Chiral color-dielectric model with perturbative quantum pions and gluons

L. R. Dodd and D. E. Driscoll
Department of Physics and Mathematical Physics, University of Adelaide, Adelaide, South Australia
(Received 16 July 1992)

Pionic contributions to static nucleon properties are calculated in a chiral extension of the color-dielectric model. The pion field and residual gluon field are treated perturbatively. It is shown that with a simple choice for the energy of the scalar confining field and assuming the chiral limit, the system of equations describing the bare soliton and the perturbative pion and gluon fields may be cast in a dimensionless, parameter-free form for large glueball masses. This enables a formula for the masses of the nucleon and Δ including leading-order pionic and gluonic contributions and corrections for spurious center-of-mass motion, valid for a wide range of input parameters determining the bare-soliton solutions, to be derived. A further consequence of the scaling behavior is that pionic contributions to nucleon properties, calculated using the methods of the cloudy-bag model, are insensitive to the soliton parameters, once the size of the soliton is fixed. The model results are very similar to those of the cloudy-bag model but the predicted masses are about 20% too large, and the pionic contributions to charge radii are underestimated.

PACS number(s): 21.60.−n

I. INTRODUCTION

In their simplest form color-dielectric models (CDM) [1] describe the quark structure of hadrons by confining effective quark fields with a scalar field which represents the long-range order of the QCD vacuum. Like the MIT-bag [2] Lagrangian, the typical Lagrangian of these models at this level is not chirally symmetric, but it is well known that by introducing a suitable interaction with an elementary pion field, manifest chiral symmetry may be restored [3]. There is no unique prescription for the additional terms in the effective Lagrangian and a number of different chiral versions of nontopological soliton models have been considered by various authors (reviewed recently by Birse [1]).

Following the approach which was used to obtain the cloudy-bag model (CBM) Lagrangian [3] from the MIT-bag model, Williams and Dodd [4] investigated chiral extensions of both the Nielsen-Patkos color-dielectric model [5] and the Friedberg-Lee soliton-bag model [6]. It was found that the pion fields in the soliton were sufficiently weak that pionic contributions to nucleon properties could be calculated using perturbation theory as in the CBM work [3,7–9]. This is to be contrasted with nonperturbative approaches where the pion field is treated in the mean-field approximation using the hedgehog ansatz [10]. The numerical results of Ref. [4] for pionic corrections showed an insensitivity to the details of the unperturbed soliton solutions and, when the scale of the soliton solution was fixed to reproduce the proton charge radius, broad agreement with the results of the CBM. However, no attempt was made in this work to choose a parameter set which would also fit the nucleon and Δ masses when center-of-mass corrections and gluonic corrections, discussed below, were included.

Another refinement of the CDM, necessary for the calculation of mass splittings of the hadrons, is the retention of residual color fields left over from the coarse graining of the QCD fields. For example, the mass degeneracy of the nucleon and Δ isobars is lifted by the color-magnetic hyperfine interaction. The one-gluon exchange contribution to the nucleon-Δ mass difference has been calculated in the CDM both perturbatively and self-consistently [11,12]. However, these calculations did not take into account the contribution from pion exchange expected from the chiral models.

The aim of the present work is to test the predictions of the CDM for static nucleon properties including both pionic and gluonic contributions and with center-of-mass corrections. A similar calculation has appeared recently. Leech and Birse [14] have calculated pionic contributions using Peierls-Yoccoz projection to remove spurious center-of-mass contributions. They use a chiral version of the CDM where the pion fields are accompanied by an additional scalar field, as in the linear σ model, rather than the nonlinear realization of chiral symmetry adopted in this paper. Although their Lagrangian has chiral symmetry, the Goldberger-Treiman relation, which should be satisfied by the model, is violated by the approximations made in projecting momentum eigenstates. In our work we have chosen to preserve the Goldberger-Treiman relation at the expense of using only cruder estimates of center-of-mass corrections. In our view reliable estimates of c.m. corrections which respect the symmetries of the Lagrangian remain a problem for these models. Leech and Birse did not calculate the gluonic contribution to the nucleon-Δ mass splitting but assumed that the strength of the quark-gluon coupling could be adjusted so that a fit to the nonpionic part of the mass splitting would be achieved. Here we calculate the $M1$ color-magnetic energy explicitly to see whether consistent values of the strong-coupling constant are obtained over a range of soliton parameters.

We would like to emphasize that our model is just one of many possibilities. From a more fundamental point of view it is natural to regard the pion (and other mesons) as composites of the quark and gluon fields. For example, in the work of Banerjee, Broniowski, and Cohen [15] it is assumed that an effective low-energy chiral model can be derived from QCD by entirely eliminating the gluon degrees of freedom in favor of meson exchanges between quarks. In this approach one-gluon exchange should not be added to the quark-meson model. The Lagrangian that we use, as in the CBM, includes an additional elementary pion field to restore chiral symmetry, and within the context of the model both one-pion and one-gluon exchange are calculated.

Section II describes the chiral version of the color-dielectric model considered in this paper, how the lowest-order perturbative pionic and gluonic contributions to the soliton energy are calculated, and how the masses of the nucleon and Δ are estimated including c.m. corrections. The bare soliton solutions are characterized by three parameters, the quark mass m, the glueball mass M_χ, and the scale σ_v of the confining scalar field. The magnitude of the gluonic energy shift is determined by the strong-coupling constant α_s which is essentially a free parameter of the model. The magnitude of the pionic contributions are fixed through the Goldberger-Treiman relation of the model in terms of the pion mass, the pion decay constant, and the axial coupling constant. The latter is calculated from the bare soliton solution while the pion mass and pion decay constant are given their experimental values. Thus once the bare soliton solutions are chosen there is no further freedom in the model to vary the pionic contributions to nucleon properties.

In Sec. III, following the scaling argument of McGovern, Birse, and Spanos [13] for a large glueball mass, we are able to show that the system of equations determining the bare soliton solution and the perturbative pion and gluon fields may be cast in a dimensionless, parameter-free form in the chiral limit where the pion is massless. This enables a mass formula for the nucleon and Δ masses to be given whose numerical coefficients are determined by solving the universal equations once only. This scaling which still holds to a good approximation for quite small ratios of the glueball to quark masses and for nonvanishing pion mass explains the insensitivity of pionic corrections to the soliton parameters found in earlier work [4].

Pionic contributions to static nucleon properties are considered in Sec. IV. The formulas for charge radii and magnetic moments are essentially identical with those of the cloudy-bag model, with the CBM form factor replaced by the form factor computed from the soliton solution.

Section V contains our numerical results and conclusions.

II. THE MODEL

A. The Hamiltonian

With the notation of Ref. [4], the Hamiltonian of the chiral extension of the color-dielectric model, including gluons, to be considered here may be written as

$$H=H_{\rm NS}+H_\pi+H_I^\pi+H_g+H_I^g\equiv H_0+H_I^\pi+H_I^g\ , \qquad (2.1)$$

where the Hamiltonian for the nontopological soliton in the mean-field approximation (MFA) is

$$H_{\rm NS}=\int d^3x\left[:\bar q(i\gamma\cdot\nabla+m/\chi)q:+\frac{1}{2}\sigma_v^2(\nabla\chi)^2+\frac{1}{2}\sigma_v^2M_\chi^2\chi^2\right]\ , \qquad (2.2)$$

the pion field contribution is

$$H_\pi=\int d^3x\ \frac{1}{2}:[(\partial_0\boldsymbol{\pi})^2+(\nabla\boldsymbol{\pi})^2+m_\pi^2\boldsymbol{\pi}^2]:\ , \qquad (2.3)$$

and the interaction between quarks and pions is given by

$$H_I^\pi=\frac{i}{f_\pi}\int d^3x\frac{m}{\chi}:\bar q\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5 q:\ . \qquad (2.4)$$

The remaining terms in Eq. (2.1),

$$H_g=\int d^3x\left[\kappa(\chi)F^{0\nu a}\partial_0A_\nu^a+\frac{1}{4}\kappa(\chi)F_{\mu\nu}^aF^{\mu\nu a}\right] \qquad (2.5)$$

and

$$H_I^g=\frac{1}{2}g_s\int d^3x:\bar q\gamma^\mu\lambda^aA_\mu^aq:\ , \qquad (2.6)$$

describe the coupling of effective gluon fields A_μ^a to the color-singlet dielectric mean field χ through the dielectric function $\kappa(\chi)=\chi^4$ and the quark fields q, respectively. As we consider only single-gluon exchange between quarks, the quadratic terms in the gluon field tensor

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+g_sf^{abc}A_\mu^bA_\nu^c \qquad (2.7)$$

are dropped, so that each of the gluon fields propagates like an independent electromagnetic field in the presence of a spatially varying dielectric medium. It should be noted that in the absence of a rigorous derivation of the dielectric model from QCD, there is some arbitrariness in the details of the Hamiltonian density adopted above. Bayer, Forkel, and Weise [16] and Banerjee [17] have argued that the quark-pion coupling of Eq. (2.4) should be proportional to χ^{-2}. The question of whether residual gluon interactions, Eq. (2.6), should be included at all, has been mentioned in the Introduction. However, the work of McGovern [18] in fitting the baryon spectrum with a chiral dielectric model including perturbative gluons lends some support to the model chosen here. The fit using the inverse coupling of Eq. (2.4) was found to be more satisfactory than the fit using inverse-square coupling.

In zeroth order the interactions between quarks and pions and quarks and gluons may be ignored and the bare baryon states are eigenstates of H_0 with no gluons or pions present. The bare nucleon and Δ states are thus described by the usual MFA solutions where the mean χ field has spherical symmetry and the three quarks are all placed in the lowest 1S mode. The upper and lower radial components u and v of the quark wave functions and the quark energy eigenvalue ϵ satisfy

$$\frac{du}{dr}=-\left[\epsilon+\frac{m}{\chi}\right]v\ , \tag{2.8}$$

$$\frac{dv}{dr}=\left[\epsilon-\frac{m}{\chi}\right]-\frac{2v}{r}\ , \tag{2.9}$$

and the mean field χ is determined self-consistently from

$$\frac{d^2\chi}{dr^2}+\frac{2}{\chi}\frac{d\chi}{dr}=-\frac{3m}{\sigma_v^2\chi^2}(u^2-v^2)+M_\chi^2\chi \tag{2.10}$$

with appropriate boundary conditions. The spin and isospin states of the bare nucleon and Δ, denoted here simply by $|A_0\rangle$, are degenerate with energy

$$E_0=3\epsilon+2\pi\sigma_v^2\int_0^\infty dr\, r^2\left[\left[\frac{d\chi}{dr}\right]^2+M_\chi^2\chi^2\right]\ . \tag{2.11}$$

B. Perturbation theory

Our aim is to include perturbative corrections due to one-pion and one-gluon exchange. In the remainder of this section we consider the mass splitting of the nucleon and Δ to order $(1/f_\pi)^2$ and g_s^2. In Sec. IV pionic corrections to the static nucleon properties will be evaluated.

Working in the Schrödinger picture, we may write an exact formal equation for the dressed nucleon or Δ state $|A\rangle$ which satisfies $H|A\rangle=E_A|A\rangle$,

$$|A\rangle=(Z_2^A)^{1/2}|A_0\rangle+(E_A-H_0)^{-1}\Lambda H_I|A\rangle\ , \tag{2.12}$$

where both $|A\rangle$ and $|A_0\rangle$ are normalized to unity and Λ is the complement of the projection operator onto the space of degenerate bare nucleon and Δ states

$$\Lambda=I-\sum_{A_0}|A_0\rangle\langle A_0|\ . \tag{2.13}$$

The perturbation $H_I=H_I^g+H_I^\pi$ includes interactions with gluons as well as pions. The energy shift $\Delta_A=E_A-E_0$ is determined from

$$\Delta_A=\langle A_0|H_I|A_0\rangle +\langle A_0|H_I(E_0-H_0+\Delta_A)^{-1}\Lambda H_I|A\rangle(Z_2^A)^{-1/2}\ . \tag{2.14}$$

The second-order shift

$$\Delta_A^{(2)}=\langle A_0|H_I(E_0-H_0)^{-1}\Lambda H_I|A_0\rangle \tag{2.15}$$

$$=\Delta_A^g+\Delta_A^\pi \tag{2.16}$$

is obtained by replacing $|A\rangle(Z_2^A)^{-1/2}$ by $|A_0\rangle$ in Eq. (2.14), noting that in this case $\langle A_0|H_I|A_0\rangle$ vanishes and that the shift separates into distinct gluon and pion pieces.

C. The pion shift

A calculation of the pion shift, similar to that of Chin [19] for the MIT bag, yields

$$\Delta_A^\pi=-\frac{8\pi}{3}\frac{m^2}{f_\pi^2}\sum_{i,j}\langle\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j\rangle_A M_\pi \tag{2.17}$$

with

$$M_\pi=\int_0^\infty\int_0^\infty\frac{u(r)v(r)}{\chi(r)}\Delta(r,r')\frac{u(r')v(r')}{\chi(r')}r^2dr\, r'^2dr'\ , \tag{2.18}$$

where $\Delta(r,r')$ is the free pion propagator. (In his work Chin uses a pion propagator which excludes the pion from the bag.) The question arises whether the quark-pion self-energies given by the terms with $i=j$ should be included in the sums over the spin-isospin matrix elements in Eq. (2.17). Chin excludes the self-energies from the energy shift, grouping them with the vacuum energy of the bag. On the other hand, in cloudy-bag model calculations, they are included in order that intermediate quark states may be coupled together to give the full subspace of intermediate nucleon and Δ states. If S is the spin and T the isospin of the state A, then [20]

$$\sum_{i\neq j}\langle\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j\rangle_A=36-4S(S+1)-4T(T+1) \tag{2.19}$$

and $\sum_{i=j}\langle\ \rangle_A=27$ for both the nucleon and Δ. Thus the predicted splitting of the energy levels of the nucleon and Δ due to the pion, to the order of approximation considered here, does not depend on the pionic self-energies.

It is convenient to define

$$\Pi(r)=\int_0^\infty\Delta(r,r')\frac{u(r')v(r')}{\chi(r')}r'^2dr'\ , \tag{2.20}$$

satisfying

$$\frac{d^2\Pi}{dr^2}+\frac{2}{r}\frac{d\Pi}{dr}-\frac{2\Pi}{r^2}-m_\pi^2\Pi=\frac{uv}{\chi}\ , \tag{2.21}$$

in terms of which

$$M_\pi=\int_0^\infty\left[\left[\frac{d\Pi}{dr}\right]^2+\frac{2\Pi^2}{r^2}+m_\pi^2\Pi^2\right]r^2dr\ . \tag{2.22}$$

D. The gluon shift

The shift due to exchange of gluons in the dominant $M1$ mode is

$$\Delta_A^g=-\frac{4}{3}\pi g_s^2\sum_{i,j}\langle\lambda_i\cdot\lambda_j\sigma_i\cdot\sigma_j\rangle_A M_g \tag{2.23}$$

with

$$M_g=\int_0^\infty\int_0^\infty\frac{u(r)v(r)}{r\kappa(r)}g(r,r')\frac{u(r')v(r')}{r'\kappa(r')}r^2dr\, r'^2dr'\ , \tag{2.24}$$

where $g(r,r')$ is the static Green's function [21] for the propagation of the confined $M1$ gluon.

The matrix elements of the quark spin and color observables in Eq. (2.23) are taken with respect to the spin-isospin-color states of the nucleon or Δ. In the sum over quarks it is customary to exclude the terms with $i=j$, i.e., color-magnetic self-energies of the quarks are not included, and $\sum_{i\neq j}\langle \lambda_i\cdot\lambda_j\sigma_i\cdot\sigma_j\rangle_A=\pm 16$, the plus sign for the nucleon and the minus sign for Δ. This choice is supported by the derivation [22] of the shift using relativistic, many-body perturbation theory which suggests that the quark self-energies should be regarded as part of the vacuum energy of the soliton. However, in the present work we ignore the Dirac sea and make no attempt to calculate the Casimir energy of the soliton. As usual we assume that the color electric energies for quarks in the same spatial state sum to zero.

An equivalent expression [12] for M_g which avoids the construction of the Green's function,

$$M_g=\int_0^\infty\left[\left[\frac{dF}{dr}\right]^2+\frac{2F^2}{r^2}\right]\kappa\, dr\ , \tag{2.25}$$

uses the field function $F(r)$ which satisfies

$$\frac{d^2F}{dr^2}+\frac{1}{\kappa}\frac{d\kappa}{dr}\frac{dF}{dr}-\frac{2F}{r^2}=\frac{uvr}{\kappa}\ . \tag{2.26}$$

Equations (2.24) and (2.25) may be shown to be equivalent by using the explicit expression for the Green's function and integration by parts.

E. Center-of-mass corrections

The nucleon and Δ energies

$$E_A=E_0+\Delta_A^g+\Delta_A^\pi \tag{2.27}$$

contain contributions from the center-of-mass motion of the soliton. Our calculated masses

$$M_A=(E_A^2-\langle P^2\rangle_{A,q}-\langle P^2\rangle_{A,\chi})^{1/2} \tag{2.28}$$

include corrections for the quark momentum [23]

$$\langle P^2\rangle_{A,q}=12\pi\int_0^\infty dr\left\{r^2\left[\epsilon+\frac{m}{\chi}v\right]^2+\left[-2v+r\left[\epsilon-\frac{m}{\chi}\right]u\right]^2+2v^2\right\} \tag{2.29}$$

and the momentum of the χ field [24] (using a quantum coherent state to produce the mean χ field),

$$\langle P^2\rangle_{A,\chi}=2\pi M_\chi\sigma_v^2\int_0^\infty\left[\frac{d\chi}{dr}\right]^2 r^2dr\ . \tag{2.30}$$

In our numerical calculations the differential equations (2.8) and (2.9) for the quark wave functions, Eq. (2.10) for the χ field, Eq. (2.22) for the pion field, and Eq. (2.26) for the gluon field, together with a normalized integral for the quark wave functions, are formulated as a nonlinear boundary value problem and solved simultaneously [25]. In the next section we demonstrate that this system has interesting scaling properties leading to a formula for the Δ and nucleon masses in the chiral limit $m_\pi=0$.

III. THE NUCLEON AND Δ MASSES IN THE CHIRAL LIMIT

A. Scaling

The MFA solutions describing the degenerate bare nucleon and Δ states depend on three parameters, the quark mass m, the glueball mass M_χ, and the scale σ_v of the χ field. McGovern, Birse, and Spanos [13] have shown that for sufficiently large values of the glueball mass only two of the parameters are independent and after choosing one to fix the size of the soliton, one is left with a one-parameter family of MFA solutions. In this section we extend their arguments to find a mass formula for the nucleon and Δ which includes the color-magnetic energy, the pion interaction energy, and corrections for center-of-mass motion.

With the help of a length unit

$$r_0=(mM_\chi\sigma_v)^{-1/3}\ , \tag{3.1}$$

new dimensionless variables may be introduced:

$$r=r_0x\ , \tag{3.2}$$

$$\epsilon=r_0^{-1}\epsilon_0\ , \tag{3.3}$$

$$\chi=mr_0\chi_0\ , \tag{3.4}$$

$$u=r_0^{-3/2}u_0\ , \tag{3.5}$$

$$v=r_0^{-3/2}v_0\ , \tag{3.6}$$

$$F=m^{-4}r_0^{-4}F_0\ , \tag{3.7}$$

and

$$\Pi=m^{-1}r_0^{-2}\Pi_0\ . \tag{3.8}$$

In terms of these variables, the system to be solved is (the prime denotes differentiation with respect to $x=r/r_0$)

$$u_0'=-\left[\frac{1}{\chi_0}+\epsilon_0\right]v_0\ , \tag{3.9}$$

$$v_0'+\frac{2}{x}v_0=-\left[\frac{1}{\chi_0}-\epsilon_0\right]u_0\ , \tag{3.10}$$

$$\left[\chi_0''+\frac{2}{x}\chi_0'\right]\frac{1}{M_\chi^2r_0^2}=\chi_0-\frac{3}{\chi_0^2}(u_0^2-v_0^2)\ , \tag{3.11}$$

$$F_0''+\frac{4\chi_0'}{\chi_0}F_0'-\frac{2}{x^2}F_0=\frac{u_0v_0x}{\chi_0^4}\ , \tag{3.12}$$

$$\Pi_0''+\frac{2}{x}\Pi_0'-\frac{2}{x^2}\Pi_0-m_\pi^2r_0^2\Pi_0=\frac{u_0v_0}{\chi_0}\ , \tag{3.13}$$

with the normalization condition

$$4\pi\int_0^\infty(u_0^2+v_0^2)x^2dx=1\ . \tag{3.14}$$

For sufficiently smooth variations of the χ_0 field and large values of the glueball mass M_χ, the left-hand side of Eq.

(3.11) is negligible and χ_0 is simply determined from the quark wave function,

$$\chi_0^3=3(u_0^2-v_0^2) . \tag{3.15}$$

Furthermore if the pion mass vanishes, Eqs. (3.9), (3.10), and (3.12)–(3.15) constitute a *dimensionless, parameter-free system* which needs to be solved only once to determine the quark wave functions, the χ field, and the pion and gluon fields for all values of m, σ_v, and M_χ, provided $M_\chi r_0$ is large.

B. Mass formula

Evaluation of the energy of the nucleon in terms of the scaled variables gives

$$E_N=r_0^{-1}[3\epsilon_0+c_1+c_2(M_\chi r_0)^{-2}+c_3 g_s^2(mr_0)^{-4}+c_4(f_\pi r_0)^{-2}] , \tag{3.16}$$

where $\epsilon_0, c_1, c_2, c_3$, and c_4 are the following constants:

$$\epsilon_0=2.426 , \tag{3.17}$$

$$c_1=2\pi\int \chi_0^2 x^2 dx=1.456 , \tag{3.18}$$

$$c_2=2\pi\int (\chi_0')^2 dx=6.68 , \tag{3.19}$$

$$c_3=-\frac{256}{3}\pi^2\int \left[(F_0')^2+\frac{2}{x^2}F_0^2\right]\chi_0^4 dx =-0.046\,17 , \tag{3.20}$$

and

$$c_4=-80\pi\int \left[(\Pi_0')^2+\frac{2}{x^2}\Pi_0^2\right]x^2 dx =-0.1469 , \tag{3.21}$$

determined from numerical solution of the soliton equations in the limit where Eq. (3.15) is satisfied. A typical solution is shown in Fig. 1. In the expression for the energy, Eq. (3.16), the first term is the quark energy, the second the potential energy of the χ field, the third the kinetic energy of χ, the fourth the color-magnetic energy, and the fifth the pion field energy (in the chiral limit). The energy of Δ is also given by Eq. (3.16) with c_3 replaced by $-c_3$ and c_4 replaced by $c_4/5$.

The color-magnetic energy appears to be strongly dependent on the quark mass. However, as McGovern [18] points out, the definitions of the strong-coupling constant α_s and the dielectric function are interdependent and there is no unique value of the quark-gluon coupling. From Eqs. (2.5), (2.6), and (2.7) we see that a change $\kappa\to\lambda^4\kappa$ is compensated by the changes $A_\mu^a\to\lambda^{-2}A_\mu^a$ and $g_s\to\lambda^2 g_s$. We will fix the definition of the strong-coupling constant by choosing $\kappa=1$ at the center of the soliton. Since the value of the dielectric function is proportional to $\chi^4(0)=(mr_0)^4\chi_0^4(0)$ at the center of the soliton, the coupling constant is $4\pi\alpha_s=g_s^2[mr_0\chi_0(0)]^{-4}$ and the color-magnetic energy may be written as

$$4\pi\alpha_s c_3[\chi_0(0)]^4 r_0^{-1}=0.9050\alpha_s r_0^{-1} , \tag{3.22}$$

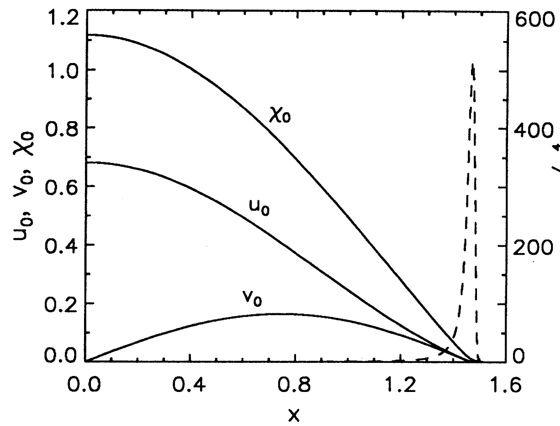

FIG. 1. Solutions of Eqs. (3.9)–(3.11) with a large value of the glueball mass, $M_\chi r_0=48.7$. The dashed curve is the gluon source term of Eq. (3.12). For $x<1.4$ these curves are indistinguishable from the solutions of Eqs. (3.9) and (3.10) with the approximation (3.15).

which shows the expected dependence on the soliton parameters. Of course, defining the coupling constant and gluon potentials in this way does not remove the sensitivity of the color-magnetic energy to the χ field inside the soliton. Once the scale of the χ field is set variations of the field inside solitons with different quark content will produce large relative changes in the gluonic energy.

The corrections, Eqs. (2.29) and (2.30), to the energy due to the center-of-mass motion also scale:

$$\langle P^2\rangle_q=c_5 r_0^{-2} \tag{3.23}$$

with

$$c_5=12\pi\int \{[(\epsilon_0+\chi_0^{-1})v_0]^2+[-2v_0/x+(\epsilon_0-\chi_0^{-1})u_0]^2+2v_0^2/x^2\}x^2 dx , =16.12 \tag{3.24}$$

and

$$\langle P^2\rangle_\chi=c_2(M_\chi r_0^3)^{-1} . \tag{3.25}$$

The nucleon and Δ masses in the model are found from Eq. (2.28), using Eq. (3.16) for the energies and Eqs. (3.23) and (3.24) for the momentum corrections.

The parameter r_0 is related to the root-mean-square radius of the quark distribution in the nucleon R by

$$R=\eta\langle x^2\rangle^{1/2}r_0 , \tag{3.26}$$

where

$$\langle x^2\rangle=4\pi\int (u_0^2+v_0^2)x^4 dx=(0.7923)^2 \tag{3.27}$$

and

$$\eta=\left[1-2\lambda+3\lambda^2+\frac{3}{2}\left[\frac{\lambda}{\epsilon_0 r_0}\right]^2\right]^{1/2} \tag{3.28}$$

TABLE I. Nucleon and Δ masses in the chiral limit. Quantities in columns labeled (a) have been calculated by solving the full set of soliton equations; the corresponding predictions from the approximate mass formula are contained in columns (b).

Quantity	$M_\chi/m=10$ $M_\chi=648.6$ MeV $\alpha_s=0.6016$ (a)	(b)	$M_\chi/m=50$ $M_\chi=1910.5$ MeV $\alpha_s=0.5699$ (a)	(b)	$M_\chi/m=150$ $M_\chi=3978.8$ MeV $\alpha_s=0.5687$ (a)	(b)
$\langle r^2\rangle^{1/2}$ (fm)	0.750	0.752	0.750	0.751	0.750	0.750
E_0 (MeV)	1600.8	1624.5	1558.3	1559.2	1553.1	1553.2
Δ_N^g (MeV)	−92.5	−95.9	−91.8	−91.5	−91.6	−91.4
Δ_N^π (MeV)	−97.9	−92.7	−95.3	−94.8	−95.3	−95.1
M_N (MeV)	1228.6	1227.1	1175.6	1165.6	1167.7	1162.1
M_Δ (MeV)	1523.6	1529.7	1470.6	1461.5	1462.7	1457.7

with

$$\lambda=\frac{\epsilon_0}{M_A r_0}\ , \tag{3.29}$$

an additional scaling factor [23] which estimates the reduction in size after removal of the motion of the center of mass.

We note that the simple scaling behavior derived here depends on our initial choice of the quadratic form of the potential energy of the χ field. In practice this means that the MFA solutions are one-phase solutions in the nomenclature of Ref. [26]. Unlike the usual bag models, there is no bag pressure, the energy of the χ field having the same $1/r_0$ dependence as the quark energy. For two-phase solutions, possible in quartic potentials, where there is rapid variation in χ between the interior and exterior of the soliton, the kinetic energy of the χ field is not negligible, and the above scaling does not hold. Of course, in this case the full equations may be solved numerically for a given parameter set which may include a bag pressure, but the simplicity of the energy formula equation (3.16) is lost.

From the work of this section, we see that the masses of the nucleon and Δ are essentially determined by the length scale $r_0=(mM_\chi\sigma_v)^{-1/3}$ and the strong-coupling constant α_s, the pion decay constant $f_\pi=93$ MeV being taken from experiment, and the small corrections due to the kinetic energy of the χ field being of order $(M_\chi r_0)^{-1}$. If r_0 is fixed by fitting the isoscalar charge radius of the nucleon and α_s by fitting the nucleon-Δ mass splitting, the predicted masses of the nucleon and Δ in the model show little variation for a wide range of quark and glueball masses. In Table I the predictions for the masses using the approximation equation (3.15) are compared with those given by numerical solution of the full system, Eqs. (3.9)–(3.14), for three parameter sets. The input parameters have been fixed by requiring a nucleon-Δ mass splitting of 295 MeV and an isoscalar nucleon radius of 0.75 fm in the full numerical calculations. Even for ratios of the glueball mass to the quark mass as small as $M_\chi/m=10$ the approximate formula is remarkably accurate.

IV. PIONIC CORRECTIONS TO NUCLEON PROPERTIES

Previous work [4] has shown that it is consistent to treat the weak pion field in chiral nontopological solitons perturbatively, as is done in the cloudy-bag model. With an appropriately modified pionic form factor, the CBM expressions may be applied to evaluate pionic contributions to nucleon properties in the present model.

The vertex functions for the absorption or emission of a pion are found by expanding the pion field in a plane-wave basis and taking matrix elements of the interaction (2.4) between the bare soliton states $|A_0\rangle$. In particular, the vertex function $v_j^{AB}(\mathbf{k})$ for the absorption of a pion with isospin j and momentum $\mathbf{k}$ on the bare baryon state $|B_0\rangle$ to produce the baryon state $|A_0\rangle$ may be written as [4]

$$v_j^{AB}(\mathbf{k})=-i\frac{f^{AB}}{m_\pi}\frac{\mu(k)}{(2\pi)^{3/2}(2\omega_k)^{1/2}}\sum_{m,n}\langle S_B,s_B,1,m|S_A,s_A\rangle\langle T_B,t_B,1,n|T_A,t_B\rangle k_m^* e_{j,n}^*\ , \tag{4.1}$$

where S_A and s_A denote the spin and third component of spin for A (and similarly T_A and t_A for isospin), k_m and $e_{j,n}$ are the spherical tensor components of the momentum $\mathbf{k}$ and the vector $\mathbf{e}_j$, respectively, and $\omega_k^2=k^2+m_\pi^2$. The CBM form factor [7]

$$\mu'(kR)=3j_1(kR)/kR\ , \tag{4.2}$$

where j_1 is the spherical Bessel function of order one and

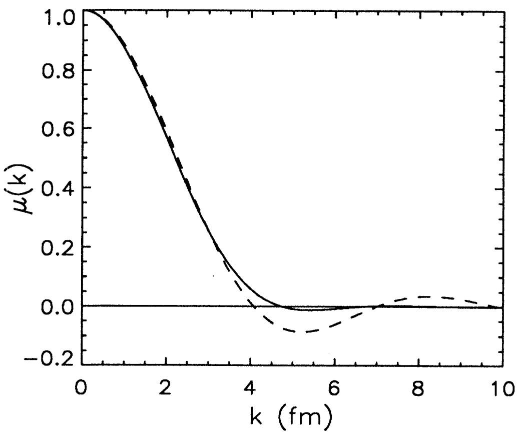

FIG. 2. Comparison of the soliton model (solid line) and CBM (dashed line) pion form factors for the parameter set of column 2 of Table II. The CBM radius, $R=1.1$ fm, minimizes the root-mean-square difference of the form factors.

R is the bag radius, is replaced in Eq. (4.1) by the soliton form factor

$$\mu(k)=\frac{\int dr\, r^3[m/\chi(r)]u(r)v(r)\mu'(kr)}{\int dr\, r^3[m/\chi(r)]u(r)v(r)} , \tag{4.3}$$

defined so that $\mu(0)=1$. The form factors are compared in Fig. 2.

From Eqs. (1.8) and (1.9) it is easy to establish [4,27] that the denominator in (4.3) is proportional to the bare axial vector coupling constant

$$\begin{aligned} g_A^b &= \frac{5}{3}\int d^3r \left[u^2(r)-\frac{1}{3}v^2(r)\right] \\ &= \frac{20}{9}4\pi \int dr\, r^3 \frac{m}{\chi} u(r)v(r) \end{aligned} \tag{4.4}$$

and hence that the nucleon-nucleon transition coupling constant in (4.1) is

$$f^{NN}=\frac{3}{2}\frac{m_\pi}{f_\pi}g_A \tag{4.5}$$

and that the other relevant couplings have the usual CBM ratios,

$$f^{NN}:f^{\Delta\Delta}:f^{N\Delta}:f^{\Delta N}=5:5:4\sqrt{2}:2\sqrt{2} . \tag{4.6}$$

In terms of the usual πNN coupling constant, $f^{NN}=(3m_\pi/2m_N)g_{\pi NN}=3\sqrt{4\pi}f_{\pi NN}$ and Eq. (4.5) is an expression of the Goldberger-Treiman relation.

A. Scaling

The solutions of Eqs. (2.8) and (2.9) for the quark wave functions and Eq. (2.10) for the χ field may be used to construct the form factor (4.3) and the transition coupling constants (4.6) using (4.4) and (4.5). With the coupling constants and the form factor calculated from the soliton solution replacing the CBM form factor and coupling constants, the usual CBM expressions for the pionic contributions to the nucleon and Δ self-energies, charge radii, and magnetic moments, etc., apply. Before discussing these contributions in detail, it is important to note that the pionic corrections will be largely independent of the choice of the bare soliton parameters. This can be seen by applying the transformations (3.2)–(3.6) of the previous section to the (4.3) and (4.4). The bare axial constant becomes

$$g_A^b=\frac{80\pi}{9}\int dx\, x^3\frac{u_0 v_0}{\chi_0} . \tag{4.7}$$

For a sufficiently large glueball mass, the scaled variables approach their limiting forms, $g_A^b=1.318$, and hence f^{NN}, $f^{\Delta\Delta}$, $f^{N\Delta}$, and $f^{\Delta N}$ are constant under variation of the soliton parameters. In the same limit the form factor only depends on the length scale set by $r_0=(M_\chi m\sigma_v)^{-1/3}$, since

$$\mu(k)=\frac{80\pi}{9g_A^b}\int dx\, x^3\frac{u_0 v_0}{\chi_0}\mu'(kr_0x) . \tag{4.8}$$

B. Pionic self-energies

The pionic self-energies of the nucleon and Δ are given by

$$\Sigma^A=-\frac{1}{12\pi^2}\sum_B \left[\frac{f^{AB}}{m_\pi}\right]^2\int dk\frac{k^4\mu^2(k)}{\omega_k(\omega_k+m_B-m_A)} . \tag{4.9}$$

In the cloudy-bag model the masses $m_B=m_N,m_\Delta$ are usually taken as the physical masses and renormalized perturbation theory is considered. Here, since we are only considering the leading order in a perturbative calculation, the masses m_N and m_Δ are equal to the bare soliton mass. In this case the energy shift given by Eq. (4.9) is the same as that given by Eq. (2.17), derived at the quark-pion level, provided the quark-pion self-energies [terms with $i=j$ in Eq. (2.17)] are included. If the scaling transformations are applied to (4.9), we see that the mass splitting of the Δ and nucleon due to pions has the form $(f_\pi r_0)^{-2}r_0^{-1}I(m_\pi r_0)$, where I is an integral depending on a single parameter, the product of the soliton scale, and the chiral symmetry-breaking pion mass.

C. Electric form factors and charge radii

The pionic contribution to the nucleon electric form factor is

$$G_{E,N}^\pi(q^2)=\pm\frac{1}{36\pi^3}\left[\frac{f^{NN}}{m_\pi}\right]^2\int d^3k\frac{\mu(k)\mu(k')\mathbf{k}\cdot\mathbf{k}'}{\omega_k\omega_{k'}(\omega_k+\omega_{k'})}\mp\frac{1}{72\pi^3}\left[\frac{f^{N\Delta}}{m_\pi}\right]^2\int d^3k\frac{\mu(k)\mu(k')\mathbf{k}\cdot\mathbf{k}'}{(\omega_k+\omega_{\Delta N})(\omega_{k'}+\omega_{\Delta N})(\omega_k+\omega_{k'})} , \tag{4.10}$$

where $\mathbf{k}'=\mathbf{k}+\mathbf{q}$ and $\omega_{\Delta N}=m_\Delta-m_N$. The upper sign holds for the proton and the lower sign for the neutron. Since (4.10) involves the difference of two similar terms, it turns out that the calculated values of the electric root-mean-square radii of the neutron and proton are quite sensitive to the assumed value of $\omega_{\Delta N}$. In our simple perturbative approach where m_Δ and m_N are equal to the bare soliton mass, $\omega_{\Delta N}=0$. Alternatively, we may compute the pionic correction, after the gluonic hyperfine splitting has been calculated, by setting $\omega_{\Delta N}=\Delta_\Delta^g-\Delta_N^g$ [cf. Eq. (2.23)]. Numerical results for both choices are compared in the next section.

The quark contribution to the electric form factor is proportional to the Fourier transform of the quark density,

$$G_{E,N}^{q}(q^2)=C_N\int d^3r[u^2(r)+v^2(r)]e^{i\mathbf{q}\cdot\mathbf{r}} , \tag{4.11}$$

where the constant C_N is determined from charge conservation, $G_{E,p}^{q}(0)+G_{E,p}^{\pi}(0)=1$ for the proton and $G_{E,n}^{q}(0)+G_{E,n}^{\pi}(0)=0$ for the neutron.

The charge radii are calculated from the electric form factors by

$$\langle r^2\rangle_N=-6\frac{\partial}{\partial q^2}[G_{E,N}^{q}(q^2)+G_{E,N}^{\pi}(q^2)]_{q^2=0} . \tag{4.12}$$

D. Magnetic moments

The pionic contribution to the nucleon magnetic moment is

$$\mu_N^\pi=\pm\frac{1}{27\pi^2}\left[\frac{f^{NN}}{m_\pi}\right]^2\int_0^\infty dk\, k^4\frac{\mu^2(k)}{\omega_k^4} \pm\frac{1}{216\pi^2}\left[\frac{f^{N\Delta}}{m_\pi}\right]^2\int_0^\infty dk\, k^4\frac{\mu^2(k)(\omega_{\Delta N}+2\omega_k)}{\omega_k^3(\omega_{\Delta N}+\omega_k)^2} , \tag{4.13}$$

the upper sign holding for the proton and the lower sign for the neutron.

The quark contribution involves several integrals which determine the probabilities of various components of the dressed nucleon. Define

$$P_{BC\pi}=\frac{1}{12\pi^2}Z_2\frac{f^{NB}f^{NC}}{m_\pi^2}\times\int_0^\infty dk\, k^4\frac{\mu^2(k)}{\omega_k(\omega_{BN}+\omega_k)(\omega_{CN}+\omega_k)} , \tag{4.14}$$

then for the proton,

$$\mu_p^q=\frac{\mu_0}{27}(27Z_2+P_{NN\pi}+20P_{\Delta\Delta\pi}+16\sqrt{2}P_{N\Delta\pi}) , \tag{4.15}$$

and for the neutron,

$$\mu_n^q=-\frac{\mu_0}{27}(18Z_2+4P_{NN\pi}+5P_{\Delta\Delta\pi}+16\sqrt{2}P_{N\Delta\pi}) . \tag{4.16}$$

The contribution from three bare quarks in Eqs. (4.15) and (4.16) is

$$\mu_0=\frac{2}{3}\int_0^\infty dr\, r^3u(r)v(r) , \tag{4.17}$$

and the normalization is determined by $Z_2+P_{NN\pi}+P_{\Delta\Delta\pi}=1$. The pionic and quark contribu-

TABLE II. Results for nucleon properties, including perturbative pions and gluons, for four different soliton-parameter sets compared to experiment. The soliton scale is fixed to give the experimental isoscalar charge radius and the strong-coupling constant is fixed to give the experimental value of the nucleon-Δ mass splitting in all cases. Center-of-mass corrections are included. The values in parentheses are for vanishing nucleon-Δ mass difference.

Quantity \ Set	1	2		3	4	Expt.
M_χ/m	10	50		150	50	
M_χ (MeV)	648	1908		3974	1920	
α_s	0.646	0.612		0.611	0.401	
$\langle r_0^2\rangle^{1/2}$ (fm)	0.750	0.750		0.750	0.750	0.750
E_0 (MeV)	1601	1556		1551	1566	
Δ_N^g (MeV)	-99.2	-98.5		-98.2	-65.0	
Δ_N^g (MeV)	-81.6	-79.5		-79.2	-160.0	
M_N (MeV)	1240	1185		1177	1139	939
$M_\Delta-M_N$ (MeV)	295	295		295	295	295
$f_{NN\pi}$	0.283	0.280		0.279	0.393	0.28
g_A^b	1.34	1.32		1.32	1.32	1.27
μ_p	2.28	2.29	(2.40)	2.29	2.71	2.76
μ_n	-1.72	-1.72	(-1.78)	-1.72	-2.18	-1.91
$\langle r_p^2\rangle^{1/2}$ (fm)	0.795	0.795	(0.770)	0.795	0.830	0.83
$\langle r_n^2\rangle^{1/2}$ (fm)	-0.264	-0.263	(-0.175)	-0.262	-0.355	-0.35

tions together give

$$\mu_N = \mu_N^q + \mu_N^\pi \ . \tag{4.18}$$

V. NUMERICAL RESULTS AND CONCLUSIONS

Typical results of our numerical calculations of static nucleon properties, including pionic contributions, are shown in Table II. In these calculations chiral symmetry is broken by using the experimental value of the pion mass in the field equation (3.13). Nevertheless Table II shows that the scaling behavior derived in Sec. III for massless pions persists; when the overall scale of the unperturbed soliton solution is set by matching the experimental isoscalar charge radius, there is little variation in the predicted nucleon properties over a wide range of input parameters for the soliton. As in other soliton-bag calculations, the predicted nucleon masses are too large. The nucleon mass can be made smaller by including the pion-quark self-energies, but following the discussion of Sec. II, we believe it is more consistent not to do so. The pion generates about 25% of the nucleon-Δ mass splitting; the rest is provided by the $M1$ color-magnetic splitting (α_s is adjusted to reproduce the experimental mass difference). A comparison of Tables I and II shows that the pionic energy shift is decreased by about 17% in going from the massless to the massive pion.

The strength of the pion coupling is fixed by the experimental values of the pion decay constant and the pion mass, and the value of the bare axial coupling g_A^b, which is calculated from the model. The axial coupling is independent of the soliton scale and is nearly constant over the various parameter sets. Thus the pion coupling is almost constant and as can be seen from the first three columns of Table II, once the soliton size is fixed, the pionic contributions to the nucleon properties exhibit little dependence on the details of the soliton solutions.

The model is qualitatively similar to the cloudy-bag model; the pion cloud increases the charge radius of the proton and gives the neutron a negative charge radius. In Fig. 3, we plot the neutron charge density, together with its quark and pion components, for the soliton model using the parameters of column 2, Table II ($M_\chi = 1908$ MeV). The quark and pion components for the CBM are also shown for comparison, where the bag radius $R = 1.1$ fm. This choice of R minimizes the root-mean-square difference of the soliton and CBM form factors (shown in Fig. 2). The quark (isoscalar) radius is then 20% larger in the soliton model than in the CBM. The pionic term $\langle r^2 \rangle_\pi$ is 10% smaller. The decrease in $\langle r^2 \rangle_\pi$ is attributable to the small tail of the form factor (Fig. 2). There are minor differences in the numerical values of the coupling parameters f^{NN} chosen in the two models, some second-order renormalization being taken into account in the CBM, but these differences are compensated for by the larger value for the mass difference $\omega_{\Delta N}$ in the CBM where the physical masses are assumed. Excluding center-of-mass corrections, the proton and neutron

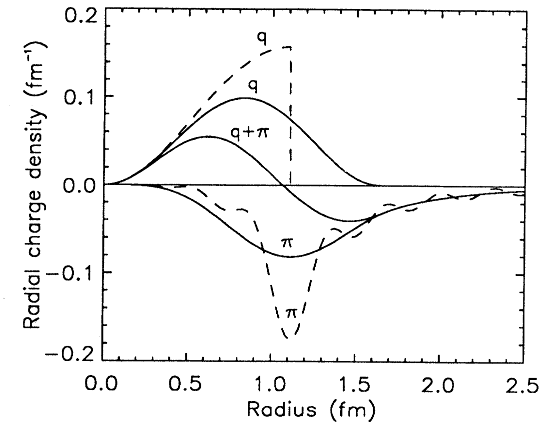

FIG. 3. Comparison of the quark and pion components of the neutron charge density in the soliton model (solid curves) and the CBM (dashed curves). The CBM radius is $R = 1.1$ fm. The total charge density is shown only for the soliton model.

charge radii are 0.93 and -0.31 fm in the soliton calculation in comparison with 0.87 and -0.34 fm in the corresponding CBM calculation [8]. The values of $f_{NN\pi}$ and g_A in Table II are bare values. They are decreased by about 5% if the renormalization procedure of the CBM is adopted.

The values for the magnetic moments and charge radii given in Table II have been calculated with the difference between the Δ and nucleon mass given by the color-magnetic energy, $\omega_{\Delta N} = 2\Delta_N^g$ in the energy denominators of Eqs. (4.10) and (4.13). For the parameter set with glueball mass $M_\chi = 1908$ MeV, the values in parentheses, calculated with a vanishing Δ-nucleon mass difference, are also given for comparison. It can be seen that the pionic contributions, particularly the neutron charge radius, are quite sensitive to the assumed value of $\omega_{\Delta N}$.

Although the πNN coupling constant agrees well with the experimental value, pionic contributions to static electromagnetic properties are somewhat underestimated by the model. From the above comparison with the CBM it appears that the pion form factor (4.3) falls off too quickly in momentum space compared with CBM form factor (4.2). This can be compensated for by regarding the pion coupling f^{NN} as an adjustable parameter rather than taking the value fixed by Goldberger-Treiman relation, Eq. (4.5), of the model. For example, the results listed in the sixth column of Table II, obtained with the pion coupling strength increased by 40%, are in excellent agreement with the experimental values. However, it should be emphasized that this procedure violates one of the attractive theoretical features of the model, the connection between the scale of the bare soliton and the magnitude of the pion cloud. In view of the simplicity of the perturbative calculation of gluonic and pionic corrections, and the crudeness of the estimates of center-of-mass effects, the quantitative results are in reasonable agreement with experiment.

ACKNOWLEDGMENTS

We wish to thank A. G. Williams for helpful discussions and the Australian Research Council for financial support. One of us (L.R.D.) gratefully acknowledges the hospitality of the Department of Physics and the Supercomputer Computations Research Institute, Florida State University, while part of this work was carried out.

---

[1] Extensive references are contained in the recent reviews by M. C. Birse, Prog. Part. Nucl. Phys. **25**, 1 (1990); and by L. Wilets, *Nontopological Solitons* (World Scientific, Singapore, 1989).

[2] A. Chodos *et al.*, Phys. Rev. D **9**, 3471 (1974); T. DeGrand *et al.*, *ibid.* **12**, 2060 (1975); C. E. DeTar and J. F. Donoghue, Annu. Rev. Nucl. Part. Sci. **33**, 235 (1983); P. Hasenfratz and J. Kuti, Phys. Rep. C **40**, 75 (1978).

[3] See, for example, the review by A. W. Thomas, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. W. Vogt (Plenum, New York, 1983), Vol. 13, p. 1.

[4] A. G. Williams and L. R. Dodd, Phys. Rev. D **37**, 1971 (1988).

[5] G. Chanfray, O. Nachtmann, and H. J. Pirner, Phys. Lett. **147B**, 249 (1984); H. B. Nielsen and A. Patkos, Nucl. Phys. **B195**, 137 (1982).

[6] R. Friedberg and T. D. Lee, Phys. Rev. D **15**, 1694 (1977); T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood, Chur, Switzerland, 1981); R. Goldflam and L. Wilets, Phys. Rev. D **25**, 1951 (1982).

[7] A. W. Thomas, S. Théberge, and G. A. Miller, Phys. Rev. D **24**, 216 (1981); S. Théberge and A. W. Thomas, Nucl. Phys. **A393**, 252 (1983); S. Théberge, Ph.D. thesis, University of British Columbia, 1981 (unpublished).

[8] S. Théberge, G. A. Miller, and A. W. Thomas, Can. J. Phys. **60**, 59 (1982).

[9] L. R. Dodd, A. W. Thomas, and R. F. Alvarez-Estrada, Phys. Rev. D **24**, 1961 (1981); R. F. Alvarez-Estrada and A. W. Thomas, J. Phys. G **9**, 161 (1983).

[10] A. Chodos and C. B. Thorn, Phys. Rev. D **12**, 2733 (1975); M. C. Birse and M. K. Bannerjee, Phys. Lett. **136B**, 284 (1984); Phys. Rev. D **31**, 118 (1985); S. Kahana, G. Ripka, and V. Soni, Nucl. Phys. **A415**, 351 (1984); H. Kitagawa, *ibid.* **A487**, 544 (1988).

[11] M. Bickeböller, M. C. Birse, and L. Wilets, Z. Phys. A **326**, 89 (1987); M. Bickeböller, M. C. Birse, M. Marshall, and L. Wilets, Phys. Rev. D **31**, 2892 (1985).

[12] L. R. Dodd and A. G. Williams, Phys. Lett. B **210**, 10 (1988).

[13] J. A. McGovern, M. C. Birse, and D. Spanos, J. Phys. G **16**, 1561 (1990).

[14] R. G. Leech and M. C. Birse, J. Phys. G **18**, 785 (1992).

[15] M. K. Banerjee, W. Broniowski, and T. D. Cohen, in *Chiral Solitons,* edited by K.-F. Liu (World Scientific, Singapore, 1987), p. 255.

[16] L. Bayer, H. Forkel, and W. Weise, Z. Phys. A **324**, 365 (1986).

[17] M. K. Banerjee, in *Quarks, Mesons, and Nuclei,* edited by W.-Y. P. Hwang and E. M. Henley (World Scientific, Singapore, 1988); C.-Y. Ren and M. K. Banerjee, Phys. Rev. C **41**, 2370 (1990); M. K. Banerjee, *ibid.* **45**, 1359 (1992).

[18] J. A. McGovern, Nucl. Phys. **A533**, 553 (1991).

[19] S. A. Chin, Nucl. Phys. **A382**, 385 (1982).

[20] A. Th. M. Aerts, P. J. G. Mulders, and J. J. deSwart, Phys. Rev. D **17**, 260 (1978).

[21] M. Bickeböller, R. Goldflam, and L. Wilets, J. Math. Phys. **26**, 1810 (1985); P. Tang and L. Wilets, *ibid.* **31**, 1661 (1990).

[22] S. A. Chin, A. K. Kerman, and X. H. Yang, MIT CTP No. 919, July 1981; S. A. Chin, Ann. Phys. (N.Y.) **108**, 301 (1977).

[23] J.-L. Dethier, R. Goldflam, E. M. Henley, and L. Wilets, Phys. Rev. D **27**, 291 (1983).

[24] E. G. Lubĕck, M. C. Birse, E. M. Henley, and L. Wilets, Phys. Rev. D **33**, 234 (1986).

[25] L. R. Dodd and M. A. Lohe, Phys. Rev. D **32**, 1816 (1985).

[26] L. R. Dodd, A. G. Williams, and A. G. Thomas, Phys. Rev. D **35**, 1040 (1987).

[27] R. Goldflam, I. Duck, and E. Umland, Phys. Lett. **127B**, 155 (1983).