

Correlation measurements in high-multiplicity events

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Requirements for correlation measurements in high-multiplicity events are discussed. Attention is focused on detection of so-called hot spots, two-particle rapidity correlations, two-particle momentum correlations (for quantum interferometry), and higher-order correlations. The signal-to-noise ratio may become large in the high-multiplicity limit, allowing meaningful single-event measurements, only if the correlations are due to collective behavior.

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I. INTRODUCTION

In the next ten years, ultrarelativistic heavy-ion collisions are planned at Brookhaven's Relativistic Heavy Ion Collider (RHIC) and CERN's Large Hadronic Collider (LHC). In both cases, experimenters expect to see multiplicities in excess of 1000 particles per unit rapidity. As a result, there has been a lot of speculation about single-event fluctuation measurements [1–3]. In this paper, I assess requirements for various single-event measurements, and calculate the number of events needed for useful measurements in cases where single-event analyses are not meaningful.

I begin with a discussion of searches for so-called hot spots (regions of unusually high particle density) in Sec. II. In Sec. III, I discuss measurements of two-particle rapidity correlation functions, assuming that high-multiplicity events are independent superpositions of lower-multiplicity events. In Sec. IV, I discuss measurements of two-particle momentum correlation functions that are commonly constructed to measure collision volumes using quantum interferometry. In Sec. V, I compare results from two-particle correlation functions with those from higher-order correlation functions. Finally, I summarize the results in Sec. VI.

II. LOOKING FOR HOT SPOTS

One common suggestion is that it may be possible to search for hot spots, or regions with unusually large numbers of pions. The basic motivation for these searches is simple: any process that creates a lot of entropy in a small rapidity bin is of interest. Thus, hot spots are commonly thought of as possible signals for interesting phenomena.

For definiteness, suppose that searches are made within

a rapidity window of size ΔY , using events with N particles in this window. If particles are randomly distributed with a flat rapidity distribution, the mean number of particles in a bin of size δy is

$$\bar{n} = \frac{N \delta y}{\Delta Y} = (dN/dy) \delta y, \quad (1)$$

where $dN/dy = N/\Delta Y$. The standard deviation is

$$\sigma_n = \sqrt{\bar{n}} - \bar{n}^2 = \bar{n} (1 - \delta y/\Delta Y) \approx \bar{n}, \quad (2)$$

if the window is large ($\Delta Y \gg \delta y$).

The central limit theorem applies in the limit $N \rightarrow \infty$, so particle number fluctuations have a Gaussian distribution. The probability that a given bin contains more than $\bar{n} + \delta n$ particles is

$$\mathcal{P}(\delta n/\bar{n}^{1/2}) \approx \left(\frac{\bar{n}}{2\pi \delta n^2} \right)^{1/2} e^{-\delta n^2/2\bar{n}}, \quad (3)$$

if $\delta n^2 \gg \bar{n}$. To achieve success in a hot spot search, the hot spots must be present significantly more often than in a random distribution:

$$f(\delta n) \gg \mathcal{P}(\delta n/\bar{n}^{1/2}), \quad (4)$$

where $f(\delta n)$ is the probability that a given bin contains a hot spot with at least δn excess particles.

For definiteness, I assume that a useful result must find hot spots at least ten times as often as expected. In this case, hot spots that occur in 10% of the bins must produce at least $2.4\bar{n}^{1/2}$ excess particles. For RHIC and LHC events, particles are produced over about ten units of rapidity, so hot spots that occur in more than 10% of bins will be seen more often than once per event, and are thus not very useful as triggers for interesting events. The larger dN/dy is, the larger the hot spots must be before they can be separated from the background fluctuations, so it is likely that hot spot searches will be most profitable in events of high energy but relatively low multiplicity.

For RHIC and LHC experiments, rapidity densities in excess of $dN/dy = 1000$ are expected. A hot spot that produces high-energy pions ($p > m$ in the hot spot rest

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frame) isotropically has a width of at least one unit of rapidity, so I take $\delta y = 1$. In this case, if hot spots occur in 10% of the bins (about one hot spot per event), they must produce 75 excess charged pions to be clearly useful as a trigger. If they occur in 1% of the bins (about one per 10 events), they must produce 100 excess pions.

One possible mechanism for visible hot spots is the production of bubbles of disordered chiral condensate [4]. These bubbles were proposed as an explanation for the so-called Centauro events observed in cosmic ray studies, in which many charged pions were produced with very few neutral particles, in contrast to typical nuclear and high-energy processes that produce equal numbers of π^+ , π^- , and π^0 mesons. A chirally disordered bubble that produces N^\pm charged pions, with no π^0 mesons, yields $N^\pm/3$ excess charged pions within one unit of rapidity of the bubble. If these bubbles are produced in less than 10% of the bins, they will be clearly visible if $N^\pm > 225$, but this is unlikely as the expected value is $N^\pm \approx 20$ [4]. Thus, hot spot searches at RHIC and LHC probably will not yield evidence for bubbles of disordered chiral condensate.

Quark-gluon plasma (QGP) droplets are typically too small to be hot spot candidates in high-multiplicity events. A typical mean radius for a QGP droplet is 1 fm, at which size the droplet should produce about 18 charged pions. There is no proposed mechanism that would produce hot spots large enough to be clearly observable at RHIC or LHC; however, if they are seen, the lack of theoretical prediction will make them even more interesting than if they were expected.

III. TWO-PARTICLE RAPIDITY CORRELATIONS

In this section, I discuss measurements of two rapidity correlation functions: the standard two-particle correlation function, R_2 , and the simplest split-bin correlation function, S_2 . The standard two-particle rapidity correlation function is [5, 6]

$$R_2(y; \Delta Y) = \frac{\rho^{(2)}(0, y)}{\rho^{(2)}(0, \Delta Y)}, \quad (5)$$

where ΔY is some large rapidity separation that is used as a reference. I assume at first that ρ is flat, and discuss the effect of corrections for nonflat distributions afterwards.

Suppose that I take some arbitrary model of particle production and analyze a single event. Consider events with N particles in a rapidity window of width ΔY , where the rapidity distribution for any given particle is $p(y) = 1/\Delta Y$. Let there be N_c correlated pairs, where typically $N_c \ll N(N-1)$, and the rapidity distribution for correlated pairs is $q(y_1 - y_2)/\Delta Y$, where q is some arbitrary function. For simplicity, I consider only events with exactly N particles in the rapidity window; generalization to events with differing multiplicities is straightforward [7].

I do not assume that all correlations are pairwise, but I neglect higher-order correlations for the moment. For ex-

ample, it is possible that the pairwise correlations result from interactions of large numbers of particles. Even in this case, however, correlation functions are dominated by pairwise correlations unless the interactions involve almost all of the particles. I discuss this in more detail in Sec. V.

Finally, I assume that a superposition of a independent events with n particles each is equivalent to a single event with an particles. This is equivalent to assuming independent nucleon collisions, or independent parton collisions. In this case I must have $N_c = kN$, where k is some unknown proportionality constant. (For example, if I combine two events I double both N and N_c , as pairs of particles from different events are clearly uncorrelated.)

I can immediately write down the two-particle density,

$$\rho^{(2)}(y_1, y_2) = \frac{[N(N-1) - N_c] + N_c \Delta Y q(y_1 - y_2)}{\Delta Y^2}. \quad (6)$$

Using Eq. (6), I obtain the two-particle correlation function,

$$R_2(y; \Delta Y) = 1 + \frac{k}{dN/dy} [q(y) - q(\Delta Y)]. \quad (7)$$

Here (and for the remainder of this paper) I drop corrections of order $1/N$ and $1/\Delta Y$ unless otherwise specified, as I am primarily interested in the analysis of high-multiplicity, high-energy events. If the mean separation for a correlated pair is y^* , then typically $q(0) \approx 1/y^*$, while $q(\Delta Y) \rightarrow 0$ for large ΔY , so the maximum value of R_2 is roughly

$$R_2^{\max} \approx 1 + \frac{k}{(dN/dy)y^*}. \quad (8)$$

I obtain a lower limit for the error in a measurement of R_2 by calculating the expected fluctuations in the absence of correlations. Consider an experimental measurement:

$$R_2(y; \Delta Y) = \frac{\sum_{i=1}^{N_{\text{ev}}} n_i(0) n_i(y)}{\sum_{i=1}^{N_{\text{ev}}} n_i(0) n_i(\Delta Y)}. \quad (9)$$

Here $n_i(z)$ is the number of particles in the i th event with rapidities between $z - \delta y/2$ and $z + \delta y/2$ (δy is thus the experimental bin size), and N_{ev} is the number of events used in the measurement.

Assuming that $\delta y \leq y$, so that the bins do not overlap, and that there are no correlations,

$$\langle n_i(0) n_i(y) \rangle = \frac{N(N-1) \delta y^2}{\Delta Y^2}, \quad (10)$$

independent of y , so clearly $\langle R_2 \rangle = 1$. The standard deviation is

$$\sigma_R = \frac{\sum_{i=1}^{N_{\text{ev}}} n_i^2(0) n_i^2(y)}{\left[\sum_{i=1}^{N_{\text{ev}}} n_i(0) n_i(y) \right]^2} + \frac{\sum_{i=1}^{N_{\text{ev}}} n_i^2(0) n_i^2(\Delta Y)}{\left[\sum_{i=1}^{N_{\text{ev}}} n_i(0) n_i(\Delta Y) \right]^2} - \frac{2}{N_{\text{ev}}}. \quad (11)$$

In the absence of correlations, the standard deviation is

$$\sigma_R = \frac{(-8N + 12)\delta y^2 + 4(N - 2)\Delta Y \delta y + 2\Delta Y^2}{N_{\text{ev}} N (N - 1) \delta y^2}. \quad (12)$$

For a good measurement, the experimental bin size must be much smaller than the total rapidity window used, so $\delta y \ll \Delta Y$. As the purpose of this paper is to consider measurements in high-multiplicity events, I take the limit $N \rightarrow \infty$, and obtain

$$\sigma_R = \frac{4\Delta Y}{N_{\text{ev}} N \delta y} \left[1 + \mathcal{O}\left(\frac{\delta y}{\Delta Y}\right) \right]. \quad (13)$$

Thus, the error in the measurement is

$$e_R = 2/\sqrt{N_{\text{ev}} (dN/dy) \delta y}, \quad (14)$$

where $dN/dy = N/\Delta Y$ is the rapidity density.

It is possible, if the fluctuations in the system are large, that the actual error is larger than given by (14). If the measured standard deviation is smaller, however, then the value of Eq. (14) is probably better to use, as this represents the error in measuring uncorrelated events. If the measured fluctuations are anomalously small, then the system is probably strongly correlated, so if $R_2 - 1$ is not significantly different from zero it is probably best to look for another correlation function that reflects these strong correlations.

Combining Eqs. (8) and (14), I obtain the signal-to-noise ratio,

$$(s/n)_R = \frac{k}{2y^*} \left(\frac{N_{\text{ev}} \delta y}{dN/dy} \right)^{1/2}. \quad (15)$$

If all particles are produced in clusters containing $n_c \gg 1$ particles, and all particles in a given cluster are pairwise correlated, then $k \approx n_c$. [This is trivial—every particle

is produced with n_c associated (correlated) particles, so there are n_c correlated pairs per particle.] Finally, for a passable measurement $\delta y < y^*$, so the best possible signal-to-noise ratio is

$$(s/n)_R = \frac{n_c}{2} \left(\frac{N_{\text{ev}}}{(dN/dy) y^*} \right)^{1/2}. \quad (16)$$

I use Eq. (16) to estimate how many events I need in order to measure R_2 well, as a function of the cluster size. For most clusters, $y^* < 1.3$, so to measure R_2 (at its peak) with better than 4σ accuracy I need

$$\frac{n_c}{2} \left(\frac{N_{\text{ev}}}{1.3 dN/dy} \right)^{1/2} > 4 \quad (17)$$

or

$$N_{\text{ev}} > 83 (dN/dy) / n_c^2. \quad (18)$$

Here I use $y^* = 1.3$; this is obtained for the most energetic clusters, and gives the most pessimistic estimates of s/n . The cluster size seen in nuclear collisions [8, 9] at 200 GeV is approximately ten charged particles [10]. If this persists up to RHIC and LHC collision energies, where the charged particle multiplicity $dN_{\text{ch}}/dy \approx 1000$, then approximately 830 events will be needed to obtain a good measurement of the peak value of R_2 .

This is, of course, a naive theorist's estimate, leaving out any possible experimental difficulties, and applies only to a measurement of the amplitude of R_2 . If I want to measure the shape of R_2 reasonably well, I should really require that $\delta y < y^*/10$, in which case I find that I need 8300 events. However, Eq. (16) does illuminate the difficulty of measuring correlation functions in high-multiplicity events: If cluster (or source) sizes are independent of dN/dy , then s/n decreases with increasing multiplicity, and accurate measurement becomes increasingly difficult. Also, Eq. (16) justifies *a posteriori* the neglect of correlations when calculating the measurement error, as for one event the correlations (the signal) are much smaller than the statistical fluctuations (the noise).

One could argue that the measurement I have outlined for R_2 does not efficiently use the available statistics. As a response, I construct split-bin correlation functions (SBCFs) [11] in order to use the available statistics with maximal efficiency. The simplest second-order SBCF is

$$S_2(\delta y; \Delta Y) = \frac{\Delta Y \sum_{j=1}^{\Delta Y/\delta y} \int_{(j-1)\delta y}^{(j-1/2)\delta y} dy_1 \int_{(j-1/2)\delta y}^{j\delta y} dy_2 \rho^{(2)}(y_1, y_2)}{\delta y \int_0^{\Delta Y/2} dy_1 \int_{\Delta Y/2}^{\Delta Y} dy_2 \rho^{(2)}(y_1, y_2)}. \quad (19)$$

Here $\Delta Y/\delta y$ must be an integer; taking $\delta y = \Delta Y/2^i$ for $i = 0, 1, 2, \dots$ uses all of the two-particle phase space without reusing any pairs of particles.

Under the assumptions that I used to calculate R_2 ,

$$S_2(\delta y; \Delta Y) = 1 + \frac{k}{dN/dy} [g(\delta y) - g(\Delta Y)], \quad (20)$$

where

$$g(z) = \frac{4}{z^2} \left[\int_0^{z/2} dx x q(x) + \int_{z/2}^z dx (z-x) q(x) \right]. \quad (21)$$

For $\delta y < y^*$, if $q(z)$ is linear in z ,

$$g(\delta y) = q(\delta y/2), \quad (22)$$

while for quadratic $q(z)$,

$$g(\delta y) = q(\delta y/\sqrt{24/7}). \quad (23)$$

Equation (23) is very close to Eq. (22), so these relations are insensitive to the shape of q and are thus robust. For $\Delta Y \gg 2y^*$,

$$g(\Delta Y) = \frac{4y^*}{\Delta Y^2} \rightarrow 0, \quad (24)$$

so for $\Delta Y \gg 2y^*$ and $\delta y < y^*$,

$$S_2(\delta y; \Delta Y) \approx R_2(\delta y/2), \quad (25)$$

independent of ΔY . The maximum value of $S_2(\Delta Y)$ is then

$$S_2^{\max}(\Delta Y) = 1 + \left[1 - \frac{4y^{*2}}{\Delta Y^2} \right] \frac{k}{(dN/dy) y^*}. \quad (26)$$

I calculate the error in the same manner as before, assuming that there are no correlations:

$$\sigma_S = \frac{4}{N_{\text{ev}} N} \left[1 + \mathcal{O} \left(\frac{\delta y}{\Delta Y} \right) \right]. \quad (27)$$

Thus, the error in the measurement is

$$e_S = 2/\sqrt{N_{\text{ev}} (dN/dy) \Delta Y}, \quad (28)$$

and the signal-to-noise ratio is

$$(s/n)_S = \frac{k}{2y^*} \left(\frac{N_{\text{ev}} \Delta Y}{dN/dy} \right)^{1/2}, \quad (29)$$

for $\Delta Y \gg 2y^*$. Setting $k = 10$, $y^* = 1.3$, $\Delta Y = 10$, and $dN/dy = 1000$ as previously, I need approximately 110 events to measure S_2 to 4σ accuracy, as compared with 830 events to determine the peak value of R_2 .

The reader should note that this is an estimate of the *minimum* requirements for a measurement of S_2 . Applying Eq. (29) to a sample of 92 central O+Em events at 200 GeV, with $y^* = 1.3$, $dN/dy \approx 40$, and $\Delta Y = 4$, I obtain $(s/n)_S \approx 12$. If I use Eq. (26) to estimate the corrections for the finite value of ΔY , I obtain $(s/n)_S \approx 7$. This is in reasonable agreement with the observed value $(s/n)_S \approx 2-3$ [12]. Most of the difference comes from the crude approximation used for R_2^{\max} , as an exact calculation [13] shows that Eq. (8) overestimates the signal by a factor of 2. Thus, the estimate of s/n is within a factor of 3, while the number of events needed for a good measurement is a factor of about 10 more than estimated.

The measurement of S_2 discussed above would give the shape of R_2 with points that are approximately 0.7 apart on a logarithmic scale, as I keep changing the bin size by

factors of 2. The previously discussed measurement of R_2 gave points approximately 0.1 apart (spacing was $y^*/10$). Duplication of this measurement using S_2 would involve seven independent measurements of S_2 , requiring approximately 800 events, as opposed to the 8300 required for the R_2 measurement. Thus, if the same number of events are used in each case, a measurement of S_2 has a bit less than one-third of the statistical noise of the corresponding measurement of R_2 .

If the single-particle distribution is not flat, the correlation functions, R_2 and S_2 , should be modified. The most useful two-particle correlation function is

$$R'_2(y; \Delta Y) = \frac{\rho^{(2)}(0, y) \rho(\Delta Y)}{\rho^{(2)}(0, \Delta Y) \rho(y)} \quad (30)$$

$$\simeq \frac{\rho^{(2)}(0, y) \langle N \rangle^2}{\rho(0) \rho(y) \langle N(N-1) \rangle}. \quad (31)$$

These rapidity correlation functions are unity whenever particles are distributed randomly in rapidity according to the single-particle distribution, independent of the multiplicity distribution. I outline the revised calculations for R_2 below.

Suppose that the distribution of cluster centers is $\pi[(y_1 + y_2)/2]$, instead of simply $1/\Delta Y$ as assumed so far. If all particles come from clusters, then the shape of the single-particle distribution is not flat. The probability that a particle is found between y and $y + dy$ is then $p(y)dy$, where

$$p(y) = \frac{\rho(y)}{N} = \int dy_c \pi(y_c) q(2y_c - 2y) \approx \pi(y), \quad (32)$$

if $\pi(y)$ varies slowly compared to q [$q(2y_c - 2y)$ occurs because the second particle of a pair has rapidity $y = 2y_c - y$]. I then obtain

$$\rho^{(2)}(y_1, y_2) = [N(N-1) - N_c] p(y_1) p(y_2) + N_c \pi \left(\frac{y_1 + y_2}{2} \right) q(y_1 - y_2), \quad (33)$$

and consequently

$$R'_2(y; \Delta Y) \simeq 1 + \frac{n_c q(y)}{dN/dy|_{y/2}}, \quad (34)$$

assuming that dN/dy varies slowly so $[dN/dy|_0][dN/dy|_y] \simeq [dN/dy|_{y/2}]^2$.

The only difference from R_2 is that dN/dy is now taken at rapidity $y/2$. The error will change slightly, as the error in determining the correction factor should now be added to the previous error. The error in determining $p(y)$ is

$$e_p/p = 1/\sqrt{N_{\text{ev}} (dN/dy)_y \delta y}, \quad (35)$$

so the error in R'_2 is naively

$$e_R = \sqrt{6/N_{\text{ev}} (dN/dy) \delta y}, \quad (36)$$

assuming that the distribution is approximately flat and that the errors are uncorrelated. This will lower the previous values of $(s/n)_R$, and raise the numbers of events required for a good measurement by about 50%.

The most useful version of S_2 has only a simple addi-

tive correction [12]:

$$S'_2(\delta y; \Delta Y) = S_2(\delta y; \Delta Y) - \Sigma_2(\delta y; \Delta Y), \quad (37)$$

where

$$\Sigma_2(\delta y; \Delta Y) = \frac{\Delta Y \sum_{j=1}^{\Delta Y/\delta y} \int_{(j-1)\delta y}^{(j-1/2)\delta y} dy_1 p(y_1) \int_{(j-1/2)\delta y}^{j\delta y} dy_2 p(y_2)}{\delta y \int_0^{\Delta Y/2} dy_1 p(y_1) \int_{\Delta Y/2}^{\Delta Y} dy_2 p(y_2)} - 1. \quad (38)$$

For the case where the particles are centered in the window, $\int_0^{\Delta Y/2} dy p(y) = \int_{\Delta Y/2}^{\Delta Y} dy p(y)$, the correction simplifies to

$$\Sigma_2(\delta y; \Delta Y) = \frac{4}{\Delta Y \delta y} \sum_{j=1}^{\Delta Y/\delta y} \int_{(j-1)\delta y}^{(j-1/2)\delta y} dy_1 \delta f(y_1) \int_{(j-1/2)\delta y}^{j\delta y} dy_2 \delta f(y_2). \quad (39)$$

Here $\delta f = p\Delta Y - 1$ is the fractional difference from the mean value. As $\delta y \rightarrow 0$, $\Sigma_2 \rightarrow \delta f^2$ from below, so if the furthest bins are within fraction f of the mean density then $\Sigma_2 < f^2$. The error in the correction factor, Σ_2 , is approximately the same size as the error in S_2 , so the experimental errors will be somewhat larger than for the uncorrected case, just as for R_2 .

IV. QUANTUM INTERFEROMETRY

Experimenters are also very interested in using Hanbury-Brown-Twiss interferometry to determine the

geometry of the collision region in high-multiplicity events [14, 15]. This technique uses species-dependent momentum correlation functions, for which particle identification is required. I briefly discuss statistical aspects of the two most common methods for identical particle interferometry, neglecting any difficulties due to errors in the identification of particles or other technical problems.

Both techniques usually use pions that are identified in a spectrometer covering $\Delta\Phi$ rad of azimuthal angle and ΔY units of pseudorapidity. For pions, pseudorapidity and rapidity are almost identical, so I do not differentiate between them in this paper. The correlation function used is

$$C_2(q) = \frac{\int_0^{\Delta\Phi} d\phi_1 d\phi_2 \int_0^{\Delta Y} dy_1 dy_2 \int dp_{T,1} dp_{T,2} \rho_{id}^{(2)} \delta(q^2 + (p_1 - p_2)^2)}{\int_0^{\Delta\Phi} d\phi_1 d\phi_2 \int_0^{\Delta Y} dy_1 dy_2 \int dp_{T,1} dp_{T,2} \rho_{uc}^{(2)} \delta(q^2 + (p_1 - p_2)^2)}, \quad (40)$$

where δ is the Dirac δ function. Here $\rho_{id}^{(2)}$ is the distribution for two identical particles, and $\rho_{uc}^{(2)}$ is an uncorrelated two-particle distribution; the two methods differ only in the prescriptions used to construct $\rho_{uc}^{(2)}$. I use standard high-energy units with $\hbar = c = 1$.

In the first method, referred to as event mixing, an uncorrelated distribution is produced by combining particles (of the same species and charge as those used in $\rho_{id}^{(2)}$) from different events. This is usually done by constructing simulated events using particles from measured events, while ensuring that no two particles come from the same events. This process is very computationally intensive for high-multiplicity events, as it is necessary to keep track of the event from which each particle is taken and check all selections to ensure that they do not

come from an event that was used earlier.

Considerable computational difficulty is removed by constructing $\rho_{uc}^{(2)}$ by convoluting the single-particle distribution obtained by averaging over all events. Using a sample of N_{ev} events, each with N particles in the spectrometer, this second procedure yields

$$\rho^{(2)}(p_1, p_2) = \frac{N-1}{N} \rho(p_1) \rho(p_2) = \rho_{uc}^{(2)} + \frac{\rho_{id}^{(2)} - \rho_{uc}^{(2)}}{N_{ev}}, \quad (41)$$

for all $p_1 \neq p_2$, where $\rho_{uc}^{(2)}$ is the value obtained by taking all convolutions with no two particles from the same event. For $N_{ev} \gg 1$, the difference between the proposed procedure and the usual one is small. This

change in procedure is even more important for measuring higher-order correlations, as the construction of ρ_{uc} by the usual event mixing quickly becomes computationally prohibitive, while the simple convolution of ρ is almost always feasible.

In the second method, referred to as charge mixing, $\rho_{uc}^{(2)}$ is constructed using particles that are identical except for charge. For example,

$$\rho_{uc}^{(2)}(p_1, p_2) = \rho^+(p_1) \rho^-(p_2), \quad (42)$$

where ρ^\pm is the π^\pm density, is often used for pion interferometry. As charge mixing does not require information from more than one event, it is the preferred normalization technique for analysis of single events.

Consider the analysis of a single event. The mean number of π^\pm mesons seen in a spectrometer covering ΔY units of rapidity and $\Delta\Phi$ rad of azimuthal angle is

$$N^\pm = \frac{(dN/dy) \Delta Y \Delta\Phi}{4\pi}, \quad (43)$$

if particles are spread randomly in y and ϕ with a uniform distribution. I imagine an ideal spectrometer that detects all pions passing through it, independent of transverse momentum, p_T . For simplicity, I assume a thermal-type p_T distribution:

$$\mathcal{P}(p_T) = \frac{4p_T}{\langle p_T \rangle^2} e^{-2p_T/\langle p_T \rangle}, \quad (44)$$

where $\mathcal{P}(p_T) dp_T$ is the probability that a given pion has transverse momentum between p_T and $p_T + dp_T$, and $\langle p_T \rangle$ is the mean p_T .

For a spectrometer large enough that edge effects are unimportant ($\Delta Y, \Delta\Phi \gg q/\langle p_T \rangle$), the standard deviation of $C_2(q)$ is

$$\sigma_C = \frac{16\pi}{(dN/dy) \Delta Y \Delta\Phi} \left\{ \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1 + \frac{\pi}{2(dN/dy) q \delta q \langle I \rangle} \right\} \quad (45)$$

in the high-multiplicity limit, where

$$\begin{aligned} \langle I^n \rangle &= \int_0^\infty dp_{T,1} \mathcal{P}(p_{T,1}) \\ &\times \left[\int_0^\infty dp_{T,2} \mathcal{P}(p_{T,2}) \int_{-\infty}^\infty dy \int_0^{2\pi} d\phi \delta(q^2 + 2m^2 + 2p_{T,1}p_{T,2} \cos \phi - 2\sqrt{(p_{T,1}^2 + m^2)(p_{T,2}^2 + m^2)} \cosh y) \right]^n. \end{aligned} \quad (46)$$

Here $(\int_{q=0}^{q'} dq^2)^n \langle I^n \rangle / (\Delta Y \Delta\Phi)^n$ is the probability that n given particles each have momenta within q' of a particle at $y = \phi = 0$, with transverse momentum distribution $\mathcal{P}(p_{T,1})$. The primary interest is in small q , and $m^2 \ll \langle p_T \rangle^2$, so I evaluate $\langle I^n \rangle$ for $q^2 = m^2 = 0$, obtaining

$$\langle I \rangle \approx \frac{[\Gamma(1/4)]^2 \sqrt{\pi}}{2m \langle p_T \rangle}, \quad (47)$$

$$\langle I^2 \rangle \approx \frac{[\Gamma(1/4)]^4}{m^2 \langle p_T \rangle^2}, \quad (48)$$

and thus

$$\sigma_C = \frac{16\pi}{(dN/dy) \Delta Y \Delta\Phi} \left\{ \frac{4}{\pi} - 1 + \frac{\sqrt{\pi} m \langle p_T \rangle}{[\Gamma(1/4)]^2 (dN/dy) q \delta q} \right\}. \quad (49)$$

The first term of Eq. (49) dominates as long as

$$q^2 \gg \frac{3\sqrt{\pi} m \langle p_T \rangle}{[\Gamma(1/4)]^2 (dN/dy)}, \quad (50)$$

where I have taken $\delta q = q$ for the smallest bin. Using $dN/dy = 1000$ and $\langle p_T \rangle = 500$ MeV, I find that the first term dominates as long as $q^2 \gg 30$ MeV², which is true for all practical measurements. Thus, the expected measurement error is

$$e_C \approx \frac{4}{\sqrt{(dN/dy) \Delta Y \Delta\Phi}}, \quad (51)$$

independent of the momentum or the bin size. As the

signal is approximately unity for $q = 0$, a 4σ determination is possible with a single event for any spectrometer with $\Delta Y \Delta\Phi > 256 (dN/dy)^{-1}$.

Quantum interference also produces two-particle rapidity correlations. These correlations are due to collective effects, so they have a different multiplicity dependence than the two-particle correlations discussed in the previous section. The momentum scale for quantum interference $q^* \approx 1/r$, where r is the size of the system, so the rapidity scale is

$$y^* \approx \frac{q^*}{\langle p_T \rangle} \approx \frac{0.4 \text{ fm}}{r}, \quad (52)$$

while the number of correlated pairs per particle is

$$k \approx \frac{q^{*2} \langle I \rangle (dN/dy)}{4\pi} \approx \frac{0.5 (dN/dy) \text{ fm}^2}{r^2}. \quad (53)$$

For simplicity, I assume that pairs are correlated if the momentum difference is less than q^* , and uncorrelated otherwise. Using Eqs. (20) and (24), I obtain

$$S_2^{\text{be}}(\delta y) \approx 1 + \frac{0.8 \text{ fm}^3}{r^3 \delta y^2}, \quad (54)$$

for $\Delta Y \gg \delta y \gg 1 \text{ fm}/r$, while Eq. (26) gives the maximum value,

$$S_2^{\text{be,max}} \approx 1 + \frac{1.25 \text{ fm}}{r}. \quad (55)$$

The two-particle correlation due to clusters is visible above S_2^{be} as long as $dN/dy < 1.25 n_c y^* r^3/\text{fm}^3$. Single-event measurements of S_2^{be} are in principle possible if $(dN/dy) \Delta Y > 40 r^2/\text{fm}^2$. Recent data [16] indicate that $r = 3 - 4 \text{ fm}$ for central Si collisions at 14.6 GeV/nucleon (approximately the Si radius); using $r = 7 \text{ fm}$ for U+U collisions at RHIC and LHC, a single-event measurement of S_2 may be possible. However, such a measurement is

very difficult technically, requiring rapidity resolution to $y^* \approx 0.06$.

V. HIGHER-ORDER CORRELATION FUNCTIONS

It is also possible that higher-order correlation functions might give better results (or more interesting results) than two-particle correlation functions. Higher-order correlation functions can in principle be used to determine three-body and higher-order interactions, and many experimenters have tried to use them for this purpose, although without much success [17, 18]. Alternatively, measuring higher-order correlation functions might provide a more accurate determination of the two-particle correlation function than can be obtained from a direct measurement.

To test these hypotheses, I construct scaled factorial moments (SFMs) [19], that also use the data more efficiently (although they reuse pairs of particles). For pedagogical purposes, I consider only the so-called exclusive SFMs,

$$F_i(\delta y; \Delta Y) = \left(\frac{\Delta Y}{\delta y} \right)^{i-1} \sum_{j=1}^{\Delta Y/\delta y} \frac{\int_{(j-1)\delta y}^{j\delta y} dy_1 \cdots dy_i \rho^{(i)}(y_1, \dots, y_i)}{N \cdots (N - i + 1)}. \quad (56)$$

Just as for S_2 , $\Delta Y/\delta y$ must be an integer; however, any two values of δy use some common phase space, so it is impossible to construct SFMs without overusing the available phase space.

To examine the feasibility of studying higher-order correlations, I extend my previous approximations to include three-body correlations, assuming N_t correlated triplets with the boost-invariant distribution $q_t(y_1 - y_2, y_1 - y_3)/\Delta Y$. For $\Delta Y \gg y^*$, the maximum value of F_i is

$$F_i^{\text{max}} \approx 1 + \left[\frac{i(i-1)}{2} \right] \frac{k}{(dN/dy)y^*} + \left[\frac{i(i-1)(i-2)}{6} \right] \frac{k_t}{[(dN/dy)y^*]^2}, \quad (57)$$

where $k_t = N_t/N$. For cluster decay, $k_t \approx n_c^2$ for $n_c \gg 1$, so

$$\left[\frac{i(i-1)(i-2)}{6} \right] \frac{k_t}{[(dN/dy)y^*]^2} \approx \frac{2(i-2)}{3i(i-1)} [F_i - 1]^2 \quad (58)$$

is the three-particle contribution to F_i , while $F_i - 1$ is the two-particle contribution.

The observed three-particle correlation decreases with increasing multiplicity as $(dN/dy)^{-2}$, even faster than the two-particle correlation. Equation (58) is apparently very general, as it holds for any values of n_c and y^* .

Given a distribution of values, corrections of order unity are likely; however, it is still probable that the difficulty of extracting the three-particle correlation increases very fast as the two-particle correlation function decreases. This is, in my opinion, the most likely reason for the failure of experimenters to extract significant three-particle correlations [17, 18] from their data, as virtually all two-particle correlations are approximately 10% or smaller.

The second hypothesis, that higher-order correlation functions might provide a more accurate determination of the two-particle correlations than direct measurement, seems to be true. Under the assumptions that I use to calculate R_2 ,

$$F_i(\delta y) = 1 + \left[\frac{i(i-1)}{2} \right] \frac{k}{dN/dy} [h(\delta y) - h(\Delta Y)], \quad (59)$$

where

$$h(z) = \frac{2}{z^2} \int_0^z dx (z-x) q(x) \approx q(z/3). \quad (60)$$

I calculate the error in the same manner as before, assuming that there are no correlations:

$$\sigma_{F_i} = \frac{i^2}{N_{\text{ev}} N} \left[1 + \mathcal{O} \left(\frac{\delta y}{\Delta Y} \right) \right]. \quad (61)$$

The statistical error in F_i is proportional to i , while the signal is proportional to $i(i-1)$, so s/n is proportional to $i-1$ and thus improves with increasing i . However, the above arguments are valid only when $(dN/dy) \delta y \gg i$, in which case most bins contain i particles, and even then apply only to statistical noise. In most cases, there is also

noise from undesired correlations produced by the detector, and this systematic noise is proportional to $i(i-1)/2$. It is thus possible that s/n is approximately independent of i .

It appears that measuring higher-order correlation functions is the most accurate way to determine two-particle correlations. Because SFMs reuse data, the cleanest approach is probably to use higher-order SBCFs [11]:

$$S_i(\delta y; \Delta Y) = \frac{\Delta Y^{i-1} \sum_{j=1}^{\Delta Y/\delta y} \prod_{n=1}^i \int_{[j-n/i]\delta y}^{[j-(n-1)/i]\delta y} dy_n \rho^{(i)}(y_1, \dots, y_i)}{\delta y^{i-1} \prod_{n=1}^i \int_{(n-1)\Delta Y/i}^{n\Delta Y/i} dy_n \rho^{(i)}(y_1, \dots, y_i)}. \quad (62)$$

Maximal use of the data without reuse is simple for S_2 , but not easily achieved for high-order SBCFs. As a result, it may be preferable to use some different generalization of S_2 for maximal efficiency. Higher-order correlation functions might also give better results for quantum interference measurements than the commonly used two-particle correlation functions.

VI. CONCLUSIONS

I have discussed four types of correlation measurements: hot spot searches, two-particle rapidity correlations, two-particle momentum correlations (for quantum interferometry), and higher-order correlation functions. Hot spot searches are most likely to be profitable in events of high energy but relatively low multiplicity. Two-particle rapidity correlations are most easily measured in events of relatively low multiplicity, if high-multiplicity events are just superpositions of lower-multiplicity events. A good measurement of S_2 at RHIC or LHC will require at least 800 events.

Single-event measurement of two-particle momentum correlations due to quantum interference is possible

in high-multiplicity events with spectrometer coverage $\Delta Y \Delta \Phi > 256 (dN/dy)^{-1}$, which should be easily attainable at RHIC and LHC. Rapidity correlations due to quantum interference are in principle measurable in single events at RHIC and LHC, but such measurements would be very difficult technically. Measuring higher-order correlation functions in high-multiplicity events gives little information about three-body and higher-order correlations. However, measuring higher-order correlation functions can give a more accurate determination of two-particle correlations than direct measurement of two-particle correlation functions.

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