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Proton capture resonance spins by multidimensional scaling: *fp* nuclei

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The method of nonmetric multidimensional scaling has been used to attribute spins to a large number of proton capture resonances in five *fp*-shell nuclei, ^{47}V, ^{51}Mn, ^{53}Mn, ^{55}Co, and ^{61}Cu, using as input information only the gamma decay branching ratios. The calibration of the method relies on the measured spins of a number of the resonances. In more than half the cases, a unique spin is found. The results of the analysis allow a reappraisal of analog states in the five nuclei.

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I. INTRODUCTION

The study of proton resonances has contributed much to the spectroscopy of light nuclei: a large body of data now exists for *sd* and *fp* nuclei. A principal objective of these studies was the identification of isobaric analog states and their characterization in terms of Coulomb energy displacement and spreading width of $T_>$ strength over a number of fragments. The assignment of spins and parities to all resonances is essential for the full realization of this objective. Such a complete spectroscopy to high energy exists in only a few cases among lighter nuclei such as ^{26}Al. From such data other phenomena have been found. For example, the ^{26}Al case has become a proving ground for theories of quantum chaos. The high resolution possible in the measurement of proton resonances is realized best in scattering experiments where very thin targets can be used. Many comprehensive studies of this kind have provided essentially complete catalogues of lower-spin resonances in the nuclei studied. However, for $l \geq 2$, the interpretation of scattering results is difficult, and complementary capture γ-ray spectroscopy becomes useful. Because (p,γ) cross sections are generally small and Ge detectors have low efficiency for high γ-ray energies, capture experiments have seldom been done at a system resolution below 1 keV. In particular, the measurement of angular distributions, on which spin assignments rely, becomes very time consuming.

A recent analysis of proton capture resonances in ^{26}Al and ^{30}P [1] was able to identify most probable spins of several resonances in each nuclide from their γ-ray branchings, using the method of nonmetric multidimensional scaling (MDS) [2]. This technique of multiple statistical correlation to generate clustering had earlier been shown to be applicable as a γ-spectroscopic tool in a few cases [3,4]. The principal advantages and properties of MDS in capture spectroscopy are the following: (i) It is applicable to weak resonances for which angular distribution measurements would be prohibitively lengthy; (ii) no *a priori* knowledge of final state spins is required; instead there must be a calibration base of known resonance spins, presumably established by conventional means; and (iii) there appears to be somewhat greater selectivity than that provided by considering the branches of single resonances to final states of known spin.

In the *sd* study [1], it was found that the spin discrimination of the MDS analysis was better in the case of ^{26}Al which offers a wider range of final state spins than ^{30}P. This is to be expected, since the physical basis for the selectivity is the low multipolarity of the principal decay branches from excited nuclear levels. In ^{26}Al, there also appears to be a small sensitivity to parity and isospin. The present study of the *fp* nuclei ^{47}V, ^{51,53}Mn, ^{55}Co, and ^{61}Cu adds to the work already done on ^{59}Cu and ^{49}V [3,4]. The data are drawn from capture studies previously reported [5–17].

II. REVIEW OF MDS FOR γ-RAY SPECTROSCOPY

MDS algorithms are designed to produce a spatial model of similarities among a group of subjects (here, proton capture resonances). Each subject is represented by a point in space and these are arranged so that if the similarities

$$C_{ij} > C_{mn} \tag{1a}$$

then the points representing subjects *i*, *j*, *m*, and *n* should be arranged so that the distances

$$d_{ij} < d_{mn} \; . \tag{1b}$$

A measure of similarity between two proton capture resonances may be constructed from their spectra. In this work, as in earlier studies,

$$C_{ij} = \sum_k a_{ik} a_{jk} \; ,$$

where a_{ik} is the square root of the branching fraction from resonance *i* to final state *k*. Other measures, such as E^{-3} weighting of intensities, have been tested with only minor changes in outcome. Metric scaling, in which the relation of inequalities (1a) and (1b) is replaced by a decreasing linear relationship between distance and similarity, also shows little difference from nonmetric scaling [2].

The program used in the present and earlier studies is MINISSA, a member of the MDS(*X*) program package developed by Roskam [18]. MINISSA uses an iterative search to obtain the spatial distribution, or map, best satisfying the conditions (1). Such a map may be constructed in any small number of dimensions.

It follows from the method of construction that maps are invariant under rotation and reflection, and the scale is arbitrary, so their axes have no physical meaning. However, resonances of greatly differing spin are unlikely to decay to the same final states, so the similarities between them are expected to be smaller than those between resonances of the same spin. One may therefore hope that the points in the map will be dispersed according to the spins of the corresponding resonances. Since the method has no knowledge of spins, some calibration is necessary, and is obtained by including in the data set some resonances of known spin. In each of the cases below, up to fifty resonances are considered simultaneously, with about half having spins known from angular distribution measurements. The remainder were uncertain, either because no angular distribution had been measured or because the measurement had left a residual ambiguity.

TABLE I. Resonances in ^{47}V.

Res. no.	E_x (MeV)	J^π (a)	J^π (b)	Res. no.	E_x (MeV)	J^π (a)	J^π (b)
1	5.636	$>\frac{1}{2}$	$\frac{1}{2},\frac{3}{2}$	17	6.240	$\frac{3}{2}$	
2	5.738	$\frac{1}{2},\frac{3}{2}$	$\frac{1}{2}$-$\frac{5}{2}$	18	6.271	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{3}{2}$
3	5.853	$\frac{1}{2}$		19	6.296	$\frac{3}{2}$	
4	5.885	$\frac{3}{2}$		20	6.351	$(\frac{3}{2})$	$\frac{3}{2}$
5	5.887	$\frac{1}{2}$		21	6.374	$(\frac{1}{2},\frac{3}{2})$	$\frac{1}{2}$
6	5.894	$\frac{1}{2}$		22	6.387	$(\frac{5}{2},\frac{7}{2})$	$\frac{5}{2}$
7	5.962	$\frac{1}{2}$		23	6.393	$\frac{5}{2}$	
8	5.994	$\frac{3}{2}$		24	6.426	$\frac{3}{2}$	
9	6.023	$\frac{1}{2}$		25	6.427	$(\frac{5}{2})$	$\frac{5}{2}$
10	6.087	$\frac{5}{2},\frac{7}{2}$	$\frac{5}{2}$	26	6.475	$\frac{5}{2}$	
11	6.122	$\frac{1}{2}$		27	6.679	$\frac{7}{2}$[c]	
12	6.132	$\frac{1}{2}$	$\frac{3}{2}$	28	6.683	$\frac{3}{2}$[d]	
13	6.157	$(\frac{3}{2}$-$\frac{7}{2})$	$\frac{5}{2}$	29	6.693	$\frac{1}{2}$	
14	6.166	$(\frac{3}{2})$	$\frac{3}{2}$	30	6.699	$\frac{3}{2}$[d]	
15	6.190	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2}$	31	6.706	$\frac{5}{2}$[d,e]	
16	6.229	$\frac{5}{2}$		32	6.953	$\frac{9}{2}$[f]	

[a] Reference [5], (p,γ) angular distributions, except as noted; spins in parentheses are from spectra.

[b] MDS, Fig. 1.

[c] A.D. allows $\frac{3}{2},\frac{7}{2}$; decay excludes $\frac{3}{2}$.

[d] Reference [6], (p,γ) angular distributions.

[e] A.D. allows $\frac{3}{2},\frac{5}{2}$; mixing ratio makes $\frac{3}{2}$ unlikely.

[f] A.D. allows $\frac{7}{2}$-$\frac{11}{2}$; decay excludes $\frac{7}{2},\frac{11}{2}$.

III. CASE STUDIES IN THE *fp* SHELL

Many proton capture experiments have been carried out over the three decades since the advent of high resolution germanium detectors. A long-standing collaboration between workers at the Central Research Institute for Physics in Budapest, King Saud University in Riyadh, Université Laval, and McMaster University has had as its objective the study of isobaric analog states. This search has yielded a large data set, some of which is drawn on here [4,7–16]. Earlier work by others is included in the case of ^{61}Cu [17]. A further case, ^{47}V, is provided by the work of de Esch and van der Leun [5], supplemented by earlier work of Schrader *et al.* [6]. The data are presented below in order of increasing *A*.

A. ^{47}V

Table I lists the 29 resonances in ^{47}V studied in detail by de Esch and van der Leun [5], with the addition of three others (28, 30, and 31) observed by Schrader *et al.* [6]. They range in proton energy from 0.48 to 1.83 MeV and cover the region expected to contain analogs of several ^{47}Ti levels. Of the 32 resonances, unambiguous spin assignments ranging from $\frac{1}{2}$ to $\frac{9}{2}$ have been made for 21 from (p,γ) angular distributions. The two-dimensional scaling map is shown in Fig. 1. Column (b) of Table I lists new spins inferred from the map.

B. ^{51}Mn

This study, whose data set is contained in Refs. [7–10], gives γ branching for 95 of the 200 resonances observed in the proton energy range 1.06 to 3.36 MeV ($6.3 < E_x < 8.6$ MeV). Of these, 19 are of unknown or ambiguous spin, 58 are of known spin, ranging from $\frac{1}{2}$ to $\frac{9}{2}$, and the remainder are known doublets, whose incomplete resolution makes spectral analysis of limited value. Table II lists 50 of the resonances, including all 19 of unknown spin, given by excitation energy for cross referencing to Refs. [7–10]. Figure 2 is their MDS map in two dimensions. Table II contains the spins which may be inferred from this.

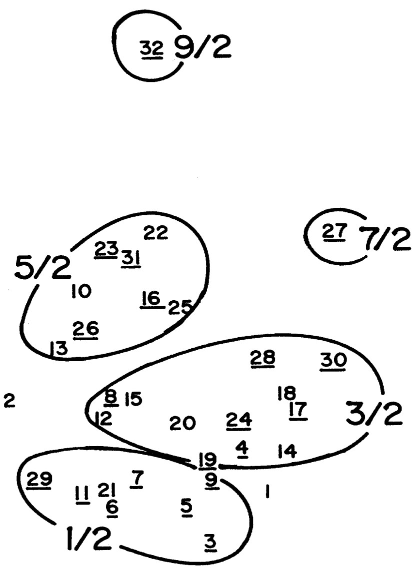

FIG. 1. MDS map for ^{47}V. The numbers are those of the resonances listed in Table I, those underlined having conventionally measured spins. Regions containing resonances of a single spin are outlined.

C. ^{53}Mn

This set of experiments, reported in Refs. [11–13], covered the proton energy range 1.38 to 2.91 MeV ($7.9 < E_x < 9.4$ MeV) as well as the $g_{9/2}$ isobaric analog state region 10.56 to 10.76 MeV. In all, 265 resonances were observed and over a hundred spectra collected. After the known doublets are removed, 50 resonances have known spins, while 60 remain unknown, or are at least uncertain. The data set was reduced to the 74 resonances shown in Table III containing 26 resonances of known spin and 48 of unknown or uncertain spin. In each of two runs, all 26 known and half of the unknown spins were analyzed. Figures 3(a) and 3(b) are the corresponding maps. The inferred spins are given in Table III.

D. ^{55}Co

The survey of ^{55}Co from 2.3 to 3.9 MeV ($7.3 < E_x < 8.9$ MeV) [14] yielded 151 resonance peaks, many of which were multiplets. In the $g_{9/2}$ analog state region, a number of resonances are overlapping. A careful study was made of these [15]. Of the 71 resonances whose spectra were measured, 18 have measured spins. Of these, many were determined from $(p,p'\gamma)$ angular distributions. Among the remaining 53, 35 are likely singlets of unknown or uncertain spin. A few of these are quite weak and near strong resonances. Table IV shows the 50 resonances used and Fig. 4 is the resulting MDS map.

TABLE II. Resonances in ^{51}Mn.

Res. no.	E_x (MeV)	J^π (a)	J^π (b)	Res. no.	E_x (MeV)	J^π (a)	J^π (b)
1	6.321	$(\frac{1}{2}\text{-}\frac{5}{2})$	$\frac{3}{2}$	26	7.669	$(\frac{5}{2},\frac{7}{2})$	$\frac{5}{2}$
2	6.694	$\frac{5}{2}^-$		27	7.699	$\frac{5}{2}$	
3	6.754	$\frac{5}{2}^-$		28	7.792	$\frac{5}{2}^-$	
4	7.045	$\frac{1}{2}^+$		29	7.849	$\frac{3}{2}^-$	
5	7.106	$\frac{5}{2}^-,\frac{7}{2}^-$	$\frac{5}{2}$	30	7.895	$(\frac{1}{2}\text{-}\frac{5}{2})$	$\frac{3}{2}$
6	7.129	$\frac{1}{2}^-$		31	7.914	$\frac{3}{2}^-$	
7	7.190	$\frac{1}{2}^-,\frac{3}{2}^-$	$\frac{3}{2}$	32	7.933	$\frac{3}{2}\text{-}\frac{7}{2}$	$\frac{3}{2},\frac{5}{2}$
8	7.210	$\frac{5}{2}$	$\frac{3}{2}$	33	7.944	$\frac{3}{2}^-$	
9	7.213	$\frac{1}{2},\frac{3}{2}$	$\frac{1}{2}$	34	8.013	$\frac{9}{2}^+$	
10	7.240	$\frac{5}{2}^+$		35	8.023	$\frac{5}{2}^+$	
11	7.314	$\frac{1}{2}$	$\frac{1}{2},\frac{3}{2}$	36	8.045	$\frac{3}{2}^-$	
12	7.339	$\frac{1}{2}$		37	8.064	$\frac{3}{2}^-$	
13	7.395	$\frac{3}{2}^-,\frac{5}{2}^-$	$\frac{5}{2},\frac{7}{2}$	38	8.143	$\frac{7}{2}^-$	
14	7.450	$\frac{3}{2}^-$		39	8.147	$\frac{3}{2}\text{-}\frac{7}{2}$	$\frac{3}{2},\frac{5}{2}$
15	7.463	$(\frac{5}{2},\frac{7}{2})$	$\frac{5}{2}$	40	8.169	$\frac{3}{2}^-,\frac{5}{2}$	$\frac{5}{2}$
16	7.467	$\frac{1}{2}^-$		41	8.174	$\frac{3}{2}^{-*}$	
17	7.514	$\frac{5}{2}^-$		42	8.187	$\frac{5}{2}^+$	
18	7.546	$\frac{5}{2}$		43	8.256	$\frac{3}{2}^+,\frac{5}{2}^{-*}$	$\frac{5}{2},\frac{7}{2}$
19	7.550	$\frac{1}{2}$		44	8.260	$\frac{5}{2}\text{-}\frac{9}{2}$	$\geq\frac{7}{2}$
20	7.560	$\frac{3}{2}^-,\frac{5}{2}^-$	$\frac{3}{2}$	45	8.266	$\frac{3}{2}\text{-}\frac{7}{2}$	$\geq\frac{7}{2}$
21	7.586	$(\frac{1}{2},\frac{3}{2})$	$\frac{1}{2}$	46	8.269	$\frac{5}{2}^-$	
22	7.599	$(\frac{3}{2})$	$\frac{3}{2}$	47	8.453	$\frac{9}{2}^+$	
23	7.618	$\frac{1}{2}$		48	8.472	$\frac{9}{2}^+$	
24	7.621	$\frac{9}{2}^+$		49	8.554	$\frac{9}{2}^+$	
25	7.643	$(\frac{1}{2},\frac{3}{2})$	$\frac{1}{2},\frac{3}{2}$	50	8.555	$\frac{9}{2}^+$	

[a] References [7–10], (p,γ) angular distributions; * with $(p,p'\gamma)$; spins in parentheses are from spectra.

[b] MDS, Fig. 2.

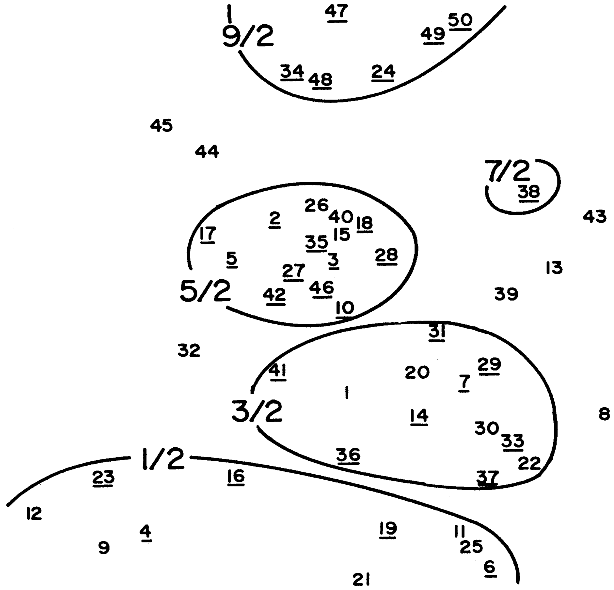

FIG. 2. MDS map for ^{51}Mn for resonances listed in Table II. Details as in Fig. 1.

E. ^{61}Cu

Measurements at McMaster consisted of a detailed study of 43 resonances in the proton energy range 3.67 to 3.83 MeV [16]. Eleven $\frac{9}{2}^+$ resonances were identified, with excitation energies from 8.45 to 8.48 MeV. For the MDS analysis, 36 other resonances from earlier work by other groups [17] were used. Of these, 14 have established spins, from $\frac{1}{2}$ to $\frac{5}{2}$. A summary is given in Table V. The corresponding MDS map is shown in Fig. 5.

IV. DISCUSSION

In all the cases studied, the separation of spin groups in the maps is fairly clear, so strong inferences can be made for many of the resonances of unknown or uncertain spin. This finding parallels that for the case of ^{59}Cu studied earlier [3]. Unlike the situation in ^{26}Al [1], no parity dependence is apparent in any of the maps, nor is there a bias seen for $T_>$ (analog) resonances.

One of the interests in the high-energy spectroscopy of fp nuclei is the identification of isobaric analog resonances. These are often fragmented into multiplets with a spreading width of several tens of keV. Using the results above, it is possible to clarify some of the analog state assignments made in the earlier reports.

A. ^{47}V

The MDS map, Fig. 1, suggests definite spins for most of the uncertain assignments of Refs. [5,6]. There is only one anomalous placing in the map, at resonance 12, assigned as spin $\frac{1}{2}$ [5], but appearing to be $\frac{3}{2}$. Analog assignments were made in Ref. [5] and in earlier work, such as that of Ref. [6], for ^{47}Ti parent states from 1.55 to 2.26 MeV. The 1.550-MeV $\frac{3}{2}^-$ state has its analog at resonance 1. Resonances 2 and 4 are too far removed to be considered likely fragments, but they could be alternates. Reference [5] proposes resonances 5 and 9 as fragments of the analog of the 1.794-MeV $\frac{1}{2}^-$ Ti state. Resonances 3, 6, and 7 are now also candidates. There is a strong $l=2$ level in ^{47}Ti at 1.825 MeV. Any of the resonances 12 to 18 are possible analogs. The only potential analog of the $\frac{1}{2}^-$ 2.163-MeV Ti level is resonance 21. The 2.260-MeV Ti level, with $J^\pi=\frac{5}{2}^+$, has possible analogs at resonances 22 and 23 (suggested in [5]) and 25 and 26. The 2.167-MeV spin-$\frac{5}{2}$ level in Ti is another possible parent. Reference [5] suggests 28 and 30 as analog fragments for the $\frac{3}{2}^-$ 2.549-MeV Ti level. Above this, in the

TABLE III. Resonances in ^{53}Mn.

Res. no.	E_x (MeV)	J^π (a)	J^π (b)	Res. no.	E_x (MeV)	J^π (a)	J^π (b)
1	7.925	$\frac{5}{2}^-$		38	8.846	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2},\frac{5}{2}$
2	7.931	$\frac{5}{2}^-$		39	8.860	$(\frac{3}{2},\frac{5}{2})$	$\frac{5}{2}$
3	8.008	$\frac{5}{2}^-$		40	8.865	$\frac{5}{2}$	
4	8.027	$\frac{5}{2}$		41	8.881	$\frac{5}{2}$	
5	8.030	$\frac{3}{2}^-,\frac{5}{2}$	$\frac{3}{2},\frac{5}{2}$	42	8.902	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$
6	8.040	$\frac{3}{2}^-,\frac{5}{2}^-$	$\frac{3}{2},\frac{5}{2}$	43	8.937	$\frac{5}{2}^-$	
7	8.136	$\frac{3}{2}^-$		44	8.946	$\frac{5}{2},\frac{7}{2}^-$	$\frac{5}{2}$
8	8.141	$\frac{3}{2}^-$		45	8.954	$(\frac{5}{2},\frac{7}{2})$	$\frac{5}{2}$
9	8.159	$\frac{5}{2}^+$		46	8.973	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$
10	8.179	$\frac{3}{2}^-,\frac{5}{2}^-$	$\frac{3}{2}$	47	8.978	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$
11	8.248	$\frac{3}{2}^-,\frac{5}{2}^-$	$\frac{5}{2}$	48	8.981	$(\frac{3}{2}-\frac{7}{2})$	$\frac{3}{2},\frac{5}{2}$
12	8.268	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2},\frac{5}{2}$	49	8.994	$\frac{5}{2},\frac{7}{2}^-$	$\frac{5}{2}$
13	8.292	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2}$	50	8.997	$(\frac{5}{2},\frac{7}{2})$	$\frac{5}{2},\frac{7}{2}$
14	8.294	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2}$	51	9.004	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$
15	8.304	$\frac{5}{2}^-$		52	9.028	$(\frac{3}{2}-\frac{7}{2})$	$\frac{3}{2},\frac{5}{2}$
16	8.327	$(\frac{5}{2},\frac{7}{2})$	$\frac{5}{2}$	53	9.098	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2},\frac{5}{2}$
17	8.336	$\frac{3}{2}^-$		54	9.115	$(\frac{3}{2}-\frac{7}{2})$	$\frac{3}{2},\frac{5}{2}$
18	8.400	$(\frac{3}{2},\frac{5}{2})$	$\frac{5}{2}$	55	9.122	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2}$
19	8.404	$\frac{3}{2}^-$		56	9.140	$(\frac{3}{2},\frac{5}{2})$	$\frac{5}{2}$
20	8.484	$\frac{3}{2}$		57	9.154	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2},\frac{5}{2}$
21	8.495	$\frac{5}{2},\frac{7}{2}$	$\frac{3}{2},\frac{5}{2}$	58	9.170	$(\frac{3}{2}-\frac{7}{2})$	$\frac{3}{2}$
22	8.502	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2}$	59	9.180	$\frac{5}{2}^-$	
23	8.507	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2},\frac{7}{2}$	60	9.209	$\frac{5}{2}^-$	
24	8.560	$\frac{3}{2}^-,\frac{5}{2}^-$	$\frac{5}{2}$	61	9.219	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$
25	8.565	$\frac{3}{2}-\frac{7}{2}$	$\frac{5}{2}$	62	9.226	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$
26	8.609	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2},\frac{5}{2}$	63	9.230	$\frac{5}{2}^-$	
27	8.613	$(\frac{3}{2}-\frac{7}{2})$	$\frac{3}{2},\frac{5}{2}$	64	9.243	$(\frac{3}{2}-\frac{7}{2})$	$\frac{3}{2},\frac{5}{2}$
28	8.654	$(\frac{3}{2})$	$\frac{5}{2},\frac{7}{2}$	65	9.246	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2},\frac{5}{2}$
29	8.674	$(\frac{3}{2})$	$\frac{5}{2}$	66	9.252	$\frac{5}{2}$	
30	8.692	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	67	9.284	$\frac{3}{2}^-$	
31	8.714	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	68	9.344	$\frac{3}{2}^-$	
32	8.732	$\frac{5}{2}^-$		69	9.348	$(\frac{3}{2}-\frac{7}{2})$	$\frac{3}{2},\frac{5}{2}$
33	8.745	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	70	9.362	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$
34	8.785	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	71	9.417	$(\frac{3}{2})$	$\frac{3}{2}$
35	8.809	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	72	10.655	$\frac{9}{2}^+$	
36	8.817	$\frac{5}{2}$		73	10.662	$\frac{9}{2}^+$	
37	8.838	$\frac{5}{2}$		74	10.733	$\frac{9}{2}^+$	

[a] References [9–11], (p,γ) angular distributions; spins in parentheses are from spectra.

[b] MDS, Fig. 3.

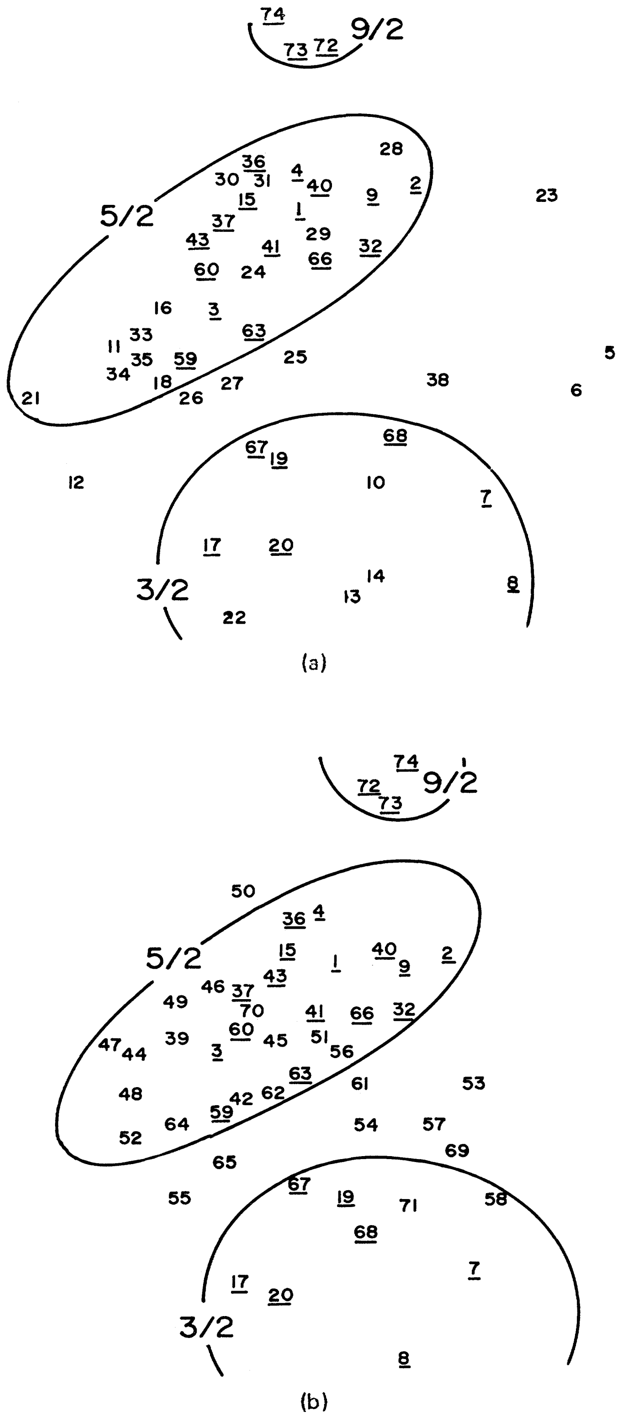

FIG. 3. MDS maps for ^{53}Mn. Maps (a) and (b) each contain the 26 resonances of known spin and, respectively, the first and second 24 unknowns from Table III. Details as in Fig. 1.

energy range relevant to this survey, few of the Ti states have well-determined spins.

B. ^{51}Mn

Of the 19 uncertain spins in Table II, the MDS analysis suggests a resolution for 14, leaving a residual uncertainty for only 5. Examination of the positions in Fig. 2 of resonances of known parity from Table II shows no dependence on parity.

Reference [7] discusses the lowest ^{51}Cr-^{51}Mn analog, and higher ones are listed in Refs. [8,9]. Resonance 1 of Table II above, at 6.321 MeV, may be a companion to the established analog at 6.309 MeV. Resonances 11 and 12 were proposed in Ref. [8] as a similarly split analog of the Cr $\frac{3}{2}^-$ state at 2.890 MeV. The spin ambiguity for resonance 11 remains, but resonance 12 does not now seem to be a possible analog fragment. The MDS analysis favors the higher spin, $\frac{5}{2}$, for resonance 13, confirming its candidacy as analog of the 2.949-MeV $\frac{5}{2}^-$ Cr state. Resonances 21 and 26, which were proposed as candidates for analogs of the 3.135-MeV $\frac{3}{2}^-$ and 3.207-MeV $\frac{7}{2}^-$ Cr levels, seem now not to qualify.

C. ^{53}Mn

The greatly fragmented analog of the $\frac{5}{2}^-$ ^{53}Cr level at 1.006 MeV has two possible components, resonances 5 and 6, whose spins remained uncertain in Ref. [11] and remain so. Reference [12] suggested resonance 16 as the analog of the 1.290 $\frac{7}{2}^-$ level. This is now unlikely. Of the three candidates as analog of the $\frac{7}{2}^-$ 1.537-MeV Cr level, one, at 8.495 MeV, was found in Ref. [12] to have spin $\frac{7}{2}^-$. The others were resonances 21 and 23 of Table III. The former, with spin $\frac{3}{2}$ or $\frac{5}{2}$, is ruled out, while the latter, still uncertain at $(\frac{5}{2},\frac{7}{2})$, remains possible. The final triplet consists of resonances 43 to 45, all $\frac{5}{2}$ and all possible analog fragments corresponding to the parent $\frac{5}{2}^-$ state in Cr at 1.974 MeV.

D. ^{55}Co

Most of the ^{55}Fe parent states in the energy range covered by Ref. [14] have spins $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$, and as a rule the latter two are not distinguished, so the assignment of their analogs is difficult. Parent $\frac{3}{2}^-$ states at 3.028, 3.552, and 3.801 MeV have analogs at 7.766 MeV (resonance 13), for which resonance 14 is a possible companion, 8.298, together with resonance 32, and 8.455, resonance 40. The $g_{9/2}$ parent at 3.814 MeV is clear. Three analog fragments were identified in Ref. [15], at 8.462, 8.466, and 8.475 MeV (resonances 41, 43, and 46 in Table IV). Of other potential candidates, this work indicates spin $\frac{9}{2}$ for resonance 42. Other spin-$\frac{9}{2}$ resonances, resonances 22, 33, and 48, are presumably also $\frac{9}{2}^+$, since $l=5$ capture is highly improbable. Their displacements from the main analog at 8.466 MeV prevent their interpretation as analog fragments. It is more likely that they belong to the class of "orphan," or core-excited states, such

TABLE IV. Resonances in ^{55}Co.

Res. no.	E_x (MeV)	J^π (a)	J^π (b)	Res. no.	E_x (MeV)	J^π (a)	J^π (b)
1	7.563	$\frac{5}{2}^-$		26	8.104	$(\frac{1}{2}-\frac{5}{2})$	$\frac{3}{2},\frac{5}{2}$
2	7.577	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	27	8.129	$\frac{5}{2}^{+*}$	
3	7.594	$(\frac{1}{2}-\frac{5}{2})$	$\frac{3}{2}$	28	8.143	$\frac{3}{2}^{+*}$	
4	7.610	$\frac{5}{2}^-$		29	8.233	$\frac{5}{2}^*$	
5	7.626	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2},\frac{7}{2}$	30	8.272	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2},\frac{7}{2}$
6	7.632	$\frac{7}{2}^-$		31	8.282	$\frac{3}{2}^{-*}$	$\frac{5}{2},\frac{7}{2}$
7	7.641	$(\frac{1}{2}^-,\frac{3}{2})$	$\frac{1}{2},\frac{3}{2}$	32	8.293	$(\frac{1}{2}-\frac{5}{2})$	$\frac{3}{2}$
8	7.649	$\frac{7}{2}^-$		33	8.358	$\frac{5}{2}-\frac{9}{2}$	$\frac{9}{2}$
9	7.662	$(\frac{1}{2}-\frac{5}{2})$	$\frac{3}{2}$	34	8.372	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2},\frac{7}{2}$
10	7.679	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	35	8.382	$\frac{5}{2},\frac{7}{2}^*$	$\frac{5}{2}$
11	7.703	$\frac{3}{2}^-,\frac{5}{2}^-$	$\frac{3}{2}$	36	8.410	$(\frac{1}{2}-\frac{5}{2})$	$\frac{3}{2}$
12	7.746	$\frac{3}{2}^-,\frac{5}{2}$	$\frac{5}{2},\frac{7}{2}$	37	8.416	$\frac{5}{2}^{+*}$	
13	7.766	$\frac{3}{2}^{-*}$		38	8.430	$\frac{3}{2}^{-*}$	
14	7.805	$(\frac{1}{2}-\frac{7}{2})$	$\frac{1}{2},\frac{3}{2}$	39	8.434	$(\frac{3}{2}-\frac{7}{2})$	$\frac{7}{2}$
15	7.815	$(\frac{3}{2},\frac{5}{2})$	$\frac{5}{2},\frac{7}{2}$	40	8.455	$(\frac{1}{2}-\frac{5}{2})$	$\frac{1}{2},\frac{3}{2}$
16	7.868	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2}$	41	8.462	$\frac{9}{2}^+$	
17	7.875	$(\frac{5}{2},\frac{7}{2})$	$\frac{5}{2},\frac{7}{2}$	42	8.464	$(\frac{3}{2}-\frac{9}{2})$	$\frac{9}{2}$
18	7.879	$(\frac{3}{2},\frac{5}{2})$	$\frac{5}{2}$	43	8.466	$\frac{9}{2}^+$	
19	7.884	$(\frac{5}{2}-\frac{9}{2})$	$\frac{5}{2},\frac{7}{2}$	44	8.467	$(\frac{5}{2},\frac{7}{2})$	$\frac{7}{2}$
20	7.937	$(\frac{3}{2}^-)$	$\frac{3}{2}$	45	8.472	$\frac{1}{2}^*$	
21	7.975	$(\frac{3}{2},\frac{5}{2})$	$\frac{7}{2},\frac{9}{2}$	46	8.475	$\frac{9}{2}^+$	
22	8.019	$\frac{5}{2}-\frac{9}{2}$	$\frac{9}{2}$	47	8.477	$(\frac{3}{2}-\frac{7}{2})$	$\frac{7}{2}$
23	8.055	$(\frac{1}{2}-\frac{5}{2})$	$\frac{1}{2},\frac{3}{2}$	48	8.702	$\frac{5}{2}-\frac{9}{2}$	$\frac{9}{2}$
24	8.065	$\frac{3}{2}^{-*}$		49	8.797	$\frac{3}{2}^{-*}$	$\frac{5}{2}$
25	8.070	$(\frac{5}{2},\frac{7}{2})$	$\frac{7}{2},\frac{9}{2}$	50	8.854	$(\frac{3}{2}-\frac{7}{2})$	$\frac{5}{2},\frac{7}{2}$

[a] References [14,15]; * $(p,p'\gamma)$, otherwise (p,γ) angular distributions; spins in parentheses are from spectra.

[b] MDS, Fig. 4.

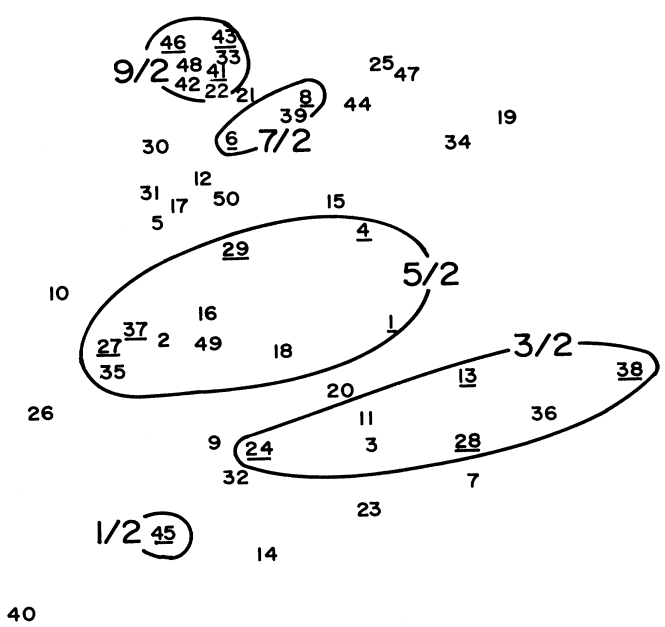

FIG. 4. MDS map for ^{55}Co. Details as in Fig. 1.

TABLE V. Resonances in ^{61}Cu.

Res. no.	E_x (MeV)	J^π (a)	J^π (b)	Res. no.	E_x (MeV)	J^π (a)	J^π (b)
1	6.362	$\frac{3}{2}$		25	8.439	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$
2	6.372	$\frac{3}{2}$		26	8.442	$(\frac{1}{2}$-$\frac{7}{2})$	$\frac{3}{2},\frac{5}{2}$
3	6.378	$\frac{3}{2}$		27	8.446	$\frac{9}{2}^+$	
4	6.393	$\frac{3}{2}$		28	8.450	$\frac{9}{2}^+$	
5	6.428	$(\frac{1}{2})$		29	8.453	$\frac{9}{2}^+$	
6	6.441	$\frac{5}{2}$		30	8.455	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2},\frac{3}{2}$
7	6.446	$\frac{5}{2}$		31	8.457	$\frac{9}{2}^+$	
8	6.457	$(\frac{3}{2}$-$\frac{7}{2})$	$\frac{5}{2}$	32	8.463	$\frac{9}{2}^+$	
9	6.466	$\frac{3}{2}$		33	8.465	$\frac{9}{2}^+$	
10	6.584	$(\frac{5}{2},\frac{7}{2})$	$\frac{3}{2}$	34	8.466	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$
11	6.619	$\frac{3}{2}$		35	8.469	$(\frac{3}{2}$-$\frac{7}{2})$	$\frac{5}{2}$
12	6.627	$\frac{1}{2}$		36	8.472	$\frac{9}{2}^+$	
13	6.645	$\frac{1}{2}$		37	8.475	$\frac{9}{2}^+$	
14	6.678	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$	38	8.477	$\frac{9}{2}^+$	
15	6.802	$(\frac{1}{2}$-$\frac{5}{1})$	$\frac{1}{2}$	39	8.480	$\frac{9}{2}^+$	
16	7.011	$\frac{3}{2}$		40	8.482	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{3}{2}$
17	7.025	$\frac{5}{2}$		41	8.484	$\frac{9}{2}^+$	
18	7.041	$\frac{1}{2}$		42	8.489	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$
19	7.107	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$	43	8.492	$(\frac{3}{2},\frac{5}{2})$	$\frac{3}{2}$
20	7.223	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$	44	8.499	$(\frac{3}{2}$-$\frac{7}{2})$	$\frac{5}{2}$
21	7.262	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2},\frac{3}{2}$	45	8.507	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$
22	7.283	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2},\frac{3}{2}$	46	8.509	$(\frac{3}{2}$-$\frac{7}{2})$	$\frac{3}{2}$
23	8.431	$(\frac{3}{2}$-$\frac{7}{2})$	$\frac{5}{2}$	47	8.522	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2},\frac{3}{2}$
24	8.436	$(\frac{1}{2}$-$\frac{5}{2})$	$\frac{1}{2}$				

[a] References [16,17], (p,γ) angular distributions; spins in parentheses are from spectra.
[b] MDS, Fig. 5.

as those observed previously in ^{59}Cu [3] and ^{51,53}Mn [19–21].

E. ^{61}Cu

The studied region of ^{61}Cu, from 6.37 to 8.48 MeV, contains possible analogs of ^{61}Ni states from 0 to 2 MeV. The spin-$\frac{3}{2}$ resonances 1 to 4, together with another at 6.351 MeV, are fragments of the analog of the Ni ground state. Resonances 6 to 8 form the analog of the $\frac{5}{2}^-$ 0.067-MeV state. The two Ni spin-$\frac{1}{2}$ states at 0.283 and 0.656 MeV have their analogs in resonances 12 to 15 and 18 to 20. The spin-$\frac{1}{2}$ resonances [30(?), 34, and 42] in the midst of the fragmented $\frac{9}{2}^+$ analog may be the analog of the $\frac{1}{2}^-$ Ni state at 2.123 MeV.

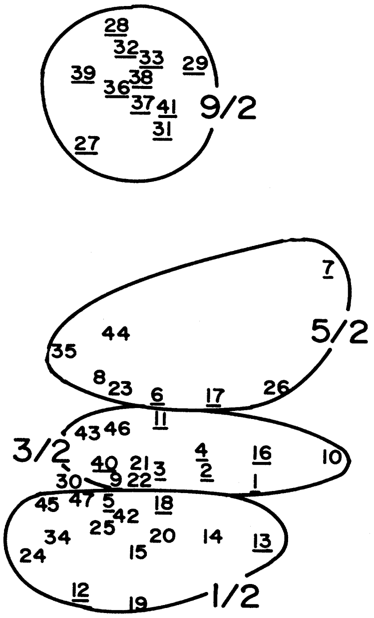

FIG. 5. MDS map for ^{61}Cu. Details as in Fig. 1.

V. CONCLUSION

Multidimensional scaling appears to provide a convenient method of multiple comparisons of capture γ-ray spectra whereby spins of parent states at high excitation can be found. Where a wide variety of initial state spins exists, distinct clustering by spin can be seen but no parity or isospin dependence is evident. The method is of particular usefulness in the case of weakly excited states for which the measurement of decay γ-ray angular distributions is difficult. A number of weaker resonances, whose spins now seem to be clearly indicated, are possible fragments of isobaric analog states.

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