ARTICLES

Quasifree pion production in the three-nucleon system

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We evaluate the total cross section for pion production in the reaction $pd \rightarrow pd\pi^0$ under the assumption that it is entirely due to the quasifree elementary process $pn \rightarrow d\pi^0$. Close to threshold only large-momentum components of the target deuteron wave function can contribute. This provides a qualitative explanation of recent measurements of a surprisingly small near-threshold cross section for the $pd \rightarrow pd\pi^0$ reaction. The final-state interaction between the proton and the deuteron in the exit channel is found to be significant. When it is taken into account the energy dependence of the measured $pd \rightarrow pd\pi^0$ reaction is reproduced. We conclude that the main properties of the $pd \rightarrow pd\pi^0$ reaction near threshold can be explained by a simple quasifree reaction model.

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I. INTRODUCTION

Recently, the total cross section of the reaction $pd \rightarrow pd\pi^0$ has been measured near threshold with the Indiana University Cooler [1]. The aim of the measurement was to investigate, in a still tractable system, to what extent pion production in a "many-body" environment can be explained by the contribution from "elementary" processes with two nucleons in the initial state. In the experiment, it was found that very near threshold this cross section is almost three orders of magnitude smaller than the cross section of the $pn \rightarrow d\pi^0$ reaction, when the two cross sections are evaluated at energies of equivalent pion center-of-mass momentum. This came as a surprise, and seemed to indicate that pion production in the three-nucleon system is more than a superposition of pion production processes involving two nucleons. In the present work we address the question of whether the experimental $pd \rightarrow pd\pi^0$ result is indeed surprising and incompatible with a quasifree reaction mechanism, or whether there is a simple explanation of the observed data once the kinematic aspects of the reaction are fully taken into account.

The possible elementary NN channels that can contribute to $pd \rightarrow pd\pi^0$ are $pn \rightarrow d\pi^0$, $pp \rightarrow pp\pi^0$, and $pn \rightarrow pn\pi^0$. Recent experiments provide us with accurate total cross-section data close to threshold for $pn \rightarrow d\pi^0$ [2], as well as $pp \rightarrow pp\pi^0$ [3], while for $pn \rightarrow pn\pi^0$ the experimental situation at low energies is still poor.

The $pn \rightarrow d\pi^0$ cross section is large in comparison to $pp \rightarrow pp\pi^0$ [4] since, in the latter, pion production via an $N\Delta$ intermediate state is suppressed [5]. Resonant production is allowed in $pn \rightarrow pn\pi^0$, so its cross section is also large. But, in this case, most of the flux may be expected to go into the four-body final state, i.e., $pd \rightarrow ppn\pi^0$. One reason for this is that a deuteron which retains its identity cannot emit a pion (even when the pion rescatters from the spectator). Furthermore, s-wave rescattering is suppressed by an order of magnitude, and is forbidden when it involves charge exchange of one of the nucleons in the deuteron. The only $pn \rightarrow pn\pi^0$ process that could contribute significantly is the one where the beam proton becomes part of the deuteron by charge exchange with the target neutron which in turn emerges as a free proton. The amplitude for this process is proportional to the deuteron form factor evaluated at the momentum transfer to the deuteron, which, close to the threshold is always large. The necessary rearrangement of the spins and isospins to form a deuteron in the final channel leads to an additional suppression of this mechanism.

From the arguments in the previous paragraph it seems justified to include in the quasifree analysis of $pd \rightarrow pd\pi^0$ only the single elementary process $pn \rightarrow d\pi^0$. This simplifies the analysis, since the inclusion of other processes would require the knowledge of the relative phases. The dominance of the $pn \rightarrow d\pi^0$ channel could be verified experimentally by studying the angular distribution of the protons from $pd \rightarrow pd\pi^0$ which, in this case, should be given by the momentum distribution in the deuteron.

Near threshold there are only a few contributing partial waves. It has been shown [2,3] that the energy dependence of the elementary (NN) cross sections is well explained in terms of the phase-space factors and final-state interactions. If it turns out that the three-nucleon process departs from this behavior, this could be taken as an indication that processes are important that arise from the complexity of the system. The near-threshold region is thus especially suited for a study of the reaction mechanism.

There have been two previous studies of pion production in the three-nucleon system, both involving the reaction $pd \rightarrow nd\pi^+$, both restricted to differential cross sections in a part of the phase space where quasifree $pp \rightarrow d\pi^+$ production is favored. The first, at a bombarding energy of 585 MeV [6], concludes that the data admit several possible reaction mechanisms, while the second,

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at 800 MeV [7], finds quantitative agreement with a spectator model.

II. DERIVATION OF THE CROSS SECTION

A. Introduction

We assume that in process I ($pd \rightarrow pd\pi^0$) the beam proton interacts with the neutron in the target deuteron to form the final deuteron and the π^0 (process II, $pn \rightarrow d\pi^0$), as indicated in Fig. 1. Our goal is then to express the total cross section $\sigma_{\rm I}$ ($pd \rightarrow pd\pi^0$) in terms of the total cross section $\sigma_{\rm II}$ ($pn \rightarrow d\pi^0$). This task can be divided into two parts. First, the amplitude for reaction I has to be related to the amplitude of the subprocess II (Sec. II B). Second, the integration over the phase space has to be carried out, starting with the square of these amplitudes (Sec. II C). Spin and isospin parts of the wave functions are ignored throughout the derivation of $\sigma_{\rm I}$; they will be discussed separately at the end of Sec. II C.

B. Quasifree amplitude

For the present discussion we present the three-nucleon momenta by $\mathbf{k}_i$ in the initial state and by $\mathbf{k}_i'$ in the final state, as shown in Fig. 1(a). The pion momentum is denoted by $\mathbf{q}$.

Ignoring, for the time being, distortions and spin-isospin degrees of freedom, the initial-state wave function is

$$\Psi_i(1,2,3)=\Psi_d(\mathbf{r}_{12})e^{i\mathbf{K}_{12}\cdot\mathbf{R}_{12}}e^{\mathbf{k}_3\cdot\mathbf{r}_3}/(2\pi)^3 , \quad (1)$$

where $\mathbf{K}_{12}=\mathbf{k}_1+\mathbf{k}_2$ and $\mathbf{R}_{12}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ are the momentum and the center-of-mass coordinate of the initial deuteron, and where $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$ is the relative coordinate of the particles forming the deuteron. The normalization of the wave function is chosen to conform with the expression for the incident flux given in Refs. [8] and [9]. The only differences between the initial and the final state is that nucleons have changed their roles as bound particles or free nucleons, so that the wave function for the final nucleonic state becomes (with the normalization consistent with Ref. [9])

$$\Psi_f(1,2,3)=e^{i\mathbf{k}_1'\cdot\mathbf{r}_1}\Psi_d(\mathbf{r}_{23})e^{\mathbf{K}_{23}'\cdot\mathbf{R}_{23}} . \quad (2)$$

As in Ref. [10], the pion wave function is contained in

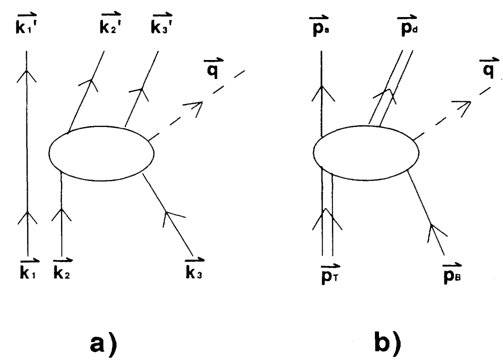

FIG. 1. (a) Notation for the nucleon momenta in the spectator model for the reaction $pd \rightarrow pd\pi^0$. (b) Notation for the momenta of the external particles in the evaluation of the phase-space integrals.

the operator for pion production from the nucleon pair (2,3). This operator, in its simplest form, can be written as [10].

$$H_\pi(2,3)=(f/\mu)(\mathbf{q}\cdot\boldsymbol{\sigma}_2\tau_2\cdot\boldsymbol{\phi}e^{-i\mathbf{q}\cdot\mathbf{r}_2}+\mathbf{q}\cdot\boldsymbol{\sigma}_3\tau_3\cdot\boldsymbol{\phi}e^{-i\mathbf{q}\cdot\mathbf{r}_3})$$
$$=e^{-i\mathbf{q}\cdot\mathbf{R}_{23}}h_\pi(\mathbf{q},\mathbf{r}_{23}) . \quad (3)$$

In order to separate the coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$, and to make the transition to the pair (2,3) plus a spectator possible, we carry out a Fourier decomposition of the initial deuteron wave function into momentum eigenstates

$$\Psi_i(1,2,3)=\frac{1}{(2\pi)^6}\int d^3\mathbf{k}_1 d^3\mathbf{k}_2\Phi_d\left[\frac{\mathbf{k}_1-\mathbf{k}_2}{2}\right]$$
$$\times\delta(\mathbf{K}_{12}-\mathbf{k}_1-\mathbf{k}_2)$$
$$\times e^{i\mathbf{k}_1\cdot\mathbf{r}_1+i\mathbf{k}_2\cdot\mathbf{r}_2+i\mathbf{k}_3\cdot\mathbf{r}_3} . \quad (4)$$

Here, the Fourier transform of the deuteron wave function $\Psi_d(\mathbf{r})$ is given by

$$\Phi_d(\boldsymbol{\kappa})=\int d^3\mathbf{r}\,\Psi_d(\mathbf{r})e^{\pi\boldsymbol{\kappa}\cdot\mathbf{r}} , \quad (5)$$

with the normalization condition

$$\int|\Phi_d(\boldsymbol{\kappa})|^2 d^3\boldsymbol{\kappa}=(2\pi)^3 . \quad (6)$$

The transition matrix element then becomes

$$\langle\Psi_f|H(2,3)|\Psi_i\rangle=\frac{1}{(2\pi)^6}\int d^3\mathbf{k}_1 d^3\mathbf{k}_2\Phi_d\left[\frac{\mathbf{k}_1-\mathbf{k}_2}{2}\right]\delta(\mathbf{K}_{12}-\mathbf{k}_1-\mathbf{k}_2)$$
$$\times\int d^3\mathbf{r}_1 e^{i(\mathbf{k}_1-\mathbf{k}_1')\cdot\mathbf{r}_1}\int d^3\mathbf{r}_2 d^3\mathbf{r}_3\Psi_d(\mathbf{r}_{23})e^{-i\mathbf{K}_{23}'\cdot\mathbf{R}_{23}-i\mathbf{q}\cdot\mathbf{R}_{23}}h_\pi(\mathbf{q},\mathbf{r}_{23})e^{i\mathbf{k}_2\cdot\mathbf{r}_2+i\mathbf{k}_3\cdot\mathbf{r}_3} . \quad (7)$$

The integration over $\mathbf{r}_1$ results in a δ function which forces the conservation of the spectator momentum, $\mathbf{k}_1=\mathbf{k}_1'$, and, after a coordinate change from $\mathbf{r}_2$ and $\mathbf{r}_3$ to $\mathbf{R}_{23}$ and $\mathbf{r}_{23}$, the integral over R_{23} ensures momentum conservation in the (2,3) subsystem. After integrating over momenta, we obtain

$$\langle\Psi_f|H_\pi(2,3)|\Psi_i\rangle=\delta(\mathbf{K}_{12}+\mathbf{k}_3-\mathbf{k}_1'-\mathbf{K}_{23}'-\mathbf{q})\Phi_d(\mathbf{k}_1'-\tfrac{1}{2}\mathbf{K}_{12})\cdot\int d^3\mathbf{r}_{23}\Psi_d(\mathbf{r}_{23})h_\pi(\mathbf{q},\mathbf{r}_{23})e^{i(\mathbf{K}_{12}-\mathbf{k}_1'-\mathbf{k}_3)\cdot\mathbf{r}_{23}/2} . \quad (8)$$

The remaining integral is the off-shell amplitude $T_2(s_2,q,k_{23})$ for the elementary reaction $pn \to d\pi^0$ in the (2,3) subsystem. In principle, this amplitude can be calculated from a model, as is done, for instance, in Ref. [10]. Here, however, we relate the square of this amplitude to the measured on-shell cross section of the elementary reaction. The amplitude depends only on the total energy $\sqrt{s_2}$ in the subsystem and the angle between the incident proton and the pion in the final state. It is advantageous to evaluate this amplitude in a frame where the $d\pi$ system [or the initial nucleon pair (2,3)] is at rest where $\mathbf{K}'_{23}+\mathbf{q}=\mathbf{k}_2+\mathbf{k}_3=0$. The matrix element for pion production becomes

$$\langle \Psi_f | H_\pi(2,3) | \Psi_i \rangle = \delta(\mathbf{K}_{12}+\mathbf{k}_3-\mathbf{k}'_1-\mathbf{K}'_{23}-\mathbf{q}) \times \Phi_d[(\mathbf{k}'_1+\mathbf{k}_3)/2] T_2(s_2,\mathbf{q},\mathbf{k}_{23}) \ . \quad (9)$$

At this stage, it would be possible to relax the assumptions made in deriving this expression. In the amplitude T_2 initial-state correlations, including $N\Delta$ admixtures, could be introduced, and the pion production operator could be generalized to contain terms required by Galilean invariance, as well as the effect of s-wave pion-nucleon rescattering [10]. In fact, by determining T_2 from the experimental cross section, such effects are included in the (2,3) system, but interactions between the beam proton and the spectator are still neglected as is required by the assumptions of the model.

C. Phase-space integrals

In the reaction $pd \to pd\pi^0$, the externally observable four-momenta of the participating particles are denoted by $P_i=(E_i,\mathbf{p}_i)$ and the final-state pion by $Q=(\omega,\mathbf{q})$, with the corresponding labels defined in Fig. 1(b). The square of the total center-of-mass energy, s, is related to the bombarding energy, $T_{\rm inc}$, by

$$s=(m_p+m_d)^2+2m_d T_{\rm inc} \ .$$

In the following, kinematic quantities without a prime are Lorentz invariant. There are two *specific* reference frames used. The first, ${}^2R_{\pi d}$, is the center-of-mass system of the $pn \to d\pi^0$ reaction. The second, ${}^3R_{\pi d}$, is the frame in the $pd \to pd\pi^0$ reaction where the final-state $\pi+d$ system is at rest. These two frames are not identical: while the final-state parameters E_d, ω, and $\mathbf{q}$ are the same in both frames, the initial-state parameters differ because the target neutron is free in ${}^2R_{\pi d}$ but bound in ${}^3R_{\pi d}$. Quantities in the ${}^2R_{\pi d}$ system are denoted by an asterisk, those in ${}^3R_{\pi d}$ are denoted by a prime. The invariant mass of the $\pi+d$ system is $\sqrt{s_2}$.

In the following we will frequently refer to the book on relativistic kinematics by Byckling and Kajantie [9]. Specific equations therein are referred to by (BK,m,n) where m is the chapter and n the equation number in Ref. [9].

The total cross section for the reaction $pd \to pd\pi^0$ is given (BK,III,2.2–2.3) as

$$\sigma_{pd\to pd\pi^0}=I_3(s)/[2(2\pi)^5\lambda^{1/2}(s,m_p^2,m_d^2)] \ . \quad (10)$$

Here, λ is the usual triangle function (BK,III,6.4), defined as

$$\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ac \ . \quad (11)$$

The denominator in Eq. (10) is the flux factor and $I_3(s)$ contains the integration over the three-body phase space (BK,III,2.4):

$$I_3(s)=\int \frac{d^3\mathbf{p}_d}{2E_d}\frac{d^3\mathbf{p}_s}{2E_s}\frac{d^3\mathbf{q}}{2\omega}\delta^4(P_B+P_T-P_s-P_d-Q)|M_I|^2, \quad (12)$$

where M_I is the invariant matrix element for $pd \to pd\pi^0$. Integrating over five of the variables in Eq. (12) leads to (BK,V,5.6)

$$I_3(s)=\frac{2\pi}{16\lambda^{1/2}(s,m_p^2,m_d^2)} \times \int_{s_2^-}^{s_2^+}\frac{ds_2}{\sqrt{s_2}}\int_{t^-}^{t^+}dt\, q'\int d\Omega_{q'}|M_I|^2 \ . \quad (13)$$

Here, the remaining two integration variables are chosen as the square s_2 of the invariant mass of the (2,3) system, and the invariant angle variable t:

$$t\equiv(P_B-P_s)^2=2m_p^2-2E'_BE'_s+2p'_Bp'_s\cos\theta'_{sB} \ . \quad (14)$$

The integration limits $s_2^\pm$ and $t^\pm$ (BK,V,5.11), are given by

$$\begin{aligned} s_2^+ &=(\sqrt{s}-m_p)^2 \ , \\ s_2^- &=(m_d+m_\pi)^2 \ , \\ t^\pm &=2m_p^2-(1/2s)(s+m_p^2-m_d^2)(s-s_2+m_p^2) \\ &\quad \pm(1/2s)\lambda^{1/2}(s,m_p^2,m_d^2)\lambda^{1/2}(s,s_2,m_p^2) \ . \end{aligned} \quad (15)$$

The cross section $\sigma_{\rm II}$ for the elementary $pn \to d\pi^0$ reaction in terms of its invariant matrix element $M_{\rm II}$ follows from phase-space considerations for a two-body final state and, when evaluated in the ${}^2R_{\pi d}$ frame (BK,IV,4.19), becomes

$$|M_{\rm II}|^2=16(2\pi)^2 s_2\frac{p_p^*}{q^*}\frac{d\sigma_{\rm II}}{d\Omega_q} \ . \quad (16)$$

The invariant matrix element $M_{\rm II}$ depends on the noninvariant amplitude T_2 for $pn \to d\pi^0$ by the usual energy-dependent normalization prescription, and $M_{\rm I}$ is related to the same T_2 by the use of Eq. (9):

$$\begin{aligned} |M_{\rm I}|^2 &=2E'_T2E'_B2E'_d2E'_s2\omega'|\Phi_d(\boldsymbol{\kappa})|^2|T_2|^2 \ , \\ |M_{\rm II}|^2 &=2E_n^*2E_p^*2E_d^*2\omega^*|T_2|^2 \ . \end{aligned} \quad (17)$$

Noting that E_d, ω, and $\mathbf{q}$ are the same in the two frames used here, and combining Eqs. (16) and (17), we obtain

$$\int |M_I|^2 q' d\Omega_{q'}=\frac{2E'_TE'_BE'_s}{E_n^*E_p^*}|\Phi_d(\boldsymbol{\kappa})|^2 16(2\pi)^2 \times s_2 p_p^* \sigma_{\rm II}(\eta_{\rm II}) \ , \quad (18)$$

where $\sigma_{\rm II}$ is the total $pn \to d\pi^0$ cross section as a function of the energy. It is customary to describe the energy dependence in terms of $\eta_{\rm II}$, the center-of-mass pion momentum in units of its rest mass with $\eta_{\rm II}=q'/m_\pi$. Inserting Eq. (18) into Eq. (13) leads to

$$\sigma_{\rm I}(\eta_{\rm I})=\frac{(2\pi)^{-2}}{\lambda(s,m_p^2,m_d^2)}\int ds_2 dt\, p_p^*\sqrt{s_2}\frac{E_T'E_B'E_s'}{E_n^*E_p^*}|\Phi_d(\kappa)|^2 \times\sigma_{\rm II}(\eta_{\rm II}) \ . \quad (19)$$

This is the final result, relating $\sigma_{\rm I}=\sigma(pd\to pd\pi^0)$ to $\sigma_{\rm II}=\sigma(pn\to d\pi^0)$. Again, it is customary to describe the energy at which $\sigma_{\rm I}$ is evaluated by $\eta_{\rm I}$, the largest center-of-mass pion momentum in units of m_π, or

$$\eta_{\rm I}=\lambda^{1/2}(s,(m_d+m_p)^2,m_\pi^2)/(2m_\pi\sqrt{s}) \ . \quad (20)$$

The limits of the integration in Eq. (19) have been defined in Eq. (15). In order to evaluate Eq. (19), the ingredients have to be calculated in terms of invariant masses and the integration variables. It is straightforward to express the following momentum and energy variables in terms of invariants:

$$\begin{aligned} p_p^* &= \lambda^{1/2}(s_2,m_p^2,m_n^2)/(2\sqrt{s_2}) \ , \\ E_n^* &= (s_2+m_n^2-m_p^2)/(2\sqrt{s_2}) \ , \\ E_p^* &= (s_2+m_p^2-m_n^2)/(2\sqrt{s_2}) \ , \\ E_s' &= (s-s_2-m_p^2)/(2\sqrt{s_2}). \end{aligned} \quad (21)$$

To obtain the remaining variables in Eq. (19), we make use of the fact that in the $^3R_{\pi d}$ frame the sum of the momenta of the target neutron and the beam proton must vanish. This leads to the condition $\mathbf{p}_s'-\mathbf{p}_B'=\mathbf{p}_T'$. Furthermore, the total energy $E'=(s+p_s^2)^{1/2}$ must add up to the sum $E_B'+E_T'$ of the beam and target energies. Together with the definition of t [Eq. (14)] this leads to

$$\begin{aligned} E_T' &= (s_2+m_d^2-t)/(2\sqrt{s_2}), \\ E_B' &= (s-m_p^2-m_d^2+t)/(2\sqrt{s_2}) \ . \end{aligned} \quad (22)$$

Finally, the nucleon momentum in the target deuteron rest frame is needed in order to evaluate the deuteron wave function $\Phi_d(\kappa)$. Making use of $\kappa=\frac{1}{2}|\mathbf{p}_s'+\mathbf{p}_B'|$, and the definition of t [Eq. (14)], one finds

$$\begin{aligned} \kappa &= \tfrac{1}{2}\sqrt{p_s'^2+p_B'^2+2p_s'p_B'\cos(\theta_{sB}')} \\ &= \tfrac{1}{2}\sqrt{(E_B'+E_s')^2-4m_p^2+t} \ . \end{aligned} \quad (23)$$

The physical input to Eq. (19), $|\Phi_d(\kappa)|^2$ and $\sigma_{\rm II}(\eta_{\rm II})$, will be discussed in the next section.

Up to now, the spin and isospin parts of the wave functions have been neglected. It is thus necessary to investigate whether the ratio $\sigma_{\rm I}/\sigma_{\rm II}$ depends on these degrees of freedom.

Averaging over the initial states introduces to the cross section a factor of $\frac{1}{4}$ in $pn\to d\pi^0$, and a factor of $\frac{1}{6}$ in $pd\to pd\pi^0$. In the sum over final states of the latter process, however, each np state appears with a weight factor $\frac{3}{2}$. For example, the np state with both nucleon spins up in the pd system arises from the magnetic quantum numbers $\mu_d=+1$, $\mu_p=+\frac{1}{2}$ with probability 1, and from $\mu_d=0$, $\mu_p=+\frac{1}{2}$ with probability $\frac{1}{2}$. In short, the spin statistical factors can be omitted in relating $\sigma_{\rm I}$ to $\sigma_{\rm II}$.

Using standard recoupling techniques for the isospin, one can relate the initial state with the nucleons 1 and 2 [see Fig. 1(a)] coupled to $T_{12}=0$ (i.e., the deuteron), to states where the nucleons 2 and 3 are coupled either to $T_{23}=0$ or 1. Since $T_{23}=0$ is not allowed as an initial state of $pn\to d\pi^0$ in the (2,3) subsystem, the initial state is set for $NN\to d\pi$ with an amplitude of $\sqrt{3}/2$. The final state of a proton and a π^0 with total isospin $\frac{1}{2}$ has the amplitude $1/\sqrt{3}$, and so the overall probability for isospin overlap is $\frac{1}{4}$. In the experimental cross section $\sigma_{\rm II}$ a factor of $\frac{1}{2}$ is already included since only one-half of the initial np system is in the $T=1$ state needed for $d\pi$. In addition, the reaction could have originated in the (1,3) pair which accounts for a factor of 2. Again, the isospin statistical factors can be omitted in relating $\sigma_{\rm I}$ to $\sigma_{\rm II}$.

D. Elementary cross section and deuteron wave function

The only dynamical quantities needed to calculate the $pd\to pd\pi^0$ cross section [Eq. (19)] are the deuteron momentum probability density $|\Phi_d|^2$ and the measured cross section $\sigma_{\rm II}$ of the elementary process, $pn\to d\pi^0$. For the final-state deuteron the full wave function is (implicitly) taken into account. The initial deuteron, however, explicitly enters the cross section [Eq. (19)] as the

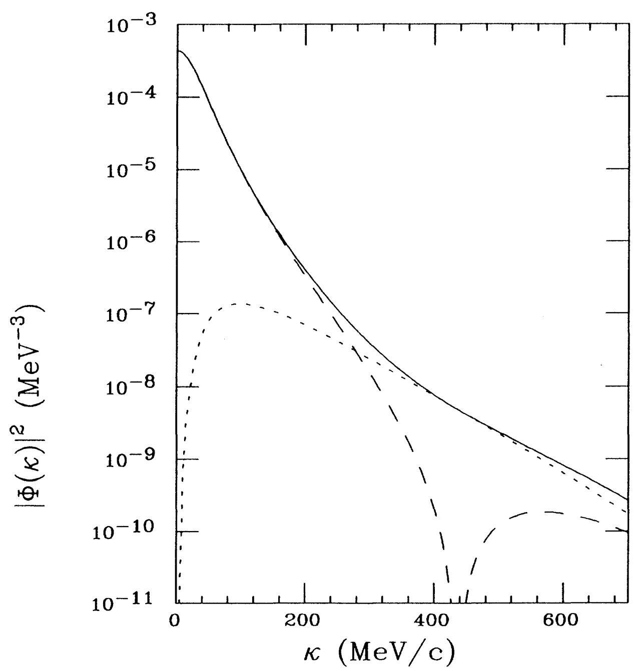

FIG. 2. Momentum probability density in the deuteron as obtained from the Bonn potential [11]. Shown are the contributions of the S state (dashed), the D state (dotted), and the sum $|\Phi_d|^2$ of the two (solid line). The normalization of $|\Phi_d|^2$ is defined in Eq. (6).

Fourier transform Φ_d of the wave function. In the present context we choose $|\Phi_d|^2$ to be the sum of the *S*- and *D*-state probability densities as a function of the magnitude of the vector argument, thereby neglecting the nonspherical aspects of the deuteron *D* state.

The deuteron momentum-space wave function used here is derived from the Bonn potential [11]. The squared wave functions corresponding to the *S* and the *D* state are given by Eq. (26) in Ref. [11]. The individual contributions of the two angular-momentum states are shown in Fig. 2 as a function of the momentum κ of a nucleon in the deuteron (dashed and dotted lines), together with their sum $|\Phi(\kappa)|^2$ (solid line).

The total cross section $\sigma_{\rm II}$ for $pn \rightarrow d\pi^0$ is parametrized as a function of $\eta_{\rm II}$, the center-of-mass pion momentum in units of its rest mass, as follows. Below $\eta_{\rm II}=1$ we use the analytic expression

$$\sigma_2 = \tfrac{1}{2}\eta_{\rm II}[184\ \mu{\rm b} + (781\ \mu{\rm b})\eta_{\rm II}^2]$$

which was obtained from a fit to a recent, accurate measurement close to threshold [2]. For $\eta_{\rm II} > 1$ a Lorentzian form is fitted to the world's supply of $pp \rightarrow d\pi^+$ cross sections [12] in this energy region. The result is then scaled according to

$$\sigma(pp \rightarrow d\pi^+)/\sigma(pn \rightarrow d\pi^0) = 2\ ,$$

a factor arising from isospin symmetry and the identity of particles in one of the channels. It needs to be stressed that the present study shows very little sensitivity to the input cross section $\sigma_{\rm II}$ for $\eta > 1$. The resulting cross section $\sigma_{\rm II}$ is shown as a dotted line in Fig. 3 together with available measurements [2,12].

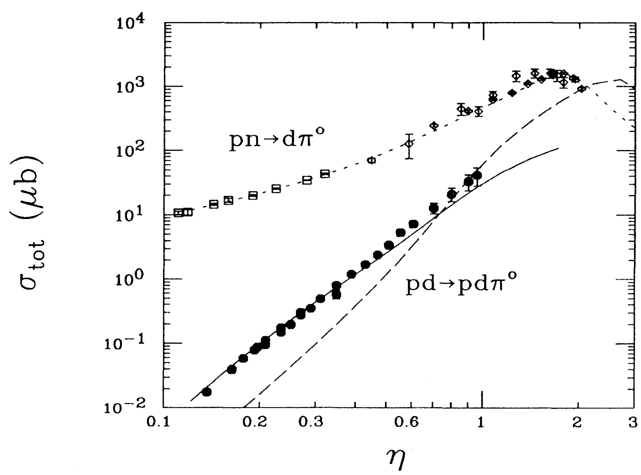

FIG. 3. Total pion production cross section as a function of $\eta = q_{\rm c.m.}/m_\pi$. Shown are the elementary input cross section $\sigma_{\rm II}(\eta) = \sigma(pn \rightarrow d\pi^0)$ (dotted line), together with available data (squares [2] diamonds [12]), and the $pd \rightarrow pd\pi^0$ cross section $\sigma_{\rm I}(\eta)$ calculated according to Eq. (19) (dashed line). The solid curve illustrates the energy dependence with the final-state interaction included. The data for $pd \rightarrow pd\pi^0$ (closed dots) are the result of a recent measurement with the Indiana Cooler [1].

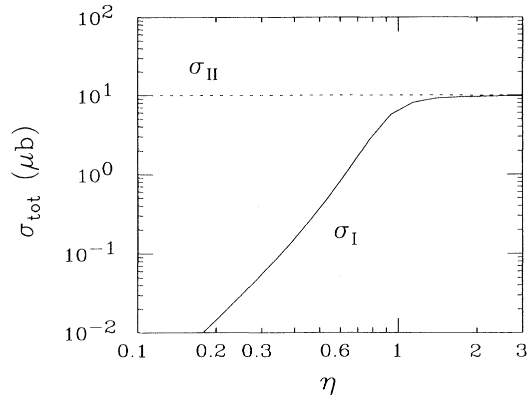

FIG. 4. The cross section $\sigma_{\rm I}$ from Eq. (19) evaluated with a constant input cross section of $\sigma_{\rm II} = 10\ \mu$b.

III. RESULTS

After numerically integrating Eq. (19) one obtains $\sigma_{\rm I}$, the $pd \rightarrow pd\pi^0$ cross section arising from quasifree production in the $pn \rightarrow d\pi^0$ channel. The result is shown as a dashed line in Fig. 3 as a function of $\eta_{\rm I}$ [Eq. (20)].

To discuss this result it is instructive to repeat the evaluation of Eq. (19), replacing the elementary cross section $\sigma_{\rm II}$ in the integral by a constant value, for example, $\sigma_{\rm II} = 10\ \mu$b. From Fig. 4 it can be seen that the calculated three-body final state approaches the input cross section for high energies ($\eta > 1$), but is strongly suppressed close to the threshold ($\eta = 0$). The suppression is due to the limited range of momenta accessible in the deuteron. The shaded area in Fig. 5 indicates the range in which the nucleon momentum κ enters the integral Eq. (19) as a function of η. It is obvious that close to threshold the reaction $pd \rightarrow pd\pi^0$ samples the deuteron momentum distribution in a narrow range around 200 MeV/c where the probability density is low. Beyond $\eta = 1$, $\kappa = 0$ can be reached and the calculated cross section approaches the input cross section.

When evaluating $\sigma_{\rm I}$ at a given $\eta_{\rm I}$ the input cross sec-

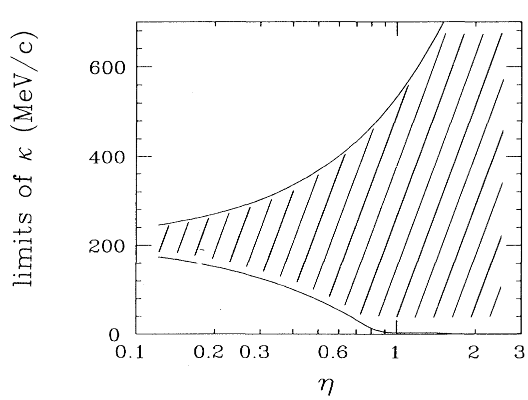

FIG. 5. Range of deuteron internal momenta that contribute to $\sigma_{\rm I}$ in Eq. (19) as a function of the energy variable η.

tion $\sigma_{\rm II}$ is needed at energies corresponding to $\eta_{\rm II}$ in the range $0<\eta_{\rm II}<\eta_{\rm I}$. Consequently, the calculated cross section below $\eta_{\rm I}=1$ is not very sensitive to the p-wave components of $\sigma_{\rm II}$, or to its value in the 3,3 resonance region.

The fact that the calculated cross section underestimates the data is not surprising. This can be due to (i) the coherent contribution of other production processes (such as $pn \to pn\pi^0$), or more complicated reaction mechanisms [6], (ii) the neglect of off-shell parts of the amplitude, or (iii) the effect of the final-state interaction between the proton and the deuteron in the exit channel. The latter effect can be shown to be significant, and will be discussed in the next section.

IV. FINAL-STATE INTERACTION

Watson [13] has suggested a procedure to separate the energy dependence due to the final-state interaction between two collision partners (in our case the proton and the deuteron) from the reaction amplitude. In essence, the procedure involves expressing the square of the matrix element as a term that varies slowly with energy times the s-wave elastic-scattering cross section of the two particles. The method is valid if the relative momentum k_f between the interacting particles is small, and the final-state interaction is strong compared with the primary reaction (here, pion production). When the integrand in the phase-space integral, Eq. (19), is multiplied by the resulting final-state weight factor $\xi_{\rm fsi}$, taken at the appropriate momentum k_f, a cross section $\sigma_{\rm I,fsi}$ results that is of arbitrary normalization but reflects the modification of the energy dependence by the final-state interaction.

Because πp scattering through a Δ is isospin forbidden, and πp s-wave scattering is weak, we only take into account the interaction between the proton and the deuteron in the exit channel. Expressing the pd scattering cross section in terms of the s-wave phase shift, one obtains for $\xi_{\rm fsi}$ in the case of pd scattering.

$$\xi_{\rm fsi}(k_f)=\frac{1}{C_0^2k_f^2}\left[\frac{2}{1+\cot^2\delta_Q}+\frac{1}{1+\cot^2\delta_D}\right] . \qquad (24)$$

where the two terms correspond to channel-spin quadruplet (Q) or doublet (D) scattering. $\delta_{Q,D}$ are the respective phase shifts, and k_f is the center-of-mass momentum in the rest system of the collision partners. The factor C_0^2 in the denominator appears because of the Coulomb interaction [14]. The phase shifts $\delta_{Q,D}$ are obtained from an effective-range expansion, which, for charged particles, becomes [13,15]

$$C_0^2k_f\cot\delta_\mu+2\gamma k_f H_\gamma=-\frac{1}{a_\mu}+\frac{1}{2}b_\mu k_f^2+c_\mu k_f^4+\cdots , \qquad (25)$$

where μ stands for either Q or D, and

$$\gamma=e^2m_{\rm red}/(\hbar^2k_f),\quad C_0^2=2\pi\gamma/(e^{2\pi\gamma}-1) ,$$
$$H_\gamma=-0.577\,22+\gamma^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+\gamma^2)}-\ln\gamma . \qquad (26)$$

Here, $m_{\rm red}$ is the reduced mass in the pd system. The expansion coefficients a_μ, b_μ, c_μ in Eq. (24) have been determined from a fit to pd elastic-scattering phase shifts [16] in the range $0<k_f<180$ MeV/c. Their respective values are [16] $a_Q=11.88$ fm, $b_Q=2.63$ fm, $c_Q=-0.54$ fm^3, $a_D=2.73$ fm, $b_D=2.27$ fm, and $c_D=0.08$ fm^3. The resulting final-state weight factor $\xi_{\rm fsi}(k_f)$ can now be calculated from Eq. (24); the result is shown in Fig. 6 as a solid line. For comparison, the same parameter is also calculated for uncharged particles (dashed line in Fig. 6). This demonstrates that the Coulomb repulsion between the final-state particles *reduces* the effect due to the strong interaction between them.

In our case, the argument k_f of the final-state weight function [Eq. (24)] is given by

$$\mathbf{k}_f=\tfrac{1}{2}(\mathbf{p}'_s-\mathbf{p}'_d)=\tfrac{1}{2}(\mathbf{p}'_s+\mathbf{q}') .$$

This involves the angle between $\mathbf{p}'_s$ and $\mathbf{q}'$ which has been integrated out between Eqs. (12) and (13). Fortuitously, it turns out (as can be shown numerically) that $\xi_{\rm fsi}$, to a very good approximation, is independent of this angle, and that k_f can be taken as

$$k_f\approx\tfrac{1}{2}\sqrt{q'^2+p_s'^2} , \qquad (27)$$

with

$$q'=\lambda^{1/2}(s_2,m_d^2,m_\pi^2)/(2\sqrt{s_2}) ,$$
$$p'_s=\lambda^{1/2}(s,s_2,m_p^2)/(2\sqrt{s_2}) . \qquad (28)$$

Multiplying the integrand in Eq. (19) by the weight factor $\xi_{\rm fsi}(k_f)$, the integral is reevaluated. The arbitrary overall normalization of the resulting cross section $\sigma_{\rm I,fsi}$ is fixed by the experimental cross section. The result is shown in Fig. 3 as a solid line. This calculation makes no statement about the magnitude of the $pd\to pd\pi^0$ total cross section but it clearly demonstrates that its *energy dependence* is explained very well if the final-state interaction is included.

The upper limit of the integration over k_f grows

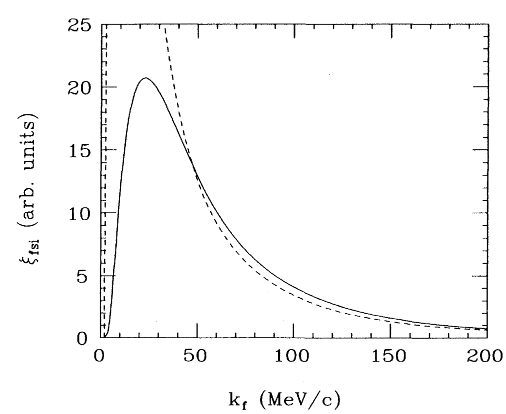

FIG. 6. Final-state interaction weight as a function of the center-of-mass momentum in the pd system (solid curve). The same parameter for uncharged particles is shown by the dashed curve.

linearly with η, and for $\eta > 1$ is beyond the validity of the amplitude parametrization of Eq. (25). Therefore, an extension of this treatment of the final-state interaction beyond the range covered by the present data would be suspect.

V. CONCLUSIONS

We have calculated the total cross section for pion production in the reaction $pd \rightarrow pd\pi^0$ under the assumption that it is entirely due to the quasifree elementary process $pn \rightarrow d\pi^0$. The absolute magnitude of the calculated cross section is in qualitative agreement with a recent near-threshold measurement. In particular, the calculation reproduces the strong suppression close to threshold of the $pd \rightarrow pd\pi^0$ cross section as compared to the contributing $pn \rightarrow d\pi^0$ process. This suppression is understood as a direct consequence of the deuteron momentum-probability distribution in conjunction with the limitations of three-body phase space.

The above calculation, although it reproduces the main features of the experimental cross section, does not give the correct energy dependence and underestimates the measurement. We find that when the interaction between the final-state *pd* pair is included, the energy dependence of the data is reproduced very well. The present treatment of the final-state interaction is approximate and only serves to demonstrate that the effect is important. For a quantitative comparison of the data with predictions of the amplitude for *s*-wave pion production, a more detailed treatment of the interaction between the particles in the exit channel is needed, using, for instance, a distorted-wave Born approximation, as outlined in Ref. [3].

In summary, it seems that the $pd \rightarrow pd\pi^0$ reaction near threshold [between threshold (208 MeV) and 300 MeV] can be explained by a simple quasifree reaction model, using as input phenomenological information on the deuteron and the elementary *NN* pion production cross section.

ACKNOWLEDGMENTS

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