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Effective number of protons for quasifree (π^-,π^0) at 500 and 400 MeV

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For the quasifree (π^-,π^0) reaction on a wide range of targets at 500 and 400 MeV, the measured effective numbers of protons are found to be in good agreement with those from a simple semiclassical attenuation model based on the nuclear density and the total cross section for free πN scattering. The A dependence for our experimental results was fitted by a power law ($\propto A^{\alpha}$), and the exponents α were found to be 0.38 ± 0.03 and 0.40 ± 0.06 for 500 and 400 MeV (π^-,π^0), respectively.

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A strongly interacting particle is unable to scatter from every nucleon within a complex nucleus because the large projectile-nucleon cross section prevents access to the interior. At large momentum transfers, such that each scattering event can be incoherent, the effect of distortions can be factored to define an effective number of nucleons available to the projectile. In this work we use the (π^-,π^0) reaction at 400 and 500 MeV to measure the effective number of protons, Z_{eff}, at large momentum transfers. We demonstrate that the factorization does indeed work by showing that the incoherent quasifree (QF) scattering cross section on carbon and copper are proportional to the free single-nucleon cross section over a range of angles, and we measure Z_{eff} for many nuclear targets.

The inclusive doubly differential cross section for QF pion single-charge exchange (SCX) is taken to be related to Z_{eff} by

$$\frac{d^2\sigma}{d\omega\, d\Omega_{\pi^0}} = Z_{\text{eff}} \frac{d\sigma}{d\Omega_{\pi^0}}\bigg|_{\text{free}} \overline{S}(q,\omega)\ , \tag{1}$$

where $d\sigma/d\Omega_{\pi^0}|_{\text{free}}$ is the singly differential cross section for free πN SCX and $\overline{S}(q,\omega)$ is the nuclear response function (as a function of momentum transfer q and energy loss ω), averaged over the non-spin-flip and spin-flip channels weighted by the corresponding singly differential cross sections for free πN SCX [1] (for pion SCX, there are only two spin-isospin channels: isovector transverse spin-flip and isovector non-spin-flip).

In this paper we first obtain Z_{eff} for a number of target nuclei from a QF pion SCX experiment and from a theoretical calculation, then make a comparison and study the A dependence. We finally compare with the results from a related previous experiment at a lower beam energy.

Pion SCX measurements with beams of 500 and 400 MeV π^- were performed at the P^3E pion channel at LAMPF using the π^0 spectrometer [2]. In order to achieve a wide energy acceptance as well as a large solid angle, the π^0 spectrometer front face was about 1 m from the beam spot on the target. Up to 12 targets, from hydrogen (from solid polyethylene) to bismuth, were used. The laboratory angles of the π^0 spectrometer were 30°, 50°, 70°, and 88.8° for 500 MeV π^- and 62.6° for 400 MeV π^- (this latter case attains the same momentum transfer as that for 500 MeV at 50°). Since the π^0 spectrometer has a wide opening angle around each scattering angle we binned the data into three angular bins of 7° each.

Our data are normalized directly to H(π^-, π^0) cross sections from phase shift calculations (from the SAID program) [3]. For all target nuclei except hydrogen, a prominent QF peak is found at the top of a smooth background in the spectra of doubly differential cross sections. The extraction of the parameters of the QF peak (peak location, peak width, and the inclusive QF cross sections $d\sigma^{\text{QF}}/d\Omega_{\pi^0}$) for each angular bin was achieved by fitting to the spectra as shown in Fig. 1. The function for the

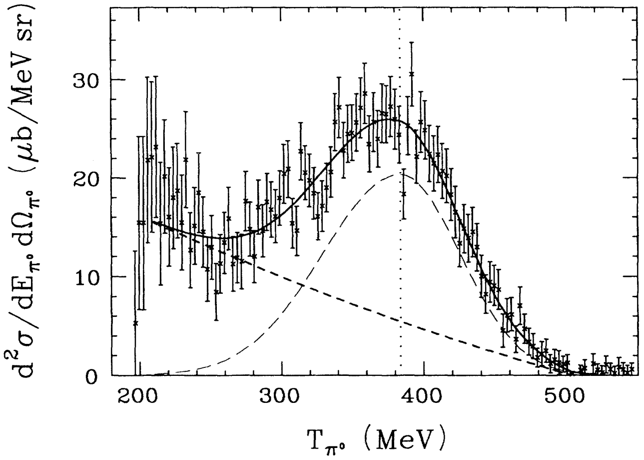

FIG. 1. The doubly differential cross section for C(π^-,π^0) at 500 MeV π^- and 50° (the angular bin is from 46.5° to 53.5°). The dotted line is at the position corresponding to free πN scattering. The solid line is the fitted result of the doubly differential cross section spectrum, and the two dashed lines correspond to the fitted QF peak and the fitted background.

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QF peak is an asymmetric Gaussian (two half Gaussians of adjustable width). The function for the background is a quadratic polynomial, which was found to be the best function to fit the spectrum at our largest scattering angle with centroid $\theta_{\pi^0} = 96.7°$ (angular bin from 93.5° to 100.5°), where the QF peak has disappeared into the background. More details can be found in Ref. [1].

At large momentum transfer we have $\int \overline{S}(q,\omega)d\omega = 1$, since there is essentially no Pauli blocking. Pauli blocking is negligible when the momentum transfer is greater than about 370 MeV/c for our experiment (corresponding to $\theta_{\pi^0} \approx 36°$ at 500 MeV). We should thus be able to fit the singly differential cross sections for free πN SCX $d\sigma/d\Omega_{\pi^0}|_{\rm free}$ [3] to the inclusive QF cross section $d\sigma^{\rm QF}/d\Omega_{\pi^0}$ to find the effective number of protons for each target at scattering angles where Pauli blocking is negligible. Figure 2 shows the data and fits for carbon and copper at 500 MeV. Note that $d\sigma^{\rm QF}/d\Omega_{\pi^0}$ is indeed proportional to $d\sigma/d\Omega_{\pi^0}|_{\rm free}$. This allows the introduction of $Z_{\rm eff}$, which is then independent of the scattering angle. For 400 MeV, only a single spectrometer setting was used, and fits for $Z_{\rm eff}$ were made for only three data points. Best-fit values of $Z_{\rm eff}$ for a number of target nuclei are listed in Table I and plotted in Fig. 3.

The effective number of protons can be expressed in a very simple semiclassical attenuation model as

$$Z_{\rm eff} = \frac{Z}{A}\int d^2b\,T(b)e^{-\sigma^T_{\pi^- N}T(b)} , \tag{2}$$

where

$$T(b) = \int dz\,\rho(r) . \tag{3}$$

Here Z is the atomic number of the target nucleus, A is its mass number, $\sigma^T_{\pi^- N}$ is the total $\pi^- N$ scattering cross section in the medium, and $\rho(r)$ is the density of nucleons inside the nucleus. $\rho(r)$ must satisfy $A = \int d^3r\rho(r)$. Note that this model includes only scattering from single nucleons and cannot include the true absorption process.

The cross section $\sigma^T_{\pi^- N}$ is calculated by averaging incoherently over all nucleons in the nucleus:

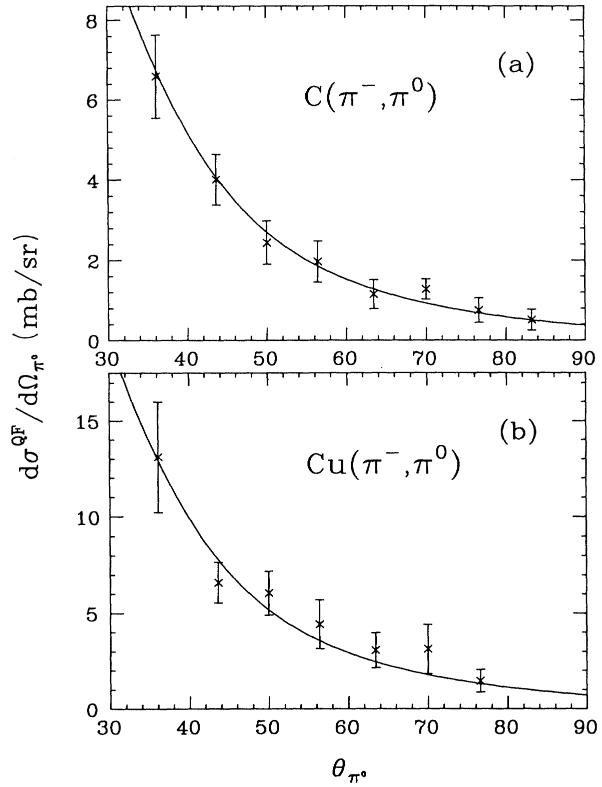

FIG. 2. Integrated (π^-,π^0) QF cross sections at 500 MeV. The solid lines are free πN SCX cross sections [3] normalized to our results.

$$\sigma^T_{\pi^- N} = \frac{Z\sigma^T_{\pi^- p} + (A-Z)\sigma^T_{\pi^- n}}{A} , \tag{4}$$

where $\sigma^T_{\pi^- p}$ and $\sigma^T_{\pi^- n}$ are the total cross sections for $\pi^- p$ and $\pi^- n$ scattering, respectively. Both $\sigma^T_{\pi^- p}$ and $\sigma^T_{\pi^- n}$ were extracted from phase shift evaluations [3].

The charge density distribution $\rho(r)$ for each target is one of the following functions: harmonic oscillator (HO),

TABLE I. The effective number of protons. Z is the atomic number and A is the mass number of each target. $Z^{\rm calc}_{\rm eff}$ and $Z^{\rm expt}_{\rm eff}$ are the effective number of protons from the calculation and the experiment, respectively. Li is pure, solid ^{7}Li; O is from water; all the other targets are natural. The distributions are described in the text.

				T_{π^-} = 500 MeV		T_{π^-} = 400 MeV	
Target	Z	A	Distribution	$Z^{\rm calc}_{\rm eff}$	$Z^{\rm expt}_{\rm eff}$	$Z^{\rm calc}_{\rm eff}$	$Z^{\rm expt}_{\rm eff}$
Li	3	7.02	HO	1.59	1.92±0.23	1.45	1.81±0.24
C	6	12.01	SG	2.22	2.79±0.22	2.03	2.26±0.48
O	8	16.00	HO	2.75	3.85±0.30	2.50	
Al	13	26.98	2pF	3.40	3.55±0.31	3.01	3.67±0.70
Fe	26	55.85	2pF	4.56	5.61±0.70	3.89	
Cu	29	63.55	2pF	5.21	5.35±0.49	4.45	4.37±0.54
Zr	40	91.22	3pG	5.41	5.85±0.63	4.46	5.04±0.83
Sn	50	118.69	3pG	5.69	5.56±0.84	4.57	6.18±1.38
Ta	73	180.95	2pF	7.53	8.39±1.40	5.99	
Bi	83	208.98	2pF	6.15	9.28±1.43	4.71	6.67±1.56

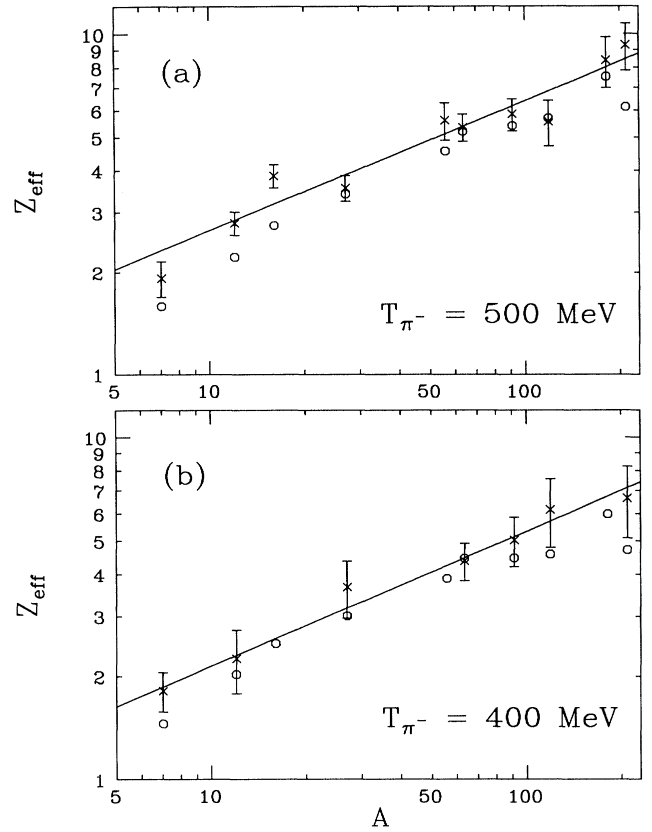

FIG. 3. The effective number of protons. The calculated and experimental results are labeled by the symbols ○ and ×, respectively. (a) shows the effective number of protons for (π^-,π^0) at 500 MeV. The experimental results are fitted by $Z_{\text{eff}}^{\text{expt}} = (1.10 \pm 0.13)A^{0.38\pm0.03}$. (b) shows the effective number of protons for (π^-,π^0) at 400 MeV. The experimental results are fitted by $Z_{\text{eff}}^{\text{expt}} = (0.86 \pm 0.18)A^{0.40\pm0.06}$.

sum of Gaussians (SG), two-parameter Fermi (2pF), or three-parameter Gaussian (3pG). These functions and their parameters are given in detail in Ref. [4]. The targets used in our experiment were generally not isotopically pure, so the density distributions have therefore been averaged according to the natural isotopic abundance [5]. The calculated results using Eq. (2) are given in Table I and Fig. 3.

The results from the experiment and from the theoretical calculation are found to be in quite good agreement for (π^-,π^0) at both 500 and 400 MeV. The calculated Z_{eff} are generally somewhat lower, which could be due to a Pauli-blocking effect: $\sigma^T_{\pi^- N}$ used in Eq. (2) should be the value in the nuclear medium, which is somewhat smaller than the value we used in the calculation, and this smaller cross section would allow somewhat better access to more protons. A large true absorption cross section, neglected in the model, would give measured Z_{eff} less than the calculated values.

The A dependence for the effective number of protons can be fitted by a power law ($\propto A^{\alpha}$) as shown in Fig. 3 to summarize our data. The exponent α, which reflects the ability of the pions to penetrate the target nuclei, is found to be 0.38 ± 0.03 and 0.40 ± 0.06 for 500 MeV and 400 MeV (π^-,π^0), respectively. Scattering from a black disk gives the limiting value of $\alpha = \frac{1}{3}$.

In a former experiment at a beam energy of 160 MeV [6], the effective number of protons (evaluated as in the present work) was found to saturate above $A \approx 100$. This is because an incident π^- interacts much more strongly with neutrons than protons at 160 MeV ($\sigma^T_{\pi^- n}$ = 195.1 mb, $\sigma^T_{\pi^- p}$ = 68.8 mb), causing the protons to be shielded by the excess neutrons in heavy nuclei. For our beam energies of 500 and 400 MeV, the incident π^- interacts somewhat more weakly with neutrons than with protons at 500 MeV ($\sigma^T_{\pi^- n}$ = 19.2 mb, $\sigma^T_{\pi^- p}$ = 33.0 mb) and somewhat more strongly at 400 MeV ($\sigma^T_{\pi^- n}$ = 30.5 mb, $\sigma^T_{\pi^- p}$ = 27.6 mb). Thus we do not expect any saturation effect in our data, and we see none.

From above, a simple proportionality is found between the QF pion SCX cross sections and the free cross sections at intermediate and large π^0 laboratory scattering angles out to a momentum transfer of about 700 MeV/c. Except for Pauli blocking, no alteration of the angular distribution for πN scattering in the nuclear medium is indicated. A very simple relation is found for the A dependence of pion distortions, and the simplest model for attenuation of the projectile is found to agree quite well with the data.

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