# Radiative corrections to the nucleon axial vector coupling constant in the chiral soliton quark model

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Second-order radiative corrections to the nucleon axial vector coupling constant from gluon, pion, and sigma meson exchange are calculated in the chiral soliton quark model. Many apparent processes are found not to contribute. The soliton is elastically decoupled from meson radiative corrections which are dominated by a gluon exchange contribution equivalent to a gluonic hybrid component of the nucleon. A 30% radiative reduction of the axial coupling strength is indicated.

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## I. INTRODUCTION

Two recent major developments make it important to reexamine critically the quark model nucleon structure calculations, especially their predictions for the axial vector coupling constant. The first of these developments is the EMC experiment [1] which radically contradicts the naive quark model expectation that the angular momentum of the nucleon is in large part due to the quark spins. As is well known, the EMC analysis leads to a nucleon quark spin close to zero [2]. In a second development, Weinberg [3] has raised the question of the reason for the success of the quark model in predicting the nucleon axial vector coupling constant $G_A$ (=1.254). In quark models of the nucleon, there are two (at least) competing points of view, characterized as the constituent quark model [4] and the current quark model. In the constituent quark model, quarks are dressed in a chiral phase transition and acquire a mass taken to be about one-third the nucleon mass. Nucleon structure calculations then involve weak binding of these constituent quarks [5]. In the current quark model, exemplified by the MIT bag model [6], relativistic quarks are confined by an *ad hoc* prescription and subject to meson interactions to restore chiral invariance violated by the confinement mechanism. Even here there is a multiplicity of opinion on, for example, the degree of dominance of the meson interactions and even on the necessity of including quarks in the first place [7].

In spite of the ambiguities, we choose the current quark point of view because it is a Lagrangian-based model, including basic QCD degrees of freedom supplemented with various meson interactions, for which we can do field theoretic perturbation calculations and, hopefully, calculate corrections to the model predictions. Furthermore, the relativistic chiral soliton quark model is well developed, especially in the work of Birse and Banerjee [8], and it is remarkably successful in its predictions of nucleon parameters including mass, magnetic moment, axial, and pion couplings. It is the quantitative success of this model which we examine by calculating perturbative corrections to the axial vector coupling constant. We concentrate specifically on the axial vector coupling, but radiative corrections to the magnetic moment are also interesting. As we will see in the conclusions, perturbation theory is quite encouraging. It gives sensible results which appear to converge where testable, but show need of improvement where otherwise suspect.

We will make frequent reference to the MIT bag model, whose relevant quark wave functions are listed in Appendix A. Together with the prescription of the SU(6) spin-isospin symmetric wave function for the $(N\Delta)$ baryon multiplet [9], it produces an axial vector coupling constant

$$G_A(\mathrm{MIT})=\langle P\uparrow|\sigma_z\tau_z|P\uparrow\rangle g_A(q) =\tfrac{5}{3}g_A(q)=1.09\ , \qquad (1.1)$$

in terms of the $1S$ quark mode value of the axial coupling

$$g_A(q)=\int (U^2-V^2/3)d^3r=0.653 \qquad (1.2)$$

for zero quark mass. $G_A$(MIT) increases to 1.25 for a quark mass equal to $1/R$ (=250 MeV for $R=0.8$ fm), and to $\frac{5}{3}$ if the quark mass is taken large and the relativistic current quark model is replaced by the nonrelativistic constituent quark model. Clearly, there is no difficulty in choosing a quark mass to produce the required result for $G_A$. We ignore this tactic, however, and use a massless current quark as a QCD degree of freedom interacting in a chirally symmetric way with the sigma-pion mesons which occupy all space in accord with the Lagrangian used by Birse [10]

$$\begin{aligned}L(x)=&\bar{q}(x)\{i\gamma_\mu\partial^\mu+g[\sigma(x)+i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma_5]\}q(x)\\&+\tfrac{1}{2}\partial_\mu\sigma(x)\partial^\mu\sigma(x)+\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}(x)\cdot\partial^\mu\boldsymbol{\pi}(x)\\&-\lambda/4[\sigma(x)^2+\boldsymbol{\pi}(x)^2-f_\pi^2]^2-f_\pi m_\pi^2\sigma(x)\ .\end{aligned} \qquad (1.3)$$

We refer to the extensive discussion of the meson-quark soliton contained in Birse's paper for technical details, and just briefly describe the results. The equations of motion for the sigma-pion-quark system separate for the hedgehog configuration where the spin and isospin are in the radial direction. The resulting soliton is a superposition of degenerate states with total angular momentum $J$ equal to total isospin $I$, with $J_z + I_z = 0$. The $N(J=\frac{1}{2}, I=\frac{1}{2})$ nucleon and the $\Delta(J=\frac{3}{2}, I=\frac{3}{2})$ $\pi N$ resonance dominate the hedgehog soliton. Matrix elements of observables, including the axial vector coupling constant, must be evaluated by projecting the hedgehog state onto the appropriate $(J,I)$ component. Birse develops the technique formally, and the intuitive results of Cohen and Broniowski [11] are very helpful also.

The hedgehog soliton is characterized by radial profile functions corresponding to $(U,V)$ of the quark states in the MIT bag, and a sigma meson profile $g(r)$ and pion profile $h(r)$. These are reproduced from Birse's results in Fig. 1. The key features of these profile functions are the following.

(1) Asymptotically $g(r) = -f_\pi$ (the pion decay constant equal to 93 MeV), and $h(r)=0$, corresponding to an asymptotic, external quark mass $M_q = g_{\sigma qq} f_\pi$ and the coupling parameter $g_{\sigma qq} = 5$ chosen for a fit by the nucleon parameters. In Birse's calculation, this asymptotic quark mass is responsible for a pseudoconfinement of the quarks so that bag confinement is rendered irrelevant. (Note, however, that even though confinement is irrelevant for the soliton ground state, it cannot be ignored when discussing excited quark states or gluon states. In these situations the soliton binding must be supplemented with a confinement mechanism. Also in so-called weak coupling, or one-phase, soliton models, the quarks are not bound by the mesons and must be confined by an MIT bag or some other mechanism.)

(2) A sigma field that turns over from $-f_\pi$ at large $r$ to $+f_\pi$ at $r=0$, with $g(r=0.5\text{ fm})=0$ corresponding to an internal quark mass that is zero at $r=0.5$ fm, giving rise to a relativistic quark bound state with Dirac radial functions $(U,V)$ similar to those of the MIT bag model.

(3) A strong potential which constrains $\sigma^2+\pi^2$ to be

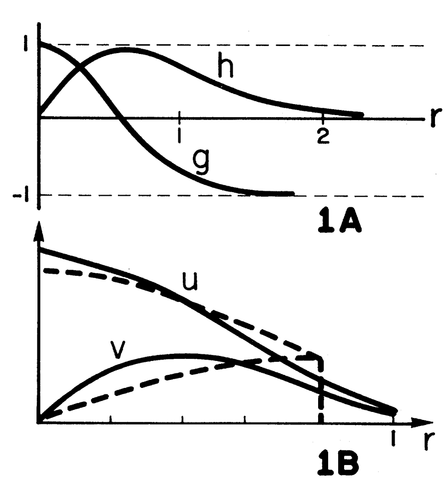


FIG. 1. (A) Birse profile functions for the hedgehog pion $h = f_\pi \sin\phi$ and for the sigma meson $g = f_\pi \cos\phi$ with $\phi = \pi \tanh(0.916r)$. (B) Birse quark radial wave functions $U$ and $V$ (solid lines) and MIT bag model, massless quark radial wave functions (dashed lines) for a bag radius 0.8 fm.

practically equal to $f_\pi^2$. We will make extensive use of the soliton calculations as a zero-order result upon which we base our perturbative corrections.

The axial vector current of this Lagrangian is

$$\mathbf{A}^\mu = \bar{q}\gamma^\mu\gamma_5 \frac{\tau}{2} q + \sigma\partial^\mu\pi - \pi\partial^\mu\sigma . \tag{1.4}$$

The axial vector coupling constant defined by the matrix element

$$G_A = 2\langle P\uparrow | \int A(z,3) d^3r | P\uparrow\rangle \tag{1.5}$$

gets quark and meson contributions which must be calculated from the hedgehog configuration by projection onto the proton spin-isospin eigenstate. The result following from Birse's Eqs. (A10) and (A18) is a quark contribution that depends on the mean number of pions in the soliton, $N_\pi$, approximately as

$$G_A(q) = \tfrac{5}{3} g_A(q) \{1 - \tfrac{2}{15} N_\pi + \tfrac{14}{225} N_\pi^2\} . \tag{1.6}$$

There is also an implicit dependence upon the meson configuration, hidden in the quark wave functions $U$ and $V$ determining $g_A(q)$. Birse obtains a value $N_\pi \simeq 1$ and a 7% reduction of the quark contribution to $G_A$ from the nominal bag model result.

The sigma-pion fields in the soliton also contribute directly to $G_A$. The projection of the hedgehog matrix element of this piece follows from Birse's Eq. (A37) as a factor of $(-\frac{1}{3})$ to give

$$G_A(\text{meson}) = -\tfrac{1}{3}\left\langle 2\int A(z,3;\text{meson}) d^3r \right\rangle \tag{1.7}$$

evaluated with the soliton profile functions for sigma and pion. Birse's soliton configuration is conveniently summarized by the profile functions

$$\sigma(r) = f_\pi \cos\phi = g(r) \tag{1.8}$$

and a radial-isospin pion soliton

$$\pi(r) = \hat{\mathbf{r}} f_\pi \sin\phi = h(r)\hat{\mathbf{r}} \tag{1.9}$$

where the profile angle

$$\phi(r) = \pi \tanh(ar) \tag{1.10}$$

varies from zero to $\pi$ for $0 < r < \text{infinity}$, with $a = 0.916$ for a radius 0.8 fm. The sigma-pion contribution to $G_A$ is

$$\begin{aligned} G_A(\text{meson}) &= \tfrac{2}{3}\int \left[\sigma \frac{d}{dz}\pi\cdot\hat{\mathbf{z}} - \pi\cdot\hat{\mathbf{z}}\frac{d}{dz}\sigma\right] d^3r \\ &= \frac{16\pi}{9}\int \left[-\frac{dg}{dr}\right] h(r) r^2 dr \end{aligned} \tag{1.11}$$

which vanishes for a constant sigma field. In terms of the angle $\phi$

$$G_A(\text{meson}) = \frac{16\pi}{9} f_\pi^2 \int \sin^2\phi \frac{d\phi}{dr} r^2 dr . \tag{1.12}$$

Birse obtains $G_A(q)=0.98$ and $G_A(m)=0.81$ for a very unsatisfactory result. It was conjectured at the time that radiative corrections might improve the model prediction, whose possibility we examine here. In a subsequent extension of the chiral soliton model to include vector mesons, Broniowski and Banerjee [12] obtained good agreement and a much reduced meson component. Stern and Clement [13] have also achieved an improved fit using different parameters for the sigma and a semiclassical projection of the hedgehog.

Before we describe the radiative corrections to the model $G_A$, let us discuss further the role of perturbative calculations in this problem. There are essential features of the problem—confinement and the spontaneously broken chiral symmetry of the vacuum state—which must be incorporated in the basis states on which the perturbation theory is erected. A rigorous theory of meson field fluctuations on the soliton background [14] seems prohibitively difficult even in the simplest model problems. The evaluation of radiative corrections for meson field modes other than simple plane waves is an intimidating task. Nonetheless, it is interesting to make even the most naive start at evaluating corrections to the underlying soliton model if for no other reason than to see perturbation theory fail explicitly, and then perhaps to fix it. Furthermore, many perturbative processes are not contained at all in the soliton results, so that even a crude estimate of their importance is useful. For example, radiative processes with quark excitation, or with antiquarks, gluons, $L=1$ sigmas and $L=0$ pions, various non-wave-function time orderings, and so on are not included in the soliton calculation. The perturbative processes which are included in the soliton results can be identified and require special attention. We consider all but these "double-counted" processes to be legitimate radiative corrections to the soliton results. The double-counted perturbative processes are among the largest, but an improved perturbation scheme on the soliton background eliminates many of these contributions. The reason is that the soliton configuration is "immunized" against elastic radiative processes. The source term for radiative modes which leave the soliton unchanged are just the soliton equations of motion, which vanish. This conclusion is made clear in Eqs. (36)–(39) of Ren and Banerjee [14]. The radiative processes that remain necessarily have excitations of the internal degrees of freedom of the soliton. However, no calculations exist on excited states of the meson-quark soliton although the required equations are also presented in Ren and Banerjee. Even if we were to solve these equations for excited states we would have to invoke a confinement mechanism for sensible results for quark and gluon modes. We avoid this quagmire in our first explorations of the problem and, without further ado, evaluate such contributions using MIT bag modes. For reference we first state results of meson radiative processes calculated with the MIT bag as a source, and then eliminate those terms which are decoupled from the soliton. In the event that the perturbative predictions were disastrous, we would be provided with strong motivation to face the real difficulties without compromise. In fact, the perturbative results seem well under control. One is reminded of the similar experience in nuclear matter calculations, where a modified perturbative approach has led to an understanding of the properties of the strongly interacting many-body problem.

In the following sections we discuss in order (1) gluon exchange-, (2) nucleon-delta multiplet pion exchange-, (3) other pion exchange-, (4) sigma-pion transition-, and finally (5) sigma exchange-radiative corrections to the axial vector coupling constant. All calculations are based on

TABLE I. Contributions of amplitudes of Figs. 2(A)–10(B) to $dG_A/G_A$ and $dZ\,sq$ of Eq. (2.3). The QCD coupling $\alpha=g^2/4\pi\simeq 0.8$. Small quantities in parentheses are contributions which survive elastic decoupling of mesons from the soliton, described in Sec. III E.

| Amplitude | $dG_A/G_A$ | $dZ\,sq$ |
|---|---|---|
| (2$A$) | (0.653) | (1.0) |
| $B$ | −0.043 | 0.129 |
| $C$ | +0.017 | 0.086 |
| $D,E,F,G$ | −0.004 | 0.004 |
| $H$ | −0.0008 | |
| $I$ | −0.005 | 0.0167 |
| $J$ | +0.036 | |
| $K$ | −0.067 | |
| Total 2 (TE) | −0.066$\alpha$ | +0.236$\alpha$ |
| 3$A$ | +0.0012 | 0.0018 |
| $B$ | +0.00006 | 0.0002 |
| $C$ | −0.0014 | |
| Total 3 (TM) | −0.00015$\alpha$ | +0.002$\alpha$ |
| 4$D$ | −0.0225 | |
| $E$ | +0.0165 | |
| $F$ | −0.0199 | |
| Total 4 (Coul) | −0.0259$\alpha$ | |
| 5$B$ | +0.019(0) | +0.173(0) |
| $C,D$ | +0.132(0) | |
| $E$ | +0.057(0) | +0.102(0) |
| Total 5 ($N\Delta\pi$) | +0.208(0) | +0.275(0) |
| 6$A$ | −0.003 | +0.024 |
| $B$ | +0.015 | |
| $C$ | −0.006 | +0.020 |
| $D$ | +0.008 | |
| Total 6 ($\pi,L=0$) | +0.014 | +0.044 |
| 7$A,B$ | +0.002 (0.0007) | +0.0095 |
| $C,D$ | +0.013 (−0.008) | |
| $E$ | +0.031 (0) | |
| $F$ | +0.018 (0) | |
| $G$ | +0.057 (0) | |
| $H$ | −0.004 | +0.027 |
| Total 7 ($\pi,L=1$) | +0.127 (−0.011) | +0.037 |
| Total 6,7 | +0.141 (0.003) | +0.081 |
| 8$A,B$ | +0.232 (0) | |
| $C,D,E$ | +0.049 | |
| Total 8 ($\sigma,\pi$) | +0.281 (0.049) | |
| 9$C,D,G$ | −0.0009 (0) | |
| $E$ | +0.0012 | +0.0019 |
| $F$ | −0.0017 | |
| $H,I$ | −0.0863 (0) | |
| $J$ | −0.0050 | |
| $K$ | +0.0028 | +0.0069 |
| Total 9 ($\sigma L=0$) | −0.0899 (−0.003) | +0.0088 |
| 10$A$ | −0.0023 | +0.0118 |
| $B$ | −0.0003 | |
| Total 10 ($\sigma,L=1$) | −0.0026 | +0.0118 |
| Total 9,10 | −0.0925 (−0.005) | +0.0206 |

ordinary time-dependent perturbation theory using energy denominators for propagators, and all time orderings for the two strong interaction vertices and the one weak interaction vertex. In a straightforward way, many such processes are recognized to make contributions that can be ignored because they are effectively absorbed into model wave functions. We omit as many calculational details as possible and simply enumerate the perturbation terms via topologically distinct Feynman diagrams, and summarize the numerical results in Table I. Section II contains the gluon radiative corrections; Sec. III contains the meson radiative corrections calculated with the MIT bag quarks as meson sources, and finally eliminates double counted meson corrections which are decoupled from the soliton; Sec. IV discusses other corrections and other work; Sec. V is a summary and conclusion.

## II. GLUON RADIATIVE CORRECTIONS

### A. TE gluon exchange

Following the work of Close and Horgan and others [15], we only briefly describe the elements required for calculating the gluon radiative corrections. The perturbation Hamiltonian is

$$H=-g_{\rm QCD}\int \bar{q}G^{\alpha}_{\mu}\gamma^{\mu}\frac{\lambda_\alpha}{2}q\,d^3r \qquad (2.1)$$

with $g^2_{\rm QCD}/4\pi=\alpha_s$ the strong coupling parameter and $\lambda_\alpha$, $\alpha=1,\ldots,8$, are the Gell-Mann SU(3) (color) matrices. The quark fields $q$ will be limited to low-lying bag modes with quantum numbers $1S$, $2S$, $1P_{1/2}$, and $1P_{3/2}$ for quarks and antiquarks. Our discussion will focus on the lowest $L=0$ TE (i.e., magnetic dipole) gluon whose bag mode wave function is also given in Appendix A. The calculation requires ($1S$ or $2S$ to $1S$ or $2S$) $q$-$q$-$g$ matrix elements and ($1S$ or $2S$ plus $1P_{1/2}$ or $1P_{3/2}$) $q$-$\bar{q}$-$g$ matrix elements which are collected in Appendix B. The effect of the lowest-lying TM gluon mode has also been explored and found to be completely negligible compared to TE. Later we discuss the contribution of the Coulomb gluon exchange which is also very small.

We summarize the required amplitudes via the Feynman diagrams of Fig. 2 and their contributions to $dG_A$, the correction to the axial vector coupling, and to $dZ\,sq$, the correction to the wave-function normalization. The wave-function renormalization is required because the perturbation mixes a $3q+g$ component into the nominal $3q$ nucleon state whose normalization must be reduced accordingly. The corrected axial vector coupling is

$$G_A+DG_A=(G_A+dG_A)/(1+dZ\,sq)\ , \qquad (2.2)$$

leading to a complete correction

$$DG_A/G_A=dG_A/G_A-dZ\,sq\ , \qquad (2.3)$$

keeping terms to second order in the strong interaction. Note that a conserved vector current gets, term by term, corrections $dG_V/G_V$ equal to $dZ\,sq$ and no radiative correction, so $DG_V/G_V=0$ as required. For the axial vector current which is only partially conserved, this cancellation does not occur exactly and the problem in perturbation theory is to choose a basis in which these partial cancellations are recognizable. In this regard, our work in the following is only a preliminary step in which the passage to a conserved axial current is not explicitly elucidated, but we are dependent on the underlying chiral-invariant model for incorporating the partial conservation.

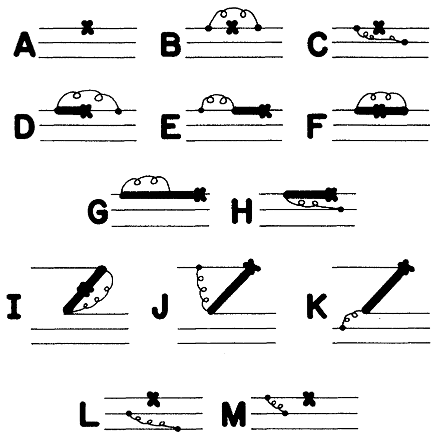


FIG. 2. Feynman diagrams for quark matrix elements of operator $X$. (A) the nominal matrix element, and (B)–(K) are transverse electric (lowest gluon bag mode) radiative corrections. (B) and (C) with no quark excitations from the $1S$ mode are gluonic nucleon contributions. In (D)–(H) heavy lines are $2S$ quark excitations. In (I)–(K) heavy lines are $1P_{1/2}$ and $1P_{3/2}$ antiquarks. All possible time orderings of vertices and all spectator interactions are implied. (L) and (M) make no contributions.

The uncorrected matrix element of Fig. 2(A) is evaluated with the SU(6) spin-isospin $3q$ wave function with model independent $1S$ quark radial functions. The processes of Figs. 2(B) and 2(C) involving $1S$ quarks and a TE gluon exchange, dominate the gluon radiative corrections. These processses constitute a perturbative description of the mixing of the $3q$ nucleon with its $3q+g$ gluonic-nucleon partner [16]. Similar processes involving a $2S$ quark excitation are indicated in Figs. 2(D)–2(H). Figure 2(H) has all possible time orderings of the gluon interaction with the spectator quarks. Only Fig. 2(F) contributes to the wave-function renormalization. Figures 2(I)–2(K) are also *a priori* interesting processes involving $1P_{1/2}$ and $1P_{3/2}$ antiquark excitations, some of which make marginally competitive contributions. Radiative processes involving only spectator quarks as in Fig. 2(L) contribute equally to $dG_A/G_A$ and $dZ\,sq$ and cancel, leaving the processes of Fig. 2(A)–2(K) to be calculated. Figure 2(M) involves only post or prior exchanges which are supposed to be absorbed into the model wave functions [17].

Gluon and other radiative contributions can sometimes be deduced qualitatively and, for example, major cancellations can be anticipated on the basis of the overall color neutrality of the nucleon. The wave-function renormalization $dZ\,sq$ is positive and can dominate the correction to $G_A$. Although it is reassuring to see these intuitive features appear, by and large we are dependent on the detailed calculations.

The contributions of the various TE gluon exchange perturbation diagrams to $dG_A$ and to $dZ\, sq$ are shown in Table I and sum to

$$G_A+dG_A=1.088-0.072\alpha_s \quad \text{and} \quad 1+dZ\, sq =1+0.236\alpha_s \tag{2.4}$$

giving a net correction

$$DG_A/G_A(\text{TE})=(-0.066-0.236)\alpha_s=-0.302\alpha_s \ . \tag{2.5}$$

### B. TM gluon exchange

The TM gluon exchange has many fewer diagrams shown in Fig. 3(A)–3(C), none of which compete with the dominant TE exchange contributions of Fig. 2. The contributions are shown in the table and sum to

$$dG_A/G_A=-0.0002\alpha_s \quad \text{and} \quad dZ\, sq=+0.0020\alpha_s \tag{2.6}$$

giving a net correction from this source of

$$DG_A/G_A(\text{TM})=-0.002\alpha_s \ . \tag{2.7}$$

### C. Coulomb gluon exchange

We calculate radiative corrections for $L=0$ and $L=1$ Coulomb gluon interactions of the quarks using the Barnes, Close, and Monaghan [18] screened Coulomb interaction for confined gluons. The interaction Hamiltonian is

$$H(\text{Coul})=\frac{\alpha_s}{2}\int d^3r\, \rho_\alpha(r)G_c(r-r')\rho_\alpha(r')d^3r' \tag{2.8}$$

with the quark color charge density

$$\rho_\alpha(r)=\bar{q}(r)\gamma^0\frac{\lambda_\alpha}{2}q(r) \tag{2.9}$$

and the screened Coulomb Green's function

$$G_c(r-r')=\begin{cases}1/(r>) & \text{for } L=0 \ ,\\ [(r<)/(r>)^2+2rr']\cos(\hat{r}\cdot\hat{r}') & \text{for } L=1 \ ,\end{cases} \tag{2.10}$$

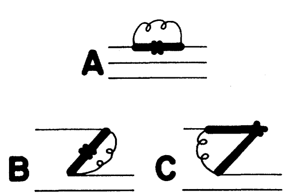


FIG. 3. Transverse magnetic gluon radiative corrections. Heavy quark lines are $1P_{1/2}$ and $1P_{3/2}$ quarks and $1S$ antiquarks.

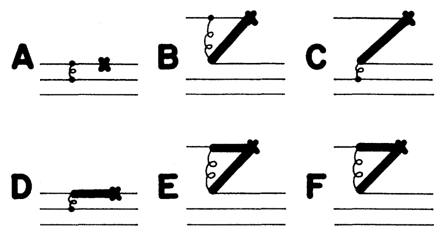


FIG. 4. Coulomb gluon radiative corrections. (A)–(C) with an $L=0$ gluon and no quark excitation make no contribution. (D) with $L=0$ gluon and $2S$ quark, (E) with $L=0$ gluon, $1P_{1/2}$ antiquark and $2S$ quark, and (F) with $L=1$ gluon, $1P_{1/2}$ and $1P_{3/2}$ quarks, and $1S,2S$ antiquarks make nonzero contributions.

where $r>$ $(r<)$ is the greater (lesser) of $r$ and $r'$, and less than the bag radius which is 1 in our units. The relevant diagrams are shown in Fig. 4. Those with post or prior $L=0$ interactions with no quark excitations are absorbed into the SU(6) wave function so Fig. 4(A) makes no contribution. The $L=0$ exchange $Z$ diagrams of Fig. 4(B) and 4(C) contribute nothing because of the color neutrality of the nucleon state. The diagrams that remain are the $L=0$ Coulomb gluon exchange with $2S$ quark excitation of Figs. 4(D) and 4(E) and also the $L=1$ exchange with $1P_{1/2}$ and $1P_{3/2}$ quark excitation of Fig. 4(F). Their calculation is straightforward and we just state the total contribution

$$DG_A/G_A(\text{Coul})=-0.026\alpha_s \ , \tag{2.11}$$

for a total one gluon exchange correction to the axial-vector coupling constant of

$$DG_A/G_A(\text{one gluon exchange})=-0.330\alpha_s \ . \tag{2.12}$$

Approximately three-quarters of this comes from the TE wave function renormalization, and the other one quarter from the explicit TE reduction of $G_A$. As already noted, this dominance leads to an interpretation in terms of a gluonic component of the nucleon which has some angular momentum carried by the gluon with a resultant depolarization of the quarks and a necessary reduction of $G_A$. For orientation, we use a value of $\alpha_s=0.8$ to give a total gluon correction of $DG_A/G_A=-0.265$. This value of $\alpha_s$ is much smaller than the frequently mentioned value of 2.2 originally introduced in bag calculations to remove the nucleon and delta degeneracy, but is close to the value of $\alpha_s=0.75$ advocated by Barnes, Close, and Monaghan [18], and to that of Golowich, Haqq, and Karl [16], and to the bag model parametrization developed by Donoghue and Johnson [19]. Stern and Clement [13] favor an even smaller value.

## III. MESON RADIATIVE CORRECTIONS

We consider the relativistic chiral quark model with confined quarks in chiral invariant interaction with scalar-isoscalar sigma mesons and with pions, for which we calculate perturbative results for comparison with hedgehog soliton predictions. We discuss in order, purely pionic, mixed pion-sigma, and purely sigma contributions. The purely pionic contribution separates naturally into $N\Delta\pi$ ($L=1$) processes which we discuss in the next Sec. III A, and other $L=0$ and $L=1$ pion reactions, involving quark excitations out of the $1S$ orbitals or excitation of antiquark states, which we postpone to Sec. III B.

### A. $N\Delta\pi$ radiative processes

The $(N\Delta\pi)$ contribution is summarized by the diagrams of Fig. 5, where the coupling strengths $G_A(NN)$ $G_A(N\Delta)$, and $G_A(\Delta\Delta)$ are readily calculated from matrix elements of $A(z,3;q)$ with the nominal SU(6) wave functions. The $(NN\pi)$, $(N\Delta\pi)$, and $(\Delta\Delta\pi)$ vertices are easily expressed [9] in terms of couplings $f(NN\pi)$, $f(N\Delta\pi)$, and $f(\Delta\Delta\pi)$ and a form factor $F(k^2)=3j_1(kR)/kR$ for an $L=1$ pion of momentum magnitude $k$.

A straightforward calculation of the axial coupling of the $\Delta$ produced from a spin-up proton by emission of a pion with ($L=1$; $L_z=1,0,-1$) and ($I_z=+1,0,-1$) produces a somewhat surprising result [20]. The $\Delta(J_z=\frac{3}{2}$, $I_z=\frac{3}{2})$ has an axial coupling of $3g_A(q)$ compared to the proton ($J_1=1/2$, $I_z=1/2$) with $\frac{5}{3}g_A(q)$. However, the $\Delta$ is produced with a probability of only $\frac{1}{4}$ for this state and all other $\Delta$ states have a smaller axial coupling. The weighted mean of the $\Delta$ axial coupling over all spin-isospin states is $\frac{5}{9}\times\frac{5}{3}$ indicating a quark spin-isospin depolarization from the initial nucleon due to the pion emission. The correction to the nucleon axial coupling will be the probability $P$ of pion emission and reabsorption, multiplied by this averaged quark depolarization of $\frac{5}{9}$ minus one from the wave-function renormalization

$$DG_A/G_A=P(\tfrac{5}{9}-1)\,, \tag{3.1}$$

which is a negative contribution from the $(\pi\Delta)$ intermediate state. This conclusion remains valid when $(N\Delta)$ weak transitions are included. One might have thought that the Adler-Weisberger sum rule [21] which is dominated by the $\Delta(++)$ resonance in $\pi N$ scattering, and which requires $G_A>1$ through the relation

$$1-1/G_A^2=[2M_N^2/(2\pi g_{\pi NN}^2)]\int\frac{d\nu}{\nu}(\sigma_+-\sigma_-) \tag{3.2}$$

(involving $g_{\pi NN}^2/4\pi=13$, $M_N$ is the nucleon mass, $\sigma_\pm$ is the $\pi_\pm$-proton cross section at $\pi$ laboratory energy $\nu$) would be qualitatively reflected in the perturbative result.

The $(N\Delta\pi)$ radiative corrections of Fig. 5 are readily calculated using the factors of Appendix B. Integrals over the virtual pion momentum get their major contributions from $\sim 2/R\sim 500$ MeV/$c$, and have converged by $\sim 4/R$. The resulting corrections, summarized in Table I, sum to

$$\begin{aligned}DG_A/G_A(N\Delta\pi)&=dG_A/G_A-dZ\,sq\\&=+0.208-0.275=-0.067\end{aligned} \tag{3.3}$$

dominated by 17% $(N\pi)$ and 10% $(\Delta\pi)$ wave-function renormalizations of the nominal three-quark nucleon state. We have used a bag radius of 0.8 fm. The corrections get rapidly more severe as the bag radius is reduced and reach $0.52-0.66=-0.14$ for $R=0.6$ fm. The individual corrections $dG_A/G_A$ and $dZ\,sq$ are no longer in any sense small and perhaps it is better to write

$$\begin{aligned}DG_A/G_A&=(1+dG_A/G_A)/(1+dZ\,sq)-1\\&=1.52/1.66-1=-0.08\end{aligned} \tag{3.4}$$

which reduces the correction to $G_A$, and leaves a perhaps believable perturbation correction. The legitimacy of these pionic radiative corrections has already been questioned because of the elastic decoupling of the pion radiative modes from the soliton. We will return to this question in the subsection following our calculations using MIT bagged quarks as meson sources. We turn now to pion radiative processes which are not included in the $(N\Delta\pi)$ system.

### B. Other pion radiative corrections

The single pion emission amplitude of Fig. 5(F) in which the virtual pion is annihilated by the axial current is known from Nambu's analysis [22] to contribute only to the induced pseudoscalar coupling, but not to the axial vector coupling because it vanishes at zero momentum transfer where $G_A$ is defined. The surviving purely pionic radiative corrections involve quark excitations, including $2S$ quarks or a $q$-$\bar{q}$ pair, plus an $L=0$ or $L=1$ pion, and are listed in Figs. 6 and 7. All these processes connect the bare nucleon to intermediate quark states outside the $(N\Delta)$ multiplet and are not included in the previous calculations, or in the soliton calculation. There are hazards of double-counting perturbative $q$-$\bar{q}$ amplitudes which should be included as part of meson states. Such a situation can be recognized when axial vector mesons are

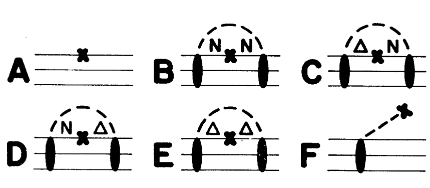


FIG. 5. One pion exchange radiative corrections within the $N\Delta\pi$ ($L=1$) system. (B)–(E) have $NN$, $N\Delta$, $\Delta N$, $\Delta\Delta$ intermediate states. (F) is the pion pole contribution to the axial vector matrix element vanishing at $k=0$. Single dashed lines are pions.

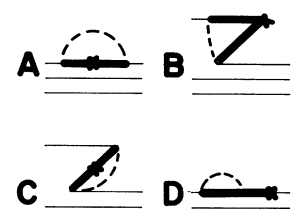


FIG. 6. One pion exchange radiative corrections with $L=0$ pions and quark excitations. Heavy lines are $1P_{1/2}$ quarks and $1S,2S$ antiquarks.

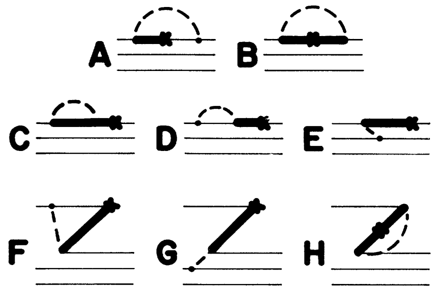


FIG. 7. One pion exchange radiative corrections with $L=1$ pions and quark excitations leading out of the $N\Delta$ system and not included in Fig. 5. Heavy lines are $2S$ quarks and $1P_{1/2}, 1P_{3/2}$ antiquarks.

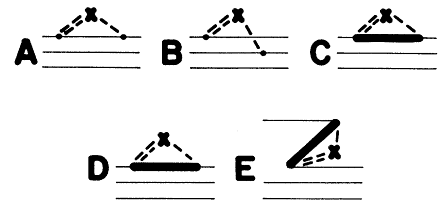


FIG. 8. Sigma-pion transition radiative corrections. (A) and (B) have $L$(sigma)$=0$, $L$(pion)$=1$, no quark excitation. (C) has a $2S$ quark. (D) and (E) have $L$(sigma)$=1$, $L$(pion)$=0$, and $1P_{1/2}$ quark and $1S$ antiquark excitations. Double dashed lines are sigmas.

considered, as in the Broniowski and Banerjee extension of the sigma-pion-quark soliton to include the vector and axial vector mesons. The contributions of individual processes to $dG_A/G_A$ and to $dZ\,sq$ are listed in Table I. We do not go into the calculational details which are straightforward, and just report the result for all the extra pion radiative corrections

$$DG_A/G_A(\text{extra pion}) = dG_A/G_A - dZ\,sq = 0.14 - 0.08 = 0.06 \;, \qquad (3.5)$$

which almost completely cancels that contained in the $(N\Delta\pi)$ processes, leaving a net pionic radiative correction of $-1\%$. This is of course much smaller than our ability to calculate. As in the previous subsection, we later select those processes of Fig. 7 which survive elastic decoupling from the soliton, as legitimate radiative corrections. We turn next to radiative processes involving both the pion and the sigma meson.

### C. Sigma-pion radiative corrections

The Birse-Banerjee chiral soliton has a vacuum value for the sigma field equal to $-f_\pi$. They settle on a coupling strength $g_{\sigma qq}=5$ which gives an external quark mass $M_q=465$ MeV. There is an essentially nonperturbative defect in the soliton interior where the sigma field switches from $+f_\pi$ to $-f_\pi$ giving an interior quark mass $M_q(r)$ which can be much smaller than $M_q$, and even negative, so the quarks are relativistic and approximate the massless MIT bag modes. Our initial sigma-pion radiative corrections will take account of fluctuations of the sigma field on the vacuum background $-f_\pi$. Such calculations are admittedly suspect, but provide necessary orientation for later calculations where the effect of the soliton field will be taken into account.

Following Birse's Eqs. (1.4) and (6.2), we have the interaction Hamiltonian for the MIT bag quark source and the sigma-pion transition axial-vector current. Another ingredient of the calculation is a mass for the asymptotic sigma field fluctuations which Birse takes as 1 GeV, fixing the meson potential strength. We now present the results of unmodified perturbation theory for sigma-pion and then sigma-sigma processes. Following these two subsections, we discuss the soliton radiative corrections which survive elastic decoupling.

Figure 8 contains the sigma-pion diagrams that contribute to $dG_A/G_A$. None of these diagrams involving a sigma-pion transition via the axial vector current have accompanying wave-function renormalizations. The processes involving all possible time-orderings and all quark contributions like Figs. 8(A) and 8(B), with an $L=0$ sigma transformed to an $L=1$ pion and no quark excitations, dominate and make a large correction

$$DG_A/G_A[\text{Figs. 8(A) and 8(B)}] = +0.232 \;. \qquad (3.6)$$

This diagram is directly analogous to the meson contribution in the soliton calculation, where the hedgehog static sigma and pion configurations contribute a large positive $G_A(m)=+0.81$. The modification of naive perturbation theory necessary for this particularly important process will be discussed in a later subsection.

Other processes which have quark excitation, $q$-$\bar{q}$ excitation, or $L=1$ sigma and $L=0$ pion with $1P_{1/2}$ quark excitation, contribute $+0.049$ for a total sigma-pion correction

$$DG_A/G_A(\sigma\text{-}\pi) = +0.281 \;. \qquad (3.7)$$

### D. Sigma radiative corrections

Radiative corrections involving only a scalar sigma meson with $L=0$ or $L=1$ and all possible $L=0$ and $L=1$ quark excitations, are listed in Figs. 9 and 10. Of these, all diagrams with an $L=0$ sigma and $1S$ quarks, Figs. 9(A) and 9(B), can be ignored because they contribute equally to $dG_A/G_A$ and to $dZ\,sq$ and therefore contribute nothing to the net correction $DG_A/G_A$. This is obvious from the spin and isospin independence of the sigma coupling, or from the fact that these sigma-exchanges can be absorbed into the $1S$ quark wave function without change of the spin or isospin or, consequently, of the axial coupling. The remaining diagrams all involve either an $L=0$ sigma and $2S$ quark or $1P$ antiquark excitations, or an $L=1$ sigma plus $1P$ quark. Diagrams with an $L=1$ sigma and $1S$ quark plus $1S$ antiquark do not contribute.

The remaining sigma diagrams are easily evaluated and we simply state the result

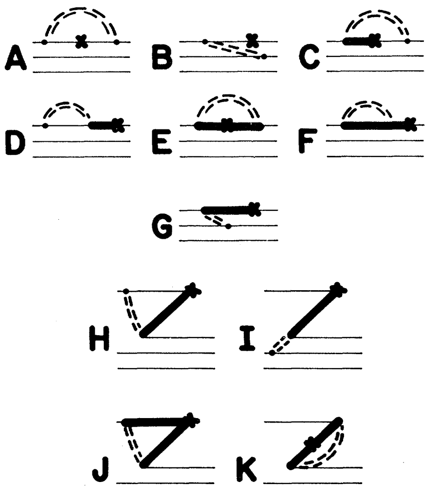

FIG. 9. Sigma exchange radiative corrections with $L$(sigma)=0. (A) and (B) do not contribute. (C)–(K) have $2S$ quark and $1P_{1/2}$ antiquark excitations.

$$DG_A/G_A(\sigma)=-0.113 \; . \tag{3.8}$$

The dominant contribution of ($-0.086$) comes from the $L=0$ sigma exchange with excitation of a $1P_{1/2}$ antiquark in the $Z$ process of Figs. 9(H) and 9(I). All the other processes contribute fractions of 1%, but the direct corrections $dG_A/G_A$ are predominantly negative, and reinforce the wave-function renormalization contributions which dominate. All these fragmentary contributions combine to ($-0.027$).

The sum of all these radiative corrections—Eqs. (2.12), (3.3), (3.5), (3.7), and (3.8)—constitutes the radiative correction to $G_A$ for the MIT bag model coupled perturbatively to QCD and to the linear ($\sigma$-$\pi$) chiral restoring interaction. The net result

$$\begin{aligned} DG_A/G_A &= -0.265(\text{gluons})-0.067(N\Delta\pi)+0.06(\pi) \\ &\quad +0.281(\sigma\pi)-0.113(\sigma) \\ &= -0.105 \end{aligned} \tag{3.9}$$

should be applied to Eq. (1.1) to reduce $G_A$ from 1.09 to 0.975. There are severe cancellations in the contributions of Eq. (3.9) whose magnitudes sum to 0.72. The cancellations can be believed qualitatively but the resulting magnitude cannot be taken seriously for at least two reasons. One is that no direct connection has been built into the gluon and meson corrections, so their cancellations must be somewhat fortuitous and can be easily destroyed by a different choice of the QCD $\alpha_s$. But also, the very existence of a soliton means that at least the sigma-quark interaction cannot be treated perturbatively.

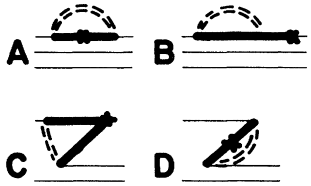

FIG. 10. Sigma exchange radiative corrections with $L$(sigma)-1 and $1P_{1/2}$, $1P_{3/2}$, $2S$ quark and $1S$ antiquark excitations. (C) and (D) do not contribute.

Before we leave the MIT bag model for the Birse-Banerjee soliton model, it is interesting to see what the successes and failures of perturbation theory are, based on Birse's nonperturbative results. It is amusing that at least the $N\Delta\pi$ contribution is very close to the 7% reduction of Eq. (1.6) and, even though they have quite different origins, the perturbative result is not misleading. Birse's result contains no semblance of the extra $\pi$ contribution of $+0.06$ or of the extra $\sigma$ contribution of $-0.113$. The $\sigma$-$\pi$ transition contribution of 0.232 from Figs. 8(A) and 8(B) is a failure of perturbation theory to produce Birse's meson contribution of $0.81/1.09=0.74$.

In the next subsection we bring our perturbative treatment of meson field fluctuations into accord with the soliton as source and, in the process, make a substantial reduction in the radiative corrections involving sigmas and pions.

### E. Radiative corrections to the soliton

Soliton matrix elements of functional derivatives of the Lagrangian with respect to the meson fields

$$\langle \text{Sol}|\partial L/\partial\phi|\text{Sol}\rangle$$

are equations of motion for the soliton and vanish for

$$\langle \text{Sol}|\phi|\text{Sol}\rangle=\phi_s \; ,$$

the soliton configuration. If we go further and investigate fluctuations $\delta\phi$ on the soliton background, the same functional derivative serves as the source of the $\delta\phi$ and vanishes in the soliton matrix element, decoupling the fluctuations from the soliton. This conclusion is also contained in the explicit equations of motion of Ren and Banerjee [14]. Correspondingly, a large number of amplitudes in the radiative corrections will vanish. Every soliton pion or sigma vertex which leaves the soliton unexcited will go to zero. Going through the figures, we see that all the amplitudes of Fig. 5 will be zero, which is only to say that $N\Delta\pi$ radiative corrections are already counted in the Birse calculation. The amplitudes of Fig. 6 remain and will be taken at their MIT bag value. Amplitudes 7(A), 7(D), 7(E), 7(F), and 7(G) with $1S$-$1S$ MIT bag vertices are set to zero, as are the previously dominant $\sigma$-$\pi$ transition amplitudes of Figs. 8(A) and 8(B) now included in Birse's (0.81), and the amplitudes 9(C), 9(D), 9(G), 9(H), and 9(I).

What remains after decoupling elastic radiative transitions from the soliton are much reduced meson radiative corrections and a correction to the axial vector coupling constant replacing the MIT bag result Eq. (3.9):

$$DG_A/G_A = -0.265(\text{gluons}) + 0(N\Delta\pi) - 0.078(\pi) + 0.049(\sigma\text{-}\pi) - 0.026(\sigma) = -0.32\ . \qquad (3.10)$$

## IV. DISCUSSION

### A. Nonleading QCD contributions

The TE gluon radiative corrections have included only the lowest energy magnetic dipole contribution with $\omega(\text{TE1}, Ng=1)=2.744$. The dominant contribution to $DG_A/G_A$ comes from the wave function renormalization necessitated by a gluonic nucleon component with a (TE1, $Ng=1$) gluon plus three $S(Ns=1)$ quarks. It is easy to check only one convergence property of the perturbation sequence without being led into prohibitive computational complexity, and that is to calculate contributions for other radial excitations with $Ng, Ns > 1$. Not surprisingly, we find that the diagonal elements in these arrays dominate the nondiagonal by an order of magnitude. The convergence is even more rapid for the $Ng=Ns=1$ leading term which dominates Figs. 2(B) and 2(C) by two orders. The $Ng=Ns=2$ term dominates the $Ng=1$, $Ns=2$ by one order and is the one quoted in Table I for the contribution of Fig. 2(F) to $dZ\,sq$. Similar conclusions apply to the $Z$ graphs of Figs. 2(J) and 2(K) where the term with $Ng=Ns=Np=1$ gives 95% of the rapidly convergent diagonal sum. To explore the contribution of higher multipoles would require the full angular momentum formalism and is not feasible with our technique of projection operators. Our general experience is that the leading diagonal term dominates other contributions at the 90% level, a precision which exceeds our ability to calculate.

### B. Other corrections

Model predictions are also subject to corrections from the current quark mass and from recoil effects [23], which can easily overwhelm the radiative corrections which are our primary interest. We quote radiative corrections only for zero quark mass because in our limited explorations we have found them to be insensitive to this parameter. We have included recoil only in the $(N\Delta\pi)$ contributions, and there only in the energy denominators. Recoil effects in the energy denominators reduce the contribution of these diagrams to $DG_A/G_A$ by thirty percent.

Recoil effects will be the only contribution to the elastically decoupled radiative modes which have been completely omitted in going from radiative corrections for the MIT bag to those for the soliton. It probably is not worth any great effort to refine the small radiative corrections involving inelastic transitions in the source, but it would be interesting to refine our treatment of elastic decoupling to include leading recoil effects.

### C. Constituent quark model

In the work of Weinberg [24], Peris [25], and Rosenfeld and Rosner [26], based on the chiral constituent quark theory of Manohar and Georgi [27], a very different view of nucleon structure leads to a calculation of the leading $1/N_c$ correction to the axial vector coupling constant. Here we point out differences between the constituent quark model and the relativistic chiral quark bag model of the nucleon. In their work a constituent quark $Q$ with mass $M_Q=360$ MeV is formed in the chiral phase transition in which the sigma field assumes the value $(-f_\pi)$ everywhere. Rosenfeld and Rosner find that the constituent quark has the Dirac coupling $g_A(Q)=1$ and a Dirac magnetic moment fit to the nucleon magnetic moment by choice of $M_Q$. Peris calculates one-loop $(\sigma,\pi)$ radiative corrections to $g_A(Q)$ in leading log approximation using the linear $\sigma$ model and finds a renormalization

$$g_A(Q)=1-\left[\frac{M_Q^2}{M_\sigma^2}\right]\ln\left[\frac{M_\sigma^2}{M_Q^2}\right] \qquad (4.1)$$

and with $M_\sigma=4\pi f_\pi$ gets $g_A(Q)=1-0.2$ and a nucleon axial coupling

$$G_A=5/3g_A(Q)=1.3\ . \qquad (4.2)$$

Weinberg uses the Adler-Weisberger sum rule for pion scattering from the constituent quark, together with the dominance of $\pi^+d$ over $\pi^+u$ scattering to argue for a similar reduction. Peris also shows that the calculations are equivalent provided $\pi Q$ scattering is dominated by $\sigma$ production through a one pion exchange Primakoff-type process. All these calculations indicate a radiative reduction of $g_A(Q)$ at the 10–20% level. There remain confinement effects and center-of-mass effects contributing further 10% reductions. In addition, Peris states that gluon exchange specifically does not renormalize $g_A(Q)$ in the leading-log approximation.

The two points of view could hardly be more different. The bag model starts with a reduction of $g_A(q)$ due to confinement, then a positive contribution from the mesons due to the reduction of $\sigma$ inside the bag, and follows with reduction due to gluon exchange involving individual quarks and quarks in interaction with spectators dominated by Figs. 2(B) and 2(C). The chiral soliton quark bag model does not manifest the Adler-Weisberger sum rule at the quark level, but only at the nucleon level. Furthermore, by working with a static source, the bag model has very soft form factors which provide low momentum cutoffs before the logarithmic divergences of Peris are ever encountered.

### D. One phase model

The chiral soliton quark model can be brought into closer accord with the constituent quark model of Manohar and Georgi and the work of Weinberg and Peris at small cost and substantial benefit, as indicated by Ren and Banerjee [28] and in earlier work [29]. At a smaller value of $g_{\sigma qq}=4$ rather than 5 as used by Birse, the interior $\sigma$ field remains near its vacuum value of $-f_\pi$ and does not flip over to $+f_\pi$ at $r=0$. As a result the quarks have almost their asymptotic mass

$Mq = g_{\sigma qq} f_\pi = 373$ MeV everywhere and must be confined by the bag. The $\sigma$ and $\pi$ fields differ little from their asymptotic values and can sensibly be treated in perturbation theory. The hedgehog soliton calculation is then replaced with a perturbatively corrected MIT bag. The radiative corrections of Eq. (3.9) are relevant with just two modifications. The meson contributions must be reduced by a factor of $(\frac{4}{5})^2$ to account for the changed coupling strength, and the gluon correction must be corrected by a factor of 0.62 to go from $Mq=0$ to $Mq=373$ MeV in the calculation of the dominant TE 1S-1S matrix element. The resulting correction to $G_A$ is

$$DG_A/G_A(Mq=373 \text{ MeV}) = -0.265(0.62) + (-0.067+0.06+0.281-0.113)(0.64) = -0.06 \,. \tag{4.3}$$

The axial matrix element $g_A$ is increased from 0.653 to 0.80 and the nucleon axial coupling becomes

$$G_A = \tfrac{5}{3} 0.8(1-0.06) = 1.25 \,. \tag{4.4}$$

If true, the benefits of being able to use constituent quarks in a bag, and perturbation theory on a simple vacuum background are enormous. The whole intractable problem of radiative corrections to the soliton would be avoided. There are still substantial differences with the work of Weinberg and of Peris, who ignore spectator interactions, do not depend on confinement for cutoffs and have no gluon radiative corrections following Manohar and Georgi who recommended a gluon coupling much smaller than that of Donoghue and Johnson, which is the one used here. The Adler-Weisberger sum rule, which depends upon PCAC and the Goldberger-Treiman relation, will be satisfied at the nucleon level but not at the quark level.

## V. CONCLUSIONS

Gluon and meson radiative corrections to the nucleon axial vector coupling constant have been calculated in the relativistic chiral quark model. Second-order perturbative processes of Figs. 2–10 have been included. The dominant contributions to the radiative corrections are made by the processes of Figs. 2(B) and 2(C). These are transverse electric gluon exchanges which mix a (three-quark plus gluon) component with the usual three-quark nucleon state with a probability of order $0.23\alpha_s$, depolarizing the quarks and reducing the axial vector coupling by $0.30\alpha_s$. Another significant process is the sigma-pion transition of Figs. 8(A) and 8(B) which is a positive contribution tending to cancel the gluon contribution. The degree of cancellation is difficult to calculate precisely not only because of the uncertainty in the gluon coupling strength but primarily because of a sensitivity of the sigma and pion couplings to the soliton backgrounds. Recent work by Mattingly and Stevenson [30] determines the gluon coupling in perturbative QCD even at very low momentum transfers and supports a value near 0.8 for $\alpha_s$ in bag calculations, which puts the gluon part of the calculation under control. A heuristic calculation of the effect of the soliton background on meson couplings indicates that meson corrections must be substantially reduced, because the soliton is elastically decoupled from meson radiative modes. The meson radiative corrections that remain involve inelastic transitions on meson emission and absorption. Calculated in the MIT bag approximation these corrections are predicted to be insignificantly small. An improved treatment of these corrections to the chiral soliton quark model leads into recoil corrections to elastic decoupling and to inelastic transition amplitudes requiring excited states of the soliton. We have showed that the relevant amplitudes are very small as calculated with MIT bag sources, leaving the dominant TE gluon correction to reduce the axial coupling as much as 30%. Agreement with experiment is only qualitative. Applied directly to Birse's result, $G_A$ is reduced from

$$G_A(q) + G_A(m) = 0.98 + 0.81$$

to perhaps

$$(0.98+0.81)(1-0.3) = 1.25 \,,$$

but it is still necessary to rationalize this correction with the Banerjee and Broniowski vector meson soliton result which already agrees with experiment.

Finally, we are led to the Manohar-Georgi constituent quarks bag-confined in a model-dependent way as the most tractable basis for calculating radiative corrections to the axial vector coupling constant. The corrections are small and the agreement is encouraging.

## ACKNOWLEDGMENTS

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## APPENDIX A: BAG MODEL DIRAC SPINORS

The bag model Dirac spinors for massless quark and anti-quark states, with bag radius equal unity, are the following.

$S$ states: $q[S] = (U, i\sigma\cdot\hat{r}V)\chi$ (for quarks), $\bar{q}[S] = (\sigma\cdot\hat{r}V, iU)\bar{\chi}$ (for antiquarks), with $\chi$ a Pauli spinor, $\bar{\chi} = i\sigma_2\chi$ an antispinor, and $U(r) = Nj_0(\omega r)$, $V(r) = Nj_1(\omega r)$ where $\omega[1S] = 2.043$, $N[1S] = 0.6403$; and $\omega[2S] = 5.396$, $N[2S] = 1.538$.

$1P_{1/2}$ state: $q[P_{1/2}]=(U,-i\sigma\cdot\hat{r}V)\sigma\cdot\hat{r}\chi$, $\bar{q}[P_{1/2}]=(\sigma\cdot\hat{r}V,iU)\sigma\cdot\hat{r}\bar{\chi}$ with $U[P_{1/2}]=1.089j_1(3.811r)$ and $V[P_{1/2}]=1.089j_0(3.811r)$.

$1P_{3/2}$ state: $q[P_{3/2}]=(U,i\sigma\cdot\hat{r}V)\hat{\varepsilon}\cdot P\chi$ and $\bar{q}[P_{3/2}]=(\sigma\cdot\hat{r}V,iU)\hat{\varepsilon}\cdot P\bar{\chi}$ with $U[P_{3/2}]=1.065j_1(3.203r)$ and $V[P_{3/2}]=1.065j_2(3.203r)$. The projection operator for the spin-$\frac{3}{2}$ vector-spinor $\hat{\varepsilon}\cdot P=(\varepsilon\cdot\hat{r}-\sigma\cdot\hat{r}\sigma\cdot\hat{\varepsilon}/3)3/\sqrt{2}$ is a convenient but redundant representation involving a three-component polarization vector $\hat{\varepsilon}$ and a two-component Pauli spinor $\chi$. The six states must be weighted with a further factor of $\frac{2}{3}$ in the $P_{3/2}$ propagator.

In addition, we note gluon TE1 and TM1 modes:

$$G(\mathrm{TE1})=(\sqrt{\alpha}\lambda_\alpha/2)(N/\sqrt{2\omega})\sqrt{3}j_1(\omega r)(\hat{r}\times\hat{\varepsilon}_g)\ ,$$

$$G(\mathrm{TM1})=(\sqrt{\alpha}\lambda_\alpha/2)(N/\sqrt{2\omega})\sqrt{3}\times\{\hat{\varepsilon}_g[\tfrac{2}{3}j_1(\omega r)-\tfrac{1}{3}j_2(\omega r)]+\hat{r}(\hat{\varepsilon}_g\cdot\hat{r})j_2(\omega r)\}\ ,$$

where $\alpha=g_{\mathrm{QCD}}^2/4\pi=0.8$ is the nominal value of the gluon coupling, $\lambda_\alpha$ are the Gell-Mann SU(3) matrices, $\hat{\varepsilon}_g$ is the gluon polarization vector, and

$$N_{\mathrm{TE1}}^2=9.097,\ \omega_{\mathrm{TE1}}=2.744;\ N_{\mathrm{TM1}}^2=21.190,\ \omega_{\mathrm{TM1}}=4.493\ .$$

## APPENDIX B: MATRIX ELEMENTS

The table of matrix elements

I. TE matrix elements with a common factor $\sqrt{\alpha}\lambda_\alpha/2$ removed, for massless quarks in a bag of unit radius. $\chi$ are Pauli spinors, $\bar{\chi}$ antispinors:

$$\langle q1S|H(\mathrm{TE})|q1S\rangle=0.4929\chi_f^{+}\sigma\cdot\hat{\varepsilon}_g\chi_i\ ,$$

$$\langle q2S|H(\mathrm{TE})|q1S\rangle=0.656\chi_f^{+}(2S)\sigma\cdot\hat{\varepsilon}_g\chi_i(1S)\ ,$$

$$\langle 0|H(\mathrm{TE})|q1S,\bar{q}P_{1/2}\rangle=-0.3255\bar{\chi}^{+}(P_{1/2})\sigma\cdot\hat{\varepsilon}_g\chi_i(1S)\ ,$$

$$\langle 0|H(\mathrm{TE})|q1S,\bar{q}P_{3/2}\rangle=-0.2093\bar{\chi}^{+}(1P_{3/2})(2\hat{\varepsilon}_p\cdot\hat{\varepsilon}_g-i\sigma\cdot\hat{\varepsilon}_p\times\hat{\varepsilon}_g)\chi_i(1S)\ .$$

II. Axial vector matrix elements with $H_w=\sigma_z\tau_z$:

$$\langle q1S|H_w|q1S\rangle=0.6530\chi_f^{+}\sigma_z\tau_z\chi_i\ ,$$

$$\langle q2S|H_w|q1S\rangle=-0.1390\chi_f^{+}(2S)\sigma_z\tau_z\chi_i(1S)\ ,$$

$$\langle q2S|H_w|q2S\rangle=0.4091\chi_f^{+}\sigma_z\tau_z\chi_i\ ,$$

$$\langle 0|H_w|q1S,\bar{q}P_{1/2}\rangle=0.4695\bar{\chi}^{+}(P_{1/2})\sigma_z\tau_z\chi_i(1S)\ ,$$

$$\langle 0|H_w|q2S,\bar{q}P_{1/2}\rangle=0.4150\bar{\chi}^{+}(P_{1/2})\sigma_z\tau_z\chi_i(2S)\ ,$$

$$\langle 0|H_w|q1S,\bar{q}P_{3/2}\rangle=0.1985\bar{\chi}^{+}(P_{3/2})(2\hat{\varepsilon}_p\cdot\hat{z}-i\sigma\cdot\hat{\varepsilon}_p\times\hat{z})\tau_z\chi(1S)\ ,$$

$$\langle 0|H_w|q2S,\bar{q}P_{3/2}\rangle=0.1265\bar{\chi}^{+}(P_{3/2})(2\hat{\varepsilon}_p\cdot\hat{z}-i\sigma\cdot\hat{\varepsilon}_p\times\hat{z})\tau_z\chi(2S)\ ,$$

$$\langle\bar{q}P_{1/2}|H_w|\bar{q}P_{1/2}\rangle=-0.2641\bar{\chi}_i^{+}(P_{1/2})\sigma_z\tau_z\bar{\chi}_f(P_{1/2})\ ,$$

$$\langle\bar{q}P_{1/2}|H_w|\bar{q}P_{3/2}\rangle=+0.2897\bar{\chi}_i^{+}(P_{3/2})(2\hat{\varepsilon}_p\cdot\hat{z}-i\sigma\cdot\hat{\varepsilon}_p\times\hat{z})\tau_z\bar{\chi}_f(P_{1/2})\ ,$$

$$\langle\bar{q}P_{3/2}|H_w|\bar{q}P_{3/2}\rangle=\bar{\chi}_i^{+}(P_{3/2})(-0.7066A-0.2923B)\tau_z\bar{\chi}_f(P_{3/2})\ ,$$

where

$$A=\langle\hat{\varepsilon}_i\cdot P^{+}\sigma_z\hat{\varepsilon}_f\cdot P\rangle\ ,$$

$$B=\langle\hat{\varepsilon}_i\cdot P^{+}\sigma\cdot\hat{r}\sigma_z\sigma\cdot\hat{r}\,\hat{\varepsilon}_f\cdot P\rangle\ ,$$

both averaged on $\hat{r}$.

[1] J. Ashman *et al.*, EMC Collaboration, Phys. Lett. B **206**, 364 (1988).

[2] S. Brodsky, J. Ellis, and M. Karliner, Phys. Lett. B **206**, 309 (1988); F. E. Close and R. G. Roberts, *ibid.* **206**, 1471 (1988); H. Dreiner, J. Ellis, and R. A. Flores, *ibid.* **221**, 167 (1989); G. Altarelli and W. J. Stirling, Particle World **1** (2), 40 (1989) for key references to this problem; A. V. Efremov, Report No. CERN-TH.6466/92, for an up to

date evaluation.

[3] S. Weinberg, Phys. Rev. Lett. **65**, 1181 (1990).

[4] F. E. Close, *Introductions to Quarks and Partons* (Academic, New York, 1979); R. K. Bhaduri, *Models of the Nucleon* (Addison-Wesley, Redwood City, CA, 1988).

[5] N. Isgur and G. Karl, Phys. Rev. D **18**, 4186 (1978); **19**, 2653 (1979); **20**, 1191 (1979).

[6] S. Theberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D **22**, 2838 (1980).

[7] V. Vento, M. Rho, E. M. Nyman, J. H. Jun, and G. E. Brown, Nucl. Phys. **A345**, 413 (1980); M. Rho, A. S. Goldhaber, and G. E. Brown, Phys. Rev. Lett. **51**, 747 (1983); E. Witten, Nucl. Phys. **B223**, 422 (1983); **B223**, 433 (1983).

[8] M. C. Birse and M. K. Banerjee, Phys. Lett. **136B**, 284 (1984); Phys. Rev. D **38**, 118 (1985).

[9] A. W. Thomas, *Advances in Nuclear Physics* (Plenum, New York, 1983), Vol. 1.

[10] M. C. Birse, Phys. Rev. D **33**, 1934 (1986).

[11] T. D. Cohen and W. Broniowski, Phys. Rev. D **34**, 3472 (1986).

[12] W. Broniowski and M. K. Banerjee, Phys. Rev. D **34**, 849 (1986).

[13] J. Stern and G. Clement, Nucl. Phys. **A504**, 621 (1989).

[14] I. Zahed, A. Wirzba, and U.-G. Meissner, Ann. Phys. **165**, 406 (1985); C.-Y. Ren and M. K. Banerjee, Phys. Rev. C **43**, 1981 (1991).

[15] F. E. Close and R. R. Horgan, Nucl. Phys. **B164**, 413 (1980); K. Ushio, Z. Phys. C **30**, 115 (1986); H. Hogaasen and F. Myhrer, Phys. Rev. D **37**, 1950 (1988).

[16] E. Golowich, Phys. Rev. D **18**, 927 (1978); T. Barnes and F. E. Close, Rutherford Appleton Laboratory Report No. RL-82-110, 1982 (unpublished); E. Golowich, E. Haqq, and G. Karl, Phys. Rev. D **28**, 160 (1983); I. Duck and E. Umland, Phys. Lett. **128B**, 221 (1983).

[17] In more detail, the justification for dropping the post- and prior-gluon exchange diagrams of Fig. 2(M) is as follows: The gluon exchange interaction is a flavor-independent, spin-symmetric amplitude which does not change the SU(6) structure of the nucleon or delta wave functions. It does contribute to the energy splitting of the $N$-$\Delta$ multiplet. If dealt with exactly it would lead to a change in the shape of the radial functions $(U,V)$ of the quark ground state mode. Dealt with perturbatively, one calculates corrections to $(U,V)$ by constructing a perturbation theory with diagonal matrix elements projected out, leaving only matrix elements connecting to excited states. In this case, these contributions are counted in Fig. 2(K). The altered shape of $(U,V)$ is built up perturbatively as a superposition of $1(1S)+\theta(g^2)(2S)+\cdots$. We include only the $(2S)$ term in our calculations. The wave-function renormalization is taken into account in Eq. (2.2).

[18] T. Barnes, F. E. Close, and S. Monaghan, Nucl. Phys. **B198**, 380 (1982).

[19] J. F. Donoghue and K. Johnson, Phys. Rev. D **21**, 1975 (1980).

[20] A. W. Schreiber and A. W. Thomas, Phys. Lett. B **215**, 141 (1992).

[21] S. Adler, Phys. Rev. **140**, B736 (1965); W. Weisberger, *ibid.* **143**, 1302 (1966).

[22] J. J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969), p. 86.

[23] A. W. Thomas, S. Theberge, and G. A. Miller, Phys. Rev. D **24**, 216 (1981); P. A. M. Guichon, Phys. Lett. **129B**, 108 (1983); O. Lie-Svendsen and H. Hogaasen, Z. Phys. C **35**, 239 (1987).

[24] S. Weinberg, Phys. Rev. Lett. **67**, 3473 (1991).

[25] S. Peris, Phys. Lett. B **268**, 415 (1991).

[26] R. Rosenfeld and J. Rosner, Phys. Rev. D **44**, 2133 (1991).

[27] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[28] C.-Y. Ren and M. K. Banerjee, Phys. Rev. C **41**, 2370 (1990); **43**, 1981 (1991).

[29] A. G. Williams and L. R. Dodd, Phys. Rev. D **37**, 1971 (1988); I. Duck, *ibid.* **34**, 1493 (1986); A. G. Williams and A. W. Thomas, Phys. Rev. C **33**, 1070 (1986).

[30] A. Mattingly and P. Stevenson, Phys. Rev. Lett. **69**, 1320 (1992).