Nucleon-nucleon bound state contribution to the proton scattering by nuclei

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Sensitivity of proton-nucleus scattering is studied to the off-shell behavior of the nucleon-nucleon (NN) effective interaction, particularly to the presence of the NN bound state. The effect is found to be negligible in a kinematical region where an optimal factorization approximation is justified. A sensitivity can be seen below 100 MeV of beam energy and for the light nuclei (^{12}C, ^{16}O). The effect is more pronounced for a tensor part of the NN t matrix which is shown in the charge-exchange cross section of proton scattering from ^{13}C.

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It has been observed in Refs. [1–3] that a nonrelativistic first-order optical potential treatment of the proton-nucleus scattering at intermediate energies is sensitive to an off-energy-shell form of a nucleon-nucleon (NN) effective interaction. One should, therefore, be careful in doing any simplification in evaluating the off-shell NN t matrix. Our aim is to investigate the role of the bound state in the NN t matrix, which is a genuine off-energy-shell effect, in the proton-nucleus scattering. Such a study can provide a useful insight on reliability of analogous calculations in situations where the number and location of elementary bound states is less known than in the NN case. The example we have in our minds is the antinucleon-nucleus scattering, where the pole structure of the antinucleon-nucleon amplitude is still rather obscure.

A straightforward way on how to get the off-shell NN t matrix is to obtain it from a NN potential by solving the Lippmann-Schwinger (LS) equation or in the case of a real potential by solving the K-matrix equation [4]. The fully off-shell t-matrix can be obtained also by solving the Low equation [5]. When the t matrix is needed at many different energies, the last method is best suited in regard to computing time consumption and we used it in our calculation. By virtue of the Low equation, we have the NN bound state pole term easily under control. This is not the case in the LS equation.

Using the Bonn potential, Elster *et al.* [1] have found that the optimum factorization procedure can simulate the full-folding integral in the first-order nucleon-nucleus optical potential very well. It has also been confirmed that even for low energies (100 MeV), the factorization can be used with a little loss of accuracy. On the other hand, Arellano *et al.* [2] have concluded that in the case of the Paris potential the on-shell approach is not an adequate approximation to the full-folding model. We have supposed, therefore, that the suitably chosen factorization scheme is reasonable for our purpose here as we are not ambitious to describe the data exactly.

The transition matrix T for the elastic scattering of a proton from a target nucleus is calculated using the nonrelativistic first-order optical potential formalism developed by Kerman, McManus, and Thaler [6, 7]. In this scheme, an auxiliary matrix $T' = (A-1)T/A$ satisfies the LS equation which, in the proton-nucleus center of mass frame (Ac.m.) reads as

$$T'(E;\mathbf{Q}',\mathbf{Q}) = U'(E;\mathbf{Q}',\mathbf{Q}) + \int \frac{d^3k}{(2\pi)^3}\frac{U'(E;\mathbf{Q}',\mathbf{k})\,T'(E;\mathbf{k},\mathbf{Q})}{E-E_0(k)+i\eta}\,. \tag{1}$$

Here A is the mass number of the nucleus, $E = E(Q)$ is the total kinetic energy, and $\mathbf{Q}$ ($\mathbf{Q}'$) is the on-shell initial (final) relative proton-nucleus momentum. We have assumed that the first-order optical potential U' is well represented by the optimal factorization approximation (OFA) [8]

$$U'(E;\mathbf{k}',\mathbf{k}) \;=\; (A-1)\; t(\epsilon;\mathbf{p}',\mathbf{p})\; F_0(\mathbf{q}), \tag{2}$$

where $F_0(\mathbf{q})$ is the nuclear ground state form factor, $\mathbf{q} = \mathbf{k}'-\mathbf{k}$ is the momentum transfer, and $\mathbf{p}$ ($\mathbf{p}'$) is the initial (final) proton-nucleon relative momentum

$$\mathbf{p} = \tfrac{1}{2}(\mathbf{k} - \mathbf{k}_{\rm eff}),\qquad \mathbf{p}' = \mathbf{k}' - \tfrac{1}{2}(\mathbf{k}+\mathbf{k}_{\rm eff}). \tag{3}$$

In Eq. (2), we have used the fact that the t matrix is Galileo invariant.

The nucleon effective momentum is chosen in the Ac.m.

$$\mathbf{k}_{\rm eff} \;=\; \frac{A-1}{2A}\,\mathbf{k}' \;-\; \frac{A+1}{2A}\,\mathbf{k}\,, \tag{4}$$

and the two-body energy in the two-body center-of-mass frame (2c.m.)

$$\epsilon = E - \frac{\mathbf{K}^2}{4m}, \qquad (5)$$

where m is the nucleon mass and the proton-nucleon total momentum

$$\mathbf{K} = \mathbf{Q} + \mathbf{k}_{\text{eff}} = \frac{A-1}{2A}(\mathbf{Q}' + \mathbf{Q}), \qquad (6)$$

is assumed on shell. Note that the energy ϵ depends on the proton-nucleus scattering angle in the Ac.m. We use the isospin formalism setting the mass of the proton equal to that of the neutron $m = 938.9$ MeV/c^2.

In order to describe a nuclear structure of even-even nuclei properly, we have employed the symmetrized Fermi densities [9] for the nuclear scalar isoscalar form factor which reproduce the nuclear charge form factors up to 3 fm^{-1} of momentum transfer. Vector and isovector form factors for spin and isospin one-half nucleus ^{13}C are evaluated using the wave function of Tiator and Wright [10] which is limited to the 1p shell.

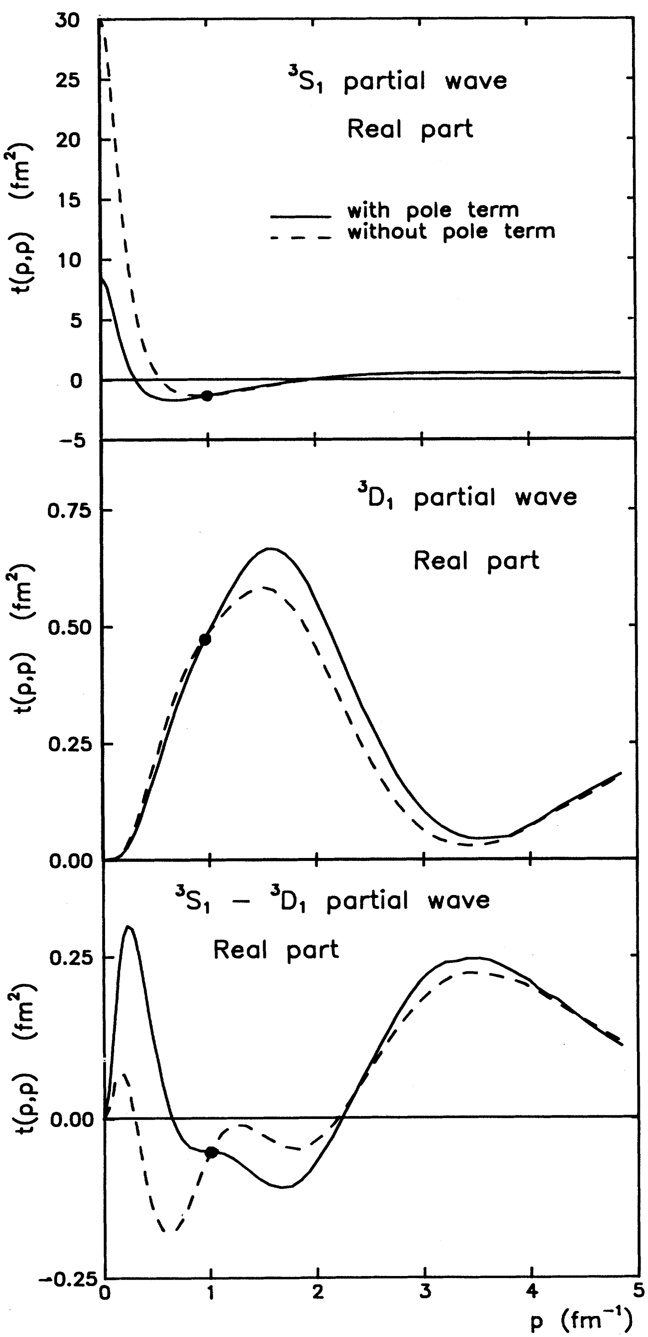

FIG. 1. The contribution of the pole term in Eq. (7) to the real part of the off-energy-shell t matrix with $p'=p$ is demonstrated for s, d, and mixed s-d partial waves. The dot represents the on-shell value for $p = 1$ fm^{-1}.

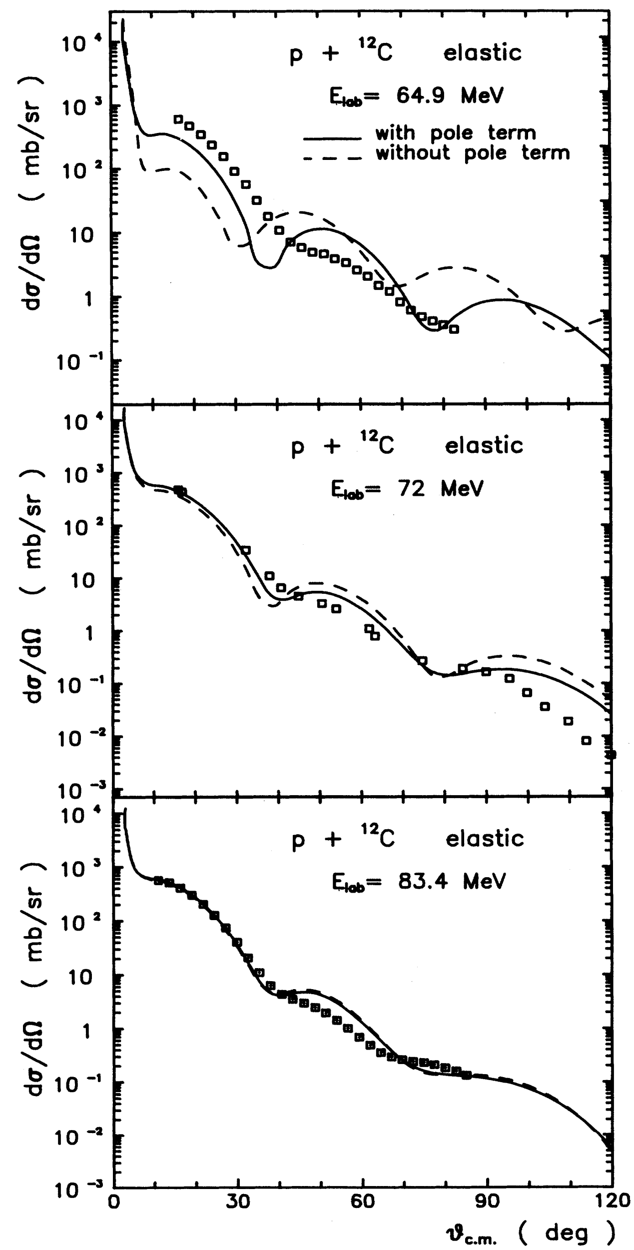

FIG. 2. The dependence of proton ^{12}C elastic differential cross section on the pole term in Eq. (7) for several energies below 100 MeV is shown. The experimental data are from Ref. [13].

In the case of the energy-independent potential, the Low equation has the form in the 2c.m.

$$t(\epsilon;\mathbf{p}',\mathbf{p}) = t(\epsilon_p;\mathbf{p}',\mathbf{p}) + \psi_0(\mathbf{p}')\psi_0^*(\mathbf{p})\frac{(\epsilon_0-\epsilon_{p'})(\epsilon-\epsilon_p)}{\epsilon-\epsilon_0} + \int \frac{d^3k}{(2\pi)^3} t(\epsilon_k;\mathbf{p}',\mathbf{k})t^*(\epsilon_k;\mathbf{p},\mathbf{k})\left[\frac{1}{\epsilon-\epsilon_k+i\eta} - \frac{1}{\epsilon_p-\epsilon_k+i\eta}\right]. \tag{7}$$

Here ϵ_0 and $\psi_0(\mathbf{p})$ are the binding energy and wave function of the deuteron, respectively, and $\epsilon_p = p^2/m$.

The half-shell t matrix needed as an input into Eq. (7) is evaluated numerically by solving the Blankenbecker-Sugar (BbS) equation with the one-meson-exchange potential (OBEPQ) [11]. Applying the minimal relativity factors [11], the covariant BbS equation can be cast into the Lippmann-Schwinger-like equation that is finally solved

$$t(\epsilon;\mathbf{p}',\mathbf{p}) = v_{\mathrm{OBE}}(\mathbf{p}',\mathbf{p}) + m\int \frac{d^3k}{(2\pi)^3}\frac{v_{\mathrm{OBE}}(\mathbf{p}',\mathbf{k})\,t(\epsilon;\mathbf{k},\mathbf{p})}{p^2-k^2+i\eta}. \tag{8}$$

The antisymmetrization of the wave function between the scattered proton and the target nucleus is neglected but the two-body t matrix is symmetrized properly like in the free NN scattering process.

The deuteron wave function and the binding energy used in Eq. (7) correspond to v_{OBE} consistently [11].

The numerical solution of Eqs. (1) and (8) is performed in the LSJ basis using the matrix inversion method. The static Coulomb interaction is included by the prescription of Vincent and Phatak [12]. The charge form factor of the nucleus is assumed in the form given by $F(q)$ normalized to the number of protons in the nucleus and multiplied

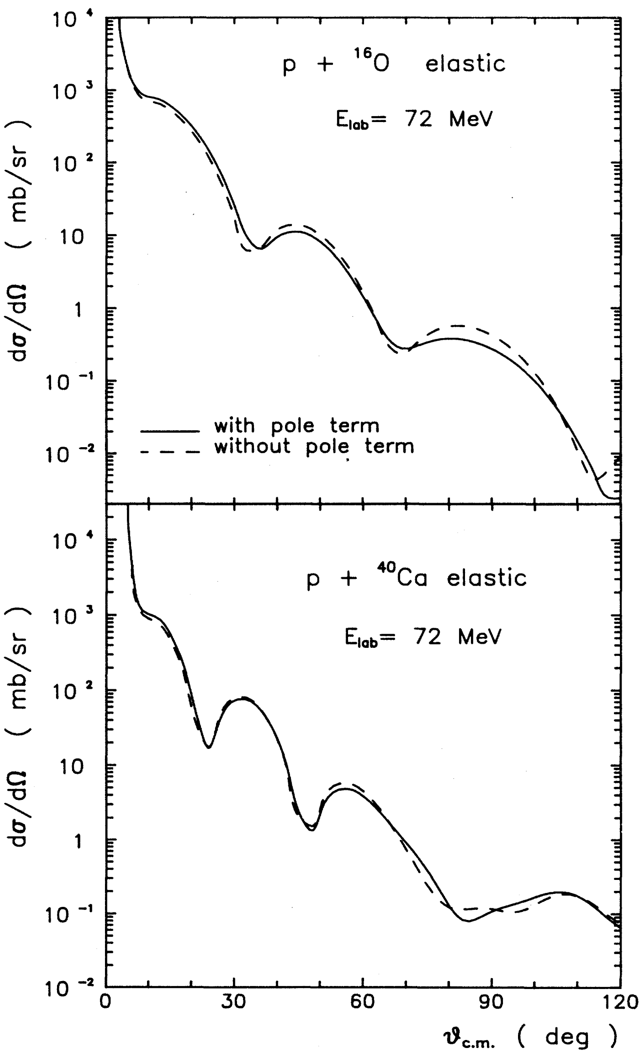

FIG. 3. The sensitivity of the elastic differential cross section to the presence of the pole term in Eq. (7) is demonstrated for proton scattering from heavier nuclei.

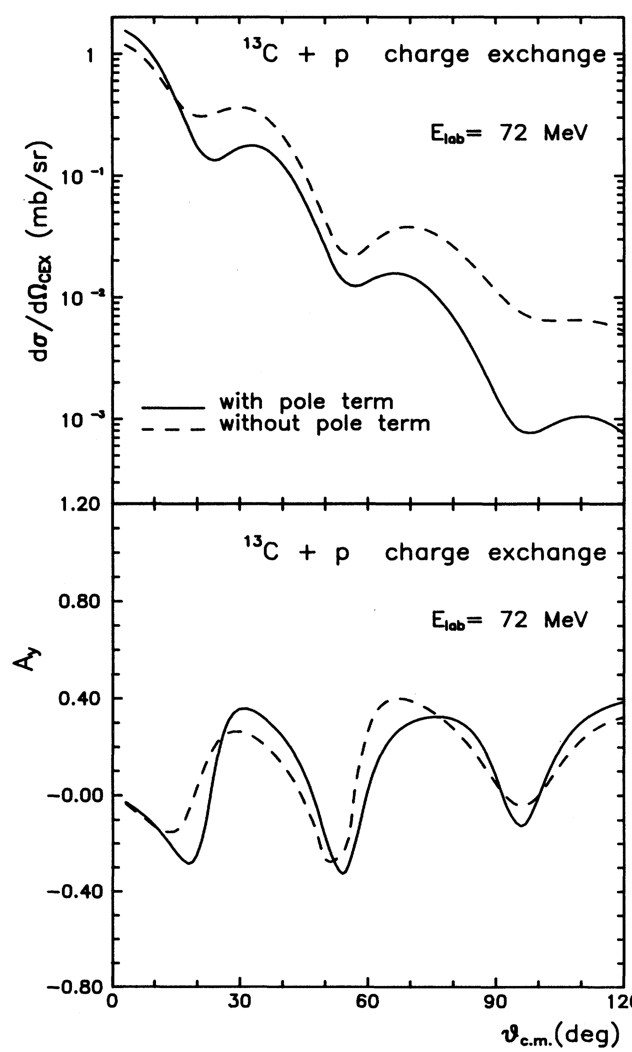

FIG. 4. The sensitivity of charge-exchange cross section and charge-exchange analyzing power to the pole term in Eq. (7) is shown for proton ^{13}C elastic scattering.

by the proton charge form factor.

We have carried out calculations of the proton elastic scattering from ^{12}C, ^{16}O, ^{40}Ca, and ^{13}C at energies around 100 MeV. The aim was to estimate a sensitivity of cross sections and analyzing powers to the pole term in Eq. (7). The results for energies higher than 83 MeV are not displayed here as they are completely insensitive to the presence of the pole.

The contribution of the pole to the s, d, and s-d partial wave NN t matrix is shown in Fig. 1 for $p' = p$. The on-shell value at $p = 1$ fm^{-1} is denoted by a dot. This value is typical for the energy range studied here. It is apparent from the Eq. (7) that the pole term is real and vanishes on shell. The influence of the pole is significant for low momenta in the s wave whereas d-wave matrix element is less altered but in a wider range (Fig. 1). The mixed s-d wave is more sensitive to the pole than the diagonal waves. As the s-d wave is produced by the tensor part of the t matrix, we can conclude that the tensor part of the t matrix is strongly affected by the pole term.

The influence of the pole on the differential cross section of proton elastic scattering from the ^{12}C at energies 65, 72, and 83 MeV is shown in Fig. 2. It is evident that the pole becomes less significant with increasing energy. The data are described only qualitatively because the medium corrections start to be relevant here, especially at 65 MeV.

Figure 3 demonstrates the effect for the heavier nuclei ^{16}O and ^{40}Ca. The significance of the pole is smaller for ^{40}Ca than for ^{16}O or ^{12}C (Fig. 2). The results displayed in Figs. 2 and 3 support the observation in Ref. [7] that the off-shell effects become less significant with increasing energy and/or increasing target mass.

A more complex situation occurs in proton scattering from the spin and isospin one-half nucleus ^{13}C which is sensitive to the tensor part of the NN t matrix. Whereas the results for elastic differential cross section and analyzing power are practically the same as those for proton ^{12}C scattering, a stronger effect appears in the charge-exchange cross section and analyzing power (Fig. 4). These observables are sensitive to the isovector tensor part of the NN t matrix where the influence of the pole is more pronounced (Fig. 1). This sensitivity, however, also vanishes for energies above 100 MeV.

We conclude that the presence of the pole term in Eq. (7) is irrelevant in the kinematical region where the OFA model is justified, i.e., above 100 MeV of the beam energy. There are, therefore, good reasons for omitting the bound state contribution to the NN off-energy-shell t matrix. One can intuitively expect a similar result also for the antiproton-nucleon t matrix where the absorption can even suppress an influence of the poles produced by the diffractive part of the interaction [14].

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