Calculation of the moment of inertia by a proper treatment of pairing correlations

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It was recently reported that the well-known discrepancy between theoretical values of the moment of inertia and observed ones could be removed by a number-projection technique for the BCS states. To examine this problem, the exact calculation of the moment of inertia in the cranking model is performed in the ^{160}Dy isotope as a typical example. The Richardson method is applied to the exact treatment of the pairing correlations and recurrence formulas for computing various matrix elements in the cranking model are developed. The numerical results in ^{160}Dy indicate that the exact treatment hardly improves the BCS values of the moment of inertia and that the discrepancy with the observed values cannot be removed by the correction for the number nonconservation in the BCS wave functions.

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I. INTRODUCTION

It has been well known that the theoretical values of the moment of inertia obtained with the cranking model are systematically smaller than the experimental ones by a factor of 10–40 % [1–4]. Many efforts [5–7] have not decisively succeeded in reducing the discrepancy between theory and experiment. The theoretical moment of inertia is usually calculated by the BCS treatment of the pairing degree of freedom but the pairing correlations play a leading role in reducing the rigid rotor value of the moment of inertia to the observed one. It is therefore necessary in the discussion on the above discrepancy to evaluate the error originated in the BCS approximation. However, this study has not been completed yet, because the exact diagonalization of the pairing Hamiltonian is impracticable due to a numerous amount of single-particle levels in deformed nuclei.

Recently, Allal and Fellah [8] studied the effects of nonconservation of the particle number in the BCS wave functions on the moment of inertia. They concluded from their calculations that the discrepancy in the moment of inertia between theory and experiment is due to the number-nonconservation effects of the BCS treatment. Yet their calculations are based on a method of successive projections of the BCS states and they have not carried out any direct comparison with exact calculations.

On the other hand, we had a preliminary result in a schematic model different from the conclusion of Allal and Fellah about the BCS error in the moment of inertia. The purpose of this paper is to investigate in a realistic case how much the moment of inertia calculated by the exact treatment of the pairing correlations is different from the BCS one and whether the discrepancy between theory and experiment is really due to the error of the BCS approximation.

The exact solution of the pairing Hamiltonian with a constant force strength can be accomplished by the Richardson method [9]. The Richardson method requires much smaller dimensions than the diagonalization method (the shell model) and is suitable for obtaining the wave functions and energies of the pair-correlated eigenstates. The calculation of the moment of inertia is then reduced to that of the matrix elements of the angular momentum J_x between the pair-correlated states in the cranking model. The computation of these matrix elements is difficult for the shell model approach because of its large dimensions. Alternatively we present manageable and useful recurrence formulas for computing the matrix elements of J_x using the Richardson solutions. We are thus able to obtain the exact value of the moment of inertia in the cranking model.

In this paper, we carry out numerical calculations of the above-mentioned exact treatment in the ^{160}Dy isotope as a typical example of deformed nucleus and compare the exact results with the BCS results in order to examine the error of the BCS approximation. Furthermore, we investigate the contributions from the fluctuations of the static pairing field (i.e., the pairing vibrational excitations) to the moment of inertia which are not included in the usual BCS treatment.

II. FORMALISM

In the cranking model, the moment of inertia is given by

$$\mathcal{J}=2\hbar^2\sum_{\mathrm{ex}}\frac{|\langle \mathrm{ex}|\hat{J}_x|\mathrm{gr}\rangle|^2}{E_{\mathrm{ex}}-E_{\mathrm{gr}}}\ , \tag{1a}$$

$$\hat{J}_x=\sum_{\mu\nu}(j_x)_{\mu\nu}(c^{\dagger}_{\mu+}c_{\nu+}-c^{\dagger}_{\nu-}c_{\mu-})\ , \tag{1b}$$

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where $|\mathrm{gr}\rangle$ and $|\mathrm{ex}\rangle$ denote the ground and excited states with the energies E_{gr} and E_{ex}, respectively. The matrix elements $(j_x)_{\mu\nu}$ are those of the x component of the angular momentum ($\hat{J}_x$) between the single-particle states $\mu\sigma$ and $\nu\sigma$ ($\sigma=\pm$ denoting the signature).

The nucleon creation operator $c^\dagger_{\nu\sigma}$ can be expressed in terms of the pair operator $S^\dagger_\nu \equiv c^\dagger_{\nu+}c^\dagger_{\nu-}$ and unpaired-nucleon operator $a^\dagger_{\nu\sigma}\equiv c^\dagger_{\nu\sigma}(1-c^\dagger_{\nu\bar{\sigma}}c_{\nu\bar{\sigma}})$ [10]:

$$c^\dagger_{\nu\sigma}=a^\dagger_{\nu\sigma}+\sigma S^\dagger_\nu a_{\nu\bar{\sigma}} \quad (\sigma=\pm) . \tag{2}$$

When we consider only the pairing correlations as the residual interactions, the ground state $|\mathrm{gr}\rangle$ does not contain the unpaired-nucleons $a^\dagger_{\nu\sigma}$ but contains only the pairs $S^\dagger_\nu$ in the expression (2). The states excited by the operator $\hat{J}_x$ from $|\mathrm{gr}\rangle$ are the states with two unpaired nucleons $a^\dagger a^\dagger$,

$$\hat{J}_x|\mathrm{gr}\rangle=\sum_{\mu\nu}(j_x)_{\mu\nu}(a^\dagger_{\mu+}a^\dagger_{\nu-}S_\nu - a^\dagger_{\mu+}a^\dagger_{\nu-}S_\mu)|S^{n/2}\gamma=0\rangle\rangle , \tag{3}$$

and hence the excited states $|\mathrm{ex}\rangle$ in Eq. (1) can be written as

$$|\mathrm{ex}(\sigma)[\mu\nu]\gamma\rangle=a^\dagger_{\mu\sigma}a^\dagger_{\nu\bar{\sigma}}|S^{(n-2)/2}[\mu\nu]\gamma\rangle\rangle , \tag{4}$$

where $|S^{(n-m)/2}[\nu_1\nu_2\cdots\nu_m]\gamma\rangle\rangle$ are pair-correlated states of $(n-m)/2$ pairs (n being the nucleon number and m being the number of unpaired nucleons) and γ is the quantum number denoting the order of energy ($\gamma=0,1,2,\ldots$). The notation $[\nu_1\nu_2\cdots\nu_m]$ means that the single-particle levels $\nu_1\nu_2\cdots\nu_m$ in the bracket [] cannot be occupied by $S^\dagger$ due to the occupation by $a^\dagger$. In this notation, $|\mathrm{gr}\rangle$ is obviously $|S^{n/2}\gamma=0\rangle\rangle$. The transformation (2) is, in a word, the number-conserving version of the Bogoliubov-Valatin transformation. After the transformation (2), our task is to determine the pair-correlated states $|S^{n/2}\gamma=0\rangle\rangle$ and $|S^{(n-2)/2}[\mu\nu]\gamma\rangle\rangle$ instead of the BCS mean field (the quasiparticle vacuum).

In the subspace excluding $a^\dagger$, the pairing Hamiltonian with a constant force strength G is written as

$$H_S=\sum_\nu 2\epsilon_\nu S^\dagger_\nu S_\nu - G\sum_{\mu\nu}S^\dagger_\mu S_\nu , \tag{5}$$

where ϵ_ν denotes the single-particle energies. The commutation relation of the operators S and $S^\dagger$ within the subspace excluding $a^\dagger$ is given by

$$[S_\nu,S^\dagger_\mu]=\delta_{\nu\mu}(1-2S^\dagger_\nu S_\nu) . \tag{6}$$

Richardson [9] proved that the eigenstates of the pairing Hamiltonian (5) are obtained in the following way:

$$|S^N[\nu_1\nu_2\cdots]\gamma\rangle\rangle\propto\prod_{i=1}^{N}\mathcal{S}^\dagger(z_i^\gamma[\nu_1\nu_2\cdots])|0\rangle , \tag{7a}$$

$$\mathcal{S}^\dagger(z_i^\gamma[\nu_1\nu_2\cdots])=\sum_{\nu(\neq)\nu_1\nu_2\cdots}\frac{1}{2\epsilon_\nu-z_i^\gamma}S^\dagger_\nu , \tag{7b}$$

where z_i^γ ($i=1,2,\ldots,N$) are the solutions of a set of the equations

$$\sum_{\nu\neq\nu_1\nu_2\cdots}\frac{1}{2\epsilon_\nu-z_i^\gamma}=\frac{1}{G}+\sum_{j(\neq i)}^{N-1}\frac{2}{z_j^\gamma-z_i^\gamma} . \tag{8}$$

The energy of a state (7a) is the sum of the solutions z_i^γ,

$$E(S^N[\nu_1\nu_2\cdots]\gamma)=\sum_i^N z_i^\gamma . \tag{9}$$

The solutions z_i^γ are generally a complex number but every complex solution has its complex conjugate one. Hence the energy (9) is always a real number. The energies of the states $|\mathrm{gr}\rangle$ and $|\mathrm{ex}(\sigma)[\mu\nu]\gamma\rangle$ appearing in Eq. (1) are given by

$$E_{\mathrm{gr}}=E(S^{n/2}\gamma=0) , \tag{10a}$$

$$E_{\mathrm{ex}(\sigma)[\mu\nu]\gamma}=\epsilon_\mu+\epsilon_\nu+E(S^{(n-2)/2}[\mu\nu]\gamma) . \tag{10b}$$

The remaining task is to calculate the matrix elements $\langle\mathrm{ex}|\hat{J}_x|\mathrm{gr}\rangle$, which are reduced to

$$\langle\mathrm{ex}(\sigma)[\mu\nu]\gamma|\hat{J}_x|\mathrm{gr}\rangle=(j_x)_{\mu\nu}\langle\langle S^{(n-2)/2}[\mu\nu]\gamma|S_\nu-S_\mu|S^{n/2}\gamma=0\rangle\rangle , \tag{11}$$

i.e., the matrix elements of the pair operator S between the pair-correlated states.

The matrix elements of S can be calculated by combining the eigenstates (7) with the commutation relation (6) in the following way:

$$\langle 0|\prod_j^{N-1}\mathcal{S}(z_j^\gamma[\nu_1\nu_2])S_\mu\prod_i^N\mathcal{S}^\dagger(z_i^0)|0\rangle=\sum_l\frac{1}{2\epsilon_\mu-z_l^0}\sum_\nu\frac{1}{2\epsilon_\nu-z_m^0}\langle 0|\prod_j^{N-1}\mathcal{S}(z_j^\gamma[\nu_1\nu_2])S^\dagger_\nu\prod_{i(\neq lm)}^{N-2}\mathcal{S}^\dagger(z_i^0)|0\rangle - 2\sum_{l<m}\frac{1}{2\epsilon_\mu-z_l^0}\frac{1}{2\epsilon_\mu-z_m^0}\langle 0|\prod_j^{N-1}\mathcal{S}(z_j^\gamma[\nu_1\nu_2])S^\dagger_\mu\prod_{i(\neq lm)}^{N-2}\mathcal{S}^\dagger(z_i^0)|0\rangle . \tag{12}$$

The matrix elements of $S^\dagger$ in the right-hand side of Eq. (12) are similarly reduced to those of S,

$$\langle 0|\prod_j^{N-1}\mathcal{S}(z_j^\gamma[\nu_1\nu_2])S^\dagger_\mu\prod_{i(\neq lm)}^{N-2}\mathcal{S}^\dagger(z_i^0)|0\rangle=\sum_p\frac{1}{2\epsilon_\mu-z_p^\gamma}\sum_\nu\frac{1}{2\epsilon_\nu-z_q^\gamma}\langle 0|\prod_{j(\neq pq)}^{N-3}\mathcal{S}(z_j^\gamma[\nu_1\nu_2])S_\nu\prod_{i(\neq lm)}^{N-2}\mathcal{S}^\dagger(z_i^0)|0\rangle - 2\sum_{p<q}\frac{1}{2\epsilon_\mu-z_p^\gamma}\frac{1}{2\epsilon_\mu-z_q^\gamma}\langle 0|\prod_{j(\neq pq)}^{N-3}\mathcal{S}(z_j^\gamma[\nu_1\nu_2])S_\mu\prod_{i(\neq lm)}^{N-2}\mathcal{S}^\dagger(z_i^0)|0\rangle . \tag{13}$$

Repeating the procedures (12) and (13), we finally reach the simple matrix elements

$$\langle 0|S_\mu \mathcal{S}^\dagger(z_i^0)|0\rangle = \frac{1}{2\epsilon_\mu - z_i^0} , \tag{14a}$$

or

$$\langle 0|\mathcal{S}(z_j^\gamma[\nu_1\nu_2])S_\mu^\dagger|0\rangle = \frac{1}{2\epsilon_\mu - z_j^\gamma} \quad (\mu \neq \nu_1, \nu_2) . \tag{14b}$$

We can regard Eqs. (12) and (13) as a set of recurrence formulas starting from the initial values (14a) or (14b).

We are thus able to calculate the matrix elements (11) and the moment of inertia (1), if we get the solutions z_i^0 and z_j^γ by solving the Richardson equations (8). The normalization of the eigenstates (7) is presented in Richardson's paper. If we want, we can alternatively evaluate the normalization constant by changing it into

$$\langle 0|\prod_j^N \mathcal{S}(z_j^\gamma[\nu_1\nu_2])\prod_i^N \mathcal{S}^\dagger(z_i^\gamma[\nu_1\nu_2])|0\rangle = \sum_{\nu \neq \nu_1\nu_2} \frac{1}{2\epsilon_\nu - z_1^\gamma}\langle 0|\prod_{j(\neq 1)}^{N-1} \mathcal{S}(z_j^\gamma[\nu_1\nu_2])S_\nu \prod_i^N \mathcal{S}^\dagger(z_i^\gamma[\nu_1\nu_2])|0\rangle . \tag{15}$$

The right-hand side of Eq. (15) can also be calculated by the recurrence formulas (12)–(14).

Our exact method of calculating the moment of inertia has an advantage that the Richardson equations (8) require only small dimensions in comparison with the shell model (the method of diagonalizing the Hamiltonian matrix). In the case of 20 nucleons distributed to 23 levels which is considered in the next section, for instance, the number of independent basis states is $23!/(10!13!) = 1144066$ for the ground state $|S^{n/2}\gamma = 0\rangle\rangle$ and is $21!/(9!12!) = 293930$ for each excited state $|S^{(n-2)/2}[\mu\nu]\gamma\rangle\rangle$, and the number of the matrix elements (11) is 1144066×293930. The shell model approach to such a large dimensional problem is impracticable even for the latest model of computer. This is the reason why the exact examination has not been performed on the problem of the moment of inertia. In contrast to this, our exact method is applicable to a considerably large space of the single-particle levels. The Richardson equations have no problem with the dimensions. When calculating the recurrence formulas (12)–(14) in the case of 20 nucleons in 23 levels, we encounter the largest matrix such as $\langle 0|\prod_j^5 \mathcal{S}(z_j^\gamma[\mu\nu])S_\nu \prod_i^6 \mathcal{S}^\dagger(z_i^0)|0\rangle$, in which the number of the different combinations of $\langle 0|\prod_j^5 \mathcal{S}(z_j^\gamma[\mu\nu])$ for fixed $\mu\nu$ is 126 and the number of the different combinations of $\prod_i^6 \mathcal{S}^\dagger(z_i^0)|0\rangle$ is 210. Thus the exact calculation of the moment of inertia becomes possible in a significantly large space of the single-particle levels, which enables us to compare the BCS results with the exact ones.

III. NUMERICAL RESULTS IN ^{160}Dy

We have carried out numerical calculations in ^{160}Dy as a typical deformed nucleus. We have fixed the single-particle states in the same manner as Nilsson *et al.* [11], using the parameters $\epsilon = 0.245$, $\kappa = 0.0637$, $\mu = 0.420$, and $\epsilon_4 = -0.015$. The matrix elements $(j_x)_{\mu\nu}$ have been obtained for these single-particle states. Obtained energy levels of the single-neutron states are shown in Fig. 1.

The number of the single-particle states considered in the calculations must be large for comparison with observed values of the moment of inertia. Our recurrence-formula method is capable of treating the single-particle space which is significantly large enough for comparison with the BCS method but still requires a limit to the dimensions on computation at present. Hence, we adopt two types of single-neutron spaces *A* and *B* illustrated in Fig. 1 for the exact calculations: *A* consists of 16 levels

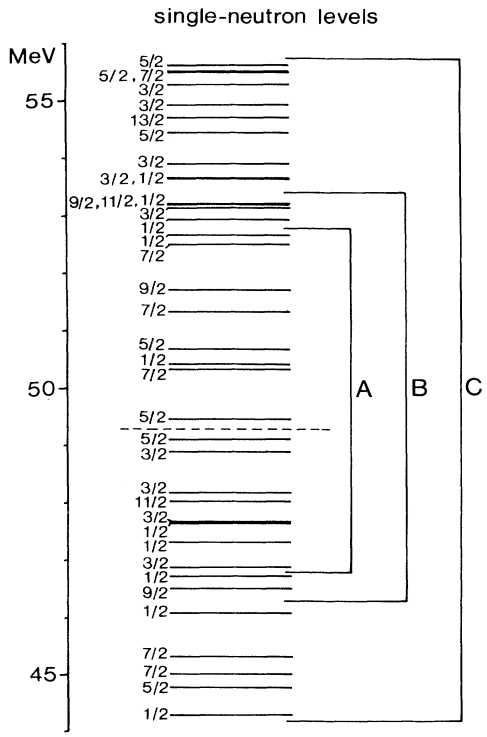

FIG. 1. Energy levels of the single-neutron states in ^{160}Dy calculated in the same manner as Nilsson *et al.* The parameters used are shown in the text. The space of the single-neutron states is truncated in the three different ways: (*A*) 16 levels with 16 neutrons; (*B*) 23 levels with 20 neutrons; (*C*) 38 levels with 30 neutrons.

with 16 neutrons and B consists of 23 levels with 20 neutrons. The space C composed of 38 levels with 30 neutrons is also considered in the BCS calculations.

Nilsson *et al.* [11] used a pairing force strength depending on the proton and neutron numbers. In this paper, we vary the pairing strength G so as to reproduce the same value of the gap $\Delta = G\sum U_\nu V_\nu$ (V_ν being the occupation probability and $U_\nu = \sqrt{1-V_\nu^2}$) in the truncated spaces A, B, and C. We employ the value $\Delta = 0.934$ MeV as a standard for neutrons of ^{160}Dy which is obtained by Ma and Tsang [7]. Namely, fixing G so as to reproduce $\Delta = 0.934$ MeV in A and B, we solve the Richardson equations and calculate the moment of inertia.

In the BCS treatment we make two types of calculation: one includes the self-energy term $-GV_\nu^2$ in the quasiparticle energy $\sqrt{(\epsilon_\nu - \lambda - GV_\nu^2)^2 + \Delta^2}$ and the other does not include it. (Note that the self-energy term $-GV_\nu^2$ is omitted in the usual numerical calculation.) We call the former the BCS1 approximation and call the latter the BCS2.

The fluctuations of the pairing mean field (i.e., the pairing vibrations) are not taken into account in the BCS calculation of the moment of inertia. Similarly we first neglect the pairing vibrational excitations with $\gamma > 0$ in the excited states $|\mathrm{ex}\rangle = |\mathrm{ex}(\sigma)[\mu\nu]\gamma\rangle$ when exactly calculating the moment of inertia, i.e., we approximate Eq. (1) to

$$\mathcal{J}_0 = 2\hbar^2 \sum_{\mu\nu\sigma} \frac{|\langle \mathrm{ex}(\sigma)[\mu\nu]\gamma = 0|\hat{J}_x|\mathrm{gr}\rangle|^2}{E_{\mathrm{ex}(\sigma)[\mu\nu]\gamma=0} - E_{\mathrm{gr}}} . \tag{16}$$

Contributions from the excited states with $\gamma > 0$ will be discussed in the next section.

Figure 2 illustrates the gap Δ and the moment of inertia $\mathcal{J}_0$ which are calculated by varying the force strength G in the three truncated spaces A, B, and C. The results of our exact method (the solid lines) are compared with those of the BCS1 (the chain lines) and the BCS2 (the broken lines). This figure indicates that the exact value of $\mathcal{J}_0$ is smaller than both the values of the BCS1 and the BCS2 for a fixed value of Δ. In other words, the number-conserving treatment of the pairing correlations never enlarges the BCS values of the moment of inertia by a factor of more than 10% in our case.

In detail, the value of the moment of inertia calculated for a fixed Δ gradually increases as the single-particle space is extended. However, the truncation of the space does not seem to harmfully damage the discussion about the comparison of the calculated moment of inertia between the exact method and the BCS. The BCS2 approximation brings the lines of the moment of inertia closer to the exact lines than the BCS1, but the difference between

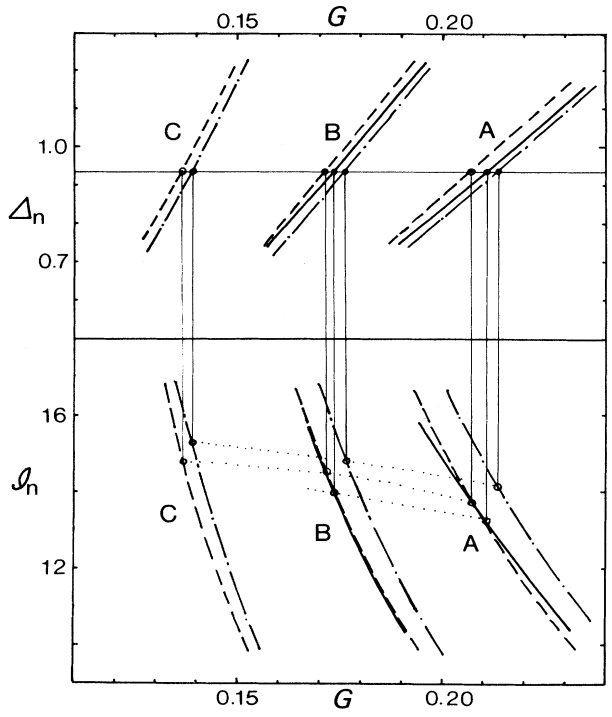

FIG. 2. The gap Δ (MeV) and the moment of inertia $\mathcal{J}_0$ ($\hbar^2\,\mathrm{MeV}^{-1}$) calculated for neutrons of ^{160}Dy. The results of our exact method are denoted by the solid lines. In the BCS treatment, the self-energy term ($-GV_\nu^2$) is included in the chain lines (BCS1) and is neglected in the broken lines (BCS2). The notations A, B, and C mean the same truncated spaces as in Fig. 1 and G (MeV) is the pairing force strength.

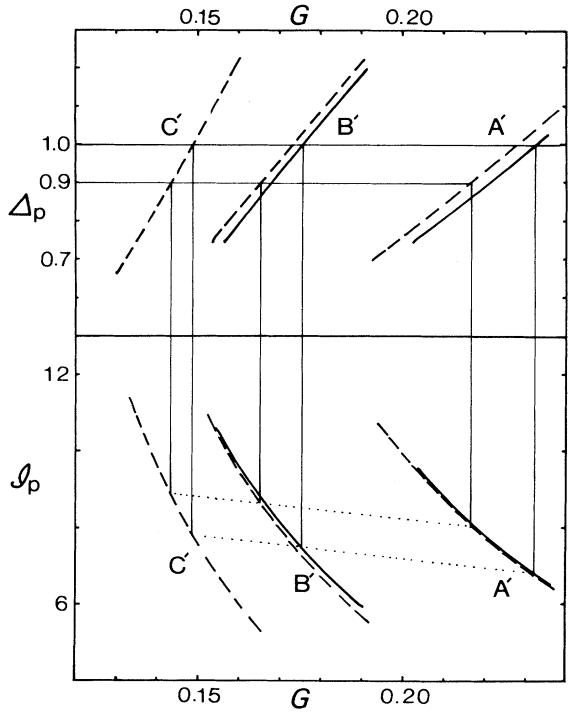

FIG. 3. The gap Δ (MeV) and the moment of inertia $\mathcal{J}_0$ ($\hbar^2\,\mathrm{MeV}^{-1}$) calculated for protons of ^{160}Dy. The single-proton levels are arranged at equal intervals in order to make the numerical solution of the Richardson equations easier. The space is truncated in the three different ways: (A') 15 levels with 16 protons; (B') 24 levels with 20 protons; (C') 42 levels with 36 protons. The solid and broken lines denote the results of the exact and BCS2 calculations, respectively.

the BCS1 and BCS2 results becomes smaller as the single-particle space is extended. If we regard Fig. 2 as the relation of $\mathcal{I}_0$ to Δ, the exact value of the moment of inertia becomes more smaller than the BCS2 value. The calculated lines of Δ suggest that the exact distribution of the pairs $S^\dagger$ in the ground state $|\mathrm{gr}\rangle$ is in an intermediate situation between the BCS1 and the BCS2.

To check the above results, we have carried out similar calculations for protons of ^{160}Dy. Because our numerical solution of the Richardson equations fails in a few specific configurations unfortunately, we have artificially arranged the single-proton levels at equal intervals. The obtained results are shown in Fig. 3, where the exact results (the solid lines) are compared with the BCS2 (the broken lines). This figure is very similar to Fig. 2 and also insists that the exact method hardly brings about a larger value of the moment of inertia than the BCS.

Accordingly we can say that the BCS calculation of the moment of inertia is a fairly good approximation to the exact calculation of $\mathcal{I}_0$, contrary to the conclusion of Allal and Fellah [8]. The discrepancy in the moment of inertia between the BCS calculation and experiment may not be removed by the number-conserving treatment of the BCS wave functions.

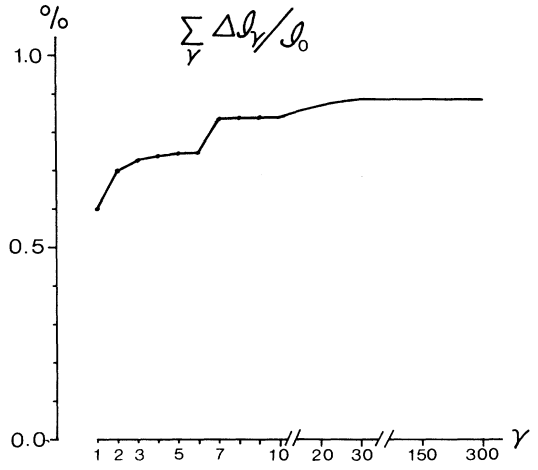

FIG. 4. Sum of the contributions from the excited states with $\gamma > 0$ to the moment of inertia in the neutron truncated space A. The single-neutron levels are arranged at equal intervals in order to make the numerical solution of the Richardson equations easier.

IV. DISCUSSION

The calculations in the previous section do not include the contributions from the excited states $|\mathrm{ex}(\sigma)[\mu\nu]\gamma > 0\rangle$ with the pairing vibrational excitations to the moment of inertia. From the cranking-model formula Eq. (1), these excited states with $\gamma > 0$ apparently contribute to $\mathcal{I}$ by the positive sign. It is a problem how much the total sum of their contributions is. Let us define the contributions from a set of the states $|\mathrm{ex}(\sigma)[\mu\nu]\gamma\rangle$ with a fixed γ to $\mathcal{I}$ such as

$$\Delta\mathcal{I}_\gamma = 2\hbar^2 \sum_{\mu\nu\sigma} \frac{|\langle \mathrm{ex}(\sigma)[\mu\nu]\gamma|\hat{J}_x|\mathrm{gr}\rangle|^2}{E_{\mathrm{ex}(\sigma)[\mu\nu]\gamma} - E_{\mathrm{gr}}} \quad (\gamma > 0) . \tag{17}$$

To find a clue to discussion, we have calculated the contributions from the three sets of states containing the lowest three pairing vibrations ($\gamma = 1$, 2, and 3) in the neutron truncated space B. The results are $\Delta\mathcal{I}_{\gamma=1}/\mathcal{I}_0 = 0.67\%$, $\Delta\mathcal{I}_{\gamma=2}/\mathcal{I}_0 = 0.05\%$, and $\Delta\mathcal{I}_{\gamma=3}/\mathcal{I}_0 = 0.15\%$. We can expect that $\Delta\mathcal{I}_\gamma$ becomes smaller as γ increases because the excitation energy in the denominator of Eq. (17) becomes larger. However, the number of the pairing vibrational states with fixed $\mu\nu$ amounts to 293 929 in the space B. So we should still estimate the total of the contributions $\sum_\gamma \Delta\mathcal{I}_\gamma$.

Although our recurrence-formula method is much less time consuming than the shell model, its computation is impracticable for such a huge number of states (293 929 × (number of combinations $[\mu\nu]$)). Therefore we have calculated $\Delta\mathcal{I}_\gamma$ up to $\gamma = 300$ in the smaller space A (in which the maximum γ is 3432 and the number of combinations $[\mu\nu]$ is 120). We have furthermore simplified the single-neutron levels to be at equal intervals in order to make the numerical solution of the Richardson equations easier. The calculated values of $\sum_\gamma \Delta\mathcal{I}_\gamma$ are plotted as a function of γ in Fig. 4. In this figure, the sum $\sum_\gamma \Delta\mathcal{I}_\gamma/\mathcal{I}_0$ almost reaches its maximum value before $\gamma \approx 30$ and hardly changes after $\gamma \approx 30$. This suggests that the corrections $\Delta\mathcal{I}_\gamma$ may be negligible for $\gamma > 300$. We can say that the total sum $\sum_\gamma \Delta\mathcal{I}_\gamma/\mathcal{I}_0$ is only 1% at most. Figure 4 also teaches us the overwhelming importance of the lowest few pairing vibrations, namely, $\sum_{\gamma \le 3} \Delta\mathcal{I}_\gamma$ is about 80% of the total correction.

Combining the value $\sum_{\gamma \le 3} \Delta\mathcal{I}_\gamma/\mathcal{I}_0 = 0.87\%$ in the space B with Fig. 4, we conclude that the total correction factor of all the excited states with $\gamma > 0$, $\sum_{\gamma > 0} \Delta\mathcal{I}_\gamma/\mathcal{I}_0$ is no more than 1–2 %. This increase in the calculated moment of inertia cannot compensate the discrepancy between theory and experiment by a factor of 10–40 %.

In conclusion, our exact treatment of the pairing correlations in ^{160}Dy indicates the fair goodness of the BCS approximation regarding the moment of inertia. Allal and Fellah's conclusion that the discrepancy in the moment of inertia between theory and experiment is due to the nonconservation of the particle number in the BCS wave functions is not certain and should be examined further. We should also investigate another origin of the discrepancy instead of the ambiguity of the BCS approximation. If it is not hopeful to readjust the parameter set within the conventional framework of a deformed potential (including various terms) plus the pairing correlations, we should search for residual correlations which have not been considered previously.

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