Orbital $M1$ versus $E2$ strength in deformed nuclei: A new energy weighted sum rule

E. Moya de Guerra
Instituto de Estructura de la Materia, Consejo Superior de Investigaciones Científicas, Serrano 119-123, 28006 Madrid, Spain

L. Zamick
Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855
(Received 23 November 1992)

Within the unified model of Bohr and Mottelson we derive the following linear energy weighted sum rule for low-energy orbital 1^+ excitations in even-even deformed nuclei $S_{\rm LE}^{\rm lew}(M_1^{\rm orb}) \cong (6/5)\epsilon[B(E2;0_1^+ \rightarrow 2_1^+ K=0)/Ze^2\langle r^2\rangle^2]\mu_N^2$ with $B(E2)$ the $E2$ strength for the transition from the ground state to the first excited state in the ground-state rotational band, $\langle r^2\rangle$ the charge rms radius squared, and ϵ the binding energy per nucleon in the nuclear ground state. It is shown that this energy weighted sum rule is in good agreement with available experimental data. The sum rule is derived using a simple ansatz for the intrinsic ground-state wave function that predicts also high-energy 1^+ strength at $2\hbar w$ carrying 50% of the total m_1 moment of the orbital $M1$ operator.

PACS number(s): 21.60.Ev

I. INTRODUCTION

Over the last years much experimental and theoretical work has been devoted to the study of orbital 1^+ excitations [1]. Thanks to the combined analyses of (e,e'), (γ,γ'), and (p,p') experiments the occurrence of low lying orbital 1^+ excitations in even-even deformed nuclei is now a well-documented and established fact. Furthermore, it has been recently found that the summed strength of the observed excitations, which typically appear concentrated around an average excitation energy ~ 3 MeV, varies quadratically with quadruple deformation [2]. This last observation has motivated theoretical work [3–8] to understand the origin of the quantitative relation between the orbital $M1$ strength and the quadruple deformation parameter β from microscopic calculations [3–5] and to derive general formulas connecting M_1 and $E2$ strengths from simplified models [6–8].

In Ref. [3] results were presented for summed orbital 1^+ strengths in Sm and Nd isotopes that had been obtained from deformed Hartree Fock (HF) and BCS calculations with the Skyrme SKIII interaction, using the angular-momentum projection formalism and subtracting spurious contributions [9,10]. The results obtained by including all possible two-quasiparticle excitations up to $E_x = 4$ MeV were found to be proportional to β^2, to agree with experimental data, and to account for about 50% of the total summed strength (obtained including all possible two-quasiparticle excitations up to $E_x \sim 25$ MeV) [3]. It was also found that for the deformed isotopes the stronger contributions occur in the range 2 MeV $\leq E_x \leq 4$ MeV and that at higher excitations the strength is much more fragmented. Because calculations neglecting pairing lead to summed strengths proportional to β rather than to β^2 it was argued that dependence on β^2 is due to the combined effect of pairing and deformation [3]. Similar results and conclusions have been reached from quasiparticle random phase approximation (QRPA) calculations based on deformed Woods-Saxon potentials with pairing, quadrupole-quadrupole, and spin-spin residual interactions [4,5]. The main difference is that the abovementioned QRPA calculations find less orbital 1^+ strengths in the high-energy region. We shall come back to this point later on.

In an attempt to establish a general connection between orbital $M1$ and $E2$ strengths, linear energy weighted sum rules (LEWSR) for orbital 1^+ excitations have been recently discussed by Heyde and De Coester [6] and by Zamick and Zheng [7]. Within the framework of the interacting boson model (IBM-2) Heyde and De Coester obtain

$$\sum_f B(M1;0_1^+ \rightarrow 1_f^+) E_x(1_f^+) = \sum_f B(E2;0_1^+ \rightarrow 2_f^+) \qquad (1)$$

with an effective $E2$ charge appropriate for boson models.

On the other hand, Zamick and Zheng [7], using a quadrupole-quadrupole interaction, find that the LEWSR for orbital $M1$ is proportional to the *difference* of summed isoscalar and summed isovector $E2$ strengths. In this work we shall show that this approach can be carried further. This will be done by introducing the mean field into the picture. We will for the first time look at both the low-energy ($\Delta N = 0$) and the high-energy ($\Delta N = 2$) contributions to LEWSR.

We shall here consider two closely related models for calculating LEWSR. First a two-body quadrupole-quadrupole interaction is used to evaluate the double commutator

$$S^{\rm lew}(M1^{\rm orb}) = \frac{3}{4\pi} \sum_f (E_f - E_{\rm g.s.}) |\langle f|\mu^{\rm orb}|{\rm g.s.}\rangle|^2 \mu_N^2$$
$$= \frac{3}{8\pi} \mu_N^2 \langle {\rm g.s.}|[\mu^{\rm orb},[H,\mu^{\rm orb}]]|{\rm g.s.}\rangle \ . \qquad (2)$$

We note that the commutator in Eq. (2) vanishes for a pure pairing interaction between like nucleons (i.e., between protons only and neutrons only) and for a pure spin-spin interaction. Therefore the LEWSR, Eq. (2), should not change when said interactions are explicitly considered. The strength of the quadrupole-quadrupole interaction is obtained from a self-consistency condition. In the second method one evaluates this same double commutator using the one-body deformed field.

II. LINEAR ENERGY WEIGHTED SUM RULE WITH A QUADRUPOLE-QUADRUPOLE INTERACTION

We write the quadrupole-quadrupole interaction as

$$H_Q=-\chi\sqrt{5}\sum_{i>j}[(r^2Y^2)i(r^2Y^2)_j]^0 . \tag{3}$$

As shown by Zheng and Zamick [7], the value of LEWSR with this interaction is (see also Ref. [11])

$$S^{\text{lew}}(M1)=\frac{9}{16\pi}\chi\left[\sum B(E2;1,1)-\sum B(E2;1,-1)\right] , \tag{4}$$

where $B(E2;e_p,e_n)$ is the value of $B(E2)$ for the hypothetical operator

$$e_p\sum_{\pi} r^2Y^2+e_n\sum_{\nu} r^2Y^2 .$$

The transition is from the $J=0^+$ ground state to excited $J=2^+$ states. Equation (4) holds if we add to the $Q\cdot Q$ interaction a pairing interaction between like nucleons.

For clarification purposes it should be noted that for an $N=Z$ nucleus with a $T=0$ ground state the value of $B(E2;1,1)$ is four times that of the true electric operator $B(E2;1,0)$ for $T=0$ final states, and is zero for $T=1$ final states. On the other hand, $B(E2;1,-1)$ will reach only $T=1$ final states and is four times the value of $B(E2)$ for the electric probe with $e_p=1$ and $e_n=0$.

A renormalized value of χ which we call χ_R can be determined, as discussed by Bes and Sorensen [12]. The interaction of a single valence particle with all the other particles, assuming they have an axially symmetric density distribution with the Z axis as a symmetry axis is

$$H_{\text{DF}}=-\chi_R\frac{5}{16\pi}Q_M(\text{core})(2z^2-x^2-y^2) , \tag{5}$$

where $Q_M(\text{core})$ is the mass quadrupole moment of the core. It is further shown that the quadrupole moment of the core is equal to that of the valence nucleons. Thus the intrinsic mass quadrupole moment Q_0^M is equal to

$$Q_0^M=Q^M(\text{core})+Q(\text{valence})\cong 2Q^M(\text{core}) . \tag{6}$$

If in contrast to the above we use a deformed oscillator Hamiltonian

$$H_{\text{DO}}=\sum\left[\frac{p^2}{2m}+\frac{1}{2}mw_x^2(x^2+y^2)+\frac{1}{2}mw_z^2z^2\right] , \tag{7}$$

then we can make it look like the above expression by introducing the oscillator deformation parameter δ_0

$$\begin{aligned}\omega_x^2&=\omega_0^2(\delta_0)\left[1+\frac{2}{3}\delta_0\right] ,\\ \omega_z^2&=\omega_0^2(\delta_0)\left[1-\frac{4}{3}\delta_0\right] ,\\ H&=\sum\left[\frac{p^2}{2m}+\frac{1}{2}m\omega_0^2r^2\right]+H_{\text{DF}} ,\end{aligned} \tag{8}$$

where the deformed field term is

$$H_{\text{DF}}=-m\omega_0^2\delta_0(2z^2-x^2-y^2)/3 . \tag{9}$$

We define a deformation parameter δ

$$\delta=\frac{3}{4}\frac{Q_0^{\pi}}{\langle R^2\rangle_{\pi}} . \tag{10}$$

This can be experimentally determined from the measured charge quadrupole moment and rms radius. By equating the two expressions for H_{DF} we obtain the following expression for χ_R:

$$\chi_R=m\omega_0^2\frac{\delta_0}{\delta}\frac{8\pi}{5\langle R^2\rangle_M}\cong\frac{m\omega_0^2 4\pi}{5\langle R^2\rangle_{\pi}}\delta_0/\delta . \tag{11}$$

However, if we use the $Q\cdot Q$ interaction among all the nucleons in the nucleus, i.e., if we allow $\Delta N=2$ mixing, then the value of χ to be used is $\chi_R/2$, as also discussed by Bes and Sorensen [12] (see pp. 143 and 144).

We further note that $B(E2;1,1)$ to an isoscalar state is *four* times that for an electric probe ($e_p=1$, $e_n=0$). Likewise with $e_p=1$, $e_n=-1$ we reach the isovector states with four times the rate of what an electric probe would give. Thus, in order to make easier comparisons with electric probes we give the sum rule in the following way:

$$S^{\text{lew}}(M1)_{\text{all particles}}=\frac{9}{10}\frac{m\omega_0^2}{\langle R_{\pi}^2\rangle}\frac{\delta_0}{\delta}\left[\frac{\sum B(E2;1,1)}{4}-\frac{\sum B(E2;1,-1)}{4}\right] . \tag{12}$$

One final remark to this section. If we had taken for our interaction $\chi/2\sum_{i,j}Q(i)\cdot Q(j)$, thus keeping $i=j$ terms, we would reach the SU(3) limit. The $i=j$ terms add a single particle term of the form r^4 to the potential and cause a splitting of single particle energies with different l. In the SU(3) limit in a calculation involving only one major shell (i.e., no $\Delta N=2$ mixing) the $E2$ operator with $e_p=1$, $e_n=1$ reaches only one 2^+ state—this can be identified as the $J=2^+$, $K=0$ member of the ground state band. The $E2$ operator with $e_p=1$, $e_n=-1$ (isovector) connects $J=2^+$, $K=1$ member of the scissors mode rotational band, as was pointed out by Retamosa *et al.* [13]. It should be noted that in-shell model calculations the scissors mode band can get fragmented into two bands, one with isospin $T=T_0$ and one with isospin

$T=T_0+1$, where T_0 is the g.s. isospin. Thus the isovector $E2$ will be fragmented into two states. Useful formulas for matrix elements of $Q\cdot Q$ in the boson SU(3) scheme have been obtained by Rosensteel [14].

III. THE DOUBLE COMMUTATOR METHOD APPLIED DIRECTLY TO THE DEFORMED FIELD

We use a simple approximation within the framework of the unified model of Bohr and Mottelson [15] for even-even axially symmetric rotational nuclei, where the properties of the ground-state rotational band are given in terms of properties of the intrinsic ground state. For the intrinsic ground state we use the anisotropic harmonic oscillator (H.O.) satisfying the self-consistency condition. The H.O. is used as an auxiliary model because ground-state expectation values have simple analytical expressions that can then be reinterpreted in terms of those of the "true" intrinsic ground state, i.e., in terms of experimentally known properties of the ground-state band.

Hence in this section we consider a rotational model picture with intrinsic Hamiltonian of the form $H_{\rm DO}$ in Eq. (7), subject to the self-consistency condition [15]

$$\Sigma_x\,\omega_x=\Sigma_y\,\omega_y=\Sigma_z\,\omega_z=\sum\omega/3\ . \tag{13}$$

In the rotational model picture the magnetic moment operator is $\mu=g_R\mathbf{I}+\mu_{\rm int}$, with $\mathbf{I}$ the total angular momentum which does not contribute to magnetic excitations and g_R the collective gyromagnetic ratio that is experimentally determined from the magnetic moment of $2^+,4^+,\ldots$ states in the ground state band. In even-even nuclei $\mu_{\rm int}$ does not contribute to static magnetic moments because it has zero expectation value in the intrinsic time even ground state. The orbital part of $\mu_{\rm int}$ is

$$\mu_{\rm int}^{\rm orb}=\sum_i(g_l^i-g_R)l_i=(1-g_R)\mathbf{L}_\pi-g_R\mathbf{L}_\nu\ . \tag{14}$$

Thus the evaluation of the double commutator in Eq. (2) amounts to the evaluation of the double commutator of $\mu_{\rm int}^{\rm orb}$ with $H_{\rm DO}$ in Eqs. (14) and (7), respectively.

Can we obtain a similar result to that obtained with a $Q\cdot Q$ interaction by applying the double commutator method directly to the deformed one-body field? Using the fact that

$$[l_i,[l_j,(2z^2-x^2-y^2)_k]]=6(2z^2-x^2-y^2)_k\delta_{ik}\delta_{jk}\ , \tag{15}$$

we readily find that

$$S^{\rm lew}(M1^{\rm orb})=\frac{3\mu_N^2}{4\pi}\delta_0 m\omega_0^2\{(1-g_R)^2Q_0^\pi+g_R^2Q_0^\nu\}\ . \tag{16}$$

Using the definition of δ in Eq. (10) and the relationship for the $B(E2)$ for the ground-state $K=0$ band of an even-even nucleus

$$B(E2;0^+\rightarrow 2^+K=0)=\frac{5}{16\pi}Q_{0\pi}^2\ . \tag{17}$$

We write Eq. (16) in the form

$$S^{\rm lew}(M1^{\rm orb})=\frac{9\mu_N^2 m\omega_0^2}{10\langle R^2\rangle_\pi}\frac{\delta_0}{\delta}B(E2;0^+\rightarrow 2^+K=0)F_{\pi\nu}\ , \tag{18}$$

where

$$F_{\pi\nu}=2\left[(1-g_R)^2+g_R^2\frac{Q_0^\nu}{Q_0^\pi}\right]\ . \tag{19}$$

We compare this result to that of the previous section for the $Q\cdot Q$ interaction. If we remember that $B(E2)_\pi\approx B(E2;1,1)/4$, the differences between Eqs. (18) and (12) are the presence of the isovector $E2$ term in the latter and also the fact that Eq. (18) involves only the $B(E2)$ to the lowest 2^+ state and contains the factor $F_{\pi\nu}$.

We note that for a nucleus with equal proton and neutron bodies ($g_R=1/2$, $Q_0^\nu/Q_0^\pi=1$) the intrinsic orbital operator in Eq. (14) is purely isovector and the factor defined in Eq. (19) takes the value $1(F_{\pi\nu}=1)$, as is for instance the case for ^{20}Ne. In this case the dominant contribution to Eq. (12) is by far the $B(E2)$ value for the transition to the lowest 2^+ state, which in the SU(3) limit is the $J=2^+$ member of the ground state ($K=0$) band and exhausts the sum rule for quadrupole isoscalar transitions from ground state.

Thus, in this particular case, one can see that the LEWSR in Eq. (18) is similar to that in Eq. (12), but in the general case one can only check by numerical comparison. One difference is of course that Eq. (18) applies to deformed nuclei with a well-developed rotational g.s. band, while Eq. (12), is, in principle, more general. We would like also to point out that the effective nucleon gyromagnetic ratios $g_{\rm eff}^i=(g_l^i-g_R)$ for dipole magnetic excitations appearing in Eq. (14) play a role analogous to the effective charges for dipole electric excitations. In the last case the effective charges result from the subtraction of the spurious center-of-mass motion, in the first case $g_{\rm eff}^i$ results from the subtraction of the spurious rotation. In what follows we shall see that within the self-consistent mean-field picture, implementation of the self-consistency condition in Eq. (13) allows one to write Eq. (18) in a more practical way for phenomenological analysis.

IV. ADDITIONAL CONSIDERATIONS

First we note that the results in Eqs. (16) and (18) can also be obtained by considering particle-hole (p.h.) contributions. But then we get additional insight. If we consider p.h. contributions to the LEWSR,

$$S^{\rm lew}(M1^{\rm orb})=\frac{3}{4\pi}\sum_{ph}(\epsilon_p-\epsilon_h)|\langle ph|\mu_{\rm int}^{\rm orb}|0\rangle|^2\mu_N^2\ , \tag{20}$$

using the selection rules for $\mu_{\rm int}^{\rm orb}$ in the H.O. basis [11], we get two sets of p.h. excitations, one set with excitation energy $|\epsilon_p-\epsilon_h|=|\hbar w_x-\hbar w_z|$ and one set with excitation energy $|\epsilon_p-\epsilon_h|=(\hbar w_x+\hbar w_z)$.

This allows us to write Eq. (20) as

$$S^{\rm lew}(M1^{\rm orb})=S_{\rm LE}+S_{\rm HE} \tag{21}$$

with $S_{\rm LE}$ the sum from all the low-energy p.h. excitations

$(|\epsilon_p-\epsilon_h|=|\hbar w_x-\hbar w_z|\simeq\hbar w_0\delta_0)$,

$$S_{\rm LE}=[(1-g_R)^2S^{\pi}_{\rm LE}+g_R^2S^{\nu}_{\rm LE}]\mu_N^2 , \tag{22}$$

$$S^{\rho}_{\rm LE}=\frac{3}{8\pi}(\hbar w_x-\hbar w_z)\left[\Sigma^{\rho}_z-\frac{\Sigma^{\rho}_x+\Sigma^{\rho}_y}{2}\right](\beta^+)^2 \tag{23}$$

with $\rho=\pi,\nu$, and $S_{\rm HE}$ the sum from all the high-energy p.h. excitations $[|\epsilon_p-\epsilon_h|=(\hbar w_x+\hbar w_z)\simeq 2\hbar w_0]$

$$S_{\rm HE}=[(1-g_R)^2S^{\pi}_{\rm HE}+g_R^2S^{\nu}_{\rm HE}]\mu_N^2 , \tag{24}$$

$$S^{\rho}_{\rm HE}=\frac{3}{8\pi}(\hbar w_x+\hbar w_z)\left[\Sigma^{\rho}_z+\frac{\Sigma^{\rho}_x+\Sigma^{\rho}_y}{2}\right](\beta^-)^2 \tag{25}$$

with

$$\beta^{\pm}=\sqrt{w_x/w_z}\pm\sqrt{w_z/w_x} .$$

Using the self-consistency condition Eq. (13) with $\Sigma=\Sigma^{\pi}+\Sigma^{\nu}$ it is a simple matter to show that

$$S^{\rho}_{\rm LE}=S^{\rho}_{\rm HE}=\frac{\hbar\Sigma^{\rho}\omega}{8\pi}\frac{(\omega_x^2-\omega_z^2)^2}{\omega_x^2\omega_z^2}$$

$$=\frac{3}{8\pi}\delta_0 m\omega_0^2Q^{\rho}_0,\rho=\pi,\nu . \tag{26}$$

Where we have used the expression for Q^{ρ}_0

$$Q^{\rho}_0=\frac{\hbar}{m}\left[2\frac{\Sigma^{\rho}_z}{\omega_z}-\frac{\Sigma^{\rho}_x+\Sigma^{\rho}_y}{\omega_x}\right]$$

$$=\frac{2}{3}\frac{\hbar\Sigma^{\rho}\omega}{m}\frac{(\omega_x^2-\omega_z^2)}{\omega_x^2\omega_z^2} , \tag{27}$$

and Eq. (8) for ω_x,ω_z. Therefore using Eqs. (20)–(26) we find the interesting result that

$$S_{\rm LE}=S_{\rm HE}=\tfrac{1}{2}S^{\rm lew}(M1^{\rm orb}) , \tag{28}$$

where $S^{\rm lew}(M1^{\rm orb})$ coincides with the value given in Eq. (16), see also Eq. (18).

Thus within this approach the strength function has only two peaks one at low energy ($E^{\rm LE}\simeq\hbar w_0\delta_0$) and one at higher energy ($E^{\rm HE}\simeq 2\hbar w_0$). A pairing interaction between like nucleons, and/or a spin-orbit interaction will cause splitting in each of these peaks but will hardly remove strength from the low-energy peak to the high-energy peak and vice versa. Therefore, we write a LEWSR for the low-energy part of the strength function taking half the value given in Eq. (16), or equivalently in Eq. (18).

To write the final expression for the energy-weighted sum rule for low orbital 1^+ excitations we substitute Eq. (18) into Eq. (28). But first we replace the H.O. parameters ω_0,δ_0 as well as Q^{ν}_0/Q^{π}_0 in Eqs. (18) and (19) by phenomenological parameters directly known from experimental electromagnetic properties of the ground-state band. To this end we use additional relations that hold within the deformed H.O. model satisfying the self-consistency condition in Eq. (13).

Contrary to Q^{π}_0, which is directly related to the experimental quadrupole moments and $B(E2)$ values of the g.s. band, there is no direct information on Q^{ν}_0. Using the definition of the collective gyromagnetic ratio $g_R=\mathcal{J}^{\pi}_{\rm cr}/\mathcal{J}_{\rm cr}$, with $\mathcal{J}_{\rm cr}=\mathcal{J}^{\pi}_{\rm cr}+\mathcal{J}^{\nu}_{\rm cr}$ the cranking moment of inertia, which in this simplified model is given by

$$\mathcal{J}^{\rho}_{\rm cr}=\tfrac{2}{3}\Sigma^{\rho}\omega\frac{(\omega_x^2+\omega_z^2)}{\hbar\omega_x^2\omega_z^2} , \tag{29}$$

we can write

$$g_R=\mathcal{J}^{\pi}_{\rm cr}/\mathcal{J}_{\rm cr}=\Sigma^{\pi}/\Sigma , \tag{30}$$

and, using Eqs. (27) and (30), we find that

$$\frac{Q^{\nu}_0}{Q^{\pi}_0}=(1-g_R)/g_R . \tag{31}$$

On the other hand, the charge rms radius squared in this model is

$$\langle r^2\rangle=\frac{\langle R^2\rangle^{\pi}}{Z}=\frac{\hbar}{Zm}\left[\frac{\Sigma_z}{\omega_z}+\frac{\Sigma_x+\Sigma_y}{\omega_z}\right]$$

$$=\frac{\hbar\Sigma^{\pi}\omega}{3mZ}\left[\frac{\omega_x^2+2\omega_z^2}{\omega_x^2\omega_z^2}\right] , \tag{32}$$

which together with Eqs. (8), (10), and (30) gives

$$\delta_0=\delta/(1+\tfrac{2}{3}\delta) \tag{33}$$

and

$$m\omega_0^2\frac{\delta_0}{\delta}=\frac{\hbar\Sigma^{\pi}\omega}{Z\langle r^2\rangle D(\delta)}=\frac{\hbar\Sigma\omega}{AD(\delta)}\frac{A}{Z}\frac{g_R}{\langle r^2\rangle} \tag{34}$$

with

$$D(\delta)=(1-\tfrac{2}{3}\delta)(1+\tfrac{4}{3}\delta)$$

and δ fixed by the relations in Eqs. (10) and (17), i.e.,

$$\delta=\frac{3}{4}\left[\frac{16\pi}{5}\right]^{1/2}\left[\frac{B(E2;0_1^+\rightarrow 2_1^+K=0)}{e^2Z^2\langle r^2\rangle^2}\right]^{1/2} . \tag{35}$$

We note that the experimental quadrupole deformation parameter β is usually defined in terms of $B(E2)$ and is related to δ in Eq. (35) by $\beta=\sqrt{\pi/5}\frac{4}{3}\delta$.

The self-consistency condition Eq. (13), which ensures that the shape of the potential follows the shape of the density, also ensures that (i) the momentum distribution is isotropic, and (ii) the energy is minimum. These intrinsic ground-state properties are satisfied by the ground-state solution of deformed HF (or HF+BCS) calculations with density-dependence effective interactions. Thus Σ^{ρ}_i,w_i, can be considered as effective quantum numbers and H.O. frequencies in the different directions corresponding to the expectation values of Q_0 and r^2 in the true intrinsic ground state [16]. Similarly $(\frac{3}{8}\hbar\Sigma\omega/A)$ can be considered as the effective H.O. energy per particle that corresponds to the true binding energy per particle $(\epsilon=|E_{\rm g.s.}|/A)$,

$$\hbar\Sigma\omega/A=8\epsilon/3 . \tag{36}$$

Substituting Eqs. (31), (34), and (36) into Eq. (18), and using Eq. (28), we can finally write a LEWSR for the

low-energy orbital 1^+ excitations as

$$S_{\rm LE}^{\rm lew}(M1^{\rm orb})=G_{\pi\nu}\frac{6}{5}\frac{Z\epsilon}{D(\delta)}\frac{B(E2;0_1^+\rightarrow 2_1^+K=0)}{(eZ\langle r^2\rangle)^2}\mu_N^2 \tag{37}$$

with

$$G_{\pi\nu}=g_R(1-g_R)2A/Z\ . \tag{38}$$

For a nucleus with an equal number of protons and neutrons we have $G_{\pi\nu}=1$ because $g_R=1/2$ and $A/Z=2$. In practice most deformed nuclei have $g_R\langle 1/2,\ A/Z\rangle 2$ but $G_{\pi\nu}$ is still close to 1. For deformed rare-earth nuclei, since g_R values are not known with high accuracy one can use the approximation $G_{\pi\nu}\simeq 1$ to evaluate the right-hand side of Eq. (37). This approximation is used in the results shown in the last column of Table I. For each nucleus in the table typically the value of $G_{\pi\nu}$ oscillates between 0.9 and 1.2, using g_R values compatible with the experimental data in Refs. [17] and [18].

In Table I we show results for several deformed nuclei obtained from Eq. (37) using $B(E2)$ values from Ref. [17], $\langle r^2\rangle$ values from Ref. [19], and binding energies from Ref. [18] approximating $G_{\pi\nu}$ by 1. The results are compared to available experimental data on several deformed nuclei. It is interesting to see that this simple approximation leads to results in good agreement with experimental data. Therefore Eq. (37) can be considered as a semiempirical LEWSR.

To the extent that more sophisticated microscopic calculations, as those discussed in Refs. [3–5], are able to reproduce the low-energy part of the strength function for the orbital $M1$ operator, they also serve as a theoretical sound basis to support Eq. (37). As mentioned in the introduction, the main difference between the QRPA results of Refs. [4,5] and the results in Ref. [3], that are obtained without inclusion of residual interaction, is that the former find less orbital 1^+ strength in the high-energy region, while the latter find $\sim 50\%$ of the scissors mode strength in the energy interval 4 MeV $\leq E_x \leq 25$ MeV strongly fragmented in many two-quasiparticle excitations. On the contrary Zawischa and Speth [20] performing QRPA calculations with Migdal interaction find that the strongest scissorslike excitations take place at $E\simeq 22$ MeV. Whether the true residual interaction tends to damp down the orbital 1^+ excitations in the high-energy region, or to collect it back into one or a few strong peaks, is still an open question that cannot be answered without experimental verification. In this context it should be mentioned that the isovector term in Eq. (12), which came from a $Q\cdot Q$ interaction, will provide some damping for $\Delta N=2$ excitations [7].

V. CONCLUSIONS

The main point of this work is to show that with a minimum of number of assumptions one can obtain a simple expression [Eq. (37)], which relates the energy-weighted sum rule for orbital magnetic dipole strength to the electric quadrupole strength and, hence, to the deformation parameter δ. The simple expression involves well-known quantities—the binding energy per particle, the mean square radius and the $B(E2)$ and g_R. Although

TABLE I. Comparison of experimental values for $\sum_x E_x B(0^+\rightarrow 1_x^+)$ with the results obtained from Eq. (37) for the low-energy linear energy-weighted sum rule of orbital 1^+ strength (see text). Also shown in the second column are the experimental $B(E2)$ values from Ref. [17] used in this work.

	$B(E2)\uparrow$	$\sum_x E_x B(0^+\rightarrow 1_x^+)(\mu_N^2\ \text{MeV})$	$S_{\rm LE}^{\rm lew}(M1^{\rm orb})(\mu_N^2\ \text{MeV})$
Nucleus	$(\text{b}\,e)^2$	Expt.: $E_x\leq 4$ MeV	Theory
^{146}Nd	0.76	2.005[a]	1.968
^{148}Nd	1.38	3.331[a]	3.339
^{150}Nd	2.75	5.95[a,b]	6.439
^{148}Sm	0.72	1.566[c]	1.719
^{150}Sm	1.35	3.085[c]	3.086
^{152}SM	3.44	7.003[c]	7.310
^{154}Sm	4.36	8.189[c]	8.968
^{156}Gd	4.64	8.039[d]	8.946
^{158}Gd	5.02	8.287[d]	9.616
^{160}Gd	5.25	7.246[d]	9.819
^{162}Dy	5.28	8.942[b]	9.926
^{164}Dy	5.60	9.678[e]	10.462

[a]Reference [21].
[b]References [22,23].
[c]Reference [2].
[d]Reference [3].
[e]Reference [24]. The value quoted has been obtained subtracting the spin strength below 4 MeV seen in (p,p') and reported in Ref. [25].

much work along these lines has been done using the interacting boson approximation (IBA) we find that we can get equally simple expressions working directly with *fermions*. This indicates that the connection between the orbital $M1$ and $B(E2)$ is quite general and quite natural.

We further emphasize that the energy-weighted orbital magnetic dipole strength should have a low-energy part, to which our formula, as seen in Table I, gives very good fits and a high-energy part ($\Delta N=2$). In this rotational model approach the relative energy-weighted strengths are easy to calculate.

Our derivation using the one-body field approach may not be as rigorous as the derivation using the two-body interaction, and it is only applicable to deformed nuclei with a rotational ground-state band. The two-body interaction approach involving an interaction $\chi Q\cdot Q$ with χ chosen self-consistently gives a slightly different expression even for $N=Z$ nuclei. In the two-body approach we get the difference between summed isoscalar and isovector $B(E2)$'s. It should, however, be noted that with a $Q\cdot Q$ interaction the isoscalar strength goes to only one state in the $\Delta N=0$ SU(3) limit—the $J=2^+$ member of the $K=0$ ground-state band. Thus if we drop the isovector term the two approaches give the same answer. The isovector term in the $\Delta N=0$ SU(3) limit consists of an $E2$ transition to the $K=1$, $J=2^+$ member of the scissors mode band. It would be of interest to try to measure this $B(E2)$ via electroexcitation. However, calculations [7] indicate that the isovector $B(E2)$ is much weaker than the isoscalar one and so indeed the main formula of this work [Eq. (37)] is quite good.

In the future it will be of interest to extend these energy-weighted sum rule techniques to states of higher multipolarity. It would also be of interest to extend the double commutator method from schematic to realistic two-body interactions.

ACKNOWLEDGMENTS

We thank Dao Chen Zheng for useful comments. This work has been supported in part by DGICYT (Spain) under Contract No. PB87/0311. One of us (L.Z.) acknowledges the support of the U.S. Department of Energy, Grant No. DE-FG05-86ER-40299.

[1] A. Richter, Nucl. Phys. **A522**, 139c (1991); **A507**, 99c (1990), and references therein.
[2] W. Ziegler, C. Rangacharyulu, A. Richter, and C. Spiler, Phys. Rev. Lett. **65**, 2515 (1990).
[3] E. Garrido, E. Moya de Guerra, P. Sarriguren, and J. M. Udias, Phys. Rev. C **44**, R1250 (1991).
[4] I. Hamamoto and C. Magnusson, Phys. Lett. B **260**, 6 (1991).
[5] P. Sarriguren, E. Moya de Guerra, R. Nojarov, and A. Faessler (unpublished).
[6] K. Heyde and C. De Coester, Phys. Rev. C **44**, R2262 (1991); **47**, 910(E) (1993).
[7] L. Zamick and D. C. Zheng, Phys. Rev. C **44**, 2522 (1991); **46**, 2106 (1992).
[8] J. N. Ginocchio, Phys. Lett. B **265**, 6 (1991).
[9] E. Moya de Guerra, Phys. Rep. **138**, 293 (1986).
[10] A. E. L. Dieperink and E. Moya de Guerra, Phys. Lett. B **189**, 267 (1987).
[11] L. Zamick, D. C. Zheng, and E. Moya de Guerra, Phys. Rev. C **39**, 2370 (1989).
[12] D. R. Bes and R. A. Sorensen, *Advances in Nuclear Physics,* edited by M. Baranger and E. Vogt (Plenum, New York, 1969), pp. 129–222.
[13] J. Retamosa, J. M. Udias, A. Poves, and E. Moya de Guerra, Nucl. Phys. **A511**, 221 (1990).
[14] G. Rosensteel, Phys. Rev. C **42**, 2463 (1990).
[15] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II.
[16] D. J. Rowe, B. Castel, and L. Zamick, Phys. Lett. B **236**, 121 (1990); E. Moya de Guerra *et al.*, Nucl. Phys. **A529**, 68 (1991).
[17] S. Raman *et al.*, At. Data Nucl. Data Tables **36**, 1 (1987); P. Raghavan, *ibid.* **42**, 189 (1989).
[18] C. M. Lederer and V. S. Shirley, *Tables of Isotopes,* 7th ed. (Wiley, New York, 1978).
[19] H. De Vries *et al.*, At. Data Nucl. Data Tables **36**, 495 (1987).
[20] D. Zawischa and J. Speth, Z. Phys. A **339**, 97 (1991).
[21] H. H. Pitz *et al.*, Nucl. Phys. **A509**, 587 (1990); U. Kneissl, private communication.
[22] H. Friedrichs *et al.*, Phys. Rev. C **45**, R892 (1992).
[23] H. H. Pitz *et al.*, Nucl. Phys. **A492**, 411 (1989).
[24] C. Wesselborg *et al.*, Phys. Lett. B **207**, 22 (1988).
[25] D. Frekers *et al.*, Phys. Lett. B **218**, 439 (1989).