac statistics of free quasinucleons endowed with the following excitation spectrum [10,14,15]:

$$E^\pm(k)=g_V V^0_{\rm eq}\pm\sqrt{\mathbf{k}^2+m^{*2}} \; , \quad (3.4)$$

so that the equilibrium Wigner function is easily calculated and reads

$$F_{\rm eq}(p)=\frac{D}{(2\pi)^3}(\gamma\cdot p^*+m^*)\delta(p^{*2}-m^{*2})\left[\frac{\theta(p^{*0})}{\exp[\beta(p^{*0}-\mu^*)]+1}+\frac{\theta(-p^{*0})}{\exp[-\beta(p^{*0}-\mu^*)]+1}-\theta(-p^{*0})\right] , \quad (3.5)$$

where D is a degeneracy factor ($D=2$, for pure neutron matter; $D=4$, for symmetric nuclear matter) and where the following notations have been used:

$$m^*=m-g_s\phi_{\rm eq} \; , \quad (3.6a)$$

$$p^{*\mu}=p^\mu-g_V V^\mu_{\rm eq} \; , \quad (3.6b)$$

$$\mu^*=\mu-g_V u_\lambda V^\lambda_{\rm eq} \; ; \quad (3.6c)$$

also $\beta\equiv 1/k_B T$ and μ is the chemical potential.

As to the equilibrium fields, they have to obey the equations of motion (2.12) or, equivalently,

$$m_S^2\phi_{\rm eq}=g_S\int d^4p\, f_{\rm eq}(p) \; , \quad (3.7)$$

$$m_V^2 V^\mu_{\rm eq}=g_V\int d^4p\, f^\mu_{\rm eq}(p) \equiv g_V n_B u^\mu \; , \quad (3.8)$$

where n_B is the baryon density and where the invariance under space-time translations of physical quantities in global equilibrium have been taken into account. The gap equation (3.7) involves $\phi_{\rm eq}$ in $f_{\rm eq}$ via Eq. (3.6a) and is, accordingly, an implicit equation for $\phi_{\rm eq}$ (see also Ref. [16]) that controls the whole thermodynamics of the system. From Eq. (3.5) one easily obtains

$$f^\mu_{\rm eq}=(p^{*\mu}/m^*)f_{\rm eq} \; , \quad (3.9)$$

$$f_{\rm eq}(p)=\frac{D}{(2\pi)^3}\delta(p^{*2}-m^{*2})\left[\frac{\theta(p^{*0})}{\exp[\beta(u_\mu p^{*\mu}-\mu^*)]+1}+\frac{\theta(-p^{*0})}{\exp[\beta(-u_\mu p^{*\mu}+\mu^*)]+1}-\theta(-p^{*0})\right] . \quad (3.10)$$

B. Conservation relations

From the above expressions the equilibrium macroscopic quantities are obtained and, in particular, the four-current and the energy-momentum tensor. They are *local* quantities via their x dependence occurring through the macroscopic (local) parameters β,μ,u^λ which must obey zeroth-order conservation relations

$$\partial_\mu J^\mu_{\rm eq}=0,\quad \partial_\mu T^{\mu\nu}_{\rm eq}=0\ , \tag{3.11}$$

which play an important role in the derivation of transport coefficients.

Using definitions (2.2) and (2.3) for the matter part (nucleons) of $J^\mu_{\rm eq}$ and $T^{\mu\nu}_{\rm eq}$, in which Eqs. (3.9) and (3.10) are introduced, one finds [17]

$$n_B=\frac{4\pi m^{*3}D}{3(2\pi)^3}\gamma^* I^*_{4,0} \tag{3.12}$$

for the baryonic density (remember that $J^\mu_{\rm eq}=n_B u^\mu$) and

$$T^{\mu\nu}_{\rm eq}=\int d^4p\,\frac{p^{*\mu}p^{*\nu}}{m^*}f_{\rm eq} -\eta^{\mu\nu}\left[-\frac{1}{2}m_s^2\left[\frac{m-m^*}{g_s}\right]^2+\frac{1}{2}\frac{g_V^2 n_B^2}{m_V^2}\right] +\frac{g_V^2 n_B^2}{m_V^2}u^\mu u^\nu \tag{3.13}$$

for the canonical energy-momentum tensor derived from the Lagrangian (1.3). In Eq. (3.12) and in the following equations use is made of the notations

$$\gamma^*\equiv m^*\beta,\alpha^*\equiv\gamma^*\mu^*/m^*=\beta(\mu-g_V^2n_B/m_V^2)\ , \tag{3.14}$$

$$i^{*\pm}_{n,m}=\int dy\,\sinh^n y\,\cosh^m y\frac{1}{\exp(\gamma^*\cosh y\pm\alpha^*)+1}\ , \tag{3.15}$$

$$I^{*\pm}_{n,m}=\int dy\,\sinh^n y\,\cosh^m y\frac{\exp(\gamma^*\cosh y\pm\alpha^*)}{[\exp(\gamma^*\cosh y\pm\alpha^*)+1]^2}\ . \tag{3.16}$$

From the general expression of $T^{\mu\nu}_{\rm eq}$,

$$T^{\mu\nu}_{\rm eq}=(\rho+P)u^\mu u^\nu-P\eta^{\mu\nu}\ , \tag{3.17}$$

and from Eq. (3.13), one obtains

$$P=\frac{4\pi m^{*4}D}{3(2\pi)^3}i^*_{4,0}-\frac{m_s^2}{2g_s^2}(m-m^*)^2+\frac{g_V^2}{2m_V^2}n_B^2\ , \tag{3.18}$$

$$\rho=\frac{4\pi m^{*4}D}{(2\pi)^3}i^*_{2,2}+\frac{m_s^2}{2g_s^2}(m-m^*)^2+\frac{g_V^2}{2m_V^2}n_B^2\ , \tag{3.19}$$

which constitute the equation of state of the system when the gap equation (3.6a), or, equivalently,

$$m^*=m-\frac{g_s^2}{m_s^2}\frac{4\pi m_s^3 D}{(2\pi)^3}i^*_{2,0} \tag{3.20}$$

is joined. It is not necessary, here, to enter into a detailed study of these last equations since they have been carefully studied elsewhere [10,14].

Let us now derive the consequences of the conservation equations (3.11). First, the conservation of the baryonic charge immediately leads to

$$\dot n_B+n_B\theta=0\ , \tag{3.21}$$

where use of the following (standard) notations

$$\dot A\equiv u^\mu\partial_\mu A,\quad \theta\equiv\partial_\mu u^\mu \tag{3.22}$$

has been made. Next, the contractions of $\partial_\mu T^{\mu\nu}_{\rm eq}=0$ with u_ν and with $\Delta_{\mu\nu}(u)\equiv\eta_{\mu\nu}-u_\mu u_\nu$ yield, respectively,

$$\dot\rho+(\rho+P)\theta=0\ , \tag{3.23}$$

$$(\rho+P)\Delta^{\nu\lambda}(u)\dot u_\nu=\Delta^{\nu\lambda}(u)\partial_\nu P\ ; \tag{3.24}$$

Eqs. (3.21) and (3.23) finally provide

$$\dot\alpha^* I^*_{2,2}-\dot\gamma^* I^*_{2,3}=-\frac{1}{3}\gamma^*\theta I^*_{4,1}\left[1+\frac{3\dot m^*}{\theta m^*}\right]\ , \tag{3.25}$$

$$\dot\alpha^* I^*_{2,1}-\dot\gamma^* I^*_{2,2}=-\frac{1}{3}\gamma^*\theta I^*_{4,0}\left[1+\frac{3\dot m^*}{\theta m^*}\right]\ . \tag{3.26}$$

These last equations are easily solved for the quantities which are needed in subsequent calculations, and one finds

$$\dot\alpha^*=\frac{\gamma^*}{3}\frac{I^*_{4,0}I^*_{2,3}-I^*_{2,2}I^*_{4,1}}{(I^*_{2,2})^2-I^*_{2,1}I^*_{2,3}}\theta^*\ , \tag{3.27}$$

$$\dot\gamma^*=\frac{\gamma^*}{3}\frac{I^*_{4,0}I^*_{2,2}-I^*_{2,1}I^*_{4,1}}{(I^*_{2,2})^2-I^*_{2,1}I^*_{2,3}}\theta^*\ . \tag{3.28}$$

These equations are formally identical to the corresponding ones obtained in the Anderson-Witting calculation [6], the only difference being that nonstarred quantities are replaced by starred ones α^*, γ^*, and $I^*_{n,m}$ and with

$$\theta^*\equiv\theta\left[1+\frac{3\dot m^*}{\theta m}\right]\ . \tag{3.29}$$

θ^* is a kind of effective divergence of the four-velocity and it is shown in Sec. V that it is actually this quantity that appears in the entropy production. Finally, after replacing the explicit expressions (3.18) and (3.19) of the energy density and of the pressure into Eq. (3.24), one arrives at

$$\Delta^{\mu\lambda}\partial_\lambda\alpha=[(\rho+P)/n_B]\Delta^{\mu\lambda}(u)(\partial_\lambda\beta+\beta\dot u_\lambda)\ , \tag{3.30}$$

which is identical to the expression given by Anderson and Witting [6] *except* that, in our case, the enthalpy

$$h=(\rho+P)/n_B \tag{3.31}$$

also contains the contribution of the average fields $\phi_{\rm eq}$ and $V^\lambda_{\rm eq}$.

C. The Chapman-Enskog expansion and the off-equilibrium quantities

Let us now solve the kinetic system (2.24) and (2.25) at order 1 in the expansion parameter ε or, equivalently, in

τ. Since its right-hand side contains spatial gradients [18], the first order can be obtained quite easily by replacing f and f^μ by their corresponding zeroth-order $f_{\rm eq}$ and $f^\mu_{\rm eq}$. In this way, one gets

$$f_{(1)}=-\frac{1}{u\cdot p^*}\left[p^*\cdot\partial-\frac{1}{2}\partial_\lambda(p^{*2}-m^{*2})\cdot\nabla^\lambda\right]f_{\rm eq}\ , \quad (3.32)$$

$$f^\mu_{(1)}=(p^{*\mu}/m^*)f_{(1)}\ . \quad (3.33)$$

It is interesting to notice that the equilibrium form for f^μ, i.e., $f^\mu_{\rm eq}=(p^{*\mu}/m^*)f_{\rm eq}$, is conserved at order 1 in τ. This property has the nice consequence that the off-equilibrium part of $T^{\mu\nu}$ is symmetric.

The calculation of $J^\mu_{(1)}$ and $T^{\mu\nu}_{(1)}$ does not present any particular difficulty although it is a bit long and uses various properties of the integrals $I^*_{n,m}$. The final expressions for these off-equilibrium quantities are

$$J^\mu_{(1)}=\tau\frac{4\pi Dm^{*4}}{3(2\pi)^3}\left[\frac{I^*_{4,1}I^*_{4,-1}}{I^*_{4,0}}-I^*_{4,0}\right]\Delta^{\mu\lambda}(u)(\partial_\lambda\beta+\beta\dot u_\lambda) \quad (3.34)$$

(note also the interesting alternative form for $J^\mu_{(1)}$

$$J^\mu_{(1)}=\tau\frac{4\pi Dm^{*4}}{3(2\pi)^3}\frac{I^*_{4,0}}{I^*_{4,1}}\left[\frac{I^*_{4,1}I^*_{4,-1}}{I^*_{4,0}}-I^*_{4,0}\right][\Delta^{\mu\lambda}(u)\partial_\lambda\alpha^*+\beta g_V F^{\mu\lambda}u_\lambda])\ ,$$

for the baryonic four-current and

$$\begin{aligned}T^{\mu\nu}_{(1)}=\tau\frac{4\pi Dm^{*4}}{(2\pi)^3}&\left[\frac{2}{15}I^*_{6,-1}\gamma^*\sigma^{\mu\nu}+\frac{\gamma^*}{9}\left[\frac{I^{*2}_{4,0}I^*_{2,3}+I^{*2}_{4,1}I^*_{2,1}-2I^*_{2,2}I^*_{4,0}I^*_{4,1}}{I^{*2}_{2,2}-I^*_{2,1}I^*_{2,3}}+I^*_{6,-1}\right]\theta^*\Delta^{\mu\nu}(u)\right]\\&+\tau\frac{4\pi Dm^{*4}}{3(2\pi)^3}\frac{g_V^2n_B}{m_V^2}[u^\mu\Delta^{\mu\lambda}(u)+u^\nu\Delta^{\mu\lambda}(u)]\left[\frac{I^*_{4,1}I^*_{4,-1}}{I^{*2}_{4,0}}-I^*_{4,0}\right](\partial_\lambda\beta+\beta\dot u_\lambda)\\&+\tau\gamma^*\frac{Dm^{*3}}{3(2\pi)^3}(m-m^*)\left[\frac{I^{*2}_{4,0}I^*_{2,3}+I^{*2}_{4,1}I^*_{2,1}-2I^*_{2,2}I^*_{4,0}I^*_{4,1}}{I^{*2}_{2,2}-I^*_{2,1}I^*_{2,3}}+I^*_{6,-1}\right]\theta^*\eta^{\mu\nu}\end{aligned} \quad (3.35)$$

where $\sigma^{\mu\nu}$ is the shear tensor

$$\sigma^{\mu\nu}\equiv[\Delta^{\mu\alpha}(u)\Delta^{\nu\beta}(u)-\tfrac{1}{3}\Delta^{\mu\nu}(u)\Delta^{\alpha\beta}(u)]\partial_{(\alpha}u_{\beta)} \quad (3.36)$$

with the following properties:

$$u_\mu\sigma^{\mu\nu}=0,\quad \Delta^{\mu\nu}(u)\sigma_{\mu\nu}=0\ . \quad (3.37)$$

From Eqs. (3.34) and (3.35) the transport coefficients of the medium are obtained and discussed in the next section.

IV. TRANSPORT COEFFICIENTS

The general structure of $J^\mu_{(1)}$ and $T^{\mu\nu}_{(1)}$ is given by Eqs. (3.34) and (3.35), i.e.,

$$J^\mu_{(1)}=K\Delta^{\mu\nu}(u)(\partial_\lambda\beta+\beta\dot u_\lambda)\ , \quad (4.1)$$

$$T^{\mu\nu}_{(1)}=2\eta\sigma^{\mu\nu}+\tfrac{1}{3}\zeta\theta^*\Delta^{\mu\nu}(u)+A^{\mu\nu}\ , \quad (4.2)$$

where $A^{\mu\nu}$ is easily obtained from Eq. (3.35). While Eq. (4.1) obeys $u_\mu J^\mu_{(1)}=0$, $T^{\mu\nu}_{(1)}$ *is not* orthogonal to u_μ. In other words, the Landau-Lifschitz matching conditions [19,20] are not obeyed by the off-equilibrium energy-momentum tensor. This is due to the collective fields $\langle\phi\rangle$ and $\langle V^\lambda\rangle$. The baryonic part of $T^{\mu\nu}_{(1)}$ can indeed by shown to obey the Landau-Lifschitz condition $u_\mu T^{\mu\nu}_{(1)\rm baryon}=0$, but this is of little use here since transport coefficients involve the total off-equilibrium energy-momentum tensor.

Usually, in order to identify the transport coefficients, one transforms the off-equilibrium quantities $J^\mu_{(1)}$ and $T^{\mu\nu}_{(1)}$ to their Eckart [21] form with a redefinition of the equilibrium four-velocity [19,20,22]: It is indeed defined up to $O(\tau)$ terms.

Another problem has to be discussed. In the preceding section an "effective" four-divergence of the average four-velocity field u^μ was introduced, namely, θ^*, and the bulk viscosity is to be defined with respect to θ^*. This has yet to be justified on the basis of an analysis of the entropy production of these dissipative processes. This is done below using *two* different definitions of the entropy, one based on the Gibbs relation and another one arising from microscopic considerations: Both lead to the same results.

A. Eckart's form and transport coefficients

In the Eckart's form of relativistic hydrodynamics [21] the off-equilibrium four-current is identically zero [up to $O(\tau^2)$ terms]. After a redefinition of the pressure and energy density.

$$\rho\to\rho+(m-m^*)\zeta\theta^*/3\ ,$$

$$P\to P-(m-m^*)\zeta\theta^*/3\ ,$$

$T^{\mu\nu}_{(1)}$ has the following form

$$T^{\mu\nu}_{(1)}=q^{(\mu}u^{\nu)}+2\eta\sigma^{\mu\nu}+\tfrac{1}{3}\zeta\theta\Delta^{\mu\nu}(u)\ . \quad (4.3)$$

This modification of the pressure and energy density as well as the use of θ^* instead of θ is justified below by the calculation of the entropy production. In Eq. (4.3) q^μ is the heat flux four-vector

$$q^\mu \equiv -\lambda \Delta^{\mu\alpha}(u)\cdot\left[\frac{\dot{u}_\alpha}{\beta}-\partial_\alpha \frac{1}{\beta}\right] . \tag{4.4}$$

η, ζ, and λ are the transport coefficients we are looking for. Changing u^μ into U^μ,

$$u^\mu = U^\mu - \xi^\mu , \tag{4.5}$$

in Eqs. (3.35) and (3.36), one arrives at the Eckart's form (4.3) with the following choice (with $n_{\text{Eck}} \equiv n_B$):

$$\xi^\mu = (K/n_B)\Delta^{\mu\lambda}(u)\cdot(\partial_\lambda \beta + \beta \dot{u}_\lambda) \tag{4.6}$$

so that, finally, one gets

$$\eta = \tau \frac{4\pi D m^{*4}}{15(2\pi)^3} I^*_{6,-1}\gamma^* \quad \text{(shear viscosity)} , \tag{4.7}$$

$$\zeta = \tau \frac{4\pi D m^{*4}}{3(2\pi)^3}\gamma^* \left[\frac{I^{*2}_{4,0}I^*_{2,3}+I^{*2}_{4,1}I^*_{2,1}-2I^*_{2,2}I^*_{4,0}I^*_{4,1}}{I^{*2}_{2,2}-I^*_{2,1}I^*_{2,3}} + I^*_{6,-1}\right] \quad \text{(bulk viscosity)} , \tag{4.8}$$

$$\lambda = \tau \frac{4\pi D m^{*3}}{3(2\pi)^3}\gamma^{*2}\frac{I^*_{4,1}}{I_{4,0}}\left[\frac{I^*_{4,1}I^*_{4,-1}}{I^*_{4,0}} - I^*_{4,0}\right] \quad \text{(thermal conductivity)} . \tag{4.9}$$

Once more, one can realize that these three quantities are *formally* identical to similar expressions obtained by Anderson and Witting [6] and they differ only by the occurrence of starred quantities. This was *a priori* not obvious although it seems natural.

B. Entropy production (1)

Here the entropy production of the off-equilibrium system is calculated on the basis of the covariant formulation of the Gibb's relation [23]

$$S^\mu = P_{\text{Eck}}\beta^\mu - \alpha J^\mu_{\text{Eck}} + \beta u_\lambda T^{\lambda\mu}_{\text{Eck}} , \tag{4.10}$$

with $\beta \equiv 1/T$ and $\alpha \equiv \mu/T$, and where S^μ is the entropy flux four-vector; the total entropy of the system is thus

$$S = \int_\Sigma d\Sigma_\mu S^\mu \tag{4.11}$$

at "time" Σ.

At order zero (i.e., in local equilibrium), Eq. (4.11) can be rewritten as

$$S^\mu_{\text{eq}} = [(\rho + P)_{\text{nucleons}}\gamma^*/m^* - \alpha^* n_B]u^\mu \tag{4.12}$$

and can be shown to obey $\partial_\mu S^\mu_{\text{eq}} = 0$, with the help of Eqs. (3.27) and (3.28), as it should be.

At order 1 in τ, it is difficult to show that Eq. (4.10) leads to

$$S^\mu = P_{\text{nucleons}}(\gamma^*/m^*)u^\mu - \alpha^* J^\mu + (\gamma^*/m^*)u_\lambda T^{\lambda\mu}_{\text{nucleons}} , \tag{4.13}$$

where the index "nucleons" refers to quantities calculated with the Wigner function of the nucleons only and not, e.g., to quantities connected to the collective fields $\langle \phi \rangle$ and $\langle V^\lambda \rangle$.

From Eq. (4.13) the entropy production rate σ is given by

$$\sigma \equiv \partial_\mu S^\mu_{(1)} \tag{4.14}$$

$$= -\partial_\mu \alpha \cdot J^\mu_{(1)} + \partial_\mu(\beta u_\lambda) T^{\lambda\mu}_{(1)} . \tag{4.15}$$

$$= -\partial_\mu \alpha^* \cdot J^\mu_{(1)} + T^{\lambda\mu}_{(1)\text{nucleons}}\cdot \partial_\mu(\gamma^* u_\lambda)/m^* - (\gamma^*/m^*)(m_S^2 \dot{\phi}_{\text{eq}}\cdot \phi_{(1)} + g_V F^{\mu\lambda}u_\lambda J_{\mu(1)}) . \tag{4.16}$$

This last expression is finally cast into a sum of squares as

$$\sigma = K\{-[\Delta^{\mu\lambda}(u)\partial_\lambda \alpha^* + \beta g_V F^{\mu\lambda}u_\lambda]^2\} + \eta(\gamma^*/m^*)\cdot(\sigma^{\mu\lambda})^2 + \tfrac{1}{3}\zeta(\gamma^*/m^*)\cdot(\theta^*)^2 , \tag{4.17}$$

where use was made of the energy-momentum conservation relation written in the form

$$\partial_\mu T^{\mu\nu}_{\text{fields}} = g_S \partial^\nu \phi \int d^4p\, f + g_V F^{\lambda\nu}J_\lambda . \tag{4.18}$$

In Eq. (4.17) the first square $[\ ]^2$ is negative, owing to the space character of the tensor involved therein and the other terms are positive: The entropy production rate σ is positive as is demanded by the second principle of thermodynamics. Furthermore, it can be written in the general form

$$\sigma = \sum_i \lambda_i \chi^{i2} , \tag{4.19}$$

where the λ^i's are the transport coefficients (4.7)–(4.9) and the χ^i's are the associated thermodynamic forces. The latter appear to be modified by the presence of the scalar and vectorial fields: For instance, θ is replaced by θ^* [see Eq. (3.29)] which involves $\dot{\phi}_{\text{eq}}$ and ϕ_{eq}. It should be noticed that had we not modified the pressure term in the energy-momentum tensor (equivalently, had we not decomposed $T^{\mu\nu}_{(1)}$ as was done), then we would not have obtained a decomposition of the entropy production rate of the general form (4.19) and, consequently, our transport coefficients would have been ill defined.

C. Entropy production (2)

From a microscopic point of view, since f plays the role of a distribution function on the mass shell $p^{*2} = m^{*2}$, the entropy four-flux S^μ can be defined as usual [3,23] and is thus given by

$$S^\mu = -\int d^4p^* \frac{p^{\mu *}}{m^*}[f \ln f + (1-f)\ln(1-f) + \bar{f}\ln\bar{f} + (1-\bar{f})\ln(1-\bar{f})] , \tag{4.20}$$

where f and $\bar{f}$ refer, loosely speaking, to nucleons and antinucleons, respectively, and where the Boltzmann constant has been taken to be unity. Using the new variable $\omega^\mu \equiv p^{\mu *}/m^*$, one can rewrite Eq. (4.20) as

$$S^\mu = -m^{*3}\int \frac{d^3\omega}{\omega_0}\omega^\mu[f\ln f + (1-f)\ln(1-f) + \bar{f}\ln\bar{f} + (1-\bar{f})\ln(1-\bar{f})] , \quad (4.21)$$

and the entropy production rate σ is given by

$$\sigma \equiv \partial_\mu S^\mu = 3\partial_\mu \ln(m^*) S^\mu - m^{*3}\int \frac{d^3\omega}{\omega_0}\omega^\mu \left[\ln\left[\frac{f}{1-f}\right]\partial_\mu f + \ln\left[\frac{\bar{f}}{1-\bar{f}}\right]\partial_\mu \bar{f} \right] . \quad (4.22)$$

Using the expansions of f and $\bar{f}$ in powers of τ

$$f = f_{\rm eq} + f_{(1)}, \quad \bar{f} = \bar{f}_{\rm eq} + \bar{f}_{(1)} , \quad (4.23)$$

one obtains

$$\ln\left[\frac{f}{1-f}\right] \sim \ln\left[\frac{f_{\rm eq}}{1-f_{\rm eq}}\right] + \frac{f_{(1)}}{f_{\rm eq}(1-f_{\rm eq})} \quad (4.24)$$

and a similar expression for $\bar{f}$. In the calculation of σ we must also use

$$\begin{aligned} \partial_\mu f_{\rm eq} &= [\partial_\mu \alpha^* - \partial_\mu(\gamma^* u_\lambda)\cdot\omega^\lambda] f_{\rm eq}(1-f_{\rm eq}) , \\ \partial_\mu \bar{f}_{\rm eq} &= -[\partial_\mu \alpha^* + \partial_\mu(\gamma^* u_\lambda)\cdot\omega^\lambda] \bar{f}_{eq}(1-\bar{f}_{eq}) , \end{aligned} \quad (4.25)$$

so that one finally obtains

$$\sigma = 3\partial_\mu \ln(m^*)\left[S^\mu + \alpha^* J^\mu - \frac{4}{3}\frac{\gamma^*}{m^*}u_\lambda T^{\lambda\mu}_{\rm nucleons}\right] - \frac{\gamma^*}{m^*}u^\lambda\left[\frac{1}{2}m_s^2\partial^\lambda\phi^2 + g_V F^{\mu\lambda}J_\mu\right] - \partial_\mu\alpha^*\cdot J^\mu_{(1)} + \partial_\mu(\gamma^* u_\lambda)(T^{\lambda\mu}_{(1)\rm nucleons}/m^*) . \quad (4.26)$$

At order zero, $\sigma = 0$ as it should be, while the first-order entropy four-flux reduces to

$$S^\mu_{(1)} = -\alpha^* J^\mu_{(1)} + (\gamma^*/m^*) u_\lambda T^{\mu\lambda}_{(1)\rm nucleons} \quad (4.27)$$

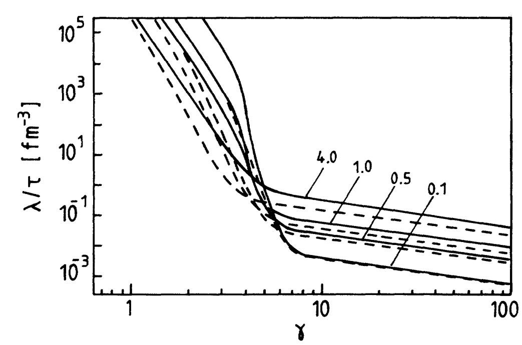

FIG. 1. Thermal conductivity λ/τ at constant baryonic number, as a function of the "temperature parameter" $\gamma \equiv mc^2/k_B T$ (pure neutron matter). The dashed lines represent the Anderson-Witting case (no collective effects) while the continuous ones correspond to the Walecka model in the Hartree approximation.

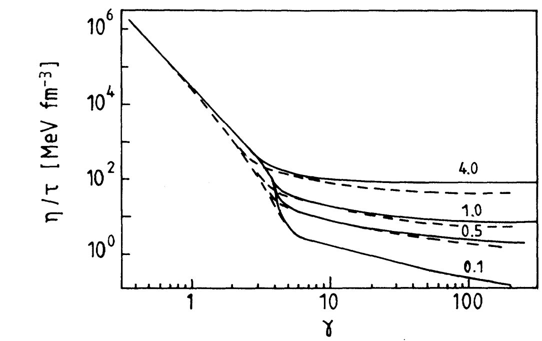

FIG. 2. Shear viscosity η/τ at constant baryonic number, as a function of γ (pure neutron matter).

and the total entropy production rate $\sigma = \partial_\mu(S^\mu_{\rm eq} + S^\mu_{(1)})$ finally gives rise to Eq. (4.17).

V. NUMERICAL RESULTS

In this section the general form of the transport coefficients is calculated, discussed, and compared to the results obtained by Anderson and Witting [6] in the absence of any collective effects whatsoever. In the subsequent calculations the relaxation time τ is considered as given by a more sophisticated theory or evaluated roughly, so that we compare the various transport coefficients divided by τ, i.e., λ/τ, η/τ, and ζ/τ. Note also that the results referring to Anderson and Witting, given in this paper, differ slightly from the ones found in Ref. [6]. In particular, this appears at high densities and/or temperatures: This is due to the fact that our calculations (with and without collective effects) take account of pair production.

A number of results are exhibited in Figs. 1–6. The continuous lines refer to our results while the dashed ones correspond to the Anderson-Witting case. While the shear viscosity coefficient still possesses roughly the same behavior and numerical values in both cases, the bulk viscosity and the thermal conductivity are significantly different.

For the sake of brevity, only the calculations performed for pure neutron matter are given below. Also, in

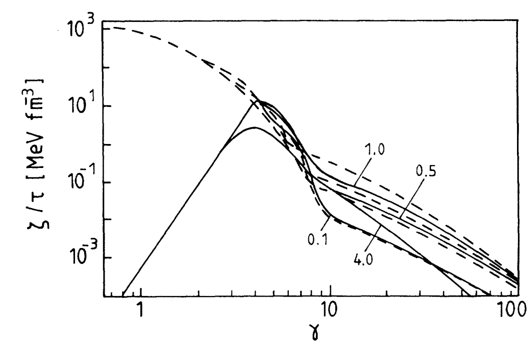

FIG. 3. Bulk viscosity ζ/τ at constant baryonic number, as a function of γ (pure neutron matter).

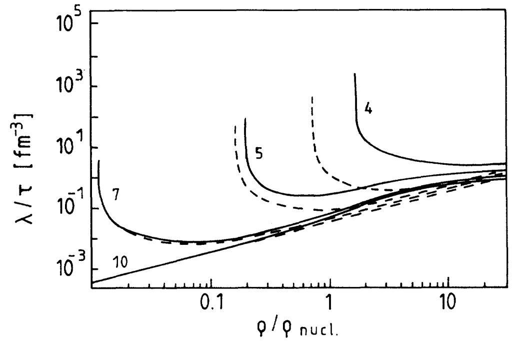

FIG. 4. Thermal conductivity λ/τ at constant temperature as a function of the dimensionless energy density $\rho/\rho_{\rm nucl}$ (pure neutron matter).

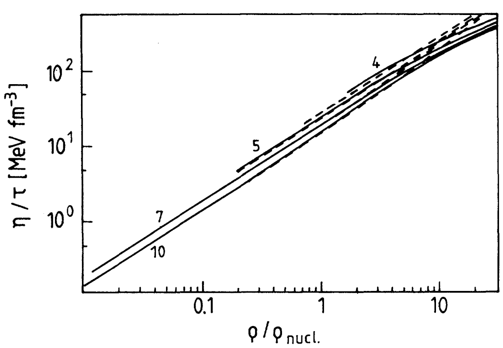

FIG. 5. Shear viscosity η/τ at constant temperature as a function of the dimensionless energy density $\rho/\rho_{\rm nucl}$ (pure neutron matter).

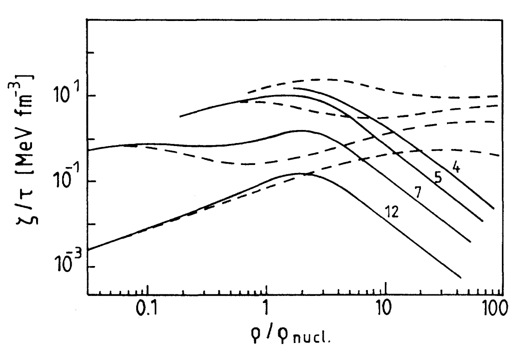

FIG. 6. Bulk viscosity ζ/τ at constant temperature as a function of the dimensionless energy density $\rho/\rho_{\rm nucl}$ (pure neutron matter).

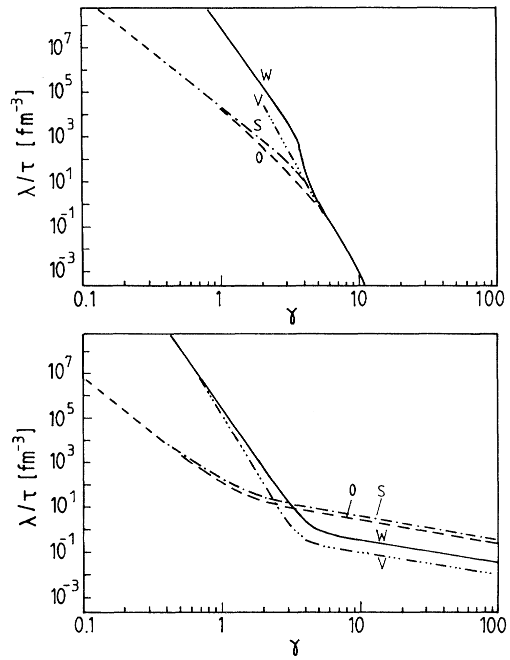

FIG. 7. The effects of the various collective fields is indicated in the case of thermal conductivity (pure neutron matter) for two different chemical potentials, as a function of γ. *W*: Walecka model; *V*: vector field only; *S*: scalar field only; *O*: Andersen-Witting case.

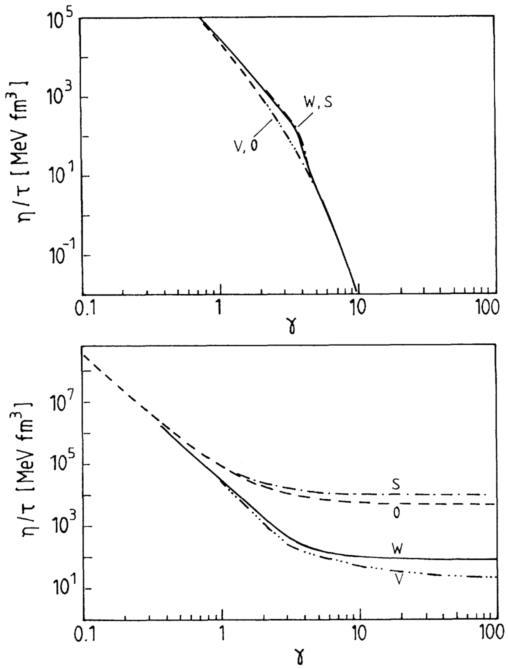

FIG. 8. The effects of the various collective fields is indicated in the case of shear viscosity (pure neutron matter) for two different chemical potentials, as a function of γ.

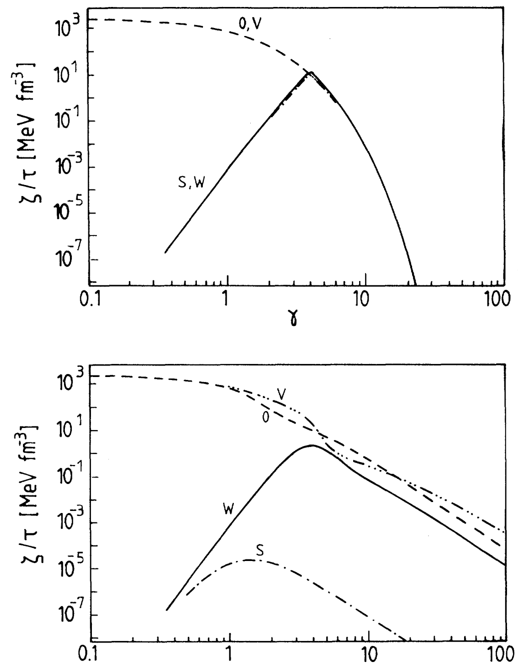

FIG. 9. The effects of the various collective fields is indicated in the case of bulk viscosity (pure neutron matter) for two different chemical potentials, as a function of γ.

order to evaluate the effects of the different collective fields, the corresponding coupling constants have been switched off and the result is compared to the other cases (Figs. 7–9). Therefore, each figure exhibits four different curves (at least when they are not superposed): (i) collective effects $c_s \neq 0$, $c_v \neq 0$, (ii) Anderson-Witting $c_s = c_v = 0$, (iii) vector field effects $c_v \neq 0$, $c_s = 0$, and (iv) scalar field effects $c_s \neq 0$, $c_v = 0$. Furthermore, asymptotic regimes prove quite helpful to study. Thus we begin by evaluating the various quantities at hand in such asymptotic cases. Also, the different constants are taken for the canonical Walecka model [9,10]; for instance, $c_s^2 \equiv g_s^2(m^2/m_s^2) = 267.1$ and $c_v^2 = g_v^2(m^2/m_v^2) = 195.9$.

A. Low-temperature—degenerate case: $\gamma^* \gg 1$, $\alpha^* \gg \gamma^*$

In this case a Sommerfeld expansion is performed on the various integrals $I^*_{n,m}$ and one finds

$$I^*_{2,0} \to \frac{u}{\gamma^*} - \frac{\pi^2}{6\gamma^{*3}}\frac{1}{u^3} - \frac{7\pi^4}{120\gamma^{*5}}\frac{4x^2+1}{u^7} + \cdots ,$$

$$I^*_{2,1} \to \frac{xu}{\gamma^*} + \frac{\pi^2}{6\gamma^{*3}}\frac{2x^3-3x}{u^3} - \frac{7\pi^4}{120\gamma^{*5}}\frac{5x}{u^7} + \cdots ,$$

$$I^*_{2,2} \to \frac{x^2u}{\gamma^*} + \frac{\pi^2}{6\gamma^{*3}}\frac{6x^4-9x^2+2}{u^3} - \frac{7\pi^4}{120\gamma^{*5}}\frac{x^2+4}{u^7} + \cdots ,$$

$$I^*_{2,3} \to \frac{x^3u}{\gamma^*} + \frac{\pi^2}{6\gamma^{*3}}\frac{12x^5-19x^3+6x}{u^3} + \frac{7\pi^4}{120\gamma^{*5}}\frac{8x^7-28x^5+35x^3-20x}{u^7} + \cdots ,$$

$$I^*_{4,-1} \to \frac{u^3}{x\gamma^*} + \frac{\pi^2}{6\gamma^{*3}}\frac{2x^4-x^4+2}{x^3u} + \frac{7\pi^4}{120\gamma^{*5}}\frac{8x^4-20x^2+15}{x^5u^5} + \cdots ,$$

$$I^*_{4,0} \to \frac{u^3}{\gamma^*} + \frac{\pi^2}{6\gamma^{*3}}\frac{6x^2-3}{u} + \frac{7\pi^4}{120\gamma^{*5}}\frac{3}{u^5} + \cdots ,$$

$$I^*_{4,1} \to \frac{xu^3}{\gamma^*} + \frac{\pi^2}{6\gamma^{*3}}\frac{12x^3-9x}{u} + \frac{7\pi^4}{120\gamma^{*5}}\frac{8x^5-20x^3+15x}{u^7} + \cdots ,$$

where $u \equiv \mu^*/m^*$ and $x = \sqrt{u^2-1}$. The thermodynamic parameters are thus given by

$$P \to \frac{4\pi m^{*4}D}{3(2\pi)^3}\left[\frac{xu}{8}(2x^2-5) + \frac{3}{8}\ln(x+u) + \frac{\pi^2}{2\gamma^{*2}}xu\right] - \frac{m_S^2}{2g_S^2}(m-m^*)^2 + \frac{g_V^2}{2m_V^2}n^2 ,$$

$$\rho \to \frac{4\pi m^{*4}D}{(2\pi)^3}\left[\frac{xu}{8}(2x^2-1) - \frac{1}{8}\ln(x+u) + \frac{\pi^2}{6\gamma^{*2}}\frac{3x^3-2x}{u}\right] + \frac{m_S^2}{2g_S^2}(m-m^*)^2 + \frac{g_V^2}{2m_V^2}n^2 ,$$

$$n_B \to \frac{4\pi m^{*3}D}{3(2\pi)^3}\left[u^3 + \frac{\pi^2}{6\gamma^{*2}}\frac{6x^2-3}{u}\right] ,$$

$$m^* \to m - \frac{g_S^2}{m_S^2}\frac{4\pi m^{*4}D}{(2\pi)^3}\left[\frac{1}{2}xu - \frac{1}{2}\ln(x+u) + \frac{\pi^2}{6\gamma^{*2}}\frac{x}{u}\right] ,$$

$$S \to \frac{4\pi m^{*3}D}{3(2\pi)^3}\left[\frac{\pi^2}{\gamma^*}xu + \frac{7\pi^4}{120\gamma^{*3}}\frac{8x^5-23x^3+18x}{u^7}\right] ,$$

while the transport coefficients (per unit relaxation time) are

$$\frac{\lambda}{\tau} \to \frac{4\pi m^{*3}D}{3(2\pi)^3}\left[\frac{\pi^2}{9\gamma^*}\frac{u^3}{x} + \frac{\pi^4}{90\gamma^{*3}}\frac{14-27x^2+4x^4}{ux^3}\right] ,$$

$$\frac{\eta}{\tau} \to \frac{4\pi m^{*4}D}{15(2\pi)^3}\left[\frac{u^5}{x} + \frac{\pi^2}{6\gamma^{*2}}(12x^4+x^2+2)\frac{u}{x^3}\right] ,$$

$$\frac{\zeta}{\tau} \to \frac{4\pi m^{*4}D}{3(2\pi)^3}\left[\frac{16\pi^4}{45\gamma^{*4}}\frac{u}{x^5}\right] .$$

The collision time τ can be estimated by the simple formula $\tau = 1/(\sigma n_T \bar{v})$ where σ is the cross section in the medium, $\bar{v}$ is a mean velocity, and n_T represents the number of holes available in the Fermi sea,

$$n_T = n_B(\gamma^*) - n_B(0) \to [4\pi m^{*4} D / 3(2\pi)^3](\pi^2/6\gamma^{*2})[(6x^2-3)/u] .$$

The mean velocity is defined (see, e.g., Ref. [24]) by

$$\bar{v} = |\bar{p}|/E \sim \sqrt{(\rho/n_B)^2 - m^2}/(\rho/n_B) .$$

The exact value of the cross section corrected for collective effects in a dense hadronic medium is still a debated issue. In any case, the temperature dependence brought about by the cross section is not very important compared to the T^2 factor from n_T, so that for practical purposes, and considering the other uncertainties stemming from the imperfection of the model we may take it as being constant and of the order of 40–100 mb. Replacing this estimate of τ in the reduced transport coefficients λ/τ, η/τ, and ζ/τ indicates that they respectively vary, at low temperatures, as T^{-1}, T^{-2}, and T^2. This is the standard result expected for relativistic Fermi systems [25].

B. High-temperature—nondegenerate case: $\gamma^* \ll 1, \alpha^* \gg \gamma^*$

In this case, the Fermi-Dirac factors can be expanded in geometric series that lead to rapidly converging series of modified Bessel (i.e., Kelvin) functions [26]; i.e., one has

$$\frac{1}{e^{-\gamma^*[\cosh(x) - \mu^*/m^*]} + 1} = \sum_{k=0}^{\infty} (-1)^k e^{-k\gamma^*[\cosh(x) - \mu^*/m^*]} ,$$

$$\frac{1}{(e^{-\gamma^*[\cosh(x) - \mu^*/m^*]} + 1)^2} = \sum_{k=0}^{\infty} (-1)^k (k+1) e^{-k\gamma^*[\cosh(x) - \mu^*/m^*]} ,$$

which leads to

$$I^*_{2,0} \sim \frac{2}{\gamma^*} \sum_{k=1}^{\infty} (-1)^{k-1} \sinh\left[\frac{k\gamma^*\mu^*}{m^*}\right] K_1(k\gamma^*) ,$$

$$I^*_{2,1} \sim \frac{2}{\gamma^*} \sum_{k=1}^{\infty} (-1)^{k-1} \cosh\left[\frac{k\gamma^*\mu^*}{m^*}\right] K_2(k\gamma^*) ,$$

$$I^*_{2,2} \sim \frac{1}{2\gamma^*} \sum_{k=1}^{\infty} (-1)^{k-1} \sinh\left[\frac{k\gamma^*\mu^*}{m^*}\right] \times [3K_3(k\gamma^*) + K_1(k\gamma^*)] ,$$

$$I^*_{2,3} \sim \frac{1}{\gamma^*} \sum_{k=1}^{\infty} (-1)^{k-1} \cosh\left[\frac{k\gamma^*\mu^*}{m^*}\right] \times [K_4(k\gamma^*) + K_2(k\gamma^*)] ,$$

$$I^*_{4,0} \sim \frac{6}{\gamma^{*2}} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \sinh\left[\frac{k\gamma^*\mu^*}{m^*}\right] K_2(k\gamma^*) ,$$

$$I^*_{4,1} \sim \frac{6}{\gamma^{*2}} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \cosh\left[\frac{k\gamma^* m^*}{m^*}\right] K_3(k\gamma^*) .$$

Replacing further the Bessel functions by their high-temperature approximation [26]

$$K_n(x) \sim \tfrac{1}{2}(n-1)!(2/x)^n ,$$

we find the high-temperature form of the integrals

$$I^*_{2,-1} \sim \frac{1}{\gamma^*} ,$$

$$I^*_{2,0} \sim \frac{\mu^*}{m^*}\frac{1}{\gamma^*} - \frac{1}{2}\frac{\mu^*}{m^*} ,$$

$$I^*_{2,1} \sim \frac{\pi^2}{3\gamma^{*3}} + \frac{1}{\gamma^*}\left[\frac{\mu^{*2}}{m^{*2}} - 1\right] + \frac{1}{3} ,$$

$$I^*_{2,2} \sim \frac{\mu^*}{m^*}\frac{\pi^2}{\gamma^{*3}} + \frac{1}{2\gamma^*}\left[2\frac{\mu^{*3}}{m^{*3}} - \frac{\mu^*}{m^*}\right] - \frac{1}{4}\frac{\mu^*}{m^*} ,$$

$$I^*_{2,3} \sim \frac{7\pi^4}{15\gamma^{*5}} + \left[12\frac{\mu^{*2}}{m^{*2}} - 1\right]\frac{\pi^2}{6\gamma^{*2}} + \frac{1}{2\gamma^*}\left[2\frac{\mu^{*4}}{m^{*4}} - \frac{\mu^{*2}}{m^{*2}} - 1\right] + \frac{7}{30} .$$

As a consequence, the thermodynamical parameters are given in this region by

$$P \to \left[\frac{7(m^*)^4 D\pi^2}{360} - \frac{g_S^2}{m_S^2}\frac{(m^*)^6 D^2}{288} + \frac{g_V^2}{m_V^2}\frac{(m^*)^6 D^2}{72}\frac{\mu^{*2}}{m^{*2}}\right]\frac{1}{\gamma^{*4}} ,$$

$$\rho \to \left[\frac{7(m^*)^4 D\pi^2}{120} + \frac{g_S^2}{m_S^2}\frac{(m^*)^6 D^2}{288} + \frac{g_V^2}{m_V^2}\frac{(m^*)^6 D^2}{72}\frac{\mu^{*2}}{m^{*2}}\right]\frac{1}{\gamma^{*4}} ,$$

$$n_B \to \frac{(m^*)^3 D}{6}\frac{\mu^*}{m^*}\frac{1}{\gamma^{*2}} ,$$

$$S \to \frac{(m^*)^3 D}{\gamma^{*3}}\frac{7\pi^2}{90} ,$$

$$m - m^* \to \frac{g_S^2}{m_S^2}\frac{(m^*)^3 D}{12\gamma^{*2}} ,$$

and the transport coefficients behave as

$$\frac{\lambda}{\tau} \to \frac{4\pi m^{*3} D}{3(2\pi)^3}\left\{\frac{49\pi^6}{675}\frac{m^{*2}}{\mu^{*2}}\frac{1}{\gamma^{*5}} + \frac{7\pi^4}{675}\left[\frac{m^*}{\mu^*}\left[-15 - 14\frac{\mu^*}{m^*} + 36\frac{\mu^{*2}}{m^{*2}}\right]\right]\frac{1}{\gamma^{*3}}\right\} ,$$

$$\frac{\eta}{\tau} \to \frac{4\pi m^{*4} D}{15(2\pi)^3}\left[\frac{7\pi^4}{15\gamma^{*4}} + \frac{\pi^2}{6\gamma^{*2}}\left[-5 + 12\frac{\mu^{*2}}{m^{*2}}\right]\right] ,$$

$$\frac{\zeta}{\tau} \to \frac{4\pi m^{*4} D}{3(2\pi)^3}\left[\frac{16}{21}\right] .$$

At constant baryonic charge n_B, one has

$$\mu^* \sim \gamma^2 n_B, \quad m^* \sim \gamma^2 m, \quad \lambda/\tau \sim 1/\gamma^9 = T^9,$$
$$\eta/\tau \sim 1/\gamma^4 = T^4, \quad \zeta/\tau \sim \gamma^8 = T^{-8}.$$

C. Low-temperature—nondegenerate case: $\gamma^* \gg 1$, $\alpha^* \ll \gamma^*$

The integrals are again expanded as a sum of Bessel functions and the $K_n(x)$ estimated here from their low-temperature form [26]

$$K_n(x) \to \left[\frac{\pi}{2x}\right]^{1/2} e^{-x}\left[1+\frac{4n^2-1}{8x} + \frac{(4n^2-1)(4n^2-9)}{2(8x)^2}+\cdots\right], \quad x\to\infty,$$

from which it follows that the various thermodynamical parameters are given by

$$n_B \to \frac{(m^*)^3 D}{2\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\frac{1}{\gamma^{*3/2}}e^{\gamma^*(\mu^*/m^*-1)},$$

$$P \to \frac{(m^*)^4 D}{2\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\frac{1}{\gamma^{*5/2}}e^{\gamma^*(\mu^*/m^*-1)} - \frac{g_S^2}{m_S^2}\frac{(m^*)^6 D^2}{16\pi^3}\frac{1}{\gamma^{*3}}e^{2\gamma^*(\mu^*/m^*-1)} + \frac{g_V^2}{m_V^2}\frac{(m^*)^6 D^2}{8\pi^3}\frac{1}{\gamma^{*3}}e^{2\gamma^*(\mu^*/m^*-1)},$$

$$\rho \to \frac{(m^*)^4 D}{2\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\frac{1}{\gamma^{*3/2}}e^{\gamma^*(\mu^*/m^*-1)} + \frac{g_S^2}{m_S^2}\frac{(m^*)^6 D^2}{16\pi^3}\frac{1}{\gamma^{*3}}e^{2\gamma^*(\mu^*/m^*-1)} + \frac{g_V^2}{m_V^2}\frac{(m^*)^6 D^2}{8\pi^3}\frac{1}{\gamma^{*3}}e^{2\gamma^*(\mu^*/m^*-1)},$$

$$S \to \frac{(m^*)^3 D}{2\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\left[\frac{(1-\mu^*/m^*)}{\gamma^{*1/2}}+\frac{1}{\gamma^{*3/2}}\right] \times e^{\gamma^*(\mu^*/m^*-1)},$$

$$m^* - m \to \frac{g_S^2}{m_S^2}\frac{(m^*)^3 D}{2\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\frac{1}{\gamma^{*3/2}}e^{\gamma^*(\mu^*/m^*-1)},$$

while the transport coefficients read

$$\frac{\lambda}{\tau} \to \frac{5m^{*3}D}{4\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\frac{1}{\gamma^{*5/2}}e^{\gamma^*(\mu^*/m^*-1)},$$

$$\frac{\eta}{\tau} \to \frac{(m^*)^4 D}{2\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\frac{1}{\gamma^{*5/2}}\left[1+\frac{133}{64\gamma^*}\right] \times e^{\gamma^*(\mu^*/m^*-1)},$$

$$\frac{\zeta}{\tau} \to \frac{5m^{*3}D}{4\pi^2}\left[\frac{\pi}{2}\right]^{1/2}\frac{1}{\gamma^{*9/2}}e^{\gamma^*(\mu^*/m^*-1)}.$$

D. High-temperature—general case $\gamma^* \ll 1$

In the high-temperature range, the integrals were also expanded irrespective of the value of the chemical potential, using the following method: first a change of variables in $I^*_{2,n}$ [27] was performed,

$$I^*_{2,n} = \int_0^\infty \sinh^2 x \cosh^n x \left[\frac{e^{\gamma^*\cosh x - \alpha^*}}{(1+e^{\gamma^*\cosh x-\alpha^*})^2} + (-1)^{n+1}\frac{e^{\gamma^*\cosh x+\alpha^*}}{(1+e^{\gamma^*\cosh x+\alpha^*})^2}\right]dx$$
$$= \int_1^\infty \frac{1}{\gamma^*}\frac{(n+1)y^{n+1}-ny^{n-1}}{\sqrt{y^2-1}}\left[\frac{1}{1+e^{\gamma^* y-\alpha^*}} + (-1)^{n+1}\frac{1}{1+e^{\gamma^* y+\alpha^*}}\right]dy$$
$$= \int_0^\infty \frac{2}{\gamma^*}\frac{(n+1)(1+t^2/\gamma^*)^{n+1}-n(1+t^2/\gamma^*)^{n-1}}{(2\gamma^*+t^2)^{1/2}} \times \left[\frac{1}{1+e^{t^2+\gamma^*(1-\mu^*/m^*)}} + (-1)^{n+1}\frac{1}{1+e^{t^2+\gamma^*(1+\mu^*/m^*)}}\right]dt,$$

with $y=\cosh(x)=1+t^2/\gamma^*$, and then expanded as a power series of the small parameter γ^*. For example, we have

$$I_{2,3} \sim \frac{2}{\gamma^*}\int_0^\infty \left[\frac{4t^7}{\gamma^{*4}}+\frac{12t^5}{\gamma^{*3}}\right]\left[2+2\gamma^*\frac{d}{d(t^2)}\right]\frac{1}{1+e^{t^2}}dt.$$

The results are identical to the above ones ($\gamma^* \ll 1, \alpha^* \ll \gamma^*$).

E. Discussion

The effects of the coupling to the vector and scalar fields were studied separately. This is displayed in Figs. 7–9. Figure 7 is a good illustration of the modification brought by the coupling to the vector meson field. It is seen that the presence of a vector coupling at constant chemical potential tends to increase the thermal conductivity in the high-temperature regime (it behaves now as T^9 instead of T^5) and lower it at low temperature (without changing the slope). When one remembers that the thermal conductivity is a transport of energy, an explanation for this behavior can be given: the energies are shifted by the presence of V^μ. For instance, the effective chemical potential is

$$\mu^* = \mu - g_V V^0 = \mu - (g_V^2/m_V^2) n_B \ .$$

Also at high temperatures

$$n_B/m^2 = (\mu - \mu^*)/c_V^2 \sim \mu^*/\gamma^2$$

so that the effective chemical potential vanishes as T^2 in this domain. However, one also has

$$\frac{\lambda}{\tau} \sim m^{*3} \left[\frac{\mu^*}{m^*}\right]^{-2} \left[\frac{1}{\gamma^{*5}}\right] \sim \left[\frac{1}{\gamma^5 \mu^{*2}}\right] \sim \gamma^{-9} \ .$$

At low temperatures, on the other hand,

$$\frac{\lambda}{\tau} \sim \frac{(m^{*3} u^3)}{(\gamma^* x)}$$

and $n_B \sim m^{*3} u^3$ gives

$$\frac{\lambda}{\tau} \sim \frac{n_B}{(\gamma \mu^*)} \sim \gamma^{-1} (\mu^{*2} - m^{*2})^{3/2} / \mu^* \ ,$$

with n_B and μ^* constant, but μ^* is reduced at high densities, so that λ/τ is finally lower than in the free particle case.

The presence of a scalar coupling, on the other hand, does not have such a drastic effect on the thermal conductivity. The scalar field increases λ a little in the low-temperature high-density regime. As a matter of fact, the effective mass drops to zero at high density [9,14,16]: the smaller inertia of the particles renders them more effective in transporting energy. At high temperature the scalar field enters as a second-order effect only.

Looking at the shear viscosity, collective effects prove to show up more easily at low temperatures and high densities. The scalar field enhances η a little, since its attractive properties increase the cohesion of the fluid, while the repulsive vector field lowers it by several orders of magnitude by repelling two adjacent layers of fluid. From the above results it can be checked that indeed $\eta/\tau \sim \gamma^{-4}$ at high temperature as in the free particle case. For large γ's, $\eta/\tau \sim (\mu^{*2} - m^{*2})^{5/2}/\mu^*$.

Finally, let us say a word about the bulk viscosity. Most of the differences correspond here to the high-temperature or high-density regime. This is easily explained by noticing that the effective mass drops to zero in this range whenever the scalar field is switched on: the bulk viscosity vanishes as is known to be the case for a system of massless particles. The coupling to the repulsive vector field alone favors the expansion (enhances the bulk viscosity) at low temperatures or high density when the short-range effects are expected to be predominant. The coupling to the attractive scalar field, on the other hand, counteracts the expansion (lowers the bulk viscosity).

VI. DISCUSSION AND CONCLUSION

Let us now summarize our results and discuss the various assumptions behind our calculations.

(1) A specific model had to be chosen as to the description of relativistic nuclear matter and the Walecka model was the dynamical basis of our calculations. This particular model was chosen essentially because it is "canonical" in this sense that it is used as a reference for almost all other relativistic models. Of course, it contains its own problems, such as the much too large value of the compressibility coefficient [10] of nuclear matter, but this can be cured by adding suitable terms (or fields) to the basic Lagrangian: For instance, the addition of self-couplings of the scalar field constitutes one such possibility [28]. Therefore, our choice is not unreasonable even though it is not the only possible one: our calculations can always be improved.

(2) Using the covariant Wigner function techniques developed elsewhere [11] a hierarchy, equivalent to both the statistical and the dynamical content of the model, was given. Essentially, the first equations of the hierarchy were used in the "derivation " of the transport equation: those connecting the one-particle Wigner function to averages like, e.g., $\langle F_{\rm op}\phi\rangle$ or $\langle\phi\rangle$. Next, a cluster decomposition of terms like $\langle F_{\rm op}\phi\rangle$ and $\langle F_{\rm op}V^\mu\rangle$ was done, thereby exhibiting a separation between collective terms (like $\langle F_{\rm op}\rangle\langle\phi\rangle$ and $\langle F_{\rm op}\rangle\langle V^\mu\rangle$) and irreducible correlations. The latter usually give rise, through various assumptions and reasonings, to a collision term. Since our aim was an evaluation of some collective effects the simplest collision term was chosen, a relaxation-time approximation [13] one that reduces to the Anderson-Witting [6] one in the absence of collective effects. This choice was motivated both by the necessity of comparing similar results (with and without collective effects) and by the physical content of this collision term [5,6,13].

(3) The dynamics of the system was (within this relaxation-time approximation) entirely involved in the relaxation time τ, itself to be evaluated with a more detailed analysis. However, the relaxation time could well be dependent on the collective fields ϕ and V^μ. In order to discuss this point let us limit ourselves to the case of the scalar field only. In the collision term $\tau(\phi)$ appears only as τ^{-1} so that in a first-order Chapman-Enskog expansion, one has

$$\frac{f_{(1)}}{\tau(\phi)} = \frac{f_{(1)}}{\tau(\phi_{\rm eq}) + \phi_{(1)}\tau'(\phi_{\rm eq}) + \cdots} = \frac{f_{(1)}}{\tau(\phi_{\rm eq})} + O(\tau^2) \ . \tag{6.1}$$

$f - f_{\rm eq}$ is a first-order quantity and any (new) first-order term like $\phi_{(1)}$ gives rise to a second-order term. Therefore, our results are formally not affected by the possible ϕ dependence of the relaxation time. It is clear, however, that the *numerical* value of τ has changed and that the comparison of our results with those obtained by Anderson and Witting makes sense only for those τ such that $\tau(\phi_{\rm eq}) = \tau$. In order to get a more precise idea of the influence of ϕ on τ, we obviously need a specific calculation. Nevertheless, an estimation of τ as $\tau \sim 1/n\sigma$ can give some clues in the absence of ϕ: In σ, the mass of a nucleon m has to be replaced by its effective mass $m^* \equiv m - g_s \phi_{eq}$. A simple calculation shows that the one boson exchange nucleon-nucleon total cross section is proportional to m^{-2}. This leads to $\tau \propto m^2$ and hence $\tau(\phi_{eq}) \sim (m - g\phi_{eq})^2$. It follows that ϕ makes τ *smaller:* $\tau(\phi_{eq}) \leq \tau(0)$. It should also be remarked that, in a one boson exchange calculation (or in higher-order processes), it is sufficient to use the customary vacuum boson propagator: In the domain of temperatures and densities considered both T and μ are much smaller than m_s.

(4) A few words have now to be said about the Chapman-Enskog expansion of the solution of the transport equation. This expansion was a series expansion in powers of the small parameter $\varepsilon \equiv \tau/L$ where L is a (macroscopic) hydrodynamic scale. As a matter of fact, there also exist several other scales, namely, the ones defined by the various wavelengths occurring in the system, i.e.,

$$\lambda_N = 1/m \; , \quad \lambda_S = 1/m_S \; , \quad \lambda_V = 1/m_V \; . \tag{6.2}$$

Therefore, a complete Chapman-Enskog expansion should be an expansion in powers of several dimensionless parameters, besides ε, such as $\eta \equiv 1/mL$, $\chi \equiv 1/m_s L$, $\xi \equiv 1/m_V L$. In fact, instead of η we would rather use the parameter $\eta^* \equiv 1/m^* L$. As discussed elsewhere [5] while the parameter η is negligible, it is generally not so at high densities and/or temperatures for the parameter η^* and, consequently, a multiparameter expansion should be dealt with [5]. On the other hand, the remaining parameters χ and ξ are also negligible in the approximation under study; however, when collective effects involve the consideration of quasi-bosons [30], their effective mass might lead to effective parameters whose values are not negligible (compared to unity).

(5) The problem of renormalization has now to be discussed. In our calculations no infinities occurred: This was due to the fact that, systematically, the vacuum contribution to the Wigner function, i.e., terms involving

$$F_{vac}(p) \equiv \frac{-D}{(2\pi)^3}\theta(-p^{*0})\delta(p^{*2} - m^{*2}) \; , \tag{6.3}$$

were discarded. Does this procedure make sense? The answer to this question is twofold and it depends on the fact that the system is dominated either by collisions or by collective effects. When collective effects dominate, then the thermal equilibrium state of the medium is controlled by a *renormalized* gap equation [14,30] arising from the regularization of the vacuum term occurring in the gap equation. On the other hand, when the system is dominated by collisions (as, for instance is the case of a dilute "gas"), the renormalization process reduces to the usual renormalization procedure leading to a finite cross section and hence to a finite relaxation time. However, it should be borne in mind that we are dealing with a merely phenomenological theory and also that the length scale at hand (kinetic scale) is much larger than most scales where quantum fluctuations do show up, i.e., of the order of Compton wavelengths. Accordingly, it is not necessary to take quantum fluctuations (via the vacuum Wigner function) into account (6.3). However, in order to be consistent with what is usually done in the case of thermodynamical equilibrium (in the Hartree approximation) one can use the renormalized gap equation studied elsewhere [14,29] instead of Eq. (3.20) *and* various counterterms.

(6) The limiting cases (low density and/or temperature; high density and/or temperature) can easily be understood in this model. At low densities, the effective mass m^* is close to the nucleon mass m while for the effective chemical potential μ^* one has $\mu^* = \mu + O(n_B)$. Accordingly, our results should be close to those already obtained by Anderson and Witting [6]: These properties can be checked in Figs. 4–6 where the various transport coefficients are computed as functions of the energy density expressed in units of the nuclear saturation density, for several temperatures. In the other limiting case, the effective mass of the nucleons is almost vanishing [10,14,16,30] and hence the general behavior of the transport coefficients can be obtained from the extreme relativistic limit of Anderson and Witting's results. Finally, our results mainly differ in the intermediate regime, as witnessed by the figures shown above. Note, however, the change in the slope of the curves λ and ζ when $\gamma \to 0$.

(7) Finally, we would like to warn the reader that the results contained in this paper are quite preliminary in this sense that only the self-consistent fields were taken into account in our calculations. It is clear that there exists many other kinds of collective effects that should also be taken into account. For instance, not only the collisions of quasinucleons must be dealt with—as was done here—but also there exist the quasibosons contributions (quasinucleons or quasibosons collisions) to be considered and discussed in a more "realistic" calculation. This can be done in several ways; for instance, via a Landau liquid model [31,32,15]. Such an approach is presently under active consideration [33].

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