Consistent dynamical and statistical description of fission of hot nuclei

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The modified approaches for a dynamical (Langevin) and a statistical description of fission of hot nuclei (based on entropy as the crucial quantity) proposed in a forthcoming publication are shown to give consistent values for fission rates in the framework of the liquid drop model for a wide range of excitation energies, angular momenta, and fissilities. An approximate formula for the statistical fission rate is derived which is more suitable for applications than the exact one. Contrary to the conventional approach the new statistical description takes into account the positions of the stationary points (ground state and saddle points) of the entropy (not of the potential), and the position of the scission point. The fission rate calculated with the approximate formula deviates in all cases of interest by less than 25% from the long-time limit of the dynamical (Langevin) fission rate. This is a clear improvement over the conventional model, where these deviations can be orders of magnitude.

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I. INTRODUCTION

In Ref. [1] we calculated neutron multiplicities, fission probabilities, and (HI, xn) cross sections by matching a dynamical (overdamped) Langevin equation to the standard (Bohr-Wheeler with Kramers modification) statistical model after a certain delay time. Later, we found an inconsistency in this description which we repair in a forthcoming publication [2]. Note that this inconsistency is not related to the long-time limit of the Langevin equation, which, of course, should agree with the Kramers stationary limit of the corresponding Fokker-Planck equation. The inconsistency we are dealing with has to do with the thermal properties of the system and can be made obvious by inspecting the equations of the previous dynamical approach and of the standard statistical model.

The dynamical description [1] used the overdamped Langevin equation, which reads

$$\frac{dq}{dt}=-\frac{1}{M\beta}\frac{dV}{dq}+\left[\frac{T}{M\beta}\right]^{1/2}\Gamma(t)\ . \tag{1}$$

Here q is half the distance between the centers of mass of the future fission fragments, $V(q)$ is the potential energy, M the mass, γ the friction coefficient, and $\beta=\gamma/M$ the reduced friction coefficient; β and M are treated as constants in the present investigation. The temperature is denoted by T, and $\Gamma(t)$ is a fluctuating force with $\langle\Gamma(t)\rangle=0$ and $\langle\Gamma(t)\Gamma(t')\rangle=2\delta(t-t')$. The fission rate can be calculated from Eq. (1) as

$$R_f(t)=\frac{1}{N_{\rm tot}-N_f(t)}\frac{dN_f(t)}{dt} \tag{2}$$

by counting the number of trajectories $N_f(t)$ which have reached the scission point at time t. $N_{\rm tot}$ is the total number of trajectories.

When switching to the statistical branch of the model, we used in previous work [1] the Kramers-modified Bohr-Wheeler expression $R_f^{\rm KBW}$ for the fission rate (overdamped case):

$$R_f^{\rm KBW}=\frac{\omega_{\rm sd}\omega_{\rm gs}}{\beta T}R_f^{\rm BW}\ , \tag{3}$$

where

$$R_f^{\rm BW}=\frac{1}{2\pi\hbar\rho_{\rm CN}}\int_0^{E^*_{\rm tot}-B_f}\rho_f(E^*_{\rm tot}-B_f-\epsilon)d\epsilon\ . \tag{4}$$

Here $R_f^{\rm BW}$ is the standard statistical Bohr-Wheeler fission rate expression [3], $\rho_{\rm CN}$ is the level density of the compound nucleus at the ground state (characterized by the level density parameter a_n), ρ_f is the level density at the saddle point (characterized by the parameter a_f), B_f is the fission barrier, and $E^*_{\rm tot}$ is the total excitation energy. The quantities $\omega_{\rm sd}$ and $\omega_{\rm gs}$ are the frequencies at the saddle point and at the ground state.

In the long-time limit, the rates calculated from Eqs. (1) and (3) should approach each other. However, in Eq. (1) any information on the level density parameters a_n and a_f is missing, whose ratio is often used as a fitting parameter when Eq. (3) is applied to describe data. On the other hand, there is no information on the scission point in Eq. (3). So one would expect agreement between the results of Eqs. (1) and (3) in the long-time limit only if the

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level density parameter and the scission point do not play any role, which is not generally the case.

In Ref. [2] we have removed this inconsistency and proposed an improved version of both the dynamical and statistical descriptions of the nuclear fission process and demonstrated in some model examples the superiority of the new approach as compared to the old one [1]. In order to make the present paper self-contained, we repeat in Sec. II how the inconsistency is removed. Whereas we discussed in Ref. [2] only model examples, we proceed in the present paper to the description of real nuclei. The input for calculations for real nuclei concerning the potential energy, the coordinate-dependent level density, and a derivation of an approximate formula for the modified statistical fission rate is the content of Sec. III. In Sec. IV we show and discuss the results of the calculations and draw some conclusions in Sec. V.

II. CONSISTENT DYNAMICAL AND STATISTICAL DESCRIPTION OF FISSION

In order to remove the inconsistency between the dynamical and statistical approaches discussed above, we have proposed [2] to modify the dynamical equation and the expression for the statistical decay rate in the following way.

Instead of Eq. (1) an equation of motion should be applied which is governed by the free energy F. The free energy is related to the coordinate-dependent level density parameter $a(q)$ by a formula valid for the Fermi gas model:

$$F(q,T)=V(q)-a(q)T^2 . \tag{5}$$

In this way information concerning the level density is introduced into the equation of motion, which now reads

$$\frac{dq}{dt}=-\frac{1}{M\beta}\left[\frac{\partial F}{\partial q}\right]\Bigg|_T+\left[\frac{T}{M\beta}\right]^{1/2}\Gamma(t) . \tag{6}$$

The driving force $K=-(\partial F/\partial q)|_T$ is given by the derivative of the free energy with respect to the fission coordinate at fixed temperature. We would like to stress the fact that it does not matter which thermodynamical quantities are used in the description. In the following we prefer to discuss the situation in terms of the entropy $S(E^*_{\rm tot},q)$ which is a function of q alone because of the fact that the total excitation energy $E^*_{\rm tot}(q,S)$, with its natural variables S and q, is constant. Then, using $F=E^*_{\rm tot}-TS$, we obtain the force $-\partial F/\partial q|_T = T\,\partial S/\partial q|_{E^*_{\rm tot}}$, and the Langevin equation reads

$$\frac{dq}{dt}=\frac{T}{M\beta}\left[\frac{\partial S}{\partial q}\right]\Bigg|_{E^*_{\rm tot}}+\left[\frac{T}{M\beta}\right]^{1/2}\Gamma(t) . \tag{7}$$

We now turn to the modification of the statistical model. We have derived in Ref. [2], following Ref. [19], an expression for the statistical decay rate as the inverse of a mean first-passage time (MFPT), which not only includes the coordinate-dependent level density parameter via the entropy, but also contains, contrary to Eq. (3), the position of the scission point $q_{\rm sc}$. Using Eq. (7), we repeat shortly the derivation of this formula for the fission decay rate.

The overdamped Langevin equation (7) is (for constant temperature) known to be strictly equivalent to the Smoluchowski equation for the distribution function $d(q,t)$:

$$\frac{\partial}{\partial t}d(q,t)=L(q)d(q,t) , \tag{8}$$

with the Fokker-Planck operator

$$L=-\frac{1}{M\beta}\frac{\partial}{\partial q}\left[T\frac{\partial S}{\partial q}\right]+\frac{1}{M\beta}T\frac{\partial^2}{\partial q^2} . \tag{9}$$

We obtain a rate formula from the inverse of a mean first-passage time $t_\Omega(q)$. The latter is calculated from the conditional probability density $P_\Omega(y,t;q,t=0)$ for reaching the point $y\in\Omega$ at t if the trajectory started at $t=0$ at $q\in\Omega$. Here Ω is the domain of the potential before the nucleus fissions. The probability density which defines the mean first-passage time by

$$t_\Omega(q)=\int_0^\infty dt\int_\Omega dy\,P_\Omega(y,t;q) \tag{10}$$

obeys the Smoluchowski equation (8) in the variable y for the fission process. Because we are interested in the variable q, we need the so-called backward equation which acts on the second variable q. This equation is governed by the adjoint Fokker-Planck operator L^+:

$$\frac{\partial}{\partial t}P_\Omega(y,t;q)=L^+(q)P(y,t;q) , \tag{11}$$

with

$$L^+=\frac{1}{M\beta}\left[T\frac{\partial S}{\partial q}\right]\frac{\partial}{\partial q}+\frac{1}{M\beta}T\frac{\partial^2}{\partial q^2} . \tag{12}$$

Acting with the adjoint operator L^+ on Eq. (10) and using Eq. (11) yields

$$L^+(q)t_\Omega(q)=-1 \tag{13}$$

or, explicitly,

$$\frac{dS}{dq}\frac{dt_\Omega(q)}{dq}+\frac{d^2t_\Omega(q)}{dq^2}=-\frac{M\beta}{T} . \tag{14}$$

Introducing $w(q)=dt_\Omega/dq$, the solution of the homogeneous part of the equation is given by $w(q)=C(q)\exp[-S(q)]$. The solution of the inhomogeneous equation is obtained by the method of variation of the constant $C(q)$, and applying the proper boundary conditions for fission, the reciprocal of this solution is interpreted as the fission rate $R_{\rm MFPT}$:

$$R_{\rm MFPT}=\frac{T}{\beta M}\left\{\int_{q_{\rm gs}}^{q_{\rm sc}}dy\,e^{-S(y)}\int_{-\infty}^{y}dz\,e^{S(z)}\right\}^{-1} . \tag{15}$$

The outer integration goes from the ground state $q_{\rm gs}$ to the scission point $q_{\rm sc}$ position.

By introducing the entropy in the description of hot nuclei instead of the bare potential, we have thus been able to derive a dynamical equation containing the information on the level density and a statistical decay rate formula which includes the position of the scission point. Thus the inconsistencies concerning formulas (1) and (3) are removed by replacing them by the physically more correct formulas (7) and (15).

III. INPUT FOR CALCULATIONS FOR REAL NUCLEI

In order to make calculations for real nuclei rather than for model examples as in Ref. [2], we have to specify the potential energy and the coordinate dependence of the level density parameter.

A. Potential energy surface

Because we are dealing with hot nuclei, where we think that shell corrections are of minor importance, we have chosen to use the liquid drop model in the version of Myers and Swiatecki [4]. The potential energy is given by

$$V(A,Z,L,q) = a_2(1-kI^2)A^{2/3}(B_s(q)-1) + c_3\frac{Z^2}{A^{1/3}}[B_c(q)-1]+c_rL^2A^{-5/3}B_r(q) . \quad (16)$$

Here we have dropped terms which do not depend on the deformation coordinate q. The parameters in Eq. (16) are specified to be [5]

$$a_2=17.9439 \text{ MeV}, \quad c_3=0.7053 \text{ MeV} , \quad k=1.7826, \quad c_r=34.50 \text{ MeV} . \quad (17)$$

We do not use finite-range corrections in the potential because in constructing the entropy we have to use also a formula for the level density parameter (Sec. II C), which for consistency should be on the same degree of accuracy as that for the potential, and we are not aware of level density studies taking into account finite-range effects.

Our calculations are based on the c, h, and α parametrization [6] for the quantities $B_s(q)$, $B_c(q)$, and $B_r(q)$, which depend on the deformation coordinate q. In the present paper, we only deal with symmetric fission ($\alpha=0$). If one assumes (as we do in this paper) that the motion is overdamped, the system follows the bottom of the fission valley, which then characterizes our one-dimensional potential. It turns out that the bottom of the fission valley is very close to the sequence of the saddle points of different nuclei, as shown in Fig. 1. As long as h_{sd} is a single-valued function of q_{sd}, we can parametrize B_s as a function of q in the form inserted in Fig. 2.

To find B_c we use an approximation [7] for the fission barrier B_f as function of the fissility:

$$\frac{B_f}{E_{ssp}} = \begin{cases} 0.2599-0.2151X-0.1643X^2 \\ \quad -0.0673X^3 \text{ if } X<0.6 , \\ 0.7259Y^3-0.3302Y^4+0.6387Y^5 , \\ +7.8727Y^6-12.0061Y^7 \text{ if } X>0.6 . \end{cases} \quad (18)$$

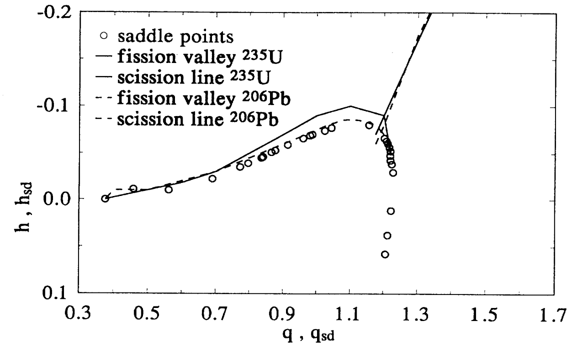

FIG. 1. Diagram shows the positions of the saddle points in the (h,q) coordinates calculated in the framework of the liquid drop model of Refs. [4,5] (open circles, from Ref. [20]). The bottom of the fission valleys and the scission lines are also shown for ^{235}U and ^{206}Pb; see Ref. [21].

Here X is the fissility parameter, $Y=1-X$, and E_{ssp} is the surface energy of a spherical nucleus with fissility X. The dependence of the saddle point position q_{sd} upon the fissility parameter, which completes our single-coordinate parametrization of the potential energy for zero angular momentum, is shown in Fig. 3.

The quantity $B_r(q)$ involved in Eq. (16) is proportional to the inverse rigid body moment of inertia and reads, in the (c,h) parametrization,

$$\begin{aligned} &B_r=J_{\parallel}^{-1} \text{ if } J_{\perp}<J_{\parallel} \text{ and } q>0.375 , \\ &B_r=J_{\perp}^{-1} \text{ in all other cases} , \\ &J_{\perp}=c^2\{1+c^{-3}+4B_{sh}[2c^3+\tfrac{4}{15}B_{sh}c^3-1]/35\}/2 , \\ &J_{\parallel}=c^2\{c^{-3}+4B_{sh}[\tfrac{4}{15}B_{sh}c^3-1]/35\} . \end{aligned} \quad (19)$$

Detailed formulas for the shape function $B_{sh}(c,h)$ and for our collective coordinate $q(c,h)$ as a function of c and h can be found in Ref. [7]. To reduce the two-dimensional

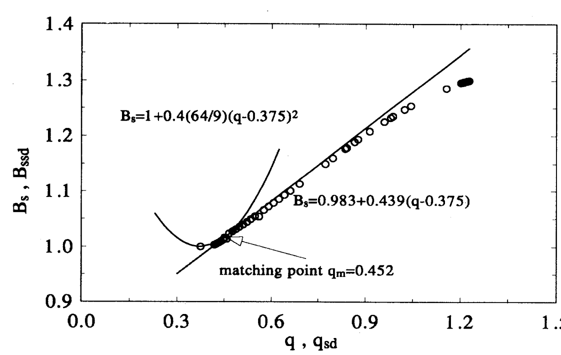

FIG. 2. Dimensionless coefficient of the surface energy for the saddle points of the liquid drop model (open circles) and the parametrization $B_s(q)$ (solid line and formulas) used in the dynamical calculation are displayed.

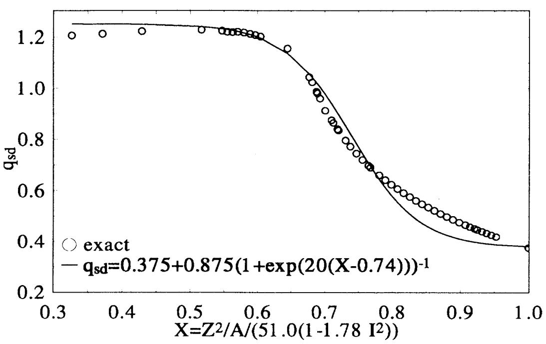

FIG. 3. Dependences of the distance between the centers of mass of the future fragments at the saddle points $q_{\rm sd}$ (open circles) on the fissility are shown; see Ref. [20]. Also shown is the corresponding parametrization (formula and solid line) used to construct B_c.

problem to our one-dimensional one, we use as simple parametrization the ansatz inserted in Fig. 4.

Looking at Figs. 1, 3, and 4, one realizes that our one-dimensional approach should break down for lighter nuclei because $h_{\rm sd}(q_{\rm sd})$, $q_{\rm sd}(X)$, and $h_{\rm sd}(c_{\rm sd})$ in this case cease to be single-valued functions. Therefore our one-dimensional parametrization works only if $X \geq 0.55$. This, however, does not cause problems in the actual calculations because such light nuclei fission mainly at high angular momenta, which effectively shifts the fissility to larger values.

The simple parametrizations above allow very quick calculations for all nuclei.

B. Coordinate dependence of the level density parameter

For an adequate description of the fission process, the coordinate-dependent level density parameter is as important as the potential energy surface. Leaving out the curvature corrections, the smooth part of the level density parameter as a function of q has the form [8]

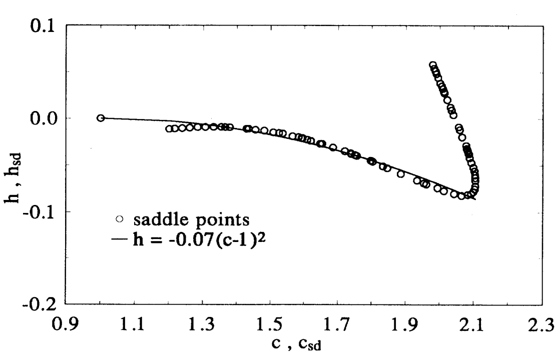

FIG. 4. $(h_{\rm sd}, c_{\rm sd})$ correlation of the saddle points (open circles, Ref. [20]) in the liquid drop model and the analytical ansatz (formula and solid line) are shown, by which we reduce the problem to a one-dimensional one.

$$a(q)=\tilde{a}_1 A+\tilde{a}_2 A^{2/3} B_s(q) . \tag{20}$$

Following Ref. [9], we select among the different possibilities the weakest coordinate dependence which is consistent with the data. This corresponds to [10]

$$\tilde{a}_1=0.073 \text{ and } \tilde{a}_2=0.095 . \tag{21}$$

For the demonstrative purpose of the present paper, the specific choice of $V(q)$ and $a(q)$ does not matter, but it will be certainly very important when we confront [11] our results with experimental data concerning fission probabilities, neutron multiplicities, and (HI, xn) cross sections following the lines of Ref. [1].

With a given choice of the level density and the potential, the entropy S is now specified as a function of mass, charge, angular momentum, coordinate, and total excitation energy:

$$S(A,Z,L,q,T)=2\sqrt{a(A,q)[E^*_{\rm tot}-V(A,Z,L,q)]} . \tag{22}$$

There is no further free adjustable parameter in the statistical and dynamical parts of the model with the exception of the reduced friction parameter β, which enters in both the statistical and dynamical approaches and which we expect to be universal for all nuclei. Thus our model goes beyond the standard macroscopic statistical model approach, in which a_n, a_f/a_n, and the fission barrier B_f are treated as adjustable parameters; for reviews, see, e.g., Refs. [12,13]. Our approach should also have consequences when comparing to former dynamical treatments; see, e.g., Refs. [1,14–17].

C. Approximate formula for the statistical fission rate

In the case of combining dynamical calculations with the statistical model, along the lines of Ref. [1], it is impossible (for reasons of computer time) to use Eq. (15) as it stands, because one would have to calculate the double integrals in Eq. (15) many times, when entering the statistical branch of the model. To avoid this we replace Eq. (15) by an approximate expression which is derived by quadratic expansions of the entropies in Eq. (15) after having extended the inner integration to infinity ($y \rightarrow \infty$) and setting in the outer integration $q_{\rm gs} \rightarrow -\infty$. The resulting approximate expression for the decay rate is

$$\begin{aligned} R_{\rm app}=&\frac{\tilde{\omega}_{\rm gs}\tilde{\omega}_{\rm sd}}{2\pi\beta}\exp[-2\{a(q_{\rm gs})(E^*_{\rm tot}-V(q_{\rm gs}))\}^{1/2}] \\ &\times\exp[2\{a(q_{\rm sd})(E^*_{\rm tot}-V(q_{\rm sd}))\}^{1/2}] \\ &\times 2\{1+{\rm erf}[(q_{\rm sc}-q_{\rm sd})\tilde{\omega}_{\rm sd}\sqrt{M/2T}\,]\}^{-1} , \end{aligned} \tag{23}$$

where

$${\rm erf}(x)=2/\sqrt{\pi}\int_0^x dt\,\exp(-t^2)$$

is the error function. The saddle point and the ground state positions are defined by the stationary points of the entropy and not, as in the conventional approach, by the potential energy. Also, the frequencies $\tilde{\omega}_{\rm gs}=\sqrt{|S''|_{\rm gs}T/M}$ and $\tilde{\omega}_{\rm sd}=\sqrt{S''_{\rm sd}T/M}$ are now calculated from the second derivative of the entropy at the stationary points. From this formula the influence of the scission point is clear: If the scission point is far away from the saddle point, the error function goes to unity and the third line of Eq. (23) also goes to unity. If saddle point and scission point coincide, the error function is zero, which leads to an enhancement by a factor of 2 as compared to the situation when the scission point is far away from the saddle point. We should mention that in the Langevin calculations we use a coordinate-dependent temperature, whereas in the approximate formula the temperature at the stationary points should be used. There is, however, no significant change in the rates if we use the temperature at the saddle or at the ground state.

The well-known expression of Kramers [18], R_K, for the overdamped case is obtained from Eq. (23) if the level density is independent of the coordinate and the scission point is far away from the saddle point and $E^*_{\rm tot} \gg V(q_{\rm sd})$:

$$R_K = \frac{\omega_{\rm gs}\omega_{\rm sd}}{2\pi\beta}\exp\{[-V(q_{\rm sd})+V(q_{\rm gs})]/T\} \,, \qquad (24)$$

where $\omega_{\rm gs}=\sqrt{V''_{\rm gs}/M}$ and $\omega_{\rm sd}=\sqrt{|V''|_{\rm sd}/M}$ are now related to the curvatures of the potential at the stationary points.

IV. RESULTS AND DISCUSSION

We start with discussing results of the calculations by inspecting Fig. 5. In the conventional theory [18,19], the crucial quantity which determines the fission rate is the difference of the potentials at saddle and ground state divided by the temperature [see Eq. (24)], whereas in the new approach this role is played by the difference in the corresponding entropies such as in the more developed statistical model [10]. Therefore we compare in Fig. 5(a) the potential energy $V(q)$ over the temperature with the negative value of the entropy $-S(q)$ (the entropy is normalized to be zero for the ground state of nonrotating nuclei) as functions of the deformation coordinate q for the example ^{219}Ac. One observes not only a change in the barrier heights, but also a shift of the stationary point (related to the saddle point configuration) of the entropy in comparison to that of the potential due to the coordinate dependence of the level density parameter. This explains (in addition to the influence of the scission point [see Eqs. (15) and (23)]) the difference of the results of the modified model [Eqs. (7) and (19) or (23)] as compared to the standard one [cf. Eq. (3)].

In Fig. 5(b) some Langevin trajectories are plotted as functions of time. In all following examples, we use a reduced friction parameter $\beta = 15\times10^{21}$ sec^{-1}, which corresponds to the overdamped case. One observes, for instance, that a fissioning trajectory (dotted line) can cross the saddle point position $q_{\rm sd}$ several times (not only once) before it decides to fission. Following Ref. [6], it turns out that the scission takes place at essentially the same position ($q_{\rm sc}\simeq 1.19$) for all nuclei. This is the position where the collective motion stops to stretch and the neck shrinks. If, as is the case for lighter nuclei (such as Pb or Os), the scission point is close to the saddle point, one can

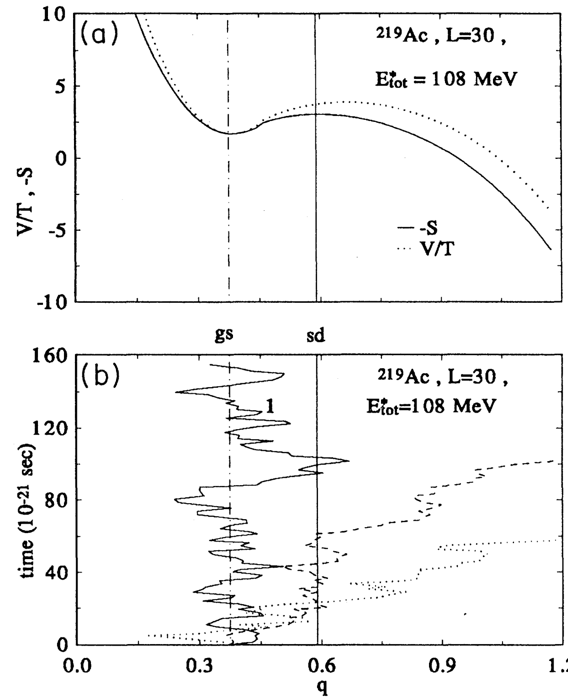

FIG. 5. (a) Potential divided by the temperature, V/T (dotted line), and the negative entropy $-S$ (solid line, normalized to be zero for the ground state of nonrotating nuclei) as a function of the separation coordinate are plotted for ^{219}Ac and angular momentum $L=30$ and $E^*_{\rm tot}=108$ MeV. (b) Three typical Langevin trajectories for this case are shown. The vertical lines characterize the positions of the ground state (gs) and saddle (sd) of the entropy.

according to the above behavior of the Langevin trajectory understand the importance of the scission point in this case. Whereas a trajectory (labeled 1 in the figure) for a light nucleus, where $q_{\rm sd}\simeq q_{\rm sc}\simeq 1.19$, now is counted as a fissioning one (because it crosses $q_{\rm sc}$), it is not counted as fissioning in the standard statistical approach. This explains why in the improved approach [see Eq. (23)] the fission rate is enhanced as compared to the conventional model in cases where the scission point is close to the saddle point.

We show as a further example the potential over the temperature and the negative entropy as function of q in Fig. 6(a) for the system ^{209}Bi. The corresponding forces governing the old $(-dV/dq)$ and modified $(T\,dS/dq)$ dynamical descriptions are also shown in Fig. 6(b). A shift of the stationary position of the saddle point configuration due to the coordinate dependence of the level density is again clearly observed.

In Fig. 7(a) the superiority of the modified description over the old one is demonstrated. In the old description, the stationary value of the dynamical rate $R_{V{\rm dyn}}$ calculated with Eq. (1) deviates an order of magnitude from the Kramers-modified Bohr-Wheeler rate $R_f^{\rm KBW}$ [Eq. (3)]. In the modified description, the stationary value of the dynamical rate $R_{\rm dyn}$ [from Eq. (7)] is close to the new statistical model rate $R_{\rm MFPT}$ [Eq. (15)]. In the present exam-

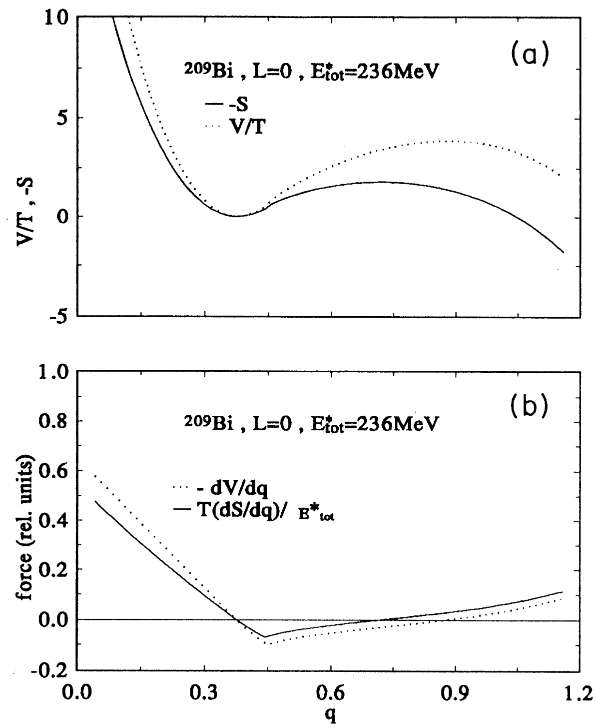

FIG. 6. (a) Same as in Fig. 5(a) but for ^{209}Bi at $L=0$ and $E^*_{\rm tot}=236$ MeV. (b) The corresponding forces calculated from the potential $(-dV/dq)$ and the entropy $(TdS/dq|_{E^*_{\rm tot}})$ as function of the separation coordinate q.

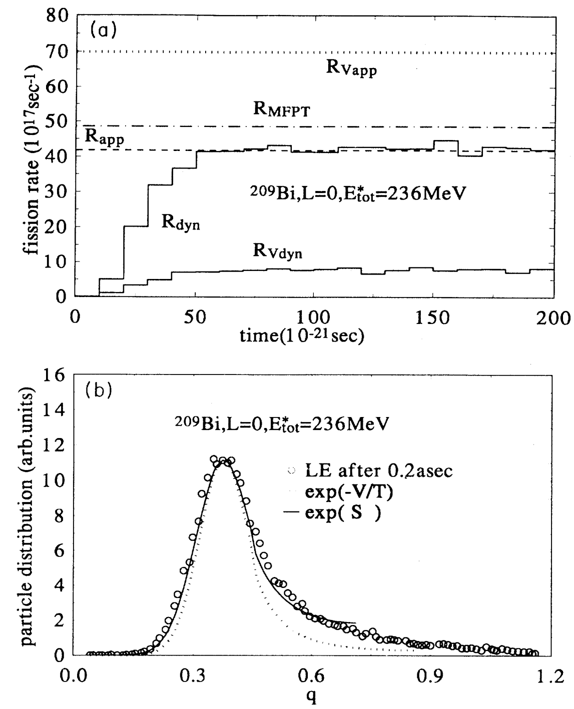

FIG. 7. (a) Fission rates of the conventional model $R_{V{\rm dyn}}$ [Eq. (2)], $R_{V{\rm app}}$ [Eq. (3)] are compared to the results of the modified model: $R_{\rm dyn}$ [Eq. (7)], $R_{\rm MFPT}$ [Eq. (15)], and $R_{\rm app}$ [Eq. (23)] for the example of Fig. 6. (b) The distribution of particles from the Langevin calculation after a long time $(2\times10^{-19}$ sec, open circles) are compared to the equilibrium distribution using the potential (dotted line) and the entropy (solid line). The curves are normalized to each other at the peak position.

ple, the approximate rate $R_{\rm app}$ [Eq. (23)] even agrees with the long-time limit of the Langevin calculation; this, however, is accidental (see the discussion following Fig. 8).

The fact that in the example above the rate $R_{\rm MFPT}$ is slightly higher than the stationary value of the dynamical rate $R_{\rm dyn}$ is somewhat surprising because we have observed in another example in Ref. [2] that $R_{\rm dyn}>R_{\rm MFTP}$ (see in this respect also Fig. 10). In order to understand at least qualitatively that both these situations can occur, let us discuss the average time of the fission process. This is composed of three times: (i) the time during which the equilibrium fluctuations around the ground state are established τ_r, (ii) the average lifetime of the system in the vicinity of the ground state, τ_q, and (iii) the average descent time from saddle to scission, τ_s. Not all fission events reach all three times, because there is a certain probability P_1 that a fission event occurs during τ_r, thus avoiding τ_q. But all fission events come through τ_s. The average fission time then is of the form

$$\langle t\rangle=\tau_r+(1-P_1)\tau_q+\tau_s\ . \tag{25}$$

If we now identify $\langle t\rangle$ with the mean first-passage time whose inverse is the rate $R_{\rm MFPT}$ [Eq. (19)] and identify the stationary value of the dynamical rate with $R_{\rm dyn}=(\tau_q+\tau_s)^{-1}$, we find

$$\frac{R_{\rm MFPT}}{R_{\rm dyn}}=\frac{\tau_q+\tau_s}{\tau_r+(1-P_1)\tau_q+\tau_s}=\frac{1}{1-(P_1-\tau_r/\tau_q)[1/(1+\tau_s/\tau_q)]}\ . \tag{26}$$

Now it is obvious that if $P_1<(\tau_r)/\tau_q$ one finds $R_{\rm MFPT}<R_{\rm dyn}$, or alternatively one has $R_{\rm MFPT}>R_{\rm dyn}$ if $P_1>(\tau_r)/\tau_q$. From this qualitative discussion, it is clear that both situations are possible (see the actual examples in Fig. 10).

In order to compare the new with the old approach, it is also instructive to look at the distribution of particles in the vicinity of the ground state after a long time $(2\times10^{-19}$ sec). In Fig. 7(b) we compare the distribution of the dynamical calculation [Eq. (5)] with the equilibrium distributions using the potential energy and entropy, respectively. Only the equilibrium distribution with the entropy is close to the dynamical one. Now we discuss the dependence of the rate of the dynamical long-time limit on the excitation energy, angular momentum and fissility. These results are compared with the "exact" [Eq. (15)] and approximate [Eq. (23)] modified statistical rate formulas.

In Fig. 8 we show the excitation energy dependence of the quasistationary limit $R_{\rm dyn}$ of the Langevin dynamics

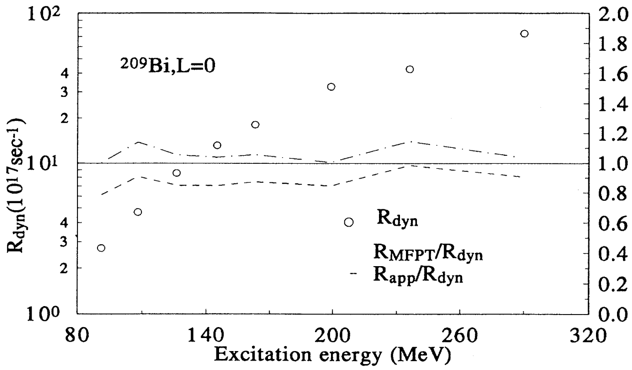

FIG. 8. Dynamically calculated fission rates $R_{\rm dyn}$ (open circles, left axis) and the ratios $R_{\rm MFPT}/R_{\rm dyn}$ and $R_{\rm app}/R_{\rm dyn}$ (lines, right axis) are shown for ^{209}Bi ($L=0$) as function of the excitation energy.

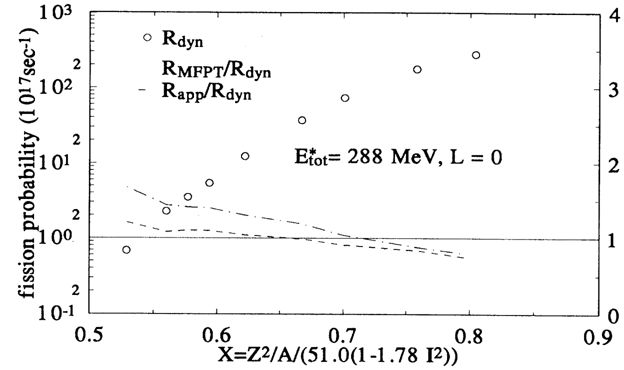

FIG. 10. Same as in Figs. 8 and 9 but as function of the fissility ($L=0$ and $E^*_{\rm tot}=288$ MeV).

which spans two orders of magnitude. We compare it to the statistical model rates $R_{\rm MFPT}$ and $R_{\rm app}$ by plotting the ratios $R_{\rm MFPT}/R_{\rm dyn}$ and $R_{\rm app}/R_{\rm dyn}$. The deviations of the statistical rates are of the order of less than 20%, and also $R_{\rm MFPT}$ and $R_{\rm app}$ deviate from each other by this amount. From Fig. 8 we can also see that the perfect agreement of $R_{\rm app}$ with $R_{\rm dyn}$ at the particular energy $E^*=236$ MeV [displayed in Fig. 7(a)] is purely accidental.

The same comparison for the different rate formulas is made in Fig. 9 for the angular momentum dependence for ^{209}Bi at $E^*=108$ MeV. Only at high angular momenta does one have more than 25% deviations. In calculations at high angular momenta along the line of Ref. [1], one would use the dynamical model alone and not switch over to the statistical branch.

In Fig. 10 we compare the different approaches as a function of the fissility using $l=0$ and $E^*=288$ MeV. For smaller fissilities we find that $R_{\rm MFPT}$ and $R_{\rm app}$ are larger than $R_{\rm dyn}$, whereas the opposite is the case for large fissilities. Again, the deviations from $R_{\rm dyn}$ for not too small fissilities are less than 25%. At small fissilities ($X \leq 0.55$), the model starts to be not very accurate, as already discussed in Sec. III A.

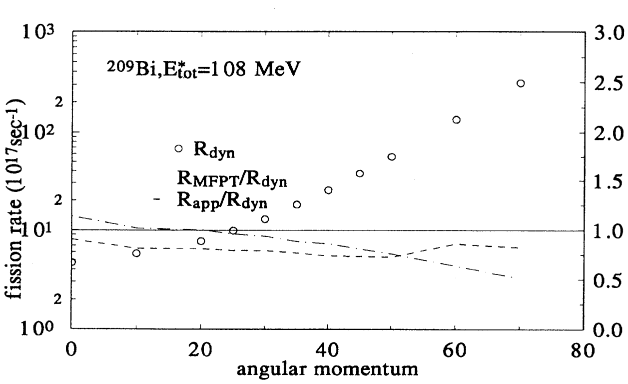

FIG. 9. Same as in Fig. 8 but as a function of angular momentum (^{209}Bi, $E^*_{\rm tot}=108$ MeV).

V. CONCLUSIONS

In order to describe heavy-ion-induced fission of hot nuclei, we propose a model that combines a dynamical Langevin description with a statistical description which is obtained as the quasistationary limit of the dynamical treatment. The idea for such a treatment and preliminary calculations were published in Ref. [1]. In Ref. [2] we removed some inconsistencies of Ref. [1] by introducing information on the level densities into the dynamical equation and by including the scission point position in the formula for the statistical decay rate. This consistent description is possible by introducing the free energy or alternatively the entropy, constructed from a coordinate-dependent level density parameter, as the crucial quantity into the theory. In Ref. [2] we showed the superiority of the modified approach by some very schematic model examples.

In the present paper, we have performed a systematic investigation of the new approach, which can be considered as a phenomenological attempt to include the level densities in the Langevin equation, by considering real nuclei. Whereas in the conventional approach (Langevin dynamics governed by a potential versus Kramers-modified Bohr-Wheeler rate), the deviation between the stationary limit of the dynamical rate and the statistical rate can be an order of magnitude, we now find within the new approach that this deviation in all cases of interest (dependences on excitation energy, angular momentum, and fissility) is less than 25%.

Therefore we propose to apply the model of the present paper to calculations such as those in Refs. [14–17] and to systematic investigations, along the lines of Ref. [1], of fission probabilities, neutron multiplicities, and (HI, *xn*) cross sections of hot fissioning nuclei produced in heavy-ion collisions. Such investigations are in progress.

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