One and two broken pairs in the interacting boson model: High-spin states in ^{190,192,194}Hg

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The structure of high-spin states in the transitional nuclei ^{190,192,194}Hg is described in the framework of the interacting boson model with one and two broken pairs. The interacting boson model is extended to the physics of high-spin states by including selective noncollective fermion states through the successive breaking of (S,D) pairs. High-spin states are described in terms of broken pairs. In ^{190,192,194}Hg, yrast states up to $J=24\hbar$, including the positions of the first and second backbending, are reproduced by the calculation. Data on $E2$ transitions in the region of the first backbending are compared with model calculations.

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I. INTRODUCTION

The interacting boson model (IBM) [1] has been remarkably successful in the description of nuclear structure phenomena related to states of relatively low angular momentum in medium heavy and heavy nuclei. To apply the model to the physics of high-spin states, one has to include, in addition to bosons, part of the original shell-model space for valence nucleons. This is done by breaking the boson pairs to form selective noncollective fermion pairs. Various extensions of the IBM have been reported that include two-fermion states (one broken pair) in addition to bosons [2–6]. In Refs. [7–9] we have further extended the IBM to include two- and four-fermion noncollective states (one and two broken pairs). Such an approach has a clear advantage over more traditional models based on the cranking approximation in that all the calculations are performed in the laboratory frame and results can be directly compared with experimental data.

In the present article, we apply the interacting boson model with one and two broken pairs to the description of yrast states in the transitional nuclei ^{190,192,194}Hg. Both the positions of the first and second backbending are known in these nuclei, as well as data on $E2$ transitions in the region of first backbending. Some properties of high-spin states in Hg isotopes have been investigated in the cranked shell model [11], in the IBM with one broken pair [2qp (two quasiparticle) states] [3], and with two broken pairs (2qp and 4qp states) [8].

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II. THE MODEL

The model is based on the simplest version of the interacting boson model: the IBM-1 [1]. In the IBM-1 model a nucleus with $2N$ valence nucleons is described as a system of N bosons of angular momentum 0 (s bosons) and 2 (d bosons). The Hamiltonian of the system contains one- and two-body interactions. The bosons can be regarded as collective fermion pair states (correlated S and D pairs) that approximate the valence nucleon pairs. No distinction between protons and neutrons is made. The model is extended to the description of high-spin states by including selective noncollective fermion states through the breaking of the correlated S and D pairs. High-spin states are described in terms of broken pairs. A boson can break to form a noncollective fermion pair. The model space for an even-even nucleus with $2N$ valence nucleons is

$$|N\ \text{bosons}\rangle \oplus |(N-1)\ \text{bosons} \otimes 1\ \text{broken pair}\rangle \oplus |(N-2)\ \text{bosons} \otimes 2\ \text{broken pairs}\rangle\ .$$

Although, in general, the fermions in broken pairs occupy all the single-particle orbitals from which the bosons have been mapped, for the description of high-spin states close to the yrast line the most important are the unique-parity high-j orbitals ($g\frac{9}{2}$, $h\frac{11}{2}$, $i\frac{13}{2}$). The one and two broken pairs are represented by two- and four-quasiparticle states, respectively. The model Hamiltonian is [7–9]

$$H = H_B + H_F + V_{BF} + V_{\text{mix}}\ , \quad (1)$$

where H_B is the IBM-1 boson Hamiltonian [1],

$$H_B=\epsilon\hat{n}_d+\sum_{L=0,2,4}\frac{1}{2}\sqrt{2L+1}C_L[(d^\dagger\times d^\dagger)^{(L)}\times(\tilde{d}\times\tilde{d})^{(L)}]^{(0)}$$
$$+\frac{1}{\sqrt{2}}V_2\{[(d^\dagger\times d^\dagger)^{(2)}\times(\tilde{d}\times s)^{(2)}]^{(0)}+\mathrm{H.c.}\}+\frac{1}{2}V_0\{[(d^\dagger\times d^\dagger)^{(0)}\times(s\times s)^{(0)}]^{(0)}+\mathrm{H.c.}\}\ . \tag{2}$$

The fermion Hamiltonian H_F contains the single-fermion energies and fermion-fermion interactions,

$$H_F=\sum_\alpha E_\alpha a_\alpha^\dagger\tilde{a}_\alpha+\frac{1}{4}\sum_{abcd}\sum_{JM}V_{abcd}^J A_{JM}^\dagger(ab)A_{JM}(cd)\ . \tag{3}$$

The boson-fermion coupling contains three terms,

$$V_{BF}=\Gamma_0\sum_{j_1j_2}(u_{j_1}u_{j_2}-v_{j_1}v_{j_2})\langle j_1\|Y_2\|j_2\rangle(a_{j_1}^\dagger\times\tilde{a}_{j_2})^{(2)}\cdot\hat{Q}_B^{(2)}$$
$$-\Lambda_0 2\sqrt{5}\sum_{j_1j_2j_3}\frac{1}{\sqrt{2j_3+1}}(u_{j_1}v_{j_3}+v_{j_1}u_{j_3})(u_{j_2}v_{j_3}+v_{j_2}u_{j_3})$$
$$\times\langle j_3\|Y_2\|j_1\rangle\langle j_3\|Y_2\|j_2\rangle:[(a_{j_1}^\dagger\times\tilde{d})^{(j_3)}\times(\tilde{a}_{j_2}\times d^\dagger)^{(j_3)}]^{(0)}:$$
$$+A_0\sqrt{5}\sum_j(2j+1)(a_j^\dagger\times\tilde{a}_j)^{(0)}\cdot(d^\dagger\times\tilde{d})^{(0)}\ , \tag{4}$$

representing the dynamical, exchange, and monopole interactions of the interacting boson-fermion model (IBFM) [10], respectively. V_{mix} is the pair-breaking interaction that mixes states with different number of fermions, conserving only the total nucleon number,

$$V_{\mathrm{mix}}=-U_0\sum_{j_1j_2}u_{j_1}u_{j_2}(u_{j_1}v_{j_2}+u_{j_2}v_{j_1})\langle j_1\|Y_2\|j_2\rangle^2\frac{1}{\sqrt{2j_2+1}}[(a_{j_2}^\dagger\times a_{j_2}^\dagger)^{(0)}\cdot s]$$
$$-U_2\sum_{j_1j_2}(u_{j_1}v_{j_2}+u_{j_2}v_{j_1})\langle j_1\|Y_2\|j_2\rangle[(a_{j_1}^\dagger\times a_{j_2}^\dagger)^{(2)}\cdot\tilde{d}]+\mathrm{H.c.} \tag{5}$$

III. HIGH-SPIN STATES IN ^{190,192,194}Hg

The isotopes ^{190,192,194}Hg are located in a transitional region which, in the IBM representation, corresponds to a change from the O(6) to the SU(3) symmetry limit. The structure of these weakly oblate and γ soft nuclei has been extensively investigated in the framework of the cranked shell model [11]. Yrast states up to $J\approx 20\hbar$ in ^{194,196,198}Hg have been described in the interacting boson model with one broken pair (2qp states) [3]. For ^{194}Hg positive-parity states up to $J=24\hbar$, including the positions of the first and second backbending, and low-lying negative-parity bands have been described in the interacting boson model with one and two broken pairs (2qp and 4qp states) [8]. Properties of yrast states of some even and odd Hg isotopes (195–198) have also been calculated by coupling one- and two-quasiparticle states to a Bohr-Mottelson core [12]. The yrast bands of the even Hg isotopes reveal an interesting anomaly around angular momenta 8^+, 10^+, and 12^+. The energies of these states are distributed in an interval of ≈ 150 keV. If one plots the moment of inertia as a function of angular velocity for the yrast states, a very strong backbending is observed. The moment of inertia increases by a factor of 20 for the triplet of states 8^+, 10^+, and 12^+ as compared to the ground-state band. This anomaly is explained by the alignment of a $i\frac{13}{2}$ neutron pair [3,8,11,12]. The experimental values of the g factors of the 12_1^+ states confirm the $(\nu i\frac{13}{2})^2$ structure. A second backbending is observed at $J=20\hbar$.

The low-lying spectra of $^{190-196}$Hg are very similar. The ground-state bands are identical to within ≈ 30 keV up to the highest observed collective states 6^+. We have therefore used the same set of parameters of the boson Hamiltonian (2) for all three isotopes ^{190,192,194}Hg. Their values are $\epsilon=0.2775$, $C_0=0.6082$, $C_2=0.2065$, $C_4=0.2013$, $V_2=-0.0219$, and $V_0=-0.2080$ (all values in MeV). The number of bosons is $N=7$ for ^{194}Hg, $N=8$ for ^{192}Hg, and $N=9$ for ^{190}Hg. The yrast two-quasiparticle structures are based on the states 8_1^+. The single-particle orbital, particularly important for the description of 2qp and 4qp states close to the yrast line, is $\nu i\frac{13}{2}$. Because of the large size of the 4qp space (more than 3000 vectors for ^{190}Hg with only $i\frac{13}{2}$ included), we have not been able to include other neutron orbitals in the model space. The occupation probabilities are obtained by a standard BCS calculation with single-particle energies taken from Kisslinger and Sorensen [13] and the pairing strength $G=22/A$ MeV. Their values are $v_{i13/2}^2(^{190}\mathrm{Hg})=0.66$, $v_{i13/2}^2(^{192}\mathrm{Hg})=0.74$, and $v_{i13/2}^2(^{194}\mathrm{Hg})=0.81$. The resulting gap parameters are $\Delta=0.93$, 0.88, and 0.81 MeV for ^{190}Hg, ^{192}Hg, and ^{194}Hg, respectively. With the Kisslinger-Sorensen choice of single-particle energies, the BCS calculation gives for the single-quasiparticle energy $E_{i13/2}\approx 1$ MeV. In order to obtain better agreement with the experimental energies, we take $E_{i13/2}=1.2$ MeV for all three isotopes. The parameters of the boson-fermion interaction [Eq. (4)] are $\Gamma_0=0.45$ MeV, $\chi=0.4$, and $A_0=0$ for all three isotopes. The strength parameter of the exchange interaction is $\Lambda_0=1.0$ MeV for ^{190,192}Hg and $\Lambda_0=2.0$ MeV for ^{194}Hg. For the residual interaction between fermions [Eq. (3)], we take the surface δ interaction with strength parameter $v_0=-0.2$ MeV. The parameters of the mixing interaction are $u_2=0.2$ MeV for all three isotopes; $u_0=1.3$ MeV for ^{190}Hg, $u_0=2.0$ MeV for ^{192}Hg, and $u_0=3.0$ MeV for ^{194}Hg.

In Fig. 1 we present the result of the model calculation of positive-parity states for ^{190}Hg. Only the few lowest calculated levels (circles) of each angular momentum J are shown in the figure. They are compared with experimental yrast states (squares). It is seen that the calculated spectrum reproduces in detail the positions of the yrast levels. The yrast states 0^+, 2^+, 4^+, and 6^+ belong to the collective ground-state band (GSB). The two-quasiparticle structure $(\nu i\frac{13}{2})^2$ is based on the triplet of isomeric states 8^+, 10^+, and 12^+. As we have shown in Ref. [8] for ^{194}Hg, these states are bandheads of three different $(\nu i\frac{13}{2})^2$ bands. The leading components in the wave functions of states belonging to these bands are

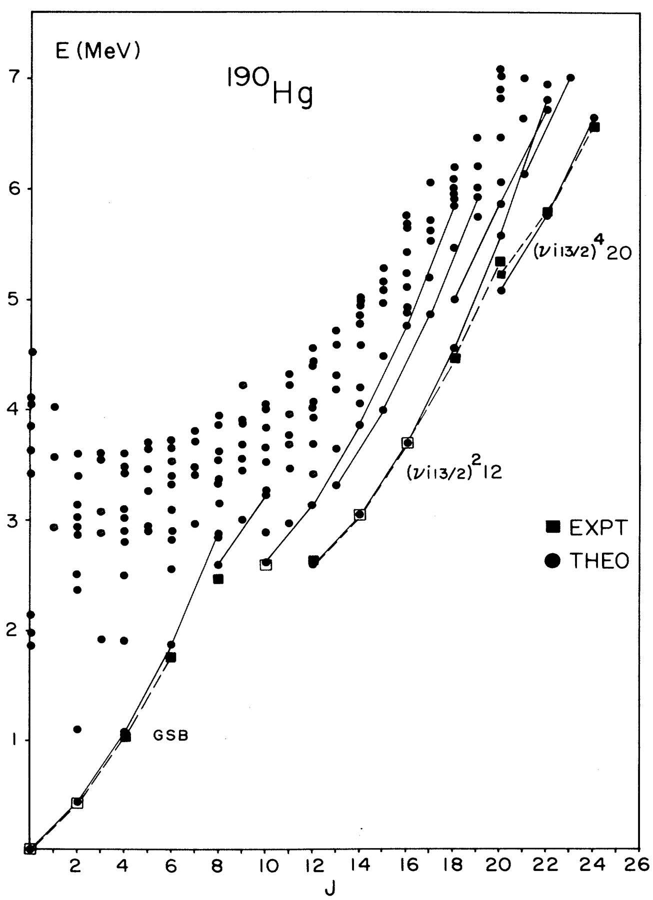

FIG. 1. Comparison between calculated (circles) and experimental (squares) positive-parity states in ^{190}Hg.

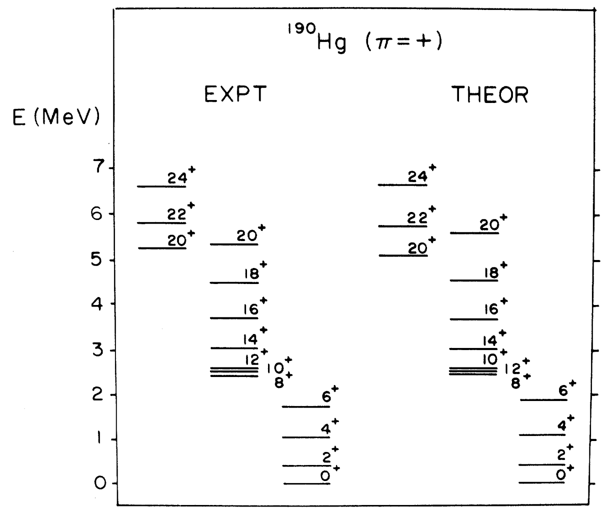

FIG. 2. Yrast levels in ^{190}Hg compared with the model calculation.

$|(\nu i\frac{13}{2})^2, J_F, J_B; J=J_F+J_B\rangle$, where J_F is the fermion pair angular momentum, $|J_B\rangle$ denotes a collective (boson) state of the ground-state band with angular momentum J_B, and J is the total angular momentum. The yrast 2qp band has $(\nu i\frac{13}{2})^2$ $J_F=12$ as the main components in the wave functions, and we also identify states that belong to the band $(\nu i\frac{13}{2})^2$ $J_F=10$. The state 8_1^+ is the bandhead of the strongly mixed band $(\nu i\frac{13}{2})^2 J_F=8$. Just above the yrast, we find the odd-spin band $|(\nu i\frac{13}{2})^2$ $J_F=12, J_B;$ $J=J_F+J_B-1\rangle$. The second backbending of the moment of inertia at $J=20_1^+$ is due to the alignment of a second pair of neutrons in the $i\frac{13}{2}$ orbital. The state 20_1^+ is the bandhead of the band $|(\nu i\frac{13}{2})^4$ $J_F=20, J_B;$ $J=20+J_B\rangle$. In Figs. 2, 3, and 4, we compare in a more usual form the results of the model calculation with ex-

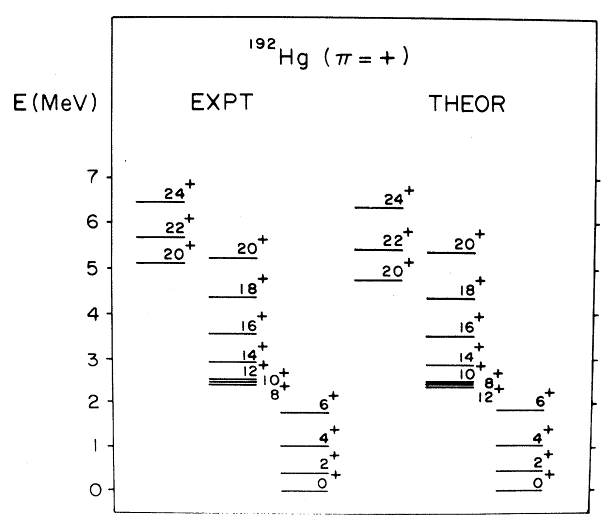

FIG. 3. Comparison between experimental and calculated yrast levels in ^{192}Hg.

perimental yrast levels for ^{190}Hg, ^{192}Hg, and ^{194}Hg, respectively. For all three isotopes, we obtain an excellent agreement with experimental data up to the highest 4qp states $J=24\hbar$. It is more difficult to obtain the correct ordering of the almost degenerate triplet 8_1^+, 10_1^+, and 12_1^+. In our calculation the dynamical boson-fermion interaction favors the state 12_1^+ to be the lowest member of this triplet.

A very sensitive test for our interpretation of the triplet of isomeric states is provided by the $B(E2)$ values for the transitions $12_1^+ \to 10_1^+$ and $10_1^+ \to 8_1^+$ [14,15]. The $E2$ transition operator has the form

$$T(E2)=\frac{3}{4\pi}e^{\mathrm{vib}}R_0^2[(d^\dagger\times\tilde{s}+s^\dagger\times\tilde{d})^{(2)}+\chi(d^\dagger\times\tilde{d})^{(2)}]$$
$$-e\frac{1}{\sqrt{5}}\sum_{j_1j_2}q_{j_1j_2}\left\{(u_{j_1}u_{j_2}-v_{j_1}v_{j_2})(a_{j_1}^\dagger\times\tilde{a}_{j_2})^{(2)}-\frac{u_{j_1}v_{j_2}}{\sqrt{N}}[(a_{j_1}^\dagger\times a_{j_2}^\dagger)^{(2)}\times\tilde{s}]^{(2)}+\frac{u_{j_2}v_{j_1}}{\sqrt{N}}[(\tilde{a}_{j_1}\times\tilde{a}_{j_2})^{(2)}\times s^\dagger]^{(2)}\right\}, \quad (6)$$

where

$$q_{j_1j_2}=\langle j_1\|r^2Y_2\|j_2\rangle$$

and we take $\langle r^2\rangle=\frac{3}{5}R_0^2$, $R_0=0.12A^{1/3}10^{-12}$ cm. For the neutron effective charge, we take $e=0.5$ and $\chi=0.4$ is the same value that is used in the boson quadrupole operator of the dynamical boson-fermion interaction [Eq. (4)]. e^{vib} should be fixed to reproduce the experimental value of $B(E2;2_1\to0_1)$. Unfortunately, $B(E2)$ values for transitions within the ground-state band are not known for ^{190,192,194}Hg. For ^{196}Hg and ^{198}Hg $B(E2;2_1\to0_1)\approx2000\ e^2\,\mathrm{fm}^4$. The ground-state bands in ^{190,192,194,196}Hg look very similar (the spectra are identical to within 30 keV) so that we can extrapolate the experimental value $B(E2;2_1\to0_1)$ from ^{196,198}Hg to ^{190,192,194}Hg. In order to obtain $B(E2;2_1\to0_1)\approx2000\ e^2\,\mathrm{fm}^4$, we take $e^{\mathrm{vib}}=1.0$ for ^{190}Hg, $e^{\mathrm{vib}}=1.5$ for ^{192}Hg, and $e^{\mathrm{vib}}=1.2$ for ^{194}Hg. In Table I we compare the experimental $B(E2)$ values [14,15] for the transitions $10_1^+\to8_1^+$ and $12_1^+\to10_1^+$ with results of the model calculation. The experimental values are much larger than those usually found in the region of backbending and especially if one thinks of them as describing interband transitions. The calculated $B(E2)$'s are of the correct order of magnitude, but are smaller than the experimental values. This indicates that in the model calculation we do not obtain enough mixing in the wave functions of these states. However, we remark that part of the discrepancy between calculated and experimental $B(E2)$'s might be accounted for by the uncertainty in $B(E2;2_1\to0_1)$. The best result is obtained for ^{192}Hg. The calculated $B(E2)$'s are just a factor of 2 smaller than the experimental values. In all three isotopes, we obtain

$$B(E2;10_1\to8_1)>B(E2;12_1\to10_1),$$

in agreement with experimental data.

In conclusion, we have analyzed the structure of high-spin states in the transitional nuclei ^{190,192,194}Hg in the framework of the interacting boson model with one and

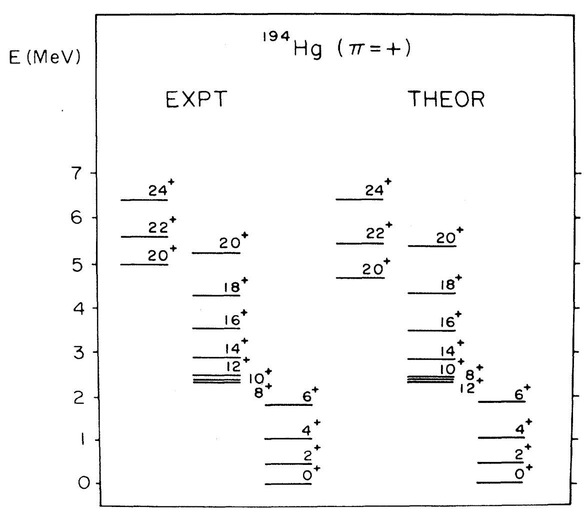

FIG. 4. Experimental yrast states in ^{194}Hg compared with results of the model calculation.

TABLE I. Reduced $E2$ transition rates in ^{190,192,194}Hg.

Transition	Isotope	Transition Energy (keV)	$B(E2)$ (e^2 fm^4) experiment [14,15]	$B(E2)$ (e^2 fm^4) theory
$12_1^+\to10_1^+$	190	23.9	640(70)	123
	192	28.4	1190(100)	523
	194	52.0	1601(133)	264
$10_1^+\to8_1^+$	190	131.9		521
	192	60.1	2750(470)	1561
	194	59.5	2068(400)	953

two broken pairs. Yrast states up to $J=24\hbar$, including the first and second backbending, are reproduced by model calculation. Comparison of $B(E2)$ values for transitions between the isomeric states 8_1^+, 10_1^+, and 12_1^+ with results of the model calculation confirms our interpretation of these states as being bandheads of three different $(\nu i\frac{13}{2})^2$ bands.

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