Diabolic effects on nuclear rotational state population in two-neutron transfer

L. F. Canto
Universidade Federal do Rio de Janeiro, Caixa Posta 68528, 21945 Rio de Janeiro, Brazil

P. Ring and Y. Sun*
Physik Department, Technische Universitaet Muenchen, D-8046 Garching, Germany

J. O. Rasmussen and S. Y. Chu
Lawrence Berkeley Laboratory, University of California, Berkeley, Berkeley, California 94720

M. A. Stoyer
Lawrence Livermore National Laboratory, Livermore, California 94550
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We refine the theoretical treatment of two-neutron transfer between deformed nuclei of the rare-earth region and a heavy spherical nucleus in the Pb region. The refinement from our previous work on effects of the Berry phase interference around the backbending region is that we now include the two lowest rotational bands of both the A and $A+2$ nucleus rather than the lowest. The inclusion of two bands, ground and aligned, was earlier done by Dasso and Winther, who found diabolic interference effects to be small. We make numerical calculations on somewhat more favorable cases for experimental study, and we include nuclear optical potential effects and finite Q_{rx} values.

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I. INTRODUCTION

Recently, Dasso and Winther [1] improved the theoretical treatment of two-neutron transfer between heavy ions. The population of rotational band structures in the spheroidal target nucleus using a spherical projectile was considered. Their enhancement over previous theoretical work was the inclusion of the yrare along with the yrast band in the spheroidal A and $A+2$ nuclei. That is, they included the aligned band in addition to the ground band in the calculations. In earlier publications [2–5], an interference in the heavy-ion two-neutron transfer was noted and associated with a Berry phase [6] that manifests itself when two-neutron transfer paths lie on both sides of a diabolic point in the plane of particle number vs angular momentum. Theoretical estimates of rotational state population patterns for such systems as Pb on rare-earth nuclei indicated a substantial dependence on this interference, but the earlier theoretical work [3, 7–9] only included the lowest (i.e., yrast) levels in the initial and final nucleus.

With inclusion of the two lowest bands in initial and final nuclei Dasso and Winther [1] find quite a difference from the previous superconducting quantum interference device (SQUID) calculations referenced above. That is, with two bands there is only a small interference effect. In this paper, we reexamine the matter, looking in particular for cases and conditions where the diabolic interference effects might be somewhat greater than in Dasso and Winther's example [1].

*Present address: Dept. Físico Téorica, C-XI Canto Blanco Univ. Autónoma de Madrid, E-28049 Madrid, Spain.

II. MODEL ASSUMPTIONS

Our computer code, as used in this section, is based on the same model and computational methods as Dasso and Winther, and for their input parameters we get the same results as they. That is, both we and they use the semiclassical method of Alder, Winther, and deBoer [10], where the time-dependent coupled Schrödinger equations in the rotational state amplitudes are numerically integrated while the nuclear centers move on a classical Rutherford trajectory. For this work we restrict ourselves, as they did, to treat only head-on (impact parameter $b = 0$) trajectories, so that only $m = 0$ amplitudes (with respect to the beam direction) need be considered.

For practical reasons, experimental studies of transfer reactions are generally made near Coulomb barrier energies. Hence, for the realistic calculations in this paper we have extended the treatment to include effects of the tail of the complex nuclear optical potential, following the methods tested and published earlier by us [9]. These extensions of ours include a modification of the trajectory such that the distance of closest approach and the force at that point take into account the real part of the nuclear optical potential as well as the repulsive Coulombic potential.

Stephens and Simon [11] first presented the essential physics of yrast backbending as a virtual crossing of ground rotational band and an $i_{13/2}$ neutron spin-aligned band, but their numerical examples required arbitrary attenuation of Coriolis matrix elements to get sharp crossing. Bengtsson, Hamamoto, and Mottelson [12] and Sun, Ring, and Nikam [5], among others, have refined the theory, calculating the mixing matrix elements' oscillatory behavior with changing chemical potential (see other ref-

TABLE I. Input properties of nuclei concerned in ^{206}Pb on ^{174}Hf.

	^{172}Hf	^{174}Hf	^{176}Hf
V_I (MeV)	±0.221	0.310	0.310
$\left(\frac{\hbar^2}{2\vartheta}\right)_g$ (MeV)	0.0159	0.0151	0.0147
$\left(\frac{\hbar^2}{2\vartheta}\right)_s$ (MeV)	0.0159	0.0151	0.0147
E_a [a] (MeV)	1.8	1.8	1.8
i_α [b] ($\hbar$)	6	6	6

[a]Aligned band energy at spin = i_α.
[b]Spin alignment.

erences in the 1983 review article of deVoigt, Dudek, and Szymanski [13]). In the deformed rare-earth region, the mixing is evidently dominated by the $i_{13/2}$ neutron orbitals. The mixing matrix elements oscillate in sign as the shell fills, with the zeros (sharp backbending) coming when the chemical potential is slightly above the energy of an $i_{13/2}$ Nilsson orbital, except for the lowest ($\Omega = 1/2$) and highest ($\Omega = 13/2$) orbitals. While the theoretical model calculations give us the pattern and general magnitude of the mixing matrix elements, the precise mixing magnitudes need experimental determination. The mixing matrix element magnitude is half the energy difference between the yrast and yrare bands at crossing.

Dasso and Winther [1] took the mixing matrix elements as 0.1 MeV for initial and 0.05 MeV for final nuclei, rather small except for the one nucleus close to a diabolic point. In the heavier rare earths the calculations of Bengtsson *et al.* [12], and of Sun *et al.* [5] show the mixing matrix elements rise to larger values half-way between diabolic points.

It is clear that the reason Dasso and Winther [1] get such a small effect between diabolic and nondiabolic transfer is that the Coulomb excitation carries the main population up and down the ground band, without getting into the aligned band sufficiently to sense the sign of the Berry phase. The most favorable cases to see an interference effect would be where the initial and final nuclei symmetrically straddle a diabolic point to maximize mixing in both nuclei.

TABLE II. Input properties of nuclei concerned in ^{206}Pb on ^{156}Gd.

	^{154}Gd	^{156}Gd	^{158}Gd
V_I (MeV)	±0.023	0.160	0.160
$\left(\frac{\hbar^2}{2\vartheta}\right)_g$ (MeV)	0.0095	0.0107	0.0102
$\left(\frac{\hbar^2}{2\vartheta}\right)_s$ (MeV)	0.0117	0.0087	0.0127
E_a [a] (MeV)	1.380	1.706	1.377
i_α [b] ($\hbar$)	6	6	6

[a]Aligned band energy at spin = i_α.
[b]Spin alignment.

TABLE III. Additional input parameters for particular reactions. ○ and △ denote the particular reaction in each figure.

	Fig. 1	Figs. 2 and 3
$E_{lab}^{(Pb)}$ (MeV)	1100	1100
V_0^a (MeV)	0	40
W_0^a (MeV)	0	40
r_0^a (fm)		1.2
a_w^a (fm)		0.65
$Q_0^{○}$ [b] (b)	7.3	7.3
$Q_0^{△}$ [b] (b)	7.3	7.3
$E_{bind}^{○}$ (MeV)	7.5	14.82
$E_{bind}^{△}$ (MeV)	7.5	14.10
$\beta_2^{○}$	0.277	0.27
$\beta_2^{△}$	0.277	0.25
$Q_{rx}^{○}$ (MeV)	0	−0.522
$Q_{rx}^{△}$ (MeV)	0	−1.5

[a]V_0, W_0, r_0, and a_w are, respectively, the real part, imaginary part, the size parameter, and the diffuseness parameter of the nuclear optical potential.
[b]Intrinsic quadrupole moment.

With this in mind we have chosen to examine the case of $2n$ transfer in both directions from ^{174}Hf. The nucleus ^{206}Pb is chosen as the partner to give large Coulomb excitation and small $|Q_{rx}|$ values for both directions. Then we make calculations for the ^{156}Gd + ^{206}Pb collision system, for which Helmer *et al.* [14] are now analyzing data. We approach the best predictions for experimental test by a succession of calculations showing the effects of various parameter changes. Input parameters are given in Tables I, II, and III.

III. RESULTS AND DISCUSSION

The calculations of Fig. 1 show the diabolic effect for ^{206}Pb on ^{174}Hf, where the Q_{rx} ($2n$-transfer reaction Q value) is set to zero and there is no nuclear optical potential—essentially the conditions of Dasso and Winther [1]. That is, we use the same magnitude of mixing strengths V_I from Fig. 7 of Ref. [5], only changing the relative signs for the diabolic case and leaving them the same for the nondiabolic case. The only essential changes from the Ref. [1] example are these larger V_I values, although Z and A are larger and the moments of inertia are slightly different, being chosen to fit experimental energy level data [15–17]. All our figures have two parts: part (a) shows yrast bands and part (b) shows yrare bands. In all figures, diabolic transfer (V_I sign change between initial and final nuclei) is indicated by a solid line and nondiabolic by a dashed line. The points are normalized to unity for the sum of populations to all rotational states in yrast and yrare bands. The yrast band shows significant differences above spin 8, while for the yrare band the differences occur mainly below spin 12. Granted the differences show up mostly in states with populations less than 20% of the low-spin states, and the

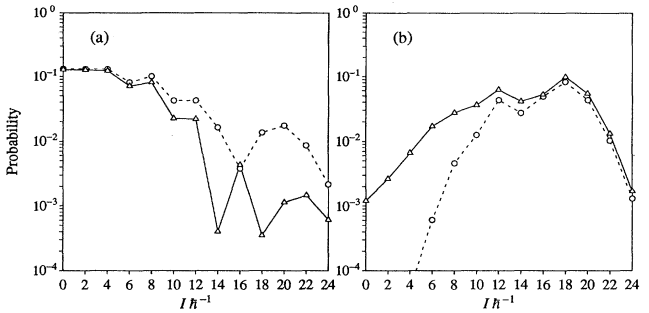

FIG. 1. $2n$-transfer probability as a function of spin for transfer to (a) yrast and (b) yrare. The calculations show the diabolic effect for ^{206}Pb on ^{174}Hf with $Q_{rx} = 0$ and without nuclear optical potential. This calculation assumes ^{174}Hf transfers to ^{172}Hf. The input parameters here (cf. Tables I and III) were chosen to compare best with Dasso and Winther [1]; only the mixing V_I, beam energy, and the target nuclei are larger. The symbol o's are the nondiabolic case and the Δ's are the diabolic calculations.

differences would be less apparent on linear plots. Nevertheless, these differences due to Berry phase interferences remain detectable.

In the next calculations, we put in the realistic Q_{rx} value and optical potential, but with the same band-mixing matrix elements as in the first example. In [18] we showed plots of these more realistic calculations, but to save space they are not shown here. They are generally similar, but there is some damping of the intensity oscillations, analogous to that caused by absorption effects in rotational inelastic scattering (cf. Fig. 2 of Ref. [9]). The Coulomb-nuclear interference from the attractive real part of the optical potential also results in a lowered intensity at highest spins, namely, a down-shift of the semiclassical rainbow maximum spin.

Finally, we calculated for the case of ^{206}Pb on ^{156}Gd, a system which has been studied experimentally [14]. This case qualitatively differs from the Hf examples above

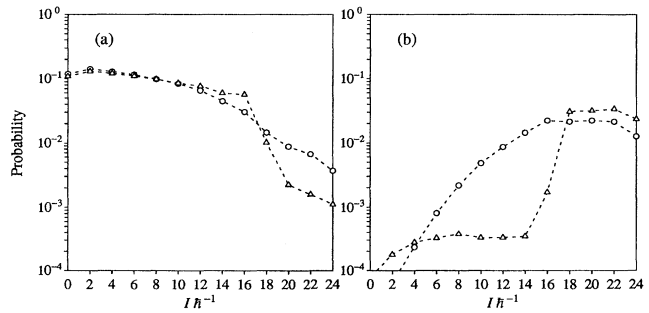

FIG. 2. $2n$-transfer probability as a function of spin for transfer to (a) yrast and (b) yrare. The calculation is for ^{206}Pb on ^{156}Gd assuming both nondiabolic $2n$ transfers to ^{154}Gd and to ^{158}Gd. For the values of parameters used in the calculation see Tables II and III. The symbol o's are for transfer to ^{158}Gd and the Δ's are for transfer to ^{154}Gd. In this figure note that both curves are dashed lines for nondiabolic cases.

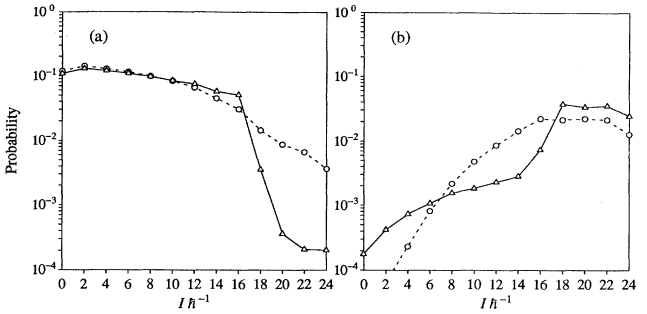

FIG. 3. Same as in Fig. 2 except the transfer to ^{154}Gd is diabolic. The symbols are the same as in Fig. 2.

mainly in that one final nucleus, ^{154}Gd is very close to a diabolic point, i.e., is a sharp backbender. The band-mixing calculations of Bengtsson *et al.* (cf. Fig. 16 of Ref. [13]) do not include Gd ($Z = 64$). For ^{154}Gd, there are extensive data on yrast and yrare bands covering below and above the crossing around $I = 16 - 18$. Therefore, we estimated the magnitude of V_I by taking half of the experimental energy splitting at closest approach between yrast and yrare bands [19]. However, it is not possible to determine the sign of V_I this way, so it is unknown if transfer to ^{154}Gd is diabolic or not. For ^{156}Gd and ^{158}Gd, we only have ground rotational bands which do not extend to spin high enough to extract $|V_I|$ and other parameters. Thus, we have used data on the Dy isotones with two additional protons. Only the yrast bands levels [20, 21] are known, but we do the best we can by fitting the second differences of the energy lev-

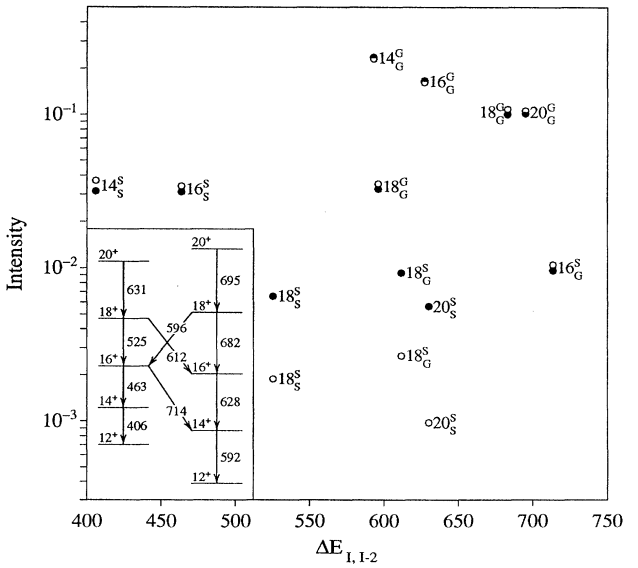

FIG. 4. Predicted relative intensities of gamma rays depopulating ^{154}Gd in the nondiabolic (•) and diabolic (○) cases. The points are labeled with the initial-state spin I and superscripts and subscripts for initial and final states, respectively. The letters G and S represent the ground and super band, respectively. The energy level inset is from the paper of Morrison *et al.* [19] with the ground band on the right and the superband (aligned band) on the left.

els and making the standard assumption of six units of spin alignment. Finally, we note the trends with Z in the V_i calculation of Bengtsson *et al.* and extrapolate to $|V_I| = 160$ keV for both ^{156}Gd and ^{158}Gd. We present our results in Fig. 2, where transfer to ^{154}Gd is assumed nondiabolic, and in Fig. 3, where transfer is diabolic. We observed some decrease in population of the yrast band above spin 16, and some increase to the yrare band below spin 12 for the diabolic transfer case relative to nondiabolic. Unfortunately, the effects of the Berry interference are small for the ^{156}Gd $\rightarrow$ ^{154}Gd reaction because the mixing matrix element is so small that its sign makes little difference, except for some of the weak transitions. Since the experimental data on gamma branching are known, it is possible to take the theoretical direct population intensities from Figs. 2 and 3, and compute the relative intensities of gamma rays. Figure 4 shows the results. The intensities of the gamma rays depopulating the 18^+ and 20^+ states of the aligned band (yrast at these spins) are very sensitive to the Berry phase, but their weakness makes them difficult to observe. There is a 10% effect on the somewhat stronger yrare $14^+ \rightarrow 12^+$ transition.

IV. CONCLUSIONS

We qualitatively agree with Dasso and Winther that Berry-phase effects in $2n$ transfer are more subtle than first thought. Inclusion of the yrare band in theory is essential. We now know that Berry interference effects are best sought where initial and final nuclei lie on opposite sides of a diabolic point but neither nucleus is too close to diabolic. The Hf cases that we first calculated have that appeal, but they have other experimental difficulties, namely, the low natural abundance of ^{174}Hf (0.162%) and the too-negative Q_{tx} value (−1.5 MeV) for the ^{206}Pb + ^{174}Hf $\rightarrow$ ^{208}Pb + ^{172}Hf diabolic transfer.

It may be that the Berry interference can be better detected in $2n$ transfer involving odd-n nuclei, such as, ^{163}Dy or ^{171}Yb, where a normal-parity orbital is blocked. The blocking should shift chemical potentials away from the $i_{13/2}$ energies and thus better span the diabolic point. This class of more challenging in-beam gamma spectroscopy may become much more feasible with the next-generation gamma detector arrays, Gammasphere, and EUROGAM.

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