Coulomb instability of hot nuclei in quantum hadrodynamics

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Mean-field theory of quantum hadrodynamics is used to study the Coulomb instability of asymmetric nuclear matter at finite temperature. The critical temperature for the liquid-gas phase transition in nuclear matter and its dependence on an asymmetry parameter are calculated. The limiting temperature $T_{\rm lim}$, which reflects the Coulomb instability of hot nuclei is studied.

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I. INTRODUCTION

In spite of its simplicity, the mean-field theory (MFT) of the quantum hadrodynamics I (QHD-I) model of the quantum hadrodynamics is very successful in describing the properties of both nuclear matter and finite nuclei [1]. In a recent paper, Su and Qian [2] investigated the thermal fluctuation effects of meson fields on quantum hadrodynamics, by means of a real-time Green's-function method with pair cutoff approximation up to the second order. It was found that the fluctuation effects on saturation energy, effective mass of nucleon, and pressure are remarkable in low baryon density regions and/or high-temperature regions. On the other hand, however, the MFT provides us with a good approximation of equation of state in nuclear matter at low temperature (≤ 25 MeV). As a result, one obtains reasonable critical temperature $T_C = 20.56$ MeV [2] for liquid-gas phase transition in symmetrical nuclear matter and $T_C = 9.1$ MeV [1] for pure neutron matter. As pointed out by Levit and Bonche [3], another temperature, namely, limiting temperature $T_{\rm lim}$, is important for a finite nuclear system with Coulomb interaction. Below the limiting temperature $T_{\rm lim}$, the nucleus can exist in equilibrium with the surrounding vapor. But above $T_{\rm lim}$, the nucleus is unstable and shall fragment. This is the so-called Coulomb instability of hot nuclei. Recently, much effort [4,5] has been devoted to studying the Coulomb instability of asymmetric nuclear matter. But most such studies are based on the nonrelativistic treatment and the effective nucleon-nucleon interactions. It is, therefore, of interest to investigate such instability of hot nuclei in a relativistic approach. It is the first purpose of this paper to study the Coulomb instability of hot nuclei in QHD models. We will calculate the limiting temperature, starting from the MFT of the QHD models. It was noticed that the interactions between nucleons and the effective nucleon mass in the QHD-I model are all independent of isospin. As a result, although the limiting temperature determined by using the QHD-I model has values similar to those given by the nonrelativistic calculation with the SkI nucleon-nucleon interaction, the asymmetry parameter in the outside vapor phase is always negative. This fact is in contrast with either the Hartree-Fock (HF) calculation [15] or the previous nonrelativistic results [4,5]. In order to overcome this difficulty we take the ρ meson degree of freedom into account in the Lagrangian, i.e., we adopt the QHD-II model with the contribution from Higgs meson ignored. It was found that the mean-field theory of the QHD-II model is more reasonable than that of the QHD-I model for describing the properties of asymmetric nuclear matter.

The second motivation is to study the asymmetry dependence of the critical temperature T_c for liquid-gas phase transition. Most of the existing calculations of the critical temperature for the liquid-gas phase transition in nuclear matter are also based on nonrelativistic theories starting from effective nucleon-nucleon interactions, such as the Skyrme interaction [6–8], the Gogny interaction [9,10], etc. [11,12]. In such studies, it was found that the critical temperature for liquid-gas phase transition in nuclear matter decreases as asymmetry parameter α increases. We shall study the asymmetry dependence of the critical temperature for the liquid-gas phase transition in the QHD approach of nuclear matter and compare the obtained result with that calculated by using nonrelativistic theory and effective nucleon-nucleon interactions.

In Sec. II, we will briefly describe the MFT of QHD-I for bulk nuclear matter. A two-phase model and the coexistence equations are described in Sec. III. The numerical results given by the QHD-I model and some discussions are presented in Sec. IV. In Sec. V, we study the mean-field theory of the QHD-II model and apply it to calculating the same properties of nuclear matter as in the QHD-I model. We then give some conclusive remarks in Sec. VI.

II. BULK MATTER IN QHD-I

The degrees of freedom in QHD-I are baryons, scalar mesons, and vector mesons. The Lagrangian density is

$$L = \bar{\psi}[\gamma_\mu(i\partial^\mu - g_v V^\mu) - (M - g_s\phi)]\psi + \tfrac{1}{2}(\partial_\mu\phi\,\partial^\mu\phi - m_s^2\phi^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_v^2 V_\mu V^\mu \,, \quad (1)$$

where

$$F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu \ , \tag{2}$$

and ϕ and V_μ are, respectively, the neutral scalar-meson field and the neutral vector-meson field. In this model, the vector mesons (ω mesons) are coupled minimally to the conserved baryon current, and the scalar mesons (σ mesons) are coupled to baryons by a Yukawa coupling. Since there is repulsion between two baryons at short distances by ω-meson exchange and attraction at large distances by σ-meson exchange, the dominant feature of nuclear force can be simulated by this model. In mean-field approximation, the scalar and vector field operators are replaced by their expectation values. Here, we will just write down the main results of the MFT of QHD-I model for nuclear matter, because the details can be found in Refs. [1] and [2].

The single nucleon spectrum in nuclear matter is given by

$$E^*(k) = \sqrt{\mathbf{k}^2 + M^{*2}} \ , \tag{3}$$

where $\mathbf{k}$ and M^* are, respectively, the momentum and the effective mass of the nucleon. The effective mass M^* of the nucleon in nuclear matter is related to its bare mass through the equation

$$M^* = M - \frac{g_s^2}{m_s^2}\frac{\gamma_n}{(2\pi)^3}\int d^3k \, \frac{M^*}{\sqrt{\mathbf{k}^2+M^{*2}}}[n_n(k)+\bar{n}_n(k)] - \frac{g_s^2}{m_s^2}\frac{\gamma_p}{(2\pi)^3}\int d^3k \, \frac{M^*}{\sqrt{\mathbf{k}^2+M^{*2}}}[n_p(k)+\bar{n}_p(k)] \ . \tag{4}$$

Here $\gamma_p = \gamma_n = 2$ is the spin degeneracy. $n_g(k)$ and $\bar{n}_q(k)$ are the baryon and antibaryon thermal distribution functions, respectively:

$$n_q(k) = (\exp\{[E^*(k) - \nu_q]/k_B T\} + 1)^{-1} \ , \tag{5a}$$

$$\bar{n}_q(k) = (\exp\{[E^*(k) + \nu_q]/k_B T\} + 1)^{-1} \quad (q = n, p) \ , \tag{5b}$$

where $\nu_q \equiv \mu_q - (g_v^2/m_v^2)\rho$ with μ_q being the usual chemical potential. ν_q or μ_q is determined by the subsidiary conditions

$$\rho_q = \frac{\gamma_q}{(2\pi)^3}\int d^3k [n_q(k) - \bar{n}_q(k)] \quad (q = n, p) \ . \tag{6}$$

The neutron density ρ_n and proton density ρ_p are related to the total density ρ and asymmetry parameter α by relations

$$\rho_n = (1+\alpha)\rho/2, \quad \rho_p = (1-\alpha)\rho/2 \ . \tag{7}$$

Equations (3)–(7) form a closed set of equations for calculating the single nucleon spectrum $E^*(k)$, effective mass M^*, and chemical potentials μ_q $(q = n, p)$. Since the nucleon-nucleon interaction in this model is independent of isospin, the resultant single nucleon spectrum and effective mass are also independent of nucleon charge.

Having obtained the single nucleon spectrum $E^*(k)$ and the chemical potentials μ_q, one can easily calculate the thermodynamical potential Ω and then calculate all other thermodynamical quantities of the system. For example, the pressure is expressed as

$$p = \frac{g_v^2}{2m_v^2}\rho - \frac{g_s^2}{2m_s^2}(M - M^*)^2 + \frac{1}{3}\frac{\gamma_n}{(2\pi)^3}\int d^3k \, \frac{\mathbf{k}^2}{\sqrt{\mathbf{k}^2+M^{*2}}}[n_n(k)+\bar{n}_n(k)] + \frac{1}{3}\frac{\gamma_p}{(2\pi)^3}\int d^3k \, \frac{\mathbf{k}^2}{\sqrt{\mathbf{k}^2+M^{*2}}}[n_p(k)+\bar{n}_p(k)] \ . \tag{8}$$

III. MODEL AND COEXISTENCE EQUATIONS

Since the main purpose of this paper is to investigate the change in the Coulomb instability of hot nuclei when the QHD-I approach is used to describe the bulk nuclear matter instead of the usual nonrelativistic approach, we will adopt the same model as used in the nonrelativistic approach. Following Refs. [3]–[5], we consider the hot nucleus as a uniformly charged drop of nuclear liquid at a given temperature T and with a sharp edge, in both thermal mechanical and chemical equilibrium with the surrounding vapor. A set of two-phase coexistence equations is, therefore, obtained by requiring equality of temperature T, pressure p, neutron chemical potential μ_n, and proton chemical potential μ_p of the liquid and vapor phases:

$$p(T,\rho_L,\alpha_L) + p_{\text{Coul}}(\rho_L) + p_{\text{surf}}(T,\rho_L) = p(T,\rho_V,\alpha_V) \ , \tag{9}$$

$$\mu_n(T,\rho_L,\alpha_L) = \mu_n(T,\rho_V,\alpha_V) \ , \tag{10}$$

$$\mu_p(T,\rho_L,\alpha_L) + \mu_{\text{Coul}}(\rho_L) = \mu_p(T,\rho_V,\alpha_V) \ , \tag{11}$$

where subscripts L and V stand for liquid and vapor, respectively. In the liquid phase, the Coulomb and surface effects have been included.

In the MFT of QHD-I model for infinite nuclear matter, the Coulomb interaction is switched off and surface effect is not considered. When Coulomb interaction is added, the single nucleon spectrum given by Eq. (3) should be added by an additional Coulomb potential energy. For simplicity, we will use an average Coulomb potential per proton in a uniformly charged sphere:

$$V_{\text{Coul}}(\rho) = \frac{6}{5}\frac{Ze^2}{R} \ , \tag{12}$$

where Z and R are the charge number and radius of the liquid droplet. The exchange term of the Coulomb interaction has been neglected. When the Coulomb interaction is switched on, the chemical potential of proton has also an additional term $\mu_{\text{Coul}} = V_{\text{Coul}}$. The contribution of the Coulomb interaction to pressure is expressed as

$$p_{\text{Coul}}(\rho) = \frac{Z^2e^2}{5AR}\rho \ , \tag{13}$$

where $A = N + Z$ is the number of nucleons in the liquid droplet.

For the liquid droplet with a surface, we should also consider the surface effect on pressure. Following Refs. [4,5], the formula for the temperature dependence of the pressure tension $\gamma(T)$ suggested by Goodman, Kapusta, and Mekjian [13] is used:

$$\gamma(T)=(1.14\ \mathrm{MeV\,fm^{-2}})\left[1+\frac{3T}{2T_c}\right]\left[1-\frac{T}{T_c}\right]^{3/2}, \quad (14)$$

where T_c is the critical temperature for infinite symmetric nuclear matter. The additional pressure given by the surface tension of the liquid droplet is then

$$p_{\mathrm{surf}}(T,\rho)=-2\gamma(T)/R\ , \quad (15)$$

where nuclear density ρ is related to nuclear radius R by relation $A=\frac{4}{3}\pi R^3\rho$ for a given nucleon number A.

IV. RESULTS AND DISCUSSIONS

By using the formalism given in Secs. II and III, we can discuss the properties of nuclear matter at finite temperature. In the numerical calculation, we choose the coupling parameters

$$C_s^2\equiv(M^2/m_s^2)g_s^2=267.1,\quad C_v^2\equiv(M^2/m_v^2)g_v^2=195.9\ ,$$

which have been taken in the QHD-I model to reproduce the equilibrium properties of nuclear matter.

A. Infinite nuclear matter

For infinite symmetric nuclear matter, the equation of state (EOS) given by pressure-density (p-ρ) isotherms is equivalent to the one given by chemical-density (μ-ρ) isotherms, as pointed by Jaqaman, Mekjiam, and Zamick [6]. Therefore, either of them can be used to calculate the critical temperature T_c for the liquid-gas phase transition. The critical temperature T_c can be determined by the condition of the inflection point of p-ρ isotherms:

$$\left[\frac{\partial p}{\partial \rho}\right]_T=0 \quad \text{and} \quad \left[\frac{\partial^2 p}{\partial \rho^2}\right]_T=0\ ; \quad (16a)$$

or by the condition of the inflection point of μ-ρ isotherms:

$$\left[\frac{\partial \mu}{\partial \rho}\right]_T=0 \quad \text{and} \quad \left[\frac{\partial^2 \mu}{\partial \rho^2}\right]_T=0\ . \quad (16b)$$

In the case of asymmetric nuclear matter, the situation changes. As mentioned by Jaqaman, Mekjian, and Zamick [6] and Su and Lin [14], the proton and neutron are not in chemical equilibrium although they may be in thermal equilibrium. So their chemical potentials are not related to each other. Since the proton and neutron have different chemical potentials, they shall also appear to have different critical temperatures T_C^p and T_C^n, respectively. But we cannot imagine that the kind of nucleons with high critical temperature can stick together after all the other kind of nucleons with lower critical temperature have boiled off. We, therefore, must choose the lower of T_C^p and T_C^n as the correct critical temperature.

Due to the definition of the single-particle spectrum $E^*(k)$ in relativistic theories such as the QHD-I model, the resultant chemical potential μ has also included the still nucleon mass M. For the convenience in the comparison between the results here and those given by the nonrelativistic theories, we define a reduced chemical potential $\tilde{\mu}$ as $\tilde{\mu}\equiv\mu-M$. We shall show $\tilde{\mu}$-ρ isotherms instead of μ-ρ isotherms. Since the difference between μ and $\tilde{\mu}$ is only a constant $M=938$ MeV, the $\tilde{\mu}$-ρ isotherm has the same behavior as the corresponding μ-ρ isotherm. Instead of μ, we will discuss the reduced chemical potential $\tilde{\mu}$ in the following. For convenience, we will just call $\tilde{\mu}$ a chemical potential. We show the $\tilde{\mu}$-ρ isotherms for infinite symmetric nuclear matter ($\alpha=0$) at various temperatures in Fig. 1. One can see that each curve except the one at $T=20.2$ MeV exhibits a typical form of two-phase coexistence, with an unphysical region (say, between points A and B in the isotherm at $T=6$ MeV). The unphysical region gets smaller as temperature T increases. At a critical temperature $T_c=20.2$ MeV, the unphysical region disappears and there appears an inflection point satisfying the condition Eq. (16b). In Fig. 2, we present the $\tilde{\mu}$-ρ isotherms for infinite asymmetric nuclear matter with asymmetry parameter $\alpha=0.4$ and at various temperatures. In the case of asymmetric nuclear matter, the proton chemical potential $\tilde{\mu}_p$ and the neutron chemical potential $\tilde{\mu}_n$ separate, with $\tilde{\mu}_n$ moving up and $\tilde{\mu}_p$

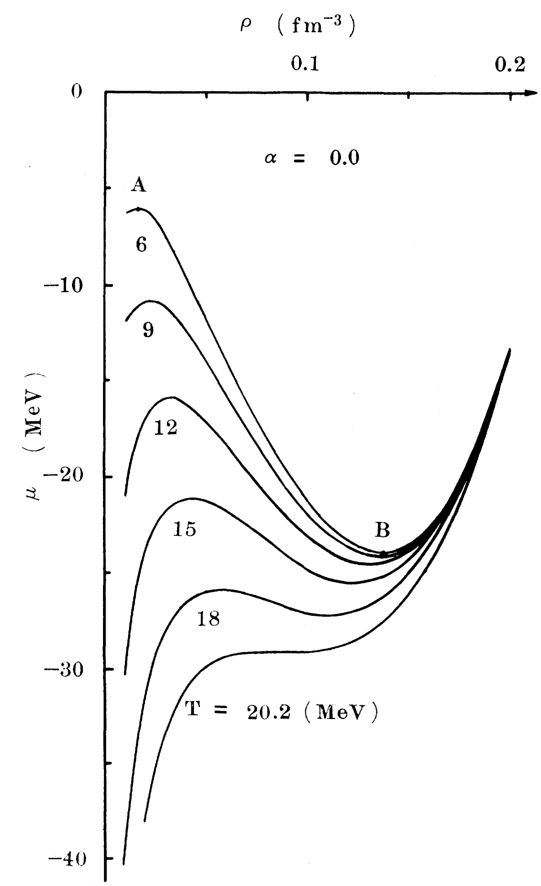

FIG. 1. $\tilde{\mu}$-ρ isotherms of infinite symmetric nuclear matter at various temperatures T (in MeV).

moving down compared to the chemical potential for symmetric nuclear matter (see Fig. 3). At lower temperatures, both of the $\tilde{\mu}_n$-ρ and $\tilde{\mu}_p$-ρ isotherms exhibit the form of two-phase coexistence. When temperature T increases, the unphysical regions in the two kinds of isotherms get smaller. We can then find a critical temperature T_c^n for neutrons and a critical temperature T_c^p for protons. The result for the asymmetry parameter $\alpha=0.4$ is $T_c^n=16.4$ MeV and $T_c^p=23.3$ MeV. As mentioned in the beginning of this subsection, we should choose the lower of the two critical temperatures, T_c^n as the correct critical temperature for asymmetric nuclear matter.

In Fig. 3, we present the $\tilde{\mu}$-ρ isotherms for infinite nuclear matter with various asymmetry parameters α and at temperature $T=10$ MeV. For symmetric nuclear matter ($\alpha=0$), the chemical potential for protons is equal to the one for neutrons, as expected. For asymmetric nuclear matter ($\alpha\neq0$), the proton chemical potential $\tilde{\mu}_p$ and neutron chemical potential $\tilde{\mu}_n$ separate, forming a gap between two curves. When asymmetry parameter α increases, the gap between these two curves also increases. It can also be seen that for not too large asymmetry parameter $\alpha(<0.92)$, the isotherms exhibit a typical two-phase coexistence form, with an unphysical region. When asymmetry parameter α increases, the unphysical

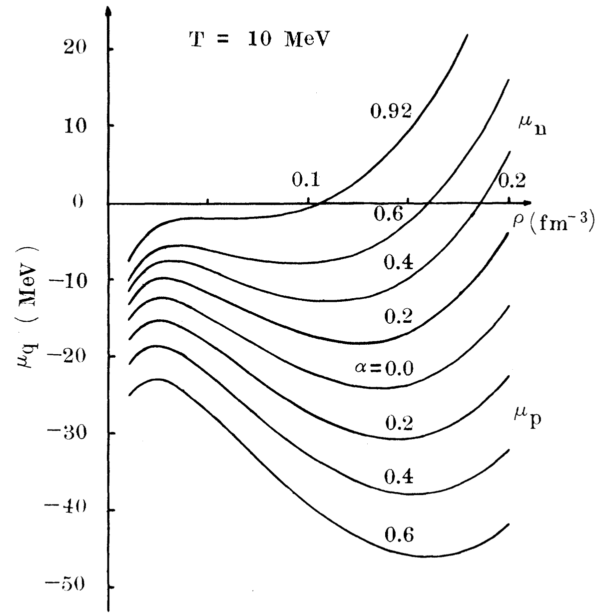

FIG. 3. $\tilde{\mu}_n$-ρ and $\tilde{\mu}_p$-ρ isotherms of infinite asymmetric nuclear matter with various asymmetry parameters and at a fixed temperature $T=10$ MeV.

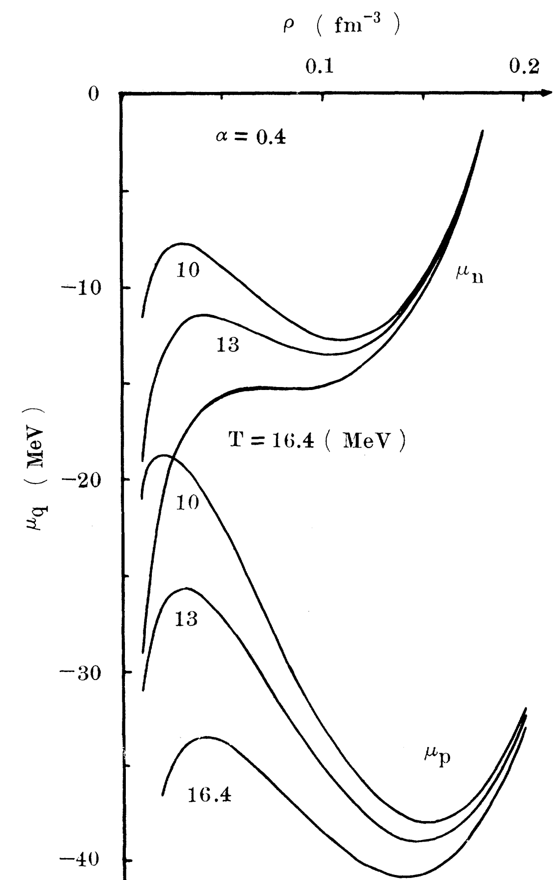

FIG. 2. $\tilde{\mu}_n$-ρ and $\tilde{\mu}_p$-ρ isotherms of infinite asymmetric nuclear matter with asymmetry parameter $\alpha=0.4$ and at various temperatures (in MeV).

region becomes smaller for $\tilde{\mu}_n$-ρ isotherms and becomes larger for $\tilde{\mu}_p$-ρ isotherms. At $\alpha=0.92$, the unphysical region in the $\tilde{\mu}_n$-ρ isotherm disappears and there appears an inflection point. We may call this asymmetry parameter a critical asymmetry for the liquid-gas phase transition in infinite nuclear matter at the fixed temperature T [8]. We can obtain a critical asymmetry parameter for each given temperature T. The resulting T_c-α_c diagram is shown in Fig. 4 with a solid curve. The phase diagram

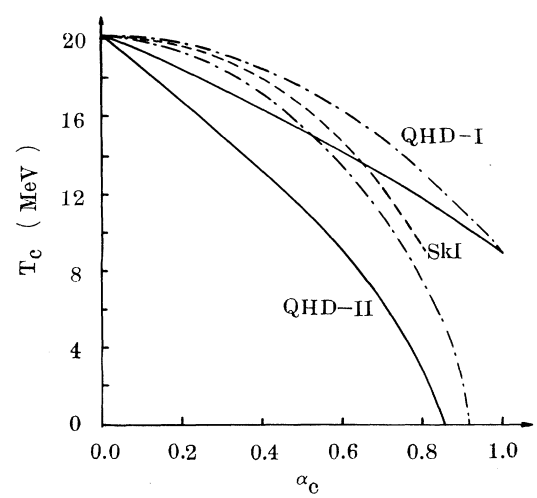

FIG. 4. The phase diagram of the critical temperature T_c vs the critical asymmetry parameter α_c for infinite nuclear matter calculated with the QHD-I and QHD-II models and the SkI nucleon-nucleon interaction. The solid curves are calculated by using the $\tilde{\mu}$-ρ isotherms, the chain curves by using the p-ρ isotherms and the dashed curve by using the p-ρ isotherms with the SkI nucleon-nucleon interaction.

separates the T-α space into two regions. In the exterior region, nuclear matter can exist in gaseous phase only, while in the interior region both liquid and gaseous phases are allowed. For example, the critical asymmetry at $T=10$ MeV is $\alpha=0.92$, above which only the gaseous phase can exist in nuclear matter. We have also shown in Fig. 4 by a dashed curve the T_c-α_c phase diagram calculated with the SkI interaction in Ref. [8], where p-ρ isotherms are used in determining the critical temperature. As a comparison, we have also calculated the critical points by using the same procedure as in Ref. [8] but in the QHD-I model, resulting in a phase diagram represented by the chain curve in Fig. 4. One can see that although the two forces give almost same critical temperature for symmetric nuclear matter ($\alpha=0$), the curve given by SkI interaction drops more quickly than that given by the QHD-I model as asymmetry parameter α increases. In other words, for large asymmetry parameter α, the temperatures predicted by the two models are quite different. In the same model (QHD-I), the two phase diagrams given by two methods deviate from each other except at the two end points of the curves. And the critical temperature given by $\tilde{\mu}_n$-ρ isotherms is lower than that given by p-ρ isotherms. The reason can be found from Fig. 3. For asymmetric nuclear matter ($\alpha\neq0$), we have $\tilde{\mu}_n > \tilde{\mu}_p$. And the unphysical region in $\tilde{\mu}_n$-ρ isotherms is smaller than that in $\tilde{\mu}_p$-ρ isotherms for a fixed set of α and T. The EOS given by p-ρ isotherms is equivalent to that given by the isotherm of average chemical potential $\tilde{\mu}$ versus density ρ, with

$$\tilde{\mu}\equiv(\tilde{\mu}_n\rho_n+\tilde{\mu}_p\rho_P)/\rho \ .$$

The width of the unphysical region in $\tilde{\mu}$-ρ isotherms is always larger than in $\tilde{\mu}_n$-ρ isotherms and smaller than that in $\tilde{\mu}_p$-ρ isotherms. So, the critical temperature determined by p-ρ (or $\tilde{\mu}$-ρ) isotherms is higher than that determined by $\tilde{\mu}_n$-ρ isotherms.

B. Finite nuclear matter

Now let us discuss the Coulomb instability of hot nuclei by calculating the limiting temperature $T_{\rm lim}$, above which the set of coexistence equations (9)–(11) has no solution.

We show in Fig. 5 by a chain curve the mass number dependence of the limiting temperature $T_{\rm lim}$ for the nuclei along the β-stability line:

$$Z=0.5A-0.3\times10^{-2}A^{5/3} \ . \qquad (17)$$

For a comparison, we also draw the curve (dashed line) calculated by using the SkI interaction [5] in Fig. 5. It is seen that the two curves have a similar trend: the limiting temperature decreases monotonously as the mass number A increases, but the rate of the decrease is smaller for large A. It is also seen that the limiting temperature given by QHD-I is higher than that given by the SkI interaction for the nuclei along the line of β stability. This result indicates that the hot nuclei described by QHD-I is more stable than that described by the nonrelativistic theory with the SkI interaction. This fact has also been reflected in the calculated critical temperatures of asymmetric nuclear matter (see Fig. 4), where the critical temperature calculated with QHD-I is higher than that given by the SkI interaction. We present the solution of the coexistence equations (9)–(11) and the equilibrium values of $\tilde{\mu}_n$, $\tilde{\mu}_p$, and p at the limiting temperature in Table I. A remarkable feature of the results is that the neutron chemical potential $\tilde{\mu}_n$ is always lower than the proton chemical potential $\tilde{\mu}_p$, which results in a negative asymmetry parameter α_V of vapor. This feature is very different from that of the results in the nonrelativistic theories, where the asymmetry parameter of vapor is always positive. The result that $\tilde{\mu}_n<\tilde{\mu}_p$ and $\alpha_V<0$ comes from the fact that the gap between $\tilde{\mu}_n$ and $\tilde{\mu}_p$ calculated with QHD-I (for example, about 30 MeV for $\alpha=0.4$ and at $\rho=0.17$ fm^{-3}) is much smaller than that calculated with the nonrelativistic theories from effective nucleon-nucleon interaction (about 50 MeV in the same condition as in QHD-I). After adding the contribution from the Coulomb interaction, we then have the opposite results: $\tilde{\mu}_n<\tilde{\mu}_p$ in the QHD-I and $\tilde{\mu}_n>\tilde{\mu}_p$ in the nonrelativistic theories. Tracing back to the starting point of the QHD-I, the unusual result may come from the fact the nucleon-nucleon interaction in QHD-I is independent of isospin. A model with the isospin dependence shall give quite a different result.

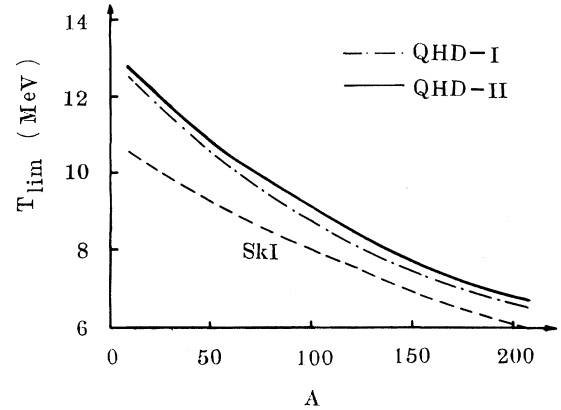

FIG. 5. The mass number dependence of limiting temperature $T_{\rm lim}$ calculated with the QHD-I (chain curve), QHD-II (solid curve) and the SkI interaction (dashed curve).

TABLE I. Equilibrium values of densities (in fm^{-3}), pressure (in MeV fm^{-3}), reduced chemical potentials (in MeV) and asymmetry parameter for the nuclei along the β-stability line at the limiting temperature, with the QHD-I model.

A	$T_{\rm lim}$	ρ_L	ρ_V	α_V	$\tilde{\mu}_n$	$\tilde{\mu}_p$	p
10	12.5	0.182	0.0334	−0.036	−17.3	−16.1	0.197
50	10.5	0.183	0.0242	−0.110	−14.8	−11.9	0.130
109	8.5	0.185	0.0205	−0.174	−12.0	−8.0	0.088
150	7.5	0.185	0.0200	−0.200	−10.5	−6.4	0.069
208	6.5	0.185	0.0143	−0.227	−8.9	−4.9	0.051

V. QHD-II MODEL

We have shown in the preceding section that although the limiting temperature calculated with the QHD-I model has reasonable values, the asymmetry parameter of the nuclear matter in the vapor phase is always negative, which is not consistent with either the Hartree-Fock (HF) results [15] or the nonrelativistic calculation for the Coulomb instability of hot nuclei [4,5], where the neutron density is larger than the proton density in the outside vapor phase. This unusual or unphysical result is caused by the shortcoming of the QHD-I model, i.e., the isospin dependence of the physical properties is not contained in the model. One possible way to improve the QHD-I model is to take into account the ρ meson degree of freedom. Then, we have the QHD-II model. In the mean-field approximation [1], the Lagrangian of QHD-II is written as

$$L^{\rm II}_{\rm MFT}=\bar{\psi}[i\gamma^{\mu}\partial_{\mu}-g_{\rho}\tfrac{1}{2}\tau_3\gamma^0 b_0-g_v\gamma^0 V_0-(M-g_s\phi_0)]\psi -\tfrac{1}{2}m_s^2\phi_0^2+\tfrac{1}{2}m_v^2V_0^2+\tfrac{1}{2}m_\rho^2 b_0^2 \,, \quad (18)$$

where ϕ_0, V_0, and b_0 are the expectation values of the neutral scalar meson, neutral vector meson, and neutral ρ meson fields, respectively. The second term in Eq. (18) is explicitly isospin dependent. With the above Lagrangian, the chemical potentials for neutron and proton become

$$\mu_n=\nu_n+\frac{g_v^2}{m_v^2}\rho-\frac{1}{2}g_\rho b_0 \,, \quad (19a)$$

$$\mu_p=\nu_p+\frac{g_v^2}{m_v^2}\rho+\frac{1}{2}g_\rho b_0 \,, \quad (19b)$$

where ν_q is determined by the condition, Eq. (6), and

$$b_0=\frac{g_\rho}{2m_\rho^2}\langle\psi^\dagger\tau_3\psi\rangle=\frac{g_\rho}{2m_\rho^2}\rho_3 \,, \quad (20)$$

with $\rho_3=\rho_p-\rho_n$. The pressure of bulk nuclear matter is now expressed as

$$p=\frac{g_v^2}{2m_v^2}\rho-\frac{g_s^2}{2m_s^2}(M-M^*)^2+\frac{g_\rho^2}{8m_\rho^2}\rho_3^2 +\frac{1}{3}\frac{\gamma_n}{(2\pi)^3}\int d^3k\frac{\mathbf{k}^2}{\sqrt{\mathbf{k}^2+M^{*2}}}[n_n(k)+\bar{n}_n(k)] +\frac{1}{3}\frac{\gamma_p}{(2\pi)^3}\int d^3k\frac{\mathbf{k}^2}{\sqrt{\mathbf{k}^2+M^{*2}}}[n_p(k)+\bar{n}_p(k)] \,. \quad (21)$$

The additional parameter $c_\rho^2=g_\rho^2M^2/m_\rho^2=54.71$, which is determined from $\rho\to2\pi$ decay. Together with the parameters C_s^2 and C_v^2 given in Sec. IV, this value predicts an accurate value for the symmetry energy coefficient [1].

By including the ρ meson degree of freedom in the Lagrangian, we have taken into account the isospin dependence of the chemical potentials μ_q and pressure p, as shown in the expressions (19) and (21). By means of these formulas, we have recalculated the same properties as in the QHD-I model. We can learn from Eqs. (19) that the neutron chemical potential μ_n shall shift up and the proton chemical potential μ_p shall shift down when the ρ meson is included. As an example, we illustrate in Fig. 6 such changes for the asymmetric nuclear matter with $\alpha=0.4$ and at temperature $T=10$ MeV, where the dashed curves are for the chemical potentials without the ρ meson and the solid curves for the chemical potential with the ρ meson. We can see that the gap between $\tilde{\mu}_n$ and $\tilde{\mu}_p$ in the QHD-II model is much larger than that in the QHD-I model. Now the value of such a gap is comparable with that in the nonrelativistic approach (for example, about 50 MeV at $\rho=0.17$ fm^{-3} in this case). Such change shall be responsible for giving a reasonable value to the asymmetry parameter of the vapor phase in the two-phase equilibrium model. Now we would like to discuss the changes in the critical phenomena. The T_c-α_c phase diagrams given by mean-field of the QHD-II model are also presented in Fig. 4, together with those predicted by the QHD-I model, where the solid curve is determined from the $\tilde{\mu}_n$-ρ isotherms and the chain curve from the p-ρ isotherms. One can easily see that the phase diagrams drop down more quickly than those given by the QHD-I model as asymmetry parameter α increases. In other words, the asymmetry effect in the QHD-II is much larger than in the QHD-I model. The inclusion of the ρ mesons increases the repulsion between nucleons, which makes the critical temperature for the liquid-gas phase transition in nuclear matter have lower value. As a result, such phase transition disappears in pure neutron matter even if at zero temperature [1]. It is also seen that the results given by the QHD-II model are closer than those by the QHD-I model to the results calculated with the SkI nucleon-nucleon interaction (dashed curve).

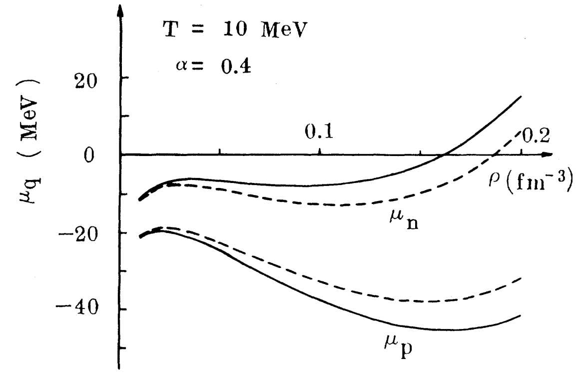

FIG. 6. The changes in the chemical potentials caused by the inclusion of the ρ mesons in the system with $\alpha=0.4$ and at $T=10$ MeV, where the dashed and solid curves stand for the results without and with the ρ mesons, respectively.

In Fig. 5, we show by a solid curve the limiting temper-

TABLE II. The same quantities as in Table I, but with the QHD-II model.

A	$T_{\rm lim}$	ρ_L	ρ_V	α_V	$\tilde{\mu}_n$	$\tilde{\mu}_p$	p
10	12.7	0.181	0.0330	0.012	−17.0	−17.1	0.203
50	10.7	0.182	0.0262	0.019	−13.3	−13.9	0.140
109	8.8	0.183	0.0220	0.070	−9.4	−11.4	0.094
150	7.7	0.183	0.0159	0.133	−7.4	−10.5	0.072
208	6.7	0.183	0.0121	0.255	−4.9	−10.1	0.052

atures for the nuclei along the β-stability line calculated with the QHD-II model. The equilibrium values of $\tilde{\mu}_n$, $\tilde{\mu}_p$, and p at the limiting temperature are presented in Table II. We can see that there is no large difference between the curves of the QHD-I and QHD-II models. But if we examine the equilibrium values α_V of the asymmetry parameter in the vapor phase, we shall find the difference between the two models is very large. In contrast to the results in the QHD-I model, the values of α_V in the QHD-II model are all positive, in consistency with the results in either the HF calculation or the nonrelativistic calculation for the Coulomb instability of hot nuclei.

VI. CONCLUSIVE REMARKS

We have studied the liquid-gas phase transition in asymmetric nuclear matter and the Coulomb instability of hot nuclei by means of the mean-field theories of the QHD-I and QHD-II models. From the results and discussions in the preceding sections, we can arrive at the following conclusions.

(1) The critical temperature for the liquid-gas phase transition in nuclear matter decreases as the asymmetry parameter of nuclear matter increases. Such an asymmetry effect in the QHD-II model is much larger than in the QHD-1 model, due to the inclusion of the ρ mesons.

(2) The QHD-II model is more reasonable than the QHD-I model when they are used to studying the properties of asymmetric nuclear matter.

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