PHYSICAL REVIEW C VOLUME 47, NUMBER 5 MAY 1993

Real part of the polarization potential for ^{11}Li-induced fusion reactions

M. S. Hussein and M. P. Pato
Nuclear Theory and Elementary Particle Phenomenology Group, Instituto de Física, Universidade de São Paulo, Caixa Postal 20516, 01498 São Paulo, São Paulo, Brazil

L. F. Canto and R. Donangelo
Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68.528, 21945 Rio de Janeiro, Rio de Janeiro, Brazil

(Received 8 September 1992)

We calculate the real part of the two-neutron-removal polarization potential for the case of $^{11}\mathrm{Li}+{}^{208}\mathrm{Pb}$ collisions at energies near and below the Coulomb barrier. The real part of V^{pol} is shown to be predominantly repulsive and to reduce the fusion cross section at above barrier energies. Below the barrier this potential changes sign and thus enhances the fusion cross section. However, this effect is considerably weaker than the hindrance produced by the imaginary part of this potential.

PACS number(s): 25.70.Jj, 27.20.+n

In two recent papers [1], we have calculated the ^{11}Li nuclear two-neutron-removal cross section. To do this, we derived the imaginary part of the polarization potential for $2n$ removal. This was initially done using the Glauber method, which is valid at high energies [1,2]. We then extended the theory by incorporating Coulomb effects [2], which become increasingly important as the collision energy decreases. This potential was used to obtain the $2n$-removal and fusion cross sections in ^{11}Li-induced reactions [3].

References to the real part of the polarization potential are scant in the literature (however, see Ref. [4]). The reason for this might be that it is small when compared to the nuclear interaction, so that its effect on the collision process is not as important as that of the imaginary part. This, and the difficulty in obtaining it through direct derivation, explains why we have not included the real part of the polarization potential in our previous studies [1,2]. However, it is well known that even small variation of the potential produce strong effects in the fusion cross section at near barrier energies. For this reason, in the present work we calculate the real part of the $2n$-removal polarization potential and study its effects on ^{11}Li-induced fusion reactions.

We begin by summarizing the derivation of the imaginary part of the $2n$-removal polarization potential of Refs. [2,6]. The polarization potential can be written

$$U(\mathbf{r},\mathbf{r}')=\langle \mathbf{r};\phi_0|vQG^{(+)}Qv|\phi_0;\mathbf{r}'\rangle\ , \tag{1}$$

where v is the coupling interaction, $G^{(+)}$ is the optical Green's function in the Q subspace, and the projection operators P and Q are defined as

$$P=|\phi_0\rangle\langle\phi_0|\ ,\quad Q=1-P\ . \tag{2}$$

Above, $\phi_0(\mathbf{x})\equiv\phi_0(x)$ represents the bound state of the $2n+{}^9\mathrm{Li}$ system while Q is a projector onto states of the $2n$ pair in the continuum. If we now introduce representations in r space and assume that the interaction v is local and separable [5], i.e., $v(\mathbf{r},\mathbf{x})\approx\overline{U}(\mathbf{r})u(x)$, where $\overline{U}(r)$ is the real part of the ^{11}Li-target optical potential and $u(x)$ is an internal excitation form factor, we obtain

$$U(\mathbf{r},\mathbf{r}')=\mathcal{F}(r)G^{(+)}(\mathbf{r},\mathbf{r}')\mathcal{F}(r')\ , \tag{3}$$

where we have defined the form factor

$$\mathcal{F}(r)=U(r)\left[\int\phi_0^2(x)u^2(x)dx\right]^{1/2}\ . \tag{4}$$

After performing partial-wave expansions for the Green's function $G^{(+)}(\mathbf{r},\mathbf{r}')$ and for the potential $U(\mathbf{r},\mathbf{r}')$, we can calculate the imaginary part of the trivially equivalent local polarization potential from the on-shell part of the Green's function:

$$\begin{aligned}\mathrm{Im}\,U_l^{\mathrm{pol}}(r)&\equiv W_l^{\mathrm{pol}}(r)\\&=-\frac{2\mu}{\hbar^2k}\mathcal{F}(r)\int_0^\infty\mathcal{F}(r')f_l^2(kr')dr'\ .\end{aligned} \tag{5}$$

Above, $f_l(kr)$ is the regular solution of the radial equation with the optical potential, which can be approximated as

$$f_l(kr)\approx\sqrt{|S_l^{(1)}|}F_l(kr)\ , \tag{6}$$

where $S_l^{(1)}$ are the l components of the optical S matrix in the intermediate (^{9}Li) channel and $F_l(kr)$ is the regular Coulomb function.

Since in the r region of interest for the breakup only the tail of $\overline{U}(r)$ matters, the form factor can be written $\mathcal{F}(r)=\mathcal{F}_0e^{-r/\alpha}$ and the polarization potential takes the form

$$W_l^{\mathrm{pol}}(r)=W_0(l,E)e^{-r/\alpha}\ , \tag{7}$$

where the strength $W_0(l,E)$ is given by

$$W_0(l,E)=\frac{-|\mathcal{F}_0|^2}{E}|S_l^{(1)}|I_l(\eta,s)\ , \tag{8}$$

in terms of the radial integral

 ©1993 The American Physical Society

$$I_l(\eta,s)=\int_0^\infty e^{-s\rho}F_l^2(\rho)d\rho\ . \tag{9}$$

The real part of $U_l^{\rm pol}$, $V_l^{\rm pol}$ is usually calculated from the off-shell part of $G^{(+)}(\mathbf{r},\mathbf{r}')$. Here, we opt for a simpler derivation using the dispersion relation. If we write

$$V_l^{\rm pol}(r)=V_0(l,E)e^{-r/\alpha}\ , \tag{10}$$

then the strength V_0 is related to the strength of the imaginary part through [6,7]

$$V_0(l,E)=\frac{1}{\pi}\mathcal{P}\int_0^\infty \frac{W_0(l,E)}{E'-E}dE'\ , \tag{11}$$

in which $\mathcal{P}$ denotes the principal value of the integral. This expression can be written, after some algebra, as

$$V_0(l,E)=\frac{1}{\pi}\left[\lim_{e\to 0}\int_e^1[W_0(l,E(1+x)) - W_0(l,E(1-x))]\frac{dx}{x} + \int_1^\infty W_0(l,E(1+x))\frac{dx}{x}\right]. \tag{12}$$

In Ref. [3], the effects of the imaginary part of the polarization potential on the fusion cross section was studied. In it, the fusion cross section was written as

$$\sigma_f=\frac{\pi}{k^2}\sum_{l=0}^\infty(2l+1)(1-T_l^{\rm bu}) \times\{\tfrac{1}{2}[T_l^f(E+F)+T_l^f(E-F)]\}\ , \tag{13}$$

where $(1-T_l^{\rm bu})$ is the breakup survival probability, given as [3]

$$1-T_l^{\rm bu}=\exp\left[\frac{-4\mathcal{F}_0^2}{E^2}|S_l^{(1)}|I_l^2(\eta,s)\right]. \tag{14}$$

The transmission factor T_l^f was approximated by the Hill-Wheeler formula

$$T_l^f=\left\{1+\exp\left[\frac{-2\pi}{\hbar\omega}\left[V_B+\frac{\hbar^2 l(l+1)}{2\mu R_B^2}-E\right]\right]\right\}^{-1}, \tag{15}$$

where V_B, R_B, and $\hbar\omega$ are the optical potential barrier parameters, giving its height, radius, and curvature, respectively. In Eq. (13), this transmission factor is calcu-

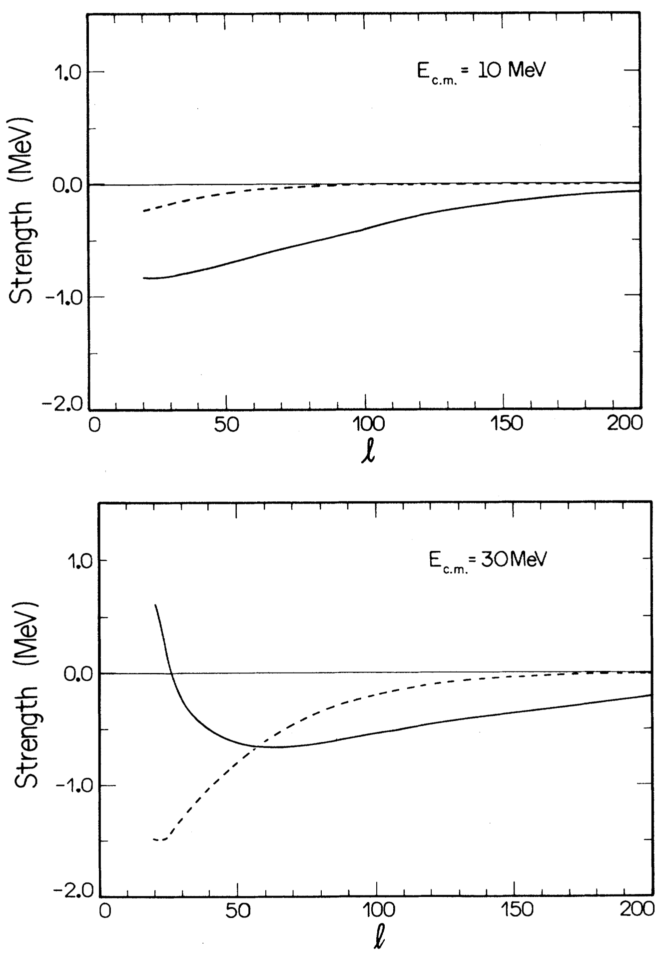

FIG. 1. Strengths of the real (solid curves) and imaginary (dashed curves) parts of the polarization potential for the ^{11}Li+ ^{208}Pb system as a function of l for different collision energies. The barrier energy is 26 MeV.

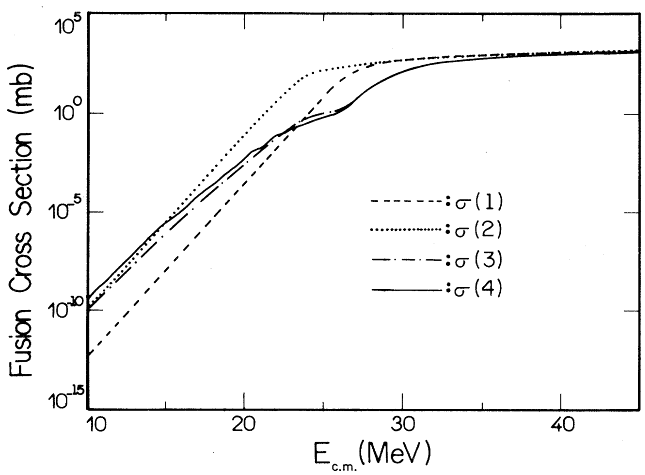

FIG. 2. Fusion cross section for the ^{11}Li+ ^{208}Pb system as a function of incident energy, showing the effect of the inclusion of the real part of the $2n$-removal polarization potential. (See text for details.)

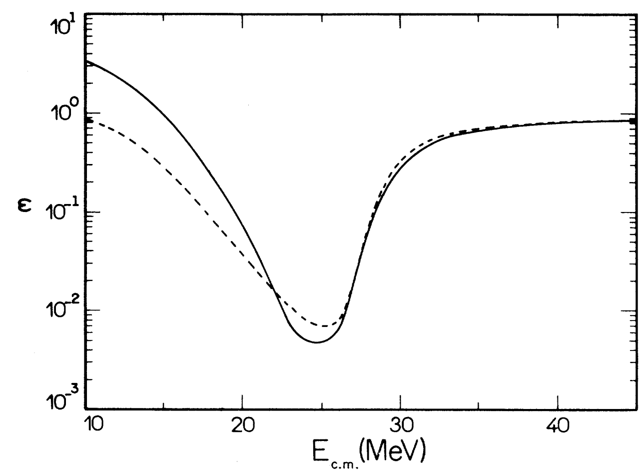

FIG. 3. The enhancement factor discussed in the text, under the same conditions as in the previous figure.

lated at the energies $E \pm F$, to take into account the enhancement arising from the coupling to the pygmy resonance. In the calculation to follow we took for $|S_l^{(1)}|$ the form $|S_l^{(1)}| = (1-T_l^f)^{1/2}$ [8].

We now estimate the effects of the real part of the polarization potential on the fusion cross section. This can be done in a simple way through the addition of an l-dependent correction to the barrier height V_B in Eq. (13):

$$V_B \rightarrow V_B + V_l^{\mathrm{pol}}(R_B) , \tag{16}$$

where V_l^{pol} is given by Eqs. (10) and (12).

We have applied this procedure to the collision ^{11}Li + ^{208}Pb. In Fig. 1 we show the real and imaginary strengths of the polarization potential, $V_0(l,E)$ and $W_0(l,E)$, as functions of the angular momentum l, at several bombarding energies. We note that the strength of the real part is dominantly repulsive at $E > E_B$ and becomes attractive at energies low enough (note the barrier height is ~ 26 MeV) when W turns negligible. This behavior is easy to understand from the dispersion relation, Eq. (11). Let us call E_{th} ($E < E_{\mathrm{th}} < E_B$) the threshold energy where W becomes zero. Then

$$V_0(E) = \frac{P}{\pi} \int_{E_{\mathrm{th}}}^{\infty} \frac{W_0(E')}{E'-E} dE' .$$

Then, since $E'-E>0$, and $W_0(E')<0 \forall E'$, we obtain

$$V_0 < 0 , \quad E < E_{\mathrm{th}} .$$

The above relation can also be understood from the statement that at very low energies the breakup coupling becomes virtual and all states that are virtually excited contribute small attraction [9].

The addition of the real potential leads to an increase in the fusion cross section at subbarrier energies. In Fig. 2 we show the bare Hill-Wheeler cross section $\sigma_F(1)$, the cross section with coupling to the pygmy resonance $\sigma_F(2)$, the cross section (2) with the effect of $\mathrm{Im} V_{\mathrm{pol}}$ [Eq. (13)], $\sigma_F(3)$ and finally the cross section (3) with $\mathrm{Re} V_{\mathrm{pol}}(R_B)$ added to V_B, $\sigma_F(4)$. At $E=10$ MeV the fusion cross section becomes larger than the one calculated with no coupling to the breakup channel (only coupling to the pygmy resonance). This is a novel effect which should be tested experimentally. This effect is better seen when considering the enhancement factor $\epsilon \equiv \sigma_F(4)/\sigma_F(2)$ shown in Fig. 3. At $E < 15$ MeV, ϵ becomes greater than 1 (solid). Also plotted as a dashed curve is $\sigma_F(3)/\sigma_F(2)$.

This work was supported in part by the Financiadora de Estudos e Pesquisas, the Conselho Nacional de Pesquisas e Desenvolvimento Científico and the Fundação de Amparo à Pesquisa do Estado de São Paulo, Brazil.

[1] L. F. Canto, R. Donangelo, and M. S. Hussein, Nucl. Phys. **A529**, 243 (1991).
[2] L. F. Canto, R. Donangelo, M. S. Hussein, and M. P. Pato, Nucl. Phys. **A542**, 131 (1992).
[3] M. S. Hussein, M. P. Pato, L. F. Canto, and R. Donangelo, Phys. Rev. C **46**, 377 (1992).
[4] K. Yabana, Y. Ogawa, and Y. Suzuki, Report No. MSUCL-821 (1990).
[5] M. S. Hussein and C. A. Bertulani, Nucl. Phys. **A524**, 306 (1991); C. A. Bertulani, L. F. Canto, and M. S. Hussein, Phys. Rev. C **45**, 2995 (1992).
[6] H. Feshbach, Ann. Phys. (N.Y.) **19**, 287 (1962).
[7] See, e.g., G. R. Satchler, Phys. Rep. **199**, 147 (1991).
[8] We remark here that Eq. (13) is an approximate one. A more precise formula would have the factor $(1-T_l^{\mathrm{bu}})$ replaced by $(1-T_l^{\mathrm{bu}})^n$ with $n \sim \frac{1}{2}$. This implies the $\mathcal{F}_0$ in Eq. (14) is changed to $\sqrt{n}\,\mathcal{F}_0$. There is certainly room for such a small variation in the value of $\mathcal{F}_0$, considering the magnitude of the error bars in the measured breakup cross section and the quality used to fix $\mathcal{F}_0$.
[9] M. S. Hussein, A. J. Baltz, and B. V. Carlson, Phys. Rep. **113**, 133 (1984).