PHYSICAL REVIEW C VOLUME 47, NUMBER 1 JANUARY 1993

# Gamow-Teller beta-decay rates for $A \leq 18$ nuclei

W.-T. Chou
*Brookhaven National Laboratory, Upton, New York 11973*
*and Clark University, Worcester, Massachusetts 01610*

E. K. Warburton
*Brookhaven National Laboratory, Upton, New York 11973*

B. A. Brown
*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, East Lansing, Michigan 48824*
(Received 30 June 1992)

A comprehensive analysis is made of the experimental information on Gamow-Teller beta decay for the light ($A \leq 18$) nuclei. Experimental data on half-lives, $Q$ values, and branching ratios are tabulated. Experimental log$ft$ values and beta-decay matrix elements are deduced from these data. The one-body-transition densities necessary to predict the beta-decay matrix elements are then calculated using a recently constructed shell-model Hamiltonian operating in the first four major shells. Using these parameters, effective Gamow-Teller operators are deduced for the $0p$ shell from a least-squares fit to 16 experimental matrix elements. The effective operators are used to calculate 83 decay matrix elements. Some specific decays are discussed.

PACS number(s): 23.40.Hc, 27.20.+n

## I. INTRODUCTION

Nuclear beta decay involves aspects of the weak, strong, and electromagnetic forces and historically has provided more information on these fundamental building blocks of the nucleus than any other nuclear process. The simplest beta-decay observable is its rate. For allowed decays the decay rate amounts to a measure of the sum of the Fermi (vector) and Gamow-Teller (axial vector) transition strengths. The Fermi transition strength is nearly independent of nuclear structure; nevertheless the study of pure Fermi superallowed $0^+ \to 0^+$ transitions has yielded extremely important tests of the conserved vector current hypothesis and the standard model [1–3]. The Gamow-Teller transition strength $B$(GT), on the other hand, is highly dependent on nuclear structure and on renormalizations of the axial-vector coupling constant due to effects of the nuclear medium. An important ongoing subject in nuclear physics is to disentangle these two effects and to understand them both. To this end, systematic studies of $B$(GT) in light nuclei have been made by Wilkinson [4–6] for $A$ = 6–21 nuclei and Brown and Wildenthal [7, 8] for $A$ = 17–39 nuclei. These studies show clearly the value of a systematic and comprehensive consideration of $B$(GT) values for a range of related nuclei. It is our purpose to extend the $1s0d$ study of Brown and Wildenthal to nuclear states with dominant shell-model configurations in the $0s$, $0p$, and $0p1s0d$ shells, where the last group excludes states which are predominantly $1s0d$ states and as such covered by the Brown-Wildenthal compilation. Our formalism and procedures follow closely after those used by Brown and Wildenthal. We collect all the experimental data necessary for the extraction of transition rates and calculate the necessary $\beta^-$ and $\beta^+$/EC phase-space factors so as to provide a consistent set of log$ft$ values and transition strengths. We consider all known decays to bound states and include decays to particle-unbound states when they can be readily extracted. This compilation has some overlap with the compilation and study of Raman *et al.* [9] which focused on 39 mixed Fermi and Gamow-Teller transitions in $A \leq 55$ nuclei. A good general reference for beta decay on which we rely — in addition to the classic papers of Wilkinson and the $1s0d$-shell studies of Brown and Wildenthal already cited — is the beautifully written and erudite book of Behrens and Bühring [10].

## II. WEAK-INTERACTION FORMALISM

### A. General

Historically, the $\beta$ decay rate — the most fundamentally important $\beta$ observable — is usually not quoted, instead the $ft$ value, or the log$ft$ value, is formed from the observed partial half-life $t$ and the phase-space factor $f$ — also called the Fermi integral. Thus [10]

$$ft = \frac{K}{[g_V^2\langle\tau\rangle^2 + g_A^2\langle\sigma\tau\rangle^2]}. \tag{2.1}$$

The partial half-life $t$ is related to the total half-life $t_{1/2}$ of the decaying body in question via

$$t = t_{1/2}/b_r, \tag{2.2}$$

where $b_r$ is the branching ratio for the level with partial half-life $t$. The constant $K$ is given by

$$K = \frac{2\pi^3(\ln\ 2)\hbar^7}{m_e^5 c^4} = 1.230\,618 \times 10^{-94}\ \mathrm{erg}^2\ \mathrm{cm}^6\ \mathrm{s}, \tag{2.3}$$

where $\hbar \equiv h/2\pi$, with $h$ being Planck's constant, $m_e$ is the electron mass, and $c$ is the speed of light. In Eq. (2.1), $g_V$ and $g_A$ are the weak-interaction vector and axial-vector coupling constants for free nucleons.

The nuclear matrix element of Eq. (2.1) for the Fermi operator is

$$\langle\tau\rangle = \langle f||\sum_k t_\pm^k||i\rangle/\sqrt{2J_i+1}, \tag{2.4}$$

and for the Gamow-Teller operator it is

$$\langle\sigma\tau\rangle = \langle f||\sum_k \sigma^k t_\pm^k||i\rangle/\sqrt{2J_i+1}, \tag{2.5}$$

where $f$ and $i$ refer to all the quantum numbers needed to specify the final and initial states, respectively, $\pm$ refers to $\beta^\pm$ decay, $t_\pm = \frac{1}{2}(\tau_x \pm i\tau_y)$ with $t_+p = n$, and $J_i$ is the nuclear initial-state spin. The sums in Eqs. (2.4) and (2.5) are over all nucleons. The matrix elements are reduced in **J** but not in **T**. The phase and reduced matrix element convention follows Edmonds [11]. It should be remarked that because of the Coulomb energy, nuclei are tilted in binding energy such that the proton-rich side is less bound than the neutron-rich side in isobaric multiplets. Thus, all Fermi decays are $\beta^+$/EC transitions.

It is convenient to define transition strengths for the Gamow-Teller and Fermi processes via

$$B(\mathrm{GT}) = (g_A/g_V)^2\langle\sigma\tau\rangle^2 \tag{2.6}$$

and

$$B(F) = \langle\tau\rangle^2 = [T(T+1) - T_{z_i}T_{z_f}]\delta_{if}(1-\delta_c), \tag{2.7}$$

where $\delta_{if}$ allows transitions between analog states only. The quantity $(1-\delta_c)$ corrects for the imperfect overlap between the initial and final states. Following recent evaluations [2] we adopt

$$\delta_c = 0.0025 \pm 0.0020. \tag{2.8}$$

For the Gamow-Teller decay, this correction is small compared to other uncertainties such as those in the nuclear wave functions and is ignored.

Our calculation of the $\beta^\pm$ phase-space factor uses the parametrization of Wilkinson and Macefield [12]. Our formalism and evaluation of electron capture (EC) follows the review article of Bambynek *et al.* [13]. Nuclear size effects and other corrections are slightly different for the Fermi and Gamow-Teller phase-space factors — now denoted $f_A$ and $f_V$, respectively [1, 12]. Taking this difference into account we arrive at our operational definition relating the transition strengths to the $f_At$ value for Gamow-Teller (axial vector) decay:

$$f_At = \frac{6139 \pm 7}{(f_V/f_A)B(F) + B(\mathrm{GT})} \tag{2.9}$$

where the constant $K/(g_V)^2 = 6139$ is a global evaluation by Dubbers [14] of all available data. We note that consideration of the eight best-known superallowed $0^+ \to 0^+$ decays gives $6147 \pm 7$ s for this constant [2].

### B. The phase-space factors for $\beta^\pm$ decay and electron capture

#### *1. Beta decay*

The phase-space integrals $f_A$ and $f_V$ are calculated using the parametrization of Wilkinson and Macfield [12] which — within the context of its definition — is accurate to 0.1%. Not included in the definition of the Wilkinson-Macefield phase-space factor $f_{\mathrm{WM}}$ for Gamow-Teller decay, but recommended by Wilkinson [1, 15] are correction factors for "outer" radiative processes [16]

$$\delta_R \approx 1 + 4.00 \times 10^{-4}Z[1 + 0.22\ln(1.36/W_0)] + 3.60 \times 10^{-6}Z^2 \tag{2.10}$$

and for the diffuseness of the actual nuclear charge distribution [15]

$$\delta_D = 1 + 1.8 \times 10^{-5}Z^{1.36} - 1.2 \times 10^{-6}ZW_0, \tag{2.11}$$

where, as is customary in $\beta$ decay, $Z$ is the atomic number of the daughter. The Fermi and Gamow-Teller phase-space integrals are related by [1]

$$\delta_V = f_V/f_A = 1 \pm \frac{2}{15}\alpha Z(W_0R) - \frac{4}{105}(W_0R)^2 \tag{2.12}$$

for $\beta^\pm$ decay. In these equations $W_0$, the maximum $\beta^\pm$ energy, and $R$, the charge radius for a uniform charge distribution, are in natural units and $R$ is given by

$$R = r_0A^{1/3}/386.16 \tag{2.13}$$

with the purely phenomenology parametrization [1]

$$r_0 = 1.614 - 0.1067(\ln A) + 0.005456(\ln A)^2 + 6.112/(A-1.76)^2\ \mathrm{fm}. \tag{2.14}$$

Then

$$f_A = \delta_R\delta_D f_{WM}, \qquad f_V = \delta_V f_A. \tag{2.15}$$

#### *2. Electron capture*

For $Z+1 \to Z$ decays we must consider both electron capture (labeled as EC or $\epsilon$) and $\beta^+$ processes. The total electron capture probability from all atomic shells is [13]

$$\lambda_c(s^{-1}) = (\ln\ 2)t^{-1} = \frac{(\ln\ 2)g_V^2}{K}\sum n_x C_x f_x^\epsilon. \tag{2.16}$$

The sum extends over all atomic subshells from which an electron can be captured. For closed shells $n_x = 1$; for a partially filled shell $n_x$ is the fractional occupancy of the shell. $C_x$ is the combination of matrix elements allowed for the subshell $x$, and $f_x^\epsilon$ has the form

$$f_x^\epsilon = \frac{\pi}{2}q_x^2\beta_x^2B_x. \tag{2.17}$$

In Eq. (2.17) $q_x$ is the neutrino energy released in the capture and is given by

$$q_x = Q(\epsilon) - E_x - E_R - E_{\text{recoil}}, \quad (2.18)$$

where $Q(\epsilon)$ is the atomic mass difference between initial and final states (and as such contains effects due to the change in electron binding energies). $E_x$ is the binding energy of an electron in the $x$ subshell in the daughter nucleus, $E_R$ is the rearrangement energy, and $E_{\text{recoil}}$ is determined from the momentum balance. All energies are in units of $m_e c^2$ (natural units), $\beta_x$ is the electron radial wave function amplitude at the nucleus ($r = 0$), and $B_x$ takes account of the effects of electron exchange and overlap. For allowed decays $C_x$ is independent of the subshell $x$ [13] and can be replaced by $B(F) + B(\text{GT})$ (it is assumed that $f^\epsilon$ is the same for Fermi and Gamow-Teller decays) and the phase-space factor becomes

$$f^\epsilon = \frac{\pi}{2}[n_K q_K^2 \beta_K^2 + n_{L1} q_{L1}^2 \beta_{L1}^2 B_{L1} + \cdots]. \quad (2.19)$$

For the light nuclei under consideration here, only the $K$ and $L1$ subshells will give non-negligible contributions and Eq. (2.19) is complete as it stands. Thus for electron capture we have the exact analog of Eq. (2.9)

$$f^\epsilon t_\epsilon = \frac{6139 \pm 7}{[\delta_V B(F) + B(\text{GT})]}, \quad (2.20)$$

where we arbitrarily assume the same relation, Eq. (2.12), between the Fermi and Gamow-Teller phase-space integrals as pertains in $\beta^+$ decay. It should be emphasized that $t_\epsilon$ is the partial half-life for the EC process to the level in question.

The above result for $f^\epsilon$, Eq. (2.19), is that given by Bambynek *et al.* However if the total EC rate is desired it is not necessary to include the $B_x$ factors — at least for light nuclei. This is so because they represent effects that redistribute the process amongst the different subshells but do not change the total rate [17].

In practice, when both are allowed, we combine the $\beta^+$ and EC processes by replacing $f_A$ in Eq. (2.9) by

$$f_A^+ = f_A(\beta^+) + f^\epsilon. \quad (2.21)$$

We then use it in conjunction with the partial half-life for the combined $\beta^+$ and EC processes.

#### *3. Decays to unbound states*

When the final state is unbound against the emission of nucleons or nucleon clusters, complexities arise which can greatly increase the difficulty of extracting $M(\text{GT})$ values from the experimental data [18–20]. Consider $\beta$ decay into a region of the continuum which, in general, consists of overlapping resonances which decay by particle emission. Because of the interference between these overlapping resonances the branching ratio, log $ft$ value, and $\beta$-decay matrix elements cannot be rigorously defined for a given level (resonance). Barker [18] developed an $R$-matrix approach which is appropriate to this situation and it was used to analyze the $^8$He [19], $^8$Li,$^8$B [20, 21], $^9$Li [22, 23], and $^{16}$N [24, 25] decays which we present. Note that the extraction of $M(\text{GT})$ for unbound states is inherently less accurate than for bound states.

In those cases where an $R$-matrix (or roughly equivalent) approach has not been made, we make a simple, very approximate estimate of the effect of the finite width of the level and use this estimate to correct $f$ and to augment its uncertainty. Penetrability factors and interference between resonances are neglected and we assume a simple Breit-Wigner shape for the resonance and an $E_\beta^{4-5}$ dependence for the integrand of $f$. Then, for a level of width $\Gamma$ and averaging $f$ over an interval of $\pm 2\Gamma$, we define a "corrected" $\bar{f} = f(1 + \Delta f_\Gamma)$ where $f$ is evaluated at the peak of the resonance ($E_\beta = E_\beta^R$) and $\Delta f_\Gamma \approx 6(\Gamma/E_\beta^R)^2$. We then calculate $M(\text{GT})$ using $\bar{f}$ and add $\Delta f_\Gamma$ to the other uncertainties in quadrature. This approximation is only useable for $\Gamma \ll E_\beta^R$, its main use is to estimate the magnitude of any possible effect due to the unbound nature of the final state.

### C. The Gamow-Teller matrix element

We follow Brown and Wildenthal [7] and define an experimental Gamow-Teller matrix element in terms of the experimental transition strength:

$$M(\text{GT})_{\text{exp}} = [(2J_i + 1)B(\text{GT})_{\text{exp}}]^{1/2}, \quad (2.22)$$

where

$$B(\text{GT})_{\text{exp}} = [(6139 \pm 7)/f_A t_{\text{exp}}] - \delta_V B(F)_{\text{th}}. \quad (2.23)$$

Our convention is that $M(\text{GT})$ is positive and experimental and theoretical quantities are denoted by subscripts of "exp" and "th," respectively. The matrix element $M(\text{GT})$ is the quantity used to relate experiment and theory. Its theoretical definition is [7]

$$M(\text{GT}) = \sqrt{12}\,(-1)^{T_f - T_{zf}} \begin{pmatrix} T_f & 1 & T_i \\ -T_{zf} & \Delta T_z & T_{zi} \end{pmatrix} \lambda \times \sum_{jj'} q_{jj'} \langle j||s||j'\rangle D_{jj'}, \quad (2.24)$$

where $\Delta T_z = T_{zf} - T_{zi}$ and

$$\lambda = |g_A/g_V| = 1.264 \pm 0.002 \quad (2.25)$$

is obtained from the same global evaluation which yielded $K/(g_A)^2$ [14]. The sum is over all possible single-particle transitions $j \to j'$ of the model space, $\langle j||s||j'\rangle$ is the reduced single-particle matrix element of $\mathbf{s}$ ($\equiv \frac{1}{2}\sigma$) for this transition calculated in the model space with the operator specific to free nucleons, and it is assumed that higher-order corrections to this matrix element can be represented by the multiplicative factor $q_{jj'}$. In general, the one-body-transition density $D_{jj'}$ is obtained from shell-model wave functions for an operator with rank in spin and isospin space of $\Delta J$ and $\Delta T$, respectively, via

$$D_{jj'} = \frac{\langle f|||\, [a^\dagger(j) \times \tilde{a}(j')]^{\Delta J, \Delta T}\, |||i\rangle}{[(2\Delta J + 1)(2\Delta T + 1)]^{1/2}}, \quad (2.26)$$

where the matrix element is doubly reduced (spin and isospin) and for Gamow-Teller decay $\Delta J = \Delta T = 1$. For a single-particle transition such as $^{15}O(\beta^+/EC)^{15}N$ taken as $(\nu p_{1/2})^{-1} \to (\pi p_{1/2})^{-1}$, $|D_{jj'}| = 1$. The single-particle matrix elements are given by Table I of Ref. [7] which, for the four possible $0p$-shell transitions, gives

| $2j$ | $2j'$ | $\sqrt{6}\ \langle j\|\|s\|\|j'\rangle$ |
|---|---|---|
| 1 | 1 | $-1$ |
| 1 | 3 | $-2\sqrt{2}$ |
| 3 | 1 | $2\sqrt{2}$ |
| 3 | 3 | $\sqrt{10}$. |

The effect of the nuclear medium can be expressed via the three independent $q_{jj'}$ ($q_{j'j} \equiv q_{jj'}$) or, equivalently, via the deviations from the free-nucleon values of the matrix elements of the operators

$$\mathbf{S} = \sum_k \mathbf{s}_k, \quad \mathbf{L} = \sum_k \mathbf{l}_k, \quad \mathbf{P} = \sum_k \mathbf{p}_k \tag{2.27}$$

defined by Brown and Wildenthal [7, 8]. In Eq. (2.27),

$$\mathbf{p} = \sqrt{8\pi}\ [Y^{(2)} \otimes \mathbf{s}]^{(1)}. \tag{2.28}$$

For free nucleons only the first of these three operators has a nonzero matrix element for the Gamow-Teller operator. The nuclear medium induces nonzero values for the other two and we can express the deviations from the free-nucleon single-particle matrix elements by [7]

$$\begin{aligned} \delta_{\frac{1}{2}\frac{1}{2}} &= q_{\frac{1}{2}\frac{1}{2}} - 1 = \delta_s - 4\delta_l + 4\delta_p, \\ \delta_{\frac{3}{2}\frac{1}{2}} &= q_{\frac{3}{2}\frac{1}{2}} - 1 = \delta_s - \ \delta_l - \tfrac{1}{2}\delta_p, \\ \delta_{\frac{3}{2}\frac{3}{2}} &= q_{\frac{3}{2}\frac{3}{2}} - 1 = \delta_s + 2\delta_l + \tfrac{2}{5}\delta_p. \end{aligned} \tag{2.29}$$

It is clear that, in general, the quenching factors $\delta_{jj'}$ for the three single-particle transitions can be expected to be different.

We expect some mass dependence in the $\delta$ correction factors since the renormalizations are theoretically somewhat different for the particle states in $A$=5 and the hole states in $A$=15 [26]. Assuming a smooth $A$ dependence for the $\delta$ correction factors, we follow Brown and Wildenthal [7, 8] and parametrize it in the form

$$\delta(A) = \delta(A = 16)\left(\frac{A}{16}\right)^P. \tag{2.30}$$

We adopt $P = 0.35$ which is the value assumed by Brown and Wildenthal in their $1s0d$-shell fit and is close to the value which would be inferred from Towner's theoretical $0p$ results for $\delta_s$ at $A$=5 and 15 which would give $P = 0.36$.

## III. THE SHELL-MODEL CALCULATIONS

Calculations of the one-body-transition densities of Eq. (2.26) were carried out with the computer code OXBASH [27]. Recently derived interactions of Warburton and Brown [28] provided the Hamiltonians. The initial and final states for the $A$ = 6–18 region under consideration are assigned either to the $0p$-shell configurations $(0s)^4(0p)^{A-4}$ or (predominantly) to the cross-shell configurations $(0s)^4(0p)^{A-4-n_{sd}}(1s0d)^{n_{sd}}$. For $A$ = 6–9, all the decays considered theoretically are $0p$-shell transitions and are treated with the interaction labeled $P$(5–16)$T$. This interaction is the result of a least-squares fit of the 15 $0p$-shell two-body matrix elements (TBME) and two single-particle energies (SPE) to 86 level binding energies in $A$ = 5–16 nuclei. The $0p$-shell decays with $A$ = 10–15 were treated with the $P$(10–16)T interaction which results from a similar fit to 51 $A$ = 10–16 level binding energies. The $D_{jj'}$ of the cross-shell $A$ = 11–18 decays were calculated with an interaction which encompasses the lowest four major shells — $0s$, $0p$, $1s0d$, and $0f1p$ — and is designed to treat a given $n\hbar\omega$ excitation of a $0\hbar\omega$ $(0s)^4(0p)^{A-4-n_{sd}}(1s0d)^{n_{sd}}$ configuration. It results from a least-squares fit to 216 level binding energies in the $A$ = 10–22 region. We shall refer to these three interactions collectively as the WBT interaction. Which of the three is actually used can be deduced from the $A$ and configurations for the transition in question.

## IV. THE EXPERIMENTAL DATA

The decaying nuclei we shall consider are listed in Table I and the experimental data from which the $M$(GT) can be extracted for specific decays are collected in Table II. Table III contains the experimental $\log f_A t$ and $M$(GT). The sixth column of Table III contains the matrix elements, $M(\mathrm{GT})_{\mathrm{free}}$, calculated with the free-nucleon operator, i.e., from Eq. (2.24) with the $q_{jj'}$ fixed at unity. The next (seventh) column contains the matrix elements, $M(\mathrm{GT})_{\mathrm{eff}}$, calculated with effective Gamow-Teller operators. We now consider the determination of these effective operators.

## V. THE EFFECTIVE GAMOW-TELLER MATRIX ELEMENTS

Empirical $0p$-shell values for the $\delta_s$, $\delta_l$, and $\delta_p$ of Eq. (2.29) were obtained from a least-squares fit of Eqs. (2.22), (2.24), and (2.30) to the 16 strongest and most reliable decays connecting states assigned to $(0s)^4(0p)^{A-4}$ configurations. The strongest is defined relative to the Gamow-Teller sum rule [47] which relates the summed $\beta^-$ and summed $\beta^+$ $B$(GT) values for a given initial state to the neutron number $N_i$ and the proton number $Z_i$ of the initial state

$$\begin{aligned} \sum_f\ & [B(\mathrm{GT}-, i \to f) - B(\mathrm{GT}+, i \to f)] \\ & = (g_A/g_V)^2 3(N_i - Z_i) \end{aligned} \tag{5.1}$$

where the sum is over all possible final states. Thus, following Brown and Wildenthal [8], the fits are made to $M$(GT) values which are normalized by dividing both experimental and theoretical values by

$$W = |g_A/g_V|[(2J_i + 1)3(N_i - Z_i)]^{1/2} \ \text{ for } N_i \neq Z_i. \tag{5.2}$$

From the sum rule of Eq. (5.1), the upper bounds on the quantities

$$R(\mathrm{GT}) = M(\mathrm{GT})/W \tag{5.3}$$

are of the order of unity. The 16 transitions included in this fit are labeled by p# (# = 1–16) in the last column of Table III. It can be seen in Table III that for six of the $\beta^-$ decays (p5–6,p10–13) included in the fit, there are mirror $\beta^+$ decays also available. There is a well-known asymmetry for mirror decays which can be traced to the different binding energies in the finite well of the neutron ($\beta^-$) and proton ($\beta^+$) [1]. The proton in $\beta^+$ decay is less bound and thus its wave function has a longer tail and its overlap with the tightly bound final state is, in general, poorer than for the neutron in $\beta^-$ decay. Thus $M(\mathrm{GT},+)$ is generally smaller than $M(\mathrm{GT},-)$ and it is more fitting to compare $M(\mathrm{GT},-)$ to calculations — such as ours — which ignore these binding energy effects.

In order to obtain some assessment of the dependence of the effective operators on the shell-model interaction used to obtain the $D_{jj'}$, the shell-model calculation and least-squares fit was also performed with the (8–16) TBME interaction of Cohen and Kurath [52]. The results of the two fits are given in Table IV. The entry $\Delta_{\mathrm{th}}$ is the root-mean-square (rms) deviation between the theoretical and experimental $R(\mathrm{GT})$. Thus there is a ~4% deviation relative to a sum rule of unity. In the final fits the quantity $\Delta_{\mathrm{th}}$ was added in quadrature to the experimental errors in order to give proper weight to the individual datum. It is seen that the two interactions give results which are equal within the uncertainties with which they are determined. From the first listed fit we see that $\delta_l$ and $\delta_p$ are consistent with zero. This can also be inferred from the uncertainties which result for the second-listed fit. They are also consistent with the "final-fit" values obtained by Brown and Wildenthal [8] from their fit to $1s0d$ shell data; these values (for $A = 16$) are $\delta_s(s\text{-}s) = -0.186(13)$, $\delta_s(d\text{-}d) = -0.204(9)$, $\delta_l(d\text{-}d) = +0.0029(22)$, $\delta_p(sd) = +0.014(7)$, where the latter is an average value for $\delta_p(s\text{-}s)$ and $\delta_p(d\text{-}d)$. Again, the uncertainties resulting from the fourth-listed fit illustrate this consistency. The fundamental calculations of Towner [26] for the $0p$ orbits give $A = 16$ results of $\delta_s = -0.189$, $\delta_l = +0.013$,

TABLE I. Decaying nuclei under consideration. All are ground states except the $^{16}$N $0^-$ state. $\Delta t_{1/2}$ and $\Delta Q$ are the uncertainties in the least significant figures of $t_{1/2}$ and $Q$, respectively. All $Q$ values are from Ref. [29]. The spin-parity and isospin assignments are from Ref. [30]. The $\beta$-decay half-lives are also from Ref. [30] unless otherwise noted in the last column.

| Decay | $2J_i^\pi$ | $2T_i$ | $t_{1/2}$ (sec) | $\Delta t_{1/2}$ | $Q$ (keV) | $\Delta Q$ | Ref. |
|---|---|---|---|---|---|---|---|
| $^1$n$(\beta^-)^1$H | $1^+$ | 1 | 6.166E+02 | 16 | 782.346 | 4 | [14, 31] |
| $^3$H$(\beta^-)^3$He | $1^+$ | 1 | 3.887E+08 | 9 | 18.596 | 2 | [32] |
| $^6$He$(\beta^-)^6$Li | $0^+$ | 2 | 8.067E−01 | 15 | 3507.76 | 90 | |
| $^7$Be$(EC)^7$Li | $3^-$ | 1 | 4.604E+06 | 6 | 861.835 | 17 | |
| $^8$He$(\beta^-)^8$Li | $0^+$ | 4 | 1.110E−01 | 15 | 10653.7 | 71 | |
| $^8$Li$(\beta^-)^8$Be | $4^+$ | 2 | 8.403E−01 | 9 | 16003.71 | 71 | [33] |
| $^8$B$(\beta^+)^8$Be | $4^+$ | 2 | 7.70E−01 | 3 | 17978.5 | 12 | |
| $^9$Li$(\beta^-)^9$Be | $3^-$ | 3 | 1.783E−01 | 4 | 13606.0 | 19 | |
| $^9$C$(\beta^+)^9$B | $3^-$ | 3 | 1.265E−01 | 9 | 16497.9 | 25 | |
| $^{10}$C$(\beta^+)^{10}$B | $0^+$ | 2 | 1.9290E+01 | 12 | 3647.82 | 10 | [35] |
| $^{11}$Li$(\beta^-)^{11}$Be | $3^-$ | 5 | 8.5E−03 | 2 | 20675 | 80 | |
| $^{11}$Be$(\beta^-)^{11}$B | $1^+$ | 3 | 1.381E+01 | 8 | 11506.1 | 64 | |
| $^{11}$C$(\beta^+)^{11}$B | $3^-$ | 1 | 1.2234E+03 | 12 | 1982.20 | 83 | |
| $^{12}$Be$(\beta^-)^{12}$B | $0^+$ | 4 | 2.13E−02 | 22 | 11707 | 15 | [36] |
| $^{12}$B$(\beta^-)^{12}$C | $2^+$ | 2 | 2.020E−02 | 20 | 13369.4 | 13 | |
| $^{12}$N$(\beta^+)^{12}$C | $2^+$ | 2 | 1.1000E−02 | 16 | 17338.0 | 10 | |
| $^{13}$B$(\beta^-)^{13}$C | $3^-$ | 3 | 1.736E−02 | 16 | 13437.2 | 11 | |
| $^{13}$N$(\beta^+)^{13}$C | $1^-$ | 1 | 5.979E+02 | 24 | 2220.45 | 27 | |
| $^{13}$O$(\beta^+)^{13}$N | $3^-$ | 3 | 8.55E−03 | 5 | 17766.2 | 95 | [37] |
| $^{14}$B$(\beta^-)^{14}$C | $4^-$ | 4 | 1.28E−02 | 8 | 20644 | 21 | [36] |
| $^{14}$C$(\beta^-)^{14}$N | $0^+$ | 2 | 1.807E+11 | 13 | 156.472 | 5 | |
| $^{14}$O$(\beta^+)^{14}$N | $0^+$ | 2 | 7.0606E+01 | 18 | 5143.064 | 80 | |
| $^{15}$C$(\beta^-)^{15}$N | $1^+$ | 3 | 2.449E+00 | 5 | 9771.68 | 80 | |
| $^{15}$O$(\beta^+)^{15}$N | $1^-$ | 1 | 1.2224E+02 | 16 | 2753.95 | 53 | |
| $^{16}$C$(\beta^-)^{16}$N | $0^+$ | 4 | 7.47E−01 | 8 | 8012.1 | 43 | |
| $^{16}$N$(\beta^-)^{16}$O | $4^-$ | 2 | 7.13E+00 | 2 | 10419.1 | 23 | |
| $^{16}$N*$(\beta^-)^{16}$O | $0^-$ | 2 | 1.571 | 69 | 10539.5 | 23 | [38] |
| $^{17}$N$(\beta^-)^{17}$O | $1^-$ | 3 | 4.174E+00 | 4 | 8680 | 15 | |
| $^{17}$Ne$(\beta^+)^{17}$F | $1^-$ | 3 | 1.093E−01 | 6 | 14536 | 50 | [39] |
| $^{18}$C$(\beta^-)^{18}$N | $0^+$ | 6 | 9.5E−02 | 10 | 11810 | 36 | [40] |
| $^{18}$N$(\beta^-)^{18}$O | $2^-$ | 4 | 6.24E−01 | 12 | 13899 | 20 | |

TABLE II. Experimental data for specific decays. Unless otherwise indicated in the last column, the spin-parity, energy, and branching ratio information is from Ref. [30]. The index $k$ orders states of given $J^\pi, T$ in energy. $\Delta E_f$ and $\Delta b_r$ are the uncertainties in the least significant figure for the energy $E_f$ of the final state and the branching ratio $b_r$ to it, respectively. $P^\epsilon$ is the percentage electron capture contributing to the decay. No entry for $\Delta f_\Gamma$ means the final state is bound, an entry of $R$ means the data was subject to an $R$-matrix analysis (and $b_r$ and $\log f_A$ are estimates), and a numerical entry indicates application of the approximate prescription discussed in the text.

| Reaction | $2J^\pi_k$,2T (i) | (f) | $E_f$ (keV) | $\Delta E_f$ | $b_r$ (%) | $\Delta b_r$ | $\log f_A$ | $P^\epsilon$ (%) | $\Delta f_\Gamma$ (%) | Ref. |
|---|---|---|---|---|---|---|---|---|---|---|
| $^1$n$(\beta^-)^1$H | $1^+$,1 | $1^+_1$,1 | 0 | 0 | 100.0 | <1 | 0.234 | 0.000 | | [31] |
| $^3$H$(\beta^-)^3$He | $1^+$,1 | $1^+_1$,1 | 0 | 0 | 100.0 | <1 | −5.533 | 0.000 | | [32] |
| $^6$He$(\beta^-)^6$Li | $0^+$,2 | $2^+_1$,0 | 0 | 0 | 100.0 | <1 | 3.003 | 0.000 | | |
| $^7$Be$(EC)^7$Li | $3^-$,1 | $3^-_1$,1 | 0 | 0 | 89.61 | 6 | −3.410 | 100.0 | | |
| | $3^-$,1 | $1^-_1$,1 | 477.612 | 3 | 10.39 | 6 | −4.112 | 100.0 | | |
| $^8$He$(\beta^-)^8$Li | $0^+$,4 | $2^+_1$,2 | 981 | 0 | 84.0 | 1 | 5.045 | 0.000 | | |
| | $0^+$,4 | $2^+_2$,2 | ~3080 | ~80 | ~7.8 | ~20 | ~4.37 | 0.000 | R | [19] |
| | $0^+$,4 | $2^+_3$,2 | ~ 5150 | ~80 | ~2.6 | ~15 | ~3.90 | 0.000 | R | [19] |
| | $0^+$,4 | $2^+_4$,2 | ~9060 | ~90 | ~2.4 | ~15 | ~2.43 | 0.000 | R | [19] |
| $^8$Li$(\beta^-)^8$Be | $4^+$,2 | $4^+_1$,0 | 3120 | 130 | ~91 | | ~5.40 | 0.000 | R | [20] |
| | $4^+$,2 | $4^+_2$,0 | ~16800 | | ~7 | | ~1.85 | 0.000 | R | [20, 41] |
| $^8$B$(\beta^+)^8$Be | $4^+$,2 | $4^+_1$,0 | 3120 | 130 | ~88 | | ~5.38 | 0.000 | R | [20] |
| | $4^+$,2 | $4^+_2$,0 | ~16800 | | ~9 | | ~2.00 | 0.000 | R | [20, 41] |
| $^9$Li$(\beta^-)^9$Be | $3^-$,3 | $3^-_1$,1 | 0 | 0 | 50.0 | 18 | 5.765 | 0.000 | | [34] |
| | $3^-$,3 | $5^-_1$,1 | 2429.4 | 13 | 29.2 | 30 | 5.355 | 0.000 | 0.00 | [22] |
| | $3^-$,3 | $1^-_1$,1 | 2780 | 120 | 15.6 | 30 | ~5.31 | 0.000 | R | [22] |
| | $3^-$,3 | $5^-_2$,1 | 7940 | 80 | 1.5 | 5 | ~4.04 | 0.000 | R | [42] |
| | $3^-$,3 | a ,1 | 11283 | 24 | 1.1 | 2 | ~3.63 | 0.000 | R | [22] |
| | $3^-$,3 | a ,1 | 11810 | 20 | 2.6 | 2 | ~2.84 | 0.000 | R | [22] |
| $^9$C$(\beta^+)^9$B | $3^-$,3 | $3^-_1$,1 | 0 | 0 | 60 | 10 | 5.950 | 0.000 | | [43] |
| | $3^-$,3 | $5^-_1$,1 | 2361 | 5 | 17 | 6 | 5.604 | 0.000 | 0.02 | [43] |
| | $3^-$,3 | $1^-_1$,1 | ~2800 | | 11 | 5 | 5.548 | 0.000 | 3.79 | [43] |
| $^{10}$C$(\beta^+)^{10}$B | $0^+$,2 | $2^+_1$,0 | 718.35 | 4 | 98.54 | 14 | 1.757 | 0.029 | | |
| | $0^+$,2 | $2^+_2$,0 | 2154.3 | 5 | <0.08 | | −0.706 | 2.154 | | |
| $^{11}$Li$(\beta^-)^{11}$Be | $3^-$,5 | $1^-_1$,3 | 320.04 | 10 | 9.2 | 7 | 6.609 | 0.000 | | |
| $^{11}$Be$(\beta^-)^{11}$B | $1^+$,3 | $1^+_1$,1 | 6791.80 | 30 | <0.03 | 0 | 3.389 | 0.000 | 0.00 | |
| | $1^+$,3 | $3^+_1$,1 | 7977.84 | 42 | 4.00 | 30 | 3.037 | 0.000 | 0.00 | |
| | $1^+$,3 | $3^+_2$,1 | 9876 | 8 | 3.1 | 4 | 1.594 | 0.000 | 2.73 | |
| $^{11}$C$(\beta^+)^{11}$B | $3^-$,1 | $3^-_1$,1 | 0 | 0 | 100.0 | 0 | 0.512 | 0.230 | | |
| $^{12}$Be$(\beta^-)^{12}$B | $0^+$,4 | $2^+_1$,2 | 0 | 0 | 99.1 | 4 | 5.462 | 0.000 | | |
| | $0^+$,4 | $2^+_2$,2 | 5000 | 20 | <0.9 | | 4.316 | 0.000 | 0.03 | |
| $^{12}$B$(\beta^-)^{12}$C | $2^+$,2 | $0^+_1$,0 | 0 | 0 | 97.22 | 30 | 5.748 | 0.000 | | |
| | $2^+$,2 | $4^+_1$,0 | 4438.91 | 31 | 1.283 | 40 | 4.912 | 0.000 | | |
| | $2^+$,2 | $0^+_2$,0 | 7654.20 | 15 | 1.5 | 3 | 4.004 | 0.000 | 0.00 | |
| | $2^+$,2 | $0^+_3$,0 | 10300 | 300 | 0.08 | 2 | 2.783 | 0.000 | b | b |
| $^{12}$N$(\beta^+)^{12}$C | $2^+$,2 | $0^+_1$,0 | 0 | 0 | 94.55 | 60 | 6.053 | 0.000 | | |
| | $2^+$,2 | $4^+_1$,0 | 4438.91 | 31 | 1.898 | 32 | 5.388 | 0.000 | | |
| | $2^+$,2 | $0^+_2$,0 | 7654.20 | 15 | 2.7 | 4 | 4.733 | 0.001 | 0.00 | |
| | $2^+$,2 | $0^+_3$,0 | 10300 | 300 | 0.46 | 15 | 3.988 | 0.002 | b | b |
| | $2^+$,2 | $2^+_1$,0 | 12710 | 6 | 0.31 | 12 | 2.967 | 0.007 | 0.00 | |
| | $2^+$,2 | $2^+_1$,2 | 15110 | 3 | 0.0044 | 15 | 0.906 | 0.190 | 0.00 | |
| $^{13}$B$(\beta^-)^{13}$C | $3^-$,3 | $1^-_1$,1 | 0 | 0 | 92.1 | 8 | 5.759 | 0.000 | | |
| | $3^-$,3 | $3^-_1$,1 | 3684.507 | 19 | 7.6 | 8 | 5.094 | 0.000 | | |
| | $3^-$,3 | $5^-_1$,1 | 7547 | 3 | 0.094 | 20 | 4.064 | 0.000 | 0.00 | |
| | $3^-$,3 | $1^-_2$,1 | 8860 | 20 | 0.16 | 3 | 3.563 | 0.000 | 0.64 | |
| | $3^-$,3 | $3^-_2$,1 | 9897 | 5 | 0.022 | 7 | 3.055 | 0.000 | 0.03 | |
| $^{13}$N$(\beta^+)^{13}$C | $1^-$,1 | $1^-_1$,1 | 0 | 0 | 100.0 | 0 | 0.895 | 0.194 | | |
| $^{13}$O$(\beta^+)^{13}$N | $3^-$,3 | $1^-_1$,1 | 0 | 0 | 89.2 | 22 | 6.099 | 0.000 | | |
| | $3^-$,3 | $3^-_1$,1 | 3502 | 2 | 9.8 | 20 | 5.606 | 0.000 | 0.01 | |
| | $3^-$,3 | $5^-_1$,1 | 7376 | 9 | 0.18 | 9 | 4.886 | 0.001 | 0.04 | |
| | $3^-$,3 | $1^-_2$,1 | 8918 | 11 | 0.61 | 14 | 4.518 | 0.001 | 0.52 | |
| | $3^-$,3 | $3^-_2$,1 | 9476 | 8 | 0.16 | 4 | 4.364 | 0.001 | 0.01 | |
| | $3^-$,3 | $5^-_2$,1 | 10360 | 50 | 0.02 | 1 | 4.099 | 0.002 | 0.01 | |
| $^{14}$B$(\beta^-)^{14}$C | $4^-$,4 | $2^-_1$,2 | 6093.8 | 2 | 81 | 9 | 5.925 | 0.000 | | |

TABLE II. (*Continued*).

| Reaction | $2J^\pi_k$,2T (i) (f) | $E_f$ (keV) | $\Delta E_f$ | $b_r$ (%) | $\Delta b_r$ | $\log f_A$ | P$^\epsilon$ (%) | $\Delta f_\Gamma$ (%) | Ref. |
|---|---|---|---|---|---|---|---|---|---|
| | $4^-,4$ $6^-_1,2$ | 6728.2 | 13 | 8.6 | 38 | 5.832 | 0.000 | | |
| | $4^-,4$ $4^-_1,2$ | 7341.4 | 31 | <11 | | 5.738 | 0.000 | | |
| $^{14}C(\beta^-)^{14}N$ | $0^+,2$ $2^+_1,0$ | 0 | 0 | 100.0 | 0 | −2.208 | 0.000 | | |
| $^{14}O(\beta^+)^{14}N$ | $0^+2$, $2^+_1,0$ | 0 | 0 | 0.61 | 1 | 3.221 | 0.007 | | |
| | $0^+2$, $2^+_2,0$ | 3948.10 | 20 | 0.054 | 2 | −1.981 | 64.2 | | |
| $^{15}C(\beta^-)^{15}N$ | $1^+,3$ $1^+_1,1$ | 5298.822 | 14 | 63.2 | 8 | 3.526 | 0.000 | | |
| | $1^+,3$ $3^+_1,1$ | 7300.83 | 2 | 0.0074 | 8 | 2.377 | 0.000 | | |
| | $1^+,3$ $1^+_2,1$ | 8312.62 | 3 | 0.041 | 5 | 1.408 | 0.000 | | |
| | $1^+,3$ $3^+_2,1$ | 8571.4 | 12 | 0.013 | 2 | 1.062 | 0.000 | | |
| | $1^+,3$ $1^+_3,1$ | 9049.71 | 7 | 0.034 | 3 | 0.191 | 0.000 | | |
| $^{15}O(\beta^+)^{15}N$ | $1^-,1$ $1^-_1,1$ | 0 | 0 | 100.0 | 0 | 1.557 | 0.099 | | |
| $^{16}C(\beta^-)^{16}N$ | $0^+,4$ $2^+_1,2$ | 3352.8 | 26 | 83.8 | 17 | 3.606 | 0.000 | 0.01 | |
| | $0^+,4$ $2^+_2,2$ | 4320.4 | 27 | 15.5 | 17 | 3.148 | 0.000 | 0.02 | |
| $^{16}N(\beta^-)^{16}O$ | $4^-,2$ $6^-_1,0$ | 6129.89 | 4 | 66.2 | 6 | 3.454 | 0.000 | | |
| | $4^-,2$ $2^-_1,0$ | 7116.85 | 14 | 4.8 | 4 | 2.943 | 0.000 | | |
| | $4^-,2$ $4^-_1,0$ | 8871.9 | 5 | 1.06 | 7 | 1.527 | 0.000 | 0.00 | |
| | $4^-,2$ $2^-_2,0$ | 9585 | 11 | 0.00120 | 5 | 0.993 | 0.000 | R | [24] |
| $^{16}N^*(\beta^-)^{16}O$ | $0^-,2$ $2^-_1,0$ | 7116.85 | 14 | 9 | 2 | 3.012 | 0.000 | | [38] |
| $^{17}N(\beta^-)^{17}O$ | $1^-,3$ $1^-_1,1$ | 3055.36 | 16 | 0.34 | 6 | 3.993 | 0.000 | | |
| | $1^-,3$ $3^-_1,1$ | 4552 | 2 | 38.0 | 13 | 3.378 | 0.000 | 0.06 | |
| | $1^-,3$ $3^-_2,1$ | 5378 | 2 | 50.1 | 13 | 2.943 | 0.000 | 0.04 | |
| | $1^-,3$ $1^-_2,1$ | 5939 | 4 | 6.9 | 5 | 2.586 | 0.000 | 0.08 | |
| $^{17}Ne(\beta^+)^{17}F$ | $1^-,3$ $1^-_1,1$ | 3104 | 3 | 0.48 | 7 | 5.087 | 0.001 | 0.00 | [44] |
| | $1^-,3$ $3^-_1,1$ | 4640 | 20 | 16.54 | 14 | 4.758 | 0.002 | 0.39 | [39] |
| | $1^-,3$ $3^-_2,1$ | 5488 | 11 | 59.2 | 4 | 4.550 | 0.002 | 0.04 | [39] |
| | $1^-,3$ $1^-_2,1$ | 6037 | 9 | 7.8 | 2 | 4.404 | 0.003 | 0.01 | [39] |
| | $1^-,3$ $3^-_3,1$ | 8075 | 10 | 7.3 | 9 | 3.758 | 0.007 | 0.20 | [39] |
| | $1^-,3$ $3^-_4,1$ | 8200 | 100 | 1.7 | 3 | 3.753 | 0.007 | 10.41 | [39],c |
| | $1^-,3$ $1^-_3,1$ | 8436 | 10 | 4.0 | 9 | 3.620 | 0.008 | 0.23 | [39] |
| | $1^-,3$ $1^-_1,3$ | 11192.9 | 23 | 0.64 | 14 | 2.087 | 0.087 | 0.00 | [39] |
| $^{18}C(\beta^-)^{18}N$ | $0^+,6$ $2^+_1,4$ | 1734.8 | 4 | 9 | 7 | 5.171 | 0.000 | | [40] |
| | $0^+,6$ $2^+_2,4$ | 2614.2 | 4 | 72 | 10 | 4.983 | 0.000 | | [40] |
| $^{18}N(\beta^-)^{18}O$ | $2^-,4$ $2^-_1,2$ | 4455.54 | 10 | 47.8 | 10 | 5.048 | 0.000 | | d |
| | $2^-,4$ $4^-_1,2$ | 5530.24 | 29 | 2.7 | 4 | 4.800 | 0.000 | | d |
| | $2^-,4$ $2^-_2,2$ | 6198.22 | 40 | 1.2 | 2 | 4.630 | 0.000 | | d |
| | $2^-,4$ $4^-_2,2$ | 6351.3 | 6 | 1.9 | 3 | 4.589 | 0.000 | 0.00 | d |
| | $2^-,4$ $0^-_1,2$ | 6880.45 | 27 | 13.0 | 8 | 4.441 | 0.000 | 0.00 | d |
| | $2^-,4$ $2^-_3,2$ | 7619 | 3 | 6.8 | 5 | 4.215 | 0.000 | 0.00 | [45],d |
| | $2^-,4$ $4^-_3,2$ | 7771.07 | 50 | 4.4 | 5 | 4.166 | 0.000 | 0.00 | d |
| | $2^-,4$ $2^-_4,2$ | 8039 | 2 | 1.8 | 2 | 4.076 | 0.000 | 0.00 | [45],d |

[a] The $\beta^-$ decay is allowed so that $2J^\pi = 1^-,3^-$, or $5^-$. This broad structure could very well be a composite due to two or more overlapping resonances.

[b] The $\Delta f_\Gamma$ correction for this decay is too large to be meaningful. A correction for the finite width of the level and any possible interference effects has not been made.

[c] $\Delta E_f$ is our estimate.

[d] See Sec. VI B 12.

and $\delta_p = +0.048$. The least-squares results are in poorer agreement with these values — especially that for $\delta_p$. However, the disagreement is not severe as can be inferred from the third-listed fit of Table IV. We use the results of the fourth-listed fit in Table IV together with the WBT interaction in our calculation of the effective $M$(GT) $M(\mathrm{GT})_{\mathrm{eff}}$ of Table III simply because they happen to reproduce the $M(\mathrm{GT})_{\mathrm{exp}}$ for $^{15}O(\beta^-)^{15}N$. This decay is strongly emphasized because the $0p_{1/2} \rightarrow 0p_{1/2}$ quenching factor of Eq. (2.29) is considerably more sensitive to $\delta_l$ and $\delta_p$ than the other two possibilities.

The $M$(GT) of the cross-shell transitions involve Gamow-Teller operators for both the $p$ and $1s0d$ shells. In principle, effective operators for both shells could be determined from a least-squares fit to the $M(\mathrm{GT})_{\mathrm{exp}}$ of Table III. However, in practice, there is not enough data of sufficient quality nor do the cross-shell interactions model the data with sufficient precision to allow such

TABLE III. Comparison of experimental and theoretical Gamow-Teller matrix elements for $A \leq 18$ decays. The notation th(free) and th(eff) refers to $M$(GT) values calculated using the "free nucleon" and "effective" Gamow-Teller operators, respectively.

| Reaction | $2J^\pi_k$,2T (i) (f) | $E_f$ (keV) | $\log f_A t$ | $M$(GT) (exp) | $M$(GT) th(free) | $M$(GT) th(eff) | Ref. or Remark |
|---|---|---|---|---|---|---|---|
| $^1$n($\beta^-$)$^1$H | $1^+$,1 $1^+_1$,1 | 0 | 3.024(1) | 3.100(7) | 3.096 | | [31],a |
| $^3$H($\beta^-$)$^3$He | $1^+$,1 $1^+_1$,1 | 0 | 3.058(1) | 2.929(5) | 3.096 | 3.096 | [32],a |
| $^6$He($\beta^-$)$^6$Li | $0^+$,2 $2^+_1$,0 | 0 | 2.910(1) | 2.748(4) | 3.031 | 2.630 | p1 |
| $^7$Be(EC)$^7$Li | $3^-$,1 $3^-_1$,1 | 0 | 3.300(1) | 2.882(4) | 3.187 | 2.747 | p2 |
| | $3^-$,1 $1^-_1$,1 | 478 | 3.534(3) | 2.678(8) | 2.901 | 2.493 | p3 |
| $^8$He($\beta^-$)$^8$Li | $0^+$,4 $2^+_1$,2 | 981 | 4.166(8) | 0.647(6) | 0.566 | 0.506 | p4 |
| | $0^+$,4 $2^+_2$,2 | ~3080 | b | 0.43(10) | 0.562 | 0.454 | [19] |
| | $0^+$,4 $2^+_3$,2 | ~5150 | b | 0.43(15) | 0.250 | 0.203 | [19] |
| | $0^+$,4 $2^+_4$,2 | ~9060 | b | 2.24(50) | 4.106 | 3.514 | [19] |
| $^8$Li($\beta^-$)$^8$Be | $4^+$,2 $4^+_1$,0 | 3120 | b | 0.364(25) | 0.507 | 0.456 | [20] |
| | $4^+$,2 $4^+_2$,0 | ~16800 | b | 6.0(25) | 4.318 | 3.694 | [20, 41] |
| $^8$B($\beta^-$)$^8$Be | $4^+$,2 $4^+_1$,0 | 3120 | b | 0.340(25) | 0.507 | 0.456 | [20] |
| | $4^+$,2 $4^+_2$,0 | ~16800 | b | 5.9(25) | 4.318 | 3.694 | [20, 41] |
| $^9$Li($\beta^-$)$^9$Be | $3^-$,3 $3^-_1$,1 | 0 | 5.310(24) | 0.347(10) | 0.573 | 0.500 | p5,a |
| | $3^-$,3 $5^-_1$,1 | 2429 | 5.064(54) | 0.425(24) | 0.519 | 0.478 | p6,a |
| | $3^-$,3 $1^-_1$,1 | 2780 | b | 0.21(5) | 0.394 | 0.333 | a |
| | $3^-$,3 $5^-_2$,1 | 7940 | b | 0.44(8) | 0.450 | 0.350 | a |
| | $3^-$,3 d,1 | 11283 | b | 2.40(40) | d | d | a,d |
| | $3^-$,3 d,1 | 11810 | b | 5.98(64) | d | d | a,d |
| $^9$C($\beta^+$)$^9$B | $3^-$,3 $3^-_1$,1 | 0 | 5.274(72) | 0.361(30) | 0.573 | 0.500 | [30, 43] |
| | $3^-$,3 $5^-_1$,1 | 2361 | 5.48(15) | 0.287(51) | 0.519 | 0.478 | [43] |
| | $3^-$,3 $1^-_1$,1 | ~2800 | 5.62(22) | 0.246(56) | 0.394 | 0.333 | [43] |
| $^{10}$C($\beta^+$)$^{10}$B | $0^+$,2 $2^+_1$,0 | 718 | 3.048(1) | 2.344(2) | 2.721 | 2.284 | p7 |
| | $0^+$,2 $2^+_2$,0 | 2154 | 3.661(3) | <1.14 | 0.891 | 0.783 | |
| $^{11}$Li($\beta^-$)$^{11}$Be | $3^-$,5 $1^-_1$,3 | 320 | 5.585(35) | 0.256(11) | 0.684 | 0.553 | |
| $^{11}$Be($\beta^-$)$^{11}$B | $1^+$,3 $1^+_1$,1 | 6792 | 5.937(30) | 0.119(4) | 0.351 | 0.292 | c1 |
| | $1^+$,3 $3^+_1$,1 | 7978 | 5.575(33) | 0.181(7) | 0.197 | 0.135 | c2 |
| | $1^+$,3 $3^+_2$,1 | 9876 | 4.231(57) | 0.838(55) | 0.259 | 0.213 | c3,a |
| $^{11}$C($\beta^+$)$^{11}$B | $3^-$,1 $3^-_1$,1 | 0 | 3.598(2) | 1.480(9) | 2.084 | 1.783 | p8 |
| $^{12}$Be($\beta^-$)$^{12}$B | $0^+$,4 $2^+_1$,2 | 0 | 3.794(45) | 0.943(18) | 1.689 | 1.395 | p9,a |
| | $0^+$,4 $2^+_2$,2 | 5000 | >4.66(18) | <0.336(75) | 0.095 | 0.047 | [34],a |
| $^{12}$B($\beta^-$)$^{12}$C | $2^+$,2 $0^+_1$,0 | 0 | 4.071(2) | 1.258(7) | 1.558 | 1.284 | p10 |
| | $2^+$,2 $4^+_1$,0 | 4439 | 5.114(14) | 0.379(6) | 0.434 | 0.400 | p11 |
| | $2^+$,2 $0^+_2$,0 | 7654 | 4.137(87) | 1.17(12) | e | e | a,e |
| | $2^+$,2 $0^+_3$,0 | 10300 | 4.19(22) | 0.43(6) | 2.135 | 1.760 | a |
| $^{12}$N($\beta^+$)$^{12}$C | $2^+$,2 $0^+_1$,0 | 0 | 4.118(3) | 1.184(4) | 1.558 | 1.284 | |
| | $2^+$,2 $4^+_1$,0 | 4439 | 5.151(7) | 0.361(3) | 0.434 | 0.400 | |
| | $2^+$,2 $0^+_2$,0 | 7654 | 4.343(64) | 0.914(68) | e | e | a,e |
| | $2^+$,2 $0^+_3$,0 | 10300 | 4.37(17) | 0.56(10) | 2.135 | 1.760 | a |
| | $2^+$,2 $2^+_1$,0 | 12710 | 3.52(17) | 2.37(46) | 2.071 | 1.756 | |
| | $2^+$,2 $2^+_1$,2 | 15110 | 3.30(15) | 1.78(88) | 0.409 | 0.322 | |
| $^{13}$B($\beta^-$)$^{13}$C | $3^-$,3 $1^-_1$,1 | 0 | 4.034(6) | 1.506(10) | 1.989 | 1.626 | p12 |
| | $3^-$,3 $3^-_1$,1 | 3685 | 4.452(46) | 0.931(49) | 0.943 | 0.797 | p13 |
| | $3^-$,3 $5^-_1$,1 | 7547 | 5.331(93) | 0.339(36) | 0.294 | 0.306 | |
| | $3^-$,3 $1^-_2$,1 | 8860 | 4.595(82) | 0.787(74) | 1.307 | 1.041 | |
| | $3^-$,3 $3^-_2$,1 | 9897 | 4.95(14) | 0.524(83) | e | e | e |
| $^{13}$N($\beta^+$)$^{13}$C | $1^-$,1 $1^-_1$,1 | 0 | 3.671(2) | 0.788(8) | 0.891 | 0.778 | p14 |
| $^{13}$O($\beta^+$)$^{13}$N | $3^-$,3 $1^-_1$,1 | 0 | 4.080(11) | 1.429(18) | 1.989 | 1.626 | |
| | $3^-$,3 $3^-_1$,1 | 3502 | 4.547(89) | 0.835(85) | 0.943 | 0.797 | |
| | $3^-$,3 $5^-_1$,1 | 7376 | 5.56(22) | 0.259(65) | 0.294 | 0.306 | |
| | $3^-$,3 $1^-_2$,1 | 8918 | 4.66(10) | 0.729(84) | 1.307 | 1.041 | |
| | $3^-$,3 $3^-_2$,1 | 9476 | 5.09(11) | 0.446(56) | e | e | e |
| | $3^-$,3 $5^-$,1 | 10360 | 5.73(22) | 0.214(54) | f | f | f |
| $^{14}$B($\beta^-$)$^{14}$C | $4^-$,4 $2^-_1$,2 | 6094 | 4.124(56) | 1.519(97) | 1.613 | 1.317 | c4 |
| | $4^-$,4 $6^-_1$,2 | 6728 | 5.01(19) | 0.55(12) | 0.529 | 0.419 | c5 |
| | $4^-$,4 $4^-_1$,2 | 7341 | 4.804(27) | 0.694(22) | 0.652 | 0.529 | c6 |
| $^{14}$C($\beta^-$)$^{14}$N | $0^+$,2 $2^+_1$,0 | 0 | 9.049(3) | 0.002(0) | 0.167 | 0.183 | |

TABLE III. (*Continued*).

| Reaction | $2J^\pi_k,2T$ (i) (f) | $E_f$ (keV) | $\log f_A t$ | $M(GT)$ (exp) | $M(GT)$ th(free) | $M(GT)$ th(eff) | Ref. or Remark |
|---|---|---|---|---|---|---|---|
| $^{14}O(\beta^+)^{14}N$ | $0^+2,\ 2^+_1,0$ | 0 | 7.284(7) | 0.018(0) | 0.167 | 0.183 | |
| | $0^+2,\ 2^+_2,0$ | 3948 | 3.138(16) | 2.119(39) | 2.665 | 2.189 | p15 |
| $^{15}C(\beta^-)^{15}N$ | $1^+,3\ 1^+_1,1$ | 5299 | 4.114(6) | 0.972(6) | 1.206 | 0.990 | c7 |
| | $1^+,3\ 3^+_1,1$ | 7301 | 6.897(47) | 0.039(2) | 0.129 | 0.056 | c8 |
| | $1^+,3\ 1^+_2,1$ | 8313 | 5.185(53) | 0.283(17) | 0.098 | 0.116 | c9 |
| | $1^+,3\ 3^+_2,1$ | 8571 | 5.337(67) | 0.238(18) | 0.019 | 0.034 | c10 |
| | $1^+,3\ 1^+_3,1$ | 9050 | 4.049(38) | 1.047(46) | 2.397 | 1.966 | c11 |
| $^{15}O(\beta^+)^{15}N$ | $1^-,1\ 1^-_1,1$ | 0 | 3.644(1) | 0.889(5) | 1.032 | 0.889 | p16 |
| $^{16}C(\beta^-)^{16}N$ | $0^+,4\ 2^+_1,2$ | 3353 | 3.556(11) | 1.306(16) | 1.461 | 1.140 | c12 |
| | $0^+,4\ 2^+_2,2$ | 4320 | 3.832(48) | 0.952(53) | 0.664 | 0.559 | c13 |
| $^{16}N(\beta^-)^{16}O$ | $4^-,2\ 6^-_1,0$ | 6130 | 4.486(5) | 1.001(5) | 0.949 | 0.820 | c14 |
| | $4^-,2\ 2^-_1,0$ | 7117 | 5.115(36) | 0.485(20) | 0.327 | 0.271 | c15 |
| | $4^-,2\ 4^-_1,0$ | 8872 | 4.355(29) | 1.165(39) | 2.001 | 1.643 | c16 |
| | $4^-,2\ 2^-_2,0$ | 9585 | b | ~0.145 | g | g | g |
| $^{16}N^*(\beta^-)^{16}O$ | $0^-,2\ 2^-_1,0$ | 7117 | 4.254(98) | 0.585(66) | 1.126 | 0.943 | [38],a |
| $^{17}N(\beta^-)^{17}O$ | $1^-,3\ 1^-_1,1$ | 3055 | 7.082(77) | 0.032(3) | 0.251 | 0.159 | c17 |
| | $1^-,3\ 3^-_1,1$ | 4552 | 4.419(17) | 0.684(13) | 2.038 | 1.634 | c18,a |
| | $1^-,3\ 3^-_2,1$ | 5378 | 3.864(15) | 1.296(22) | 0.563 | 0.439 | c18,a |
| | $1^-,3\ 1^-_2,1$ | 5939 | 4.367(33) | 0.726(28) | 0.263 | 0.201 | c19 |
| $^{17}Ne(\beta^+)^{17}F$ | $1^-,3\ 1^-_1,1$ | 3104 | 6.453(64) | 0.066(5) | 0.251 | 0.159 | c17 |
| | $1^-,3\ 3^-_1,1$ | 4640 | 4.601(23) | 0.569(9) | 2.038 | 1.634 | c18,a |
| | $1^-,3\ 3^-_2,1$ | 5488 | 3.867(15) | 1.369(22) | 0.563 | 0.439 | c18,a |
| | $1^-,3\ 1^-_2,1$ | 6037 | 4.426(17) | 0.588(13) | 0.263 | 0.201 | |
| | $1^-,3\ 1^-_3,1$ | 8075 | 3.970(21) | 1.196(79) | 2.205 | 1.777 | |
| | $1^-,3\ 1^-_4,1$ | 8200 | 3.970(21) | 0.580(58) | 0.289 | 0.247 | |
| | $1^-,3\ 1^-_5,1$ | 8436 | 3.853(27) | 1.819(91) | 0.455 | 0.378 | |
| | $1^-,3\ 1^-_1,3$ | 11193 | >3.284 | <1.087 | 0.632 | 0.537 | a |
| $^{18}C(\beta^-)^{18}N$ | $0^+,6\ 4^+_1,4$ | 1735 | >4.945 | <0.264 | | | a |
| | $0^+,6\ 2^+_1,4$ | 2614 | 4.103(76) | 0.696(61) | 0.838 | 0.651 | c20,a |
| $^{18}N(\beta^-)^{18}O$ | $2^-,4\ 2^-_1,2$ | 4456 | 5.164(10) | 0.355(4) | 0.311 | 0.258 | c21,a |
| | $2^-,4\ 4^-_1,2$ | 5530 | 6.161(57) | 0.112(8) | 0.106 | 0.101 | c22,a |
| | $2^-,4\ 2^-_2,2$ | 6198 | 6.337(63) | 0.091(7) | 0.391 | 0.333 | c23,a |
| | $2^-,4\ 4^-_2,2$ | 6351 | 6.100(60) | 0.120(9) | 0.009 | 0.045 | c24,a |
| | $2^-,4\ 0^-_1,2$ | 6880 | 5.124(26) | 0.373(12) | 0.387 | 0.324 | c25,a |
| | $2^-,4\ 2^-_3,2$ | 7619 | 5.178(34) | 0.350(14) | 0.100 | 0.086 | c26,a |
| | $2^-,4\ 4^-_3,2$ | 7771 | 5.320(45) | 0.298(16) | 0.207 | 0.187 | c27,a |
| | $2^-,4\ 2^-_4,2$ | 8039 | 5.616(50) | 0.211(12) | 0.176 | 0.168 | c28,a |

[a] See the discussion in Sec VI B.

[b] For decay to an unbound level, $\log f_A t$ is not well defined. An estimate is $6139/B(\mathrm{GT})$.

[d] The $\beta^-$ decay is allowed so that $2J^\pi = 1^-, 3^-$, or $5^-$. This broad structure could very well be a composite due to two or more overlapping resonances.

[e] This is predominantly a $2\hbar\omega$ and/or $4\hbar\omega$ state.

[f] The configuration of this state is unknown.

[g] This is predominantly a $3\hbar\omega$ state. Note that the $b_r$ and $\log f_A$ values of Table II result in a $M(\mathrm{GT})_{\mathrm{exp}}$ value about $\frac{1}{2}$ of the value of Table III, thus implying considerable interference effects in the $R$-matrix analysis of Ref. [24].

a procedure. However, to illustrate the consistency of the data with the effective operators obtained here for the $0p$ shell and previously [8] for the $1s0d$ shell, a least-squares fit was made to 44 $R(\mathrm{GT})$ with $\delta_s$ for the $0p$ shell as a variable and the other six effective operators fixed at the *sd* values. The 44 datum consisted of the 16 $0p$ shell $R(\mathrm{GT})$ used in the fits already described and 28 of the strongest and most reliable cross-shell transitions — labeled c# (# = 1–28) in Table III. The result of the fit is $\delta_s = -0.189(23)$ in satisfactory agreement with the results of Table IV. In this fit it was found that an assignment of $\Delta_{\mathrm{th}} = 0.053$ to the 28 cross-shell datum combined with $\Delta_{\mathrm{th}} = 0.038$ — determined in the $0p$-shell fit — for the 16 $0p$-shell datum gives $\chi^2 = 1.0$ for the 44 combined datum. That is, on the average, the cross-shell matrix elements are determined with an absolute error ~40% greater than the $0p$ shell $R(\mathrm{GT})$. Again, to assess the role of the interaction in this determination of the $0p$-

TABLE IV. Least squares fits to 16 $0p$ shell $M(\mathrm{GT})$.

| Fixed | WBT interaction Variable | $\Delta_{\mathrm{th}}$ | (8–16)TBME interaction Variable | $\Delta_{\mathrm{th}}$ |
|---|---|---|---|---|
| none | $\delta_s = -0.1847\pm0.0256$<br>$\delta_l = -0.0027\pm0.0193$<br>$\delta_p = 0.0075\pm0.0245$ | 0.041 | $\delta_s = -0.1708\pm0.0240$<br>$\delta_l = 0.0060\pm0.0208$<br>$\delta_p = -0.0013\pm0.0239$ | 0.038 |
| $\delta_l, \delta_p = 0.0$ | $\delta_s = -0.1821\pm0.0234$ | 0.039 | $\delta_s = -0.1727\pm0.0221$ | 0.036 |
| $\delta_l, \delta_p$ at Ref. [26] values | $\delta_s = -0.1907\pm0.0255$ | 0.042 | $\delta_s = -0.1776\pm0.0255$ | 0.042 |
| $\delta_l, \delta_p$ at sd values | $\delta_s = -0.1852\pm0.0230$ | 0.038 | $\delta_s = -0.1749\pm0.0221$ | 0.036 |

shell quenching factor, the procedure was repeated with the MK3 [53] version (also operating in the first four major shells) of the Millener-Kurath interaction [54]. The result of a least-squares fit to the same 44 $R(\mathrm{GT})$ was $\delta_s = -0.194(23)$. In this fit a $\Delta_{\mathrm{th}}$ of 0.084 for the 28 cross shell $R(\mathrm{GT})$ combined with a $\Delta_{\mathrm{th}}$ of 0.036 for the $0p$ shell to give a $\chi^2$ of 1.0; i.e., in this application, the MK3 interaction has a theoretical error about 60% larger than that of the WBT interaction.

## VI. COMPARISON OF EXPERIMENT AND THEORY

### A. General

We display omnibus comparisons between theory and experiment via plots in which the $y$ axis represents $R(\mathrm{GT})_{\mathrm{exp}}$ and the $x$ axis $R(\mathrm{GT})_{\mathrm{free}}$ or $R(\mathrm{GT})_{\mathrm{eff}}$. Thus perfect agreement is represented by a diagonal starting at $(x,y)=$ (0,0) and passing through (1,1). The first of these (Fig. 1) shows $R(\mathrm{GT})_{\mathrm{exp}}$ vs $R(\mathrm{GT})_{\mathrm{free}}$ for the 16 $0p$-shell decays discussed in the preceding section and labeled by p# in Table III. The fact that — on the average — these decays are quenched is clearly shown. The result of a one-parameter straight-line fit is indicated by the "best" fit passing through $(x,y)=$(1,0.82). In Fig. 2 we display the same 16 decays but with $R(\mathrm{GT})_{\mathrm{eff}}$ rather than $R(\mathrm{GT})_{\mathrm{free}}$ on the $y$ axis. Now the "best" fit is the diagonal and there is less scatter about it. In Fig. 3 we display a comparison of the (8–16)TBME and WBT results for these 16 points. The good agreement is evident.

We now consider the 28 cross-shell decays discussed in the preceding section and labeled in Table III. A comparison of $R(\mathrm{GT})_{\mathrm{exp}}$ and $R(\mathrm{GT})_{\mathrm{eff}}$ is given in Fig. 4.

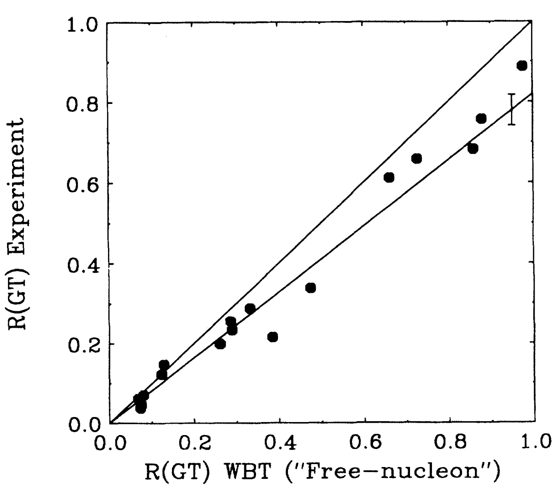


FIG. 1. Comparison of the experimental and "free nucleon" values of $R(\mathrm{GT})$ for 16 $0p$-shell decays. A diagonal line passing through the $(x,y)$ point (1,1) represents perfect agreement. The "best fit" line through the 16 points passes through (1,0.82). The error bar is the theoretical uncertainty $\Delta_{\mathrm{th}}$ which is assumed independent of $R(\mathrm{GT})$ and is discussed in the text.

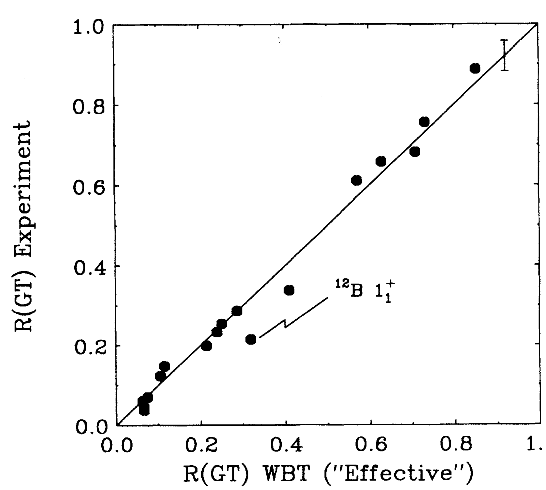


FIG. 2. Comparison of the experimental and "effective" values of $R(\mathrm{GT})$ for 16 $0p$-shell decays. A diagonal line passing through the $(x,y)$ point (1,1) represents perfect agreement. The error bar is the theoretical uncertainty $\Delta_{\mathrm{th}}$ which is assumed independent of $R(\mathrm{GT})$ and is discussed in the text.

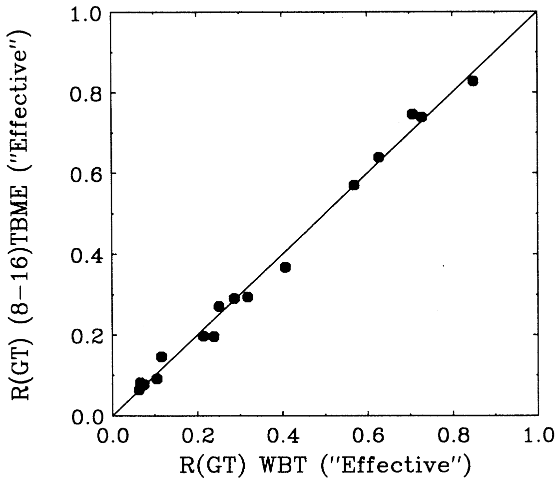


FIG. 3. Comparison of the WBT and (8–16) TBME "effective" values of $R$(GT). A diagonal line passing through the $(x, y)$ point (1,1) represents perfect agreement. The figure is a visual display of the good agreement of the two predictions.

The theoretical uncertainty of $\Delta_{\rm th} = 0.053$ is indicated. Recall that this is an absolute uncertainty assigned to all datum. Only 10 of the 28 decays have $R(\mathrm{GT})_{\rm eff}$ values considerably larger than this uncertainty. This highlights the difficulty of obtaining meaningful predictions for the majority of the cross-shell decays of Table III. However, as discussed earlier, the uncertainty $\Delta_{\rm th}$ obtained for the decays of Fig. 4 is competitive with that obtained for the significantly stronger $0p$-shell decays of Fig. 2. Our final comparison is for the MK3 and WBT predictions and is shown in Fig. 5. This figure displays generally good agreement within the uncertainties of $\Delta_{\rm th}(\mathrm{MK3}) = 0.084$ and $\Delta_{\rm th}(\mathrm{WBT}) = 0.053$. Two noticeable exceptions are

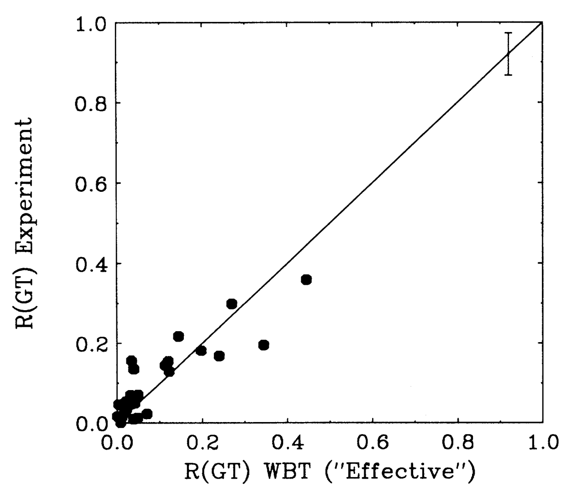


FIG. 4. Comparison of 28 cross-shell experimental and "effective" values of $R$(GT). A diagonal line passing through the $(x, y)$ point (1,1) represents perfect agreement. The error bar is the theoretical uncertainty $\Delta_{\rm th}$ which is assumed independent of $R$(GT) and is discussed in the text.

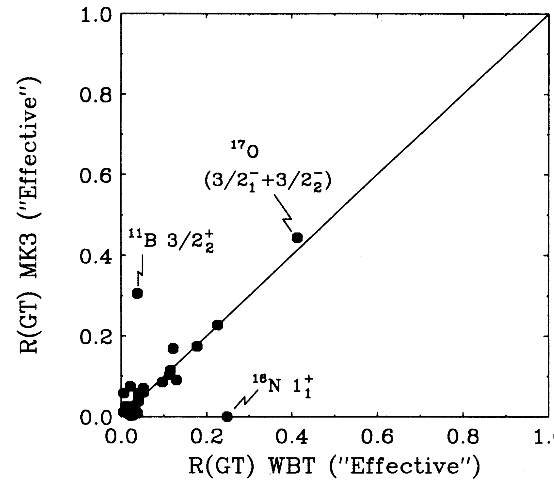


FIG. 5. Comparison of the WBT and MK3 "effective" values of $R$(GT) for 28 cross-shell decays. A diagonal line passing through the $(x, y)$ point (1,1) represents perfect agreement. The figure is a visual display of the generally good agreement of the two predictions and highlights the two cases of poor agreement. These two cases and the one labeled $^{17}$O are discussed in the text.

the two decays of $^{11}$Be and $^{16}$C. These are singled out for discussion in Sec. VI B 4 below.

### B. Discussion of some specific decays

#### *1. The neutron decay*

Our evaluation follows that of Wilkinson [31]. By choosing $K/(g_V)^2$ and $\lambda$ from a global evaluation [14] we ensure that $M(\mathrm{GT})_{\rm exp}$ and $M(\mathrm{GT})_{\rm free}$ will agree within the uncertainties. For comparison of these parameters evaluated from pure neutron data and from neutron plus $0^+ \to 0^+$ Fermi decays see Refs. [14] and [2].

#### *2. The triton decay*

Our evaluation follows that of Simpson [32] with small changes due to changes in $t_{1/2}$ and $Q$. The ratio $M(\mathrm{GT})_{\rm exp}/M(\mathrm{GT})_{\rm free}$ is $0.946 \pm 0.002$ so that the $0s_{1/2}$ correction factor $\delta_{0s}$ is $0.054 \pm 0.002$. As remarked by Simpson [32], there are a large number of calculations of this reduction and a consensus is that they get it about right — certainly within the uncertainties in the calculations. An example is the recent result of Towner [26] of $\delta_{0s} = 0.076$.

#### *3. $^8He(\beta^-)^8Li$, $^8Li(\beta^-)^8Be$, and $^8B(\beta^+)^8Be$*

For these mass-eight decays, more detailed analysis via the $R$-matrix theory have been made [18, 20, 19, 21] than for any other decays. There are two points of interest to mention here. First, we have assumed all $M$(GT) to be positive. Then the relative signs of the reduced widths amplitudes in the $R$-matrix analysis are definite and can be compared to experiment. This is discussed in Ref. [20] and utilized in Ref. [21]. Second, we have taken the values of $M(\mathrm{GT})_{\mathrm{exp}}$ for $^8$Li and $^8$B decays to the $J_k^\pi, T = 2_1^+, 0$ and $2_2^+, 0$ levels from Ref. [20]. These values are 0.36 and 0.34 for the lower level and 6.0 and 5.9 for the higher level. The values of Ref. [21] are 0.23 and 0.24 and 4.0 and 3.7, respectively. The difference is mainly due to different constraints on the $R$-matrix parameters. Associated with the Barker treatment [18, 21] are "intruder" $0^+$ and $2^+$ states at ~7 and 9 MeV, respectively. In the treatment of Ref. [20] these states appear somewhere above 20 MeV. Thus, which treatment is preferred will ultimately depend on where these — as yet unknown — states are found. In the meantime the difference between these values can be taken as a measure of the uncertainty in this particular analysis.

### *4. $^9$Li($\beta^-$)$^9$Be*

This is a complicated decay with broad, overlapping, unbound resonances as final states. An interpretation of the experimental data — $\beta^-$-delayed neutron and $\alpha$ spectra — can use some guidance from shell-model predictions. In Fig. 6 we compare the known spectrum of $^9$Be $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ states to the $P$(6-16)$T$ predictions. Also shown are the experimental $M$(GT) and the predicted $M(\mathrm{GT})_{\mathrm{eff}}$. It is on the basis of this comparison that we have *assumed* $\frac{5}{2}{}_2^-$ for the anomaly at 7940 keV. The predictions indicate a missing $\frac{3}{2}^-$ state in the 5–7 MeV range. In the $(0s)^4(0p)^5$ model space, the $^9$Be states at ~10–12 MeV have the same symmetry as the $^9$Li ground state and thus contains the bulk of the Gamow-Teller strength [54]. There seems to be ample strength seen experimentally in this energy region but it is not at all clear how this strength should be apportioned amongst the theoretical possibilities. The general agreement shown in Fig. 6 is impressive, however it should not be taken too seriously. It is expected that $2\hbar\omega$ states will commence in the 6–8 MeV region. These may well cause a considerable shift in the distribution of Gamow-Teller strength.

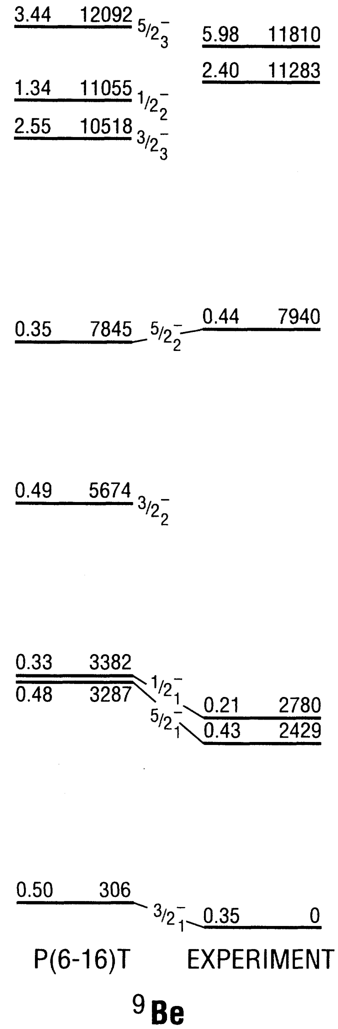


FIG. 6. Comparison of the reported Gamow-Teller decay [22] $^9$Li($\beta^-$)$^9$Be to the predictions of the WBT interaction For each level $E_x$ (in keV) and $M(\mathrm{GT})_{\mathrm{exp}}$ or $M(\mathrm{G})_{\mathrm{eff}}$ are given as well as the spin-parity and index k.

### *5. $^{11}$Be($\beta^-$)$^{11}$B and $^{16}$C($\beta^-$)$^{16}$N*

The decays to the $^{11}$B $\frac{3}{2}{}_2^+$ and $^{16}$N $1_1^+$ states are considered together here because these are the two final states for which there is a large difference between the WBT and MK3 calculations as indicated in Fig. 5. For the $^{11}$Be $\rightarrow$ $^{11}$B($\frac{3}{2}{}_2^+$) decay, $M(\mathrm{GT})_{\mathrm{eff}}$ is sensitive to the $0p$-shell part of the interaction and reasonable changes — relative to the accuracy with which the $\langle 2j_1 2j_2|V|2j_3 2j_4\rangle$ (TBME) are determined — can bring it into agreement with experiment. For instance, if the MK3 [$\equiv$ (8–16)TBME] value of the off-diagonal $\langle 31|V|33\rangle$ of 3.548 MeV is substituted for the WBT value of 2.442 MeV (determined in the least-squares fits of Ref. [28] with an accuracy of $\sim \pm 0.65$), then $M(\mathrm{GT})_{\mathrm{eff}}$ for the $^{11}$Be $\rightarrow$ $^{11}$B($\frac{3}{2}{}_2^+$) decay changes from 0.213 to 1.033 as compared to $M(\mathrm{GT})_{\mathrm{exp}}$ = 0.838. Reasonable changes in the MK3 interaction could also bring its prediction for this decay into agreement with experiment.

For the $^{16}$C $\rightarrow$ $^{16}$N($1_1^+$) decay, $M(\mathrm{GT})_{\mathrm{eff}}$ is predicted rather well by the WBT interaction and extremely poorly by the MK3 interaction. Unlike the $^{11}$Be case, it was found that this difference appears to be a complex function of both the $0p$-shell and cross-shell interactions; we have found no simple explanation for it.

This discussion of decays to the $^{11}$B $\frac{3}{2}{}_2^+$ and $^{16}$N $1_1^+$ states brings up the obvious point that it would be advantagous to include Gamow-Teller matrix elements in the least-squares fits used to determine effective interactions. Such a project — similar in spirit to the $0p$-shell fit to energy levels and electromagnetic properties of van Hees, Wolters, and Glaudemans [46] — is being contemplated.

### 6. $^{12}Be(\beta^-)^{12}B$

Experimentally, the branching ratio for decays to all bound states is 99.1 ± 0.4 % [34]. We assume decays to the $2_1^+$ and $0_1^+$ states are negligible because they are second-forbidden and isospin-forbidden decays, respectively [48]. The first-forbidden decays to the $2_1^-$ and $1_1^-$ states — the only other levels below the neutron threshold — were calculated with the WBT interaction with effective operators [49] and were found to be quite weak with branching ratios of 0.015 and 0.012 %, respectively. The unusual weakness of these decays is due to poor overlap of the $^{12}$B $1_1^-$ and $2_1^-$ states with the first-forbidden $\beta^-$ operator acting on the $^{12}$Be $0_1^+$ state (rather than to the relative kinematics of allowed and forbidden decays). The decay to the $1_1^+$ state gives the most deviate point in the comparison of Fig. 2. One possible reason for a weaker decay than predicted is that the $^{12}$Be ground state has more "intruder" admixtures of the $\geq 2\hbar\omega$ type than the $1_1^+$ state of $^{12}$B as has been previously discussed [50, 51]. This, then, would be an effect similar to the speculated deviation of the $^{11}$Li ground state from a $0\hbar\omega$ character. Note that $M(\mathrm{GT})_{\mathrm{exp}}$ for the $^{11}\mathrm{Li} \rightarrow {}^{11}\mathrm{Be}$ decay shown in Table III is also considerably less than $M(\mathrm{GT})_{\mathrm{eff}}$.

### 7. $^{14}C(\beta^-)^{14}N$

A common query of any $0p$-shell interaction is "Does it reproduce the famous vanishing [$R(\mathrm{GT}) = 6\times10^{-4}$] of the strength for the decay of $^{14}$C to the $^{14}$N ground state?". For the WBT interaction we have a predicted $M(\mathrm{GT})_{\mathrm{eff}}$ = 0.183 [Note, because of the very strong interference effects which cause this small value, $M(\mathrm{GT})_{\mathrm{eff}}$ is actually larger than $M(\mathrm{GT})_{\mathrm{free}}$.] This corresponds to $R(\mathrm{GT})_{\mathrm{eff}}$ = 0.059 which is close enough to the $\Delta_{\mathrm{th}}$ (= 0.039) obtained in the fits so that we can claim a successful passing of this test. For comparison, the (8–16)TBME interaction gives $R(\mathrm{GT})_{\mathrm{eff}} = 0.055$ for this transition.

### 8. $^{12}B(\beta^-)^{12}C$ and $^{12}N(\beta^+)^{12}C$

The deformed $^{12}$C $0_2^+$ state at 7654 keV is presumably chiefly a $4\hbar\omega$ state similar to the $^{16}$O $0_2^+$ state at 6049 keV. The large $M(\mathrm{GT})$ for the decay to this state is a challenge for the shell model which has as yet not been addressed in a quantitative way. The significant overestimation of the $M(\mathrm{GT})$ for decay to the $^{12}$C $0_3^+$ state at 10 300 keV suggests that the low-lying $0^+$ model states of $0\hbar\omega$, $2\hbar\omega$, and $4\hbar\omega$ configurations have intermixed to a considerable degree. It would be desirable to have more accurate experimental branching ratios for the decays to the $(J_k^\pi, T) = (1_1^+, 0)$ 12 710- and $(1_1^+, 1)$ 15 110-keV levels since we expect these to be relatively pure $0\hbar\omega$ states which should provide good tests of the $0p$-shell interaction.

### 9. $^{16}N^*(\beta^-)^{16}O$

Evidence for a branch of (9 ± 2)% for the $^{16}\mathrm{N}(0^-) \rightarrow {}^{16}\mathrm{O}(1_1^-)$ transition is given by Champagne *et al.* [38]. The evidence is not compelling, nevertheless we have adopted this branch in order to give a point of reference for the calculation. As seen in Table III, we actually would predict a somewhat larger branch.

### 10. $^{17}N(\beta^-)^{17}O$

The $\frac{3}{2}_1^-$ and $\frac{3}{2}_2^-$ states of $^{17}$O are only separated by 84 keV in the WBT calculation and their wave functions appear to be confused. Experimentally the $\frac{3}{2}_1^-$ state has a relatively large $^{16}\mathrm{O} + n$ spectroscopic factor, $S_n^+(1p_{3/2})$ = 0.23 and the $\frac{3}{2}_2^-$ state has $S_n^+(1p_{3/2}) \approx 0.05$ [30] while the WBT interaction reverses these two factors. The same fault is shared by the MK3 interaction. This reversal appears to be seen in the Gamow-Teller strengths also as can be seen in Table III. However the sum of the $M(\mathrm{GT})$ for these two states is fairly well reproduced by both calculations. In order to illustrate this point graphically, this sum (actually the average of the sums for $^{17}$N and the mirror decays of $^{17}$Ne) was included in the least-squares fits resulting in the plots of Figs. 4 and 5. A calculation of these decays was made as a function of the $1p_{3/2}$ SPE and indeed the $S_n^+(1p_{3/2})$ and the $M(\mathrm{GT})$ could be simultaneously reproduced quite well with a lowering of the $1p_{3/2}$ single-particle energy by $\sim$ 1 MeV.

### 11. $^{18}C(\beta^-)^{18}N$

The low-lying energy levels of the WBT $(0{+}1)\hbar\omega$ spectrum of $^{18}$N are shown in Fig. 7 together with the available experimental information [30, 40]. Pravikoff *et al.* [40] reported $\beta^-$ branches of (72 ± 10)% and (9 ± 7)% to the states at 2614 and 1735 keV (see Table II). The first of these establishes the 2614-keV level as $1^+$. If the branch to the 1735-keV level is actually as large as 9% then the 1735-keV level almost certainly has $J^\pi = 1^+$ also. This situation is labeled ⓑ in Fig. 7. On the other hand, if the $\beta^-$ branch to the 1735-keV level is zero or close to zero — as is allowed to somewhat more than one standard deviation — then its $\gamma$-ray decay modes of nearly equal branches to the first three states (see Fig. 7) would suggest the $2^+$ alternative of those offered by the comparison shown in Fig. 7. (We reject as very unlikely the possibility that the 2614-keV level is $1_2^+$ and the $1_1^+$ level is unobserved in the $\beta^-$ decay.) This alternative is labeled ⓐ in Fig. 7. For definiteness we have chosen this latter alternative in Table III where the limits on $\log f_A t$ shown for the decay to the 1735-keV level correspond to one standard deviation in the branching ratio, i.e., to $b_r$ < 16%.

Can calculations of $\gamma$-ray branching ratios with the WBT interaction help to distinquish between the alternatives ⓐ and ⓑ? To a limited extent, the answer is yes. The $B(M1)$ calculated with a free-nucleon operator is strong [1.25 Weisskopf units (W.u.)] for a $1_1^+ \rightarrow 2_1^+$ transition and weak (0.11 W.u.) for a $1_2^+ \rightarrow 1_1^+$ transition. The former is more in keeping with the $\gamma$ branch of 26% (Fig. 7) for the 2614 → 1735 transition since this

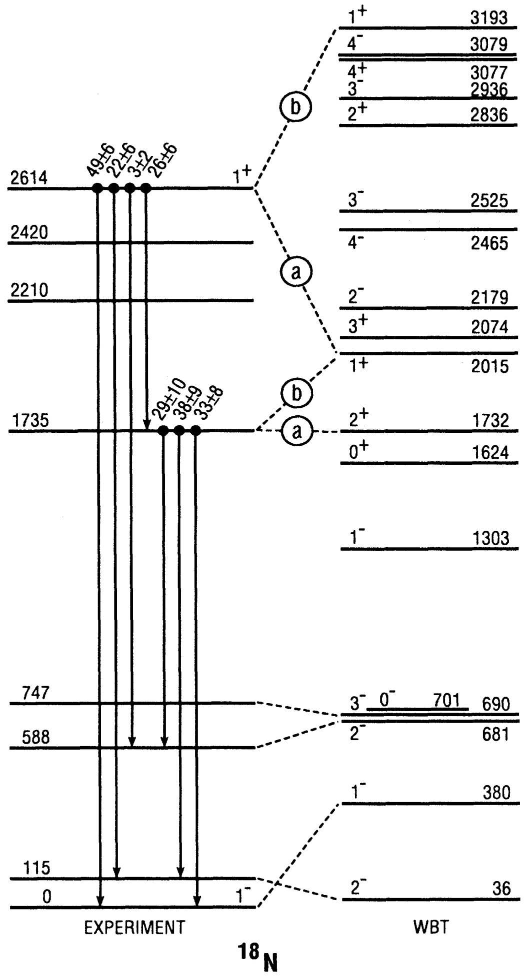


FIG. 7. Comparison of the (0+1)$\hbar\omega$ spectrum of $^{18}$N to the experimental energy-level data. The $\gamma$-ray branching ratios (from Ref. [40]) of the 1735- and 2614-keV levels are indicated. The energy-level comparison is absolute, i.e., the WBT prediction is that the $2_1^-$ state is 36 keV less bound that the $^{18}$N ground state (which however has $J^\pi = 1^-$). Our assumed identification of the experimental levels at 1735 and 2614 keV is indicated by the dashed lines labeled ⓐ. An alternate possibility is labeled ⓑ. A more accurate determination of the $\beta^-$ branching ratio to the 1735-keV level is needed to choose between these two alternatives which are discussed in the text.

would most likely need to be a strong $M1$ decay in order to compete with the higher-energy $E1$ transitions to the lower states. We do not consider these $E1$ transitions because they are notoriously difficult to calculate accurately and because we have not as yet tested the WBT interaction as to its predictive abilites for this observable.

### 12. $^{18}N(\beta^-)^{18}O$

An unknown experimental quantity in this decay is the fraction proceeding to unbound states in addition to the five listed in Table II. Using the WBT interaction, we calculate this fraction to be $P_n = 0.17$. The branching ratios for the 7619- and 8039-keV levels were derived by Zhao *et al.* [45] in a manner which is independent of this fraction. A multiplicative factor of 0.875 was ascertained and applied to the branching ratios listed in Ref. [30] for the other six listed states in order to incorporate this value of $P_n$.

## VII. SUMMARY OF THE EFFECTIVE OPERATOR

This study can be viewed as a continuation to $A = 1$ of the $A$ =17–39 Gamow-Teller study of Brown and Wildenthal [7, 8]. That study used the $W$ interaction appropriate to the $1s0d$ shell. The present study uses the WBT interaction which is an extension of the $W$ interaction appropriate to the $0p$ shell and to cross-shell $0p1s0d$ configurations as well. As discussed in Sec. V and illustrated in Fig. 2, the strong $0p$-shell decays are well reproduced with an effective operator which is essentially a simple quenching of the "free-nucleon" operator by $1 - \delta_s(p\text{-}p)(A/16)^{0.35}$. The least-squares fit gives small values for $\delta_l(p\text{-}p)$ and $\delta_p(p\text{-}p)$. These values are consistent with zero and with the values obtained for these two parameters in the $1s0d$ study [8]. With $\delta_l(p\text{-}p)$ and $\delta_p(p\text{-}p)$ fixed at the $1s0d$ values, we find $\delta_s(p\text{-}p)$ values of $-0.185 \pm 0.023$ from a fit to the $0p$-shell data and $-0.189 \pm 0.023$ from a consideration of 44 $0p1s0d$ datum. These values are not very different from the $\delta_s(s\text{-}s)$ and $\delta_s(d\text{-}d)$ values [8] of $-0.186 \pm 0.013$ and $-0.204 \pm 0.009$, respectively. In fact, a phenomenological quenching factor of

$$q_s \approx 1 - 0.19\left(\frac{A}{16}\right)^{0.35} \tag{7.1}$$

is a good approximation to both the least-squares fits and is thus applicable to the nuclei studied from $A =$ 3 to 40. We hasten to emphasize that this quenching factor is purely phenomenological. One should not at this juncture conclude anything from the $A$ dependence which, in any case, is poorly determined. To illustrate this latter point, the least-squares fit to the 16 $0p$-shell datum was repeated with $P = 0.0$ in Eq. (2.30) using the $1s0d$ values for $\delta_l(p\text{-}p)$ and $\delta_p(p\text{-}p)$. The result is $\delta_s(p\text{-}p) = -0.15\pm$ 0.02. In this fit a $\Delta_{\rm th}$ of 0.045 is obtained. This is not appreciably worse than the value of 0.039 obtained with $P = 0.35$. Perhaps the most interesting conclusion from Eq. (7.1) is that there appears to be no evidence for a "shell" effect at $A = 16$ from our analysis.

## ACKNOWLEDGMENTS

We are indebted to F. Ajzenberg-Selove whose nuclear-structure compilations were of immeasureable value to this study and to D. J. Millener for numerous consultations. Research was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016 with Brookhaven National Laboratory and Grant No. DE-FG02-88er40417 with Clark University, and in part by the National Science Foundation under Grant No. PHY-90-17077 with Michigan State University.

[1] D. H. Wilkinson, in *Ecole d'Ete de Physique Théorique*, Session XXX, edited by R. Balian, M. Rho, and G. Ripka (North-Holland, Amsterdam, 1978).
[2] J. C. Hardy, I. S. Towner, V. T. Koslowsky, E. Hagberg, and H. Schmeing, Nucl. Phys. **A509**, 429 (1990).
[3] W. E. Ormand and B. A. Brown, Phys. Rev. Lett. **62**, 866 (1989); **63**, 103 (1989).
[4] D. H. Wilkinson, Phys. Rev. C **7**, 930 (1973).
[5] D. H. Wilkinson, Nucl. Phys. **A209**, 470 (1973).
[6] D. H. Wilkinson, Nucl. Phys. **A225**, 365 (1974).
[7] B. A. Brown and B. H. Wildenthal, Phys. Rev. C **28**, 2397 (1983).
[8] B. A. Brown and B. H. Wildenthal, At. Data Nucl. Data Tables **33**, 347 (1985).
[9] S. Raman, C. A. Houser, T. A. Walkiewicz, and I. S. Towner, At. Data Nucl. Data Tables **21**, 567 (1978).
[10] H. Behrens and W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon, Oxford, 1982).
[11] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton Univ. Press, Princeton, NJ, 1960).
[12] D. H. Wilkinson and B. E. F. Macefield, Nucl. Phys. **A232**, 58 (1974).
[13] W. Bambynek, H. Behrens, M. H. Chen, B. Crasemann, M. L. Fitzpatrick, K. W. D. Ledingham, H. Genz, M. Mutterer, and R. L. Intemann, Rev. Mod. Phys. **49**, 77 (1977); **49**, 961(E) (1977).
[14] D. Dubbers, Nucl. Phys. **A527**, 239c (1991).
[15] D. H. Wilkinson, A. Gallmann, and D. E. Alburger, Phys. Rev. C **18**, 401 (1978).
[16] A. Sirlin and R. Zucchini, Phys. Rev. Lett. **57**, 1994 (1986).
[17] J. N. Bahcall, Phys. Rev. **129**, 2683 (1963); J. N. Bahcall, Rev. Mod. Phys. **50**, 881 (1978).
[18] F. C. Barker, Aust. J. Phys. **22**, 293 (1969).
[19] F. C. Barker and E. K. Warburton, Nucl. Phys. **A487**, 269 (1988).
[20] E. K. Warburton, Phys. Rev. C **33**, 303 (1986).
[21] F. C. Barker, Aust. J. Phys. **41**, 743 (1988); **42**, 25 (1989).
[22] G. Nyman *et al.*, Nucl. Phys. **A510**, 189 (1990).
[23] The analysis of Nyman *et al.* [22] is only an approximate $R$-matrix analysis; a more rigorous treatment would be desirable.
[24] F. C. Barker, Aust. J. Phys. **24**, 777 (1971).
[25] The unbound level in question here is the 9585-keV level of $^{16}$O. If we assume this level to be isolated then the results of an $R$-matrix analysis can be expressed simply in terms of an "effective" excitation energy of 9480 keV for this level, i.e., a shift of 105 keV. This procedure was followed in Table II. However note footnote g of Table III.
[26] I. S. Towner, Phys. Rep. **155**, 263 (1987).
[27] B. A. Brown, A. Etchegoyen, W. D. M. Rae, and N. S. Godwin, computer code OXBASH, 1984 (unpublished).
[28] E. K. Warburton and B. A. Brown, Phys. Rev. C **46**, 923 (1992).
[29] A. H. Wapstra and G. Audi, Nucl. Phy. **A432**, 1 (1985); midstream update, P. E. Haustein, private communication (1990).
[30] F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988); **A506**, 1 (1990); **A523**, 1 (1990); **A460**, 1 (1986); **A475**, 1 (1987).
[31] D. H. Wilkinson, Nucl. Phys. **A377**, 474 (1982).
[32] J. J. Simpson, Phys. Rev. C **35**, 752 (1987).
[33] K. E. Sale, T-F. Wang, R. N. Boyd, G. J. Mathews, D. W. Heikkinen, M. L. Roberts, M. S. Islam, and P. B. Corn, Phys. Rev. C **41**, 2418 (1990).
[34] P. L. Reeder, R. A. Warner, W. K. Hensley, D. J. Vieira, and J. M. Wouters, Phys. Rev. C **44**, 1435 (1991).
[35] P. H. Barker and G. D. Leonard, Phys. Rev. C **41**, 246 (1990).
[36] M. S. Curtin, L. H. Harwood, J. A. Nolen, B. Sherrill, Z. Q. Xie, and B. A. Brown, Phys. Rev. Lett. **56**, 34 (1986).
[37] K. Asahi, K. Matsuta, K. Takeyama, K. H. Tanaka, Y. Nojiri, and T. Minamisono, Phys. Rev. C **41**, 358 (1990).
[38] A. Champagne, G. Beaudoin, L. A. Hamel, H. Jeremie, and L. Lessard, Z. Phys. A **330**, 377 (1988).
[39] M. J. G. Borge *et al.*, Nucl. Phys. **A490**, 287 (1988).
[40] M. S. Pravikoff *et al.*, Nucl. Phys. **A528**, 225 (1991).
[41] The experimental branching ratio and $M$(GT) are given for the $T = 0$ part of the $T = 0$ and 1 isospin-mixed $J^{\pi} = 2^{+}$ states at 16.626 and 16.922 MeV in $^{8}$Be.
[42] M. Langevin *et al.*, Nucl. Phys. **A366**, 449 (1981).
[43] D. Mikolas *et al.*, Phys. Rev. C **37**, 766 (1988).
[44] J. C. Hardy, J. E. Esterl, R. G. Sextro, and J. Cerny, Phys. Rev. C **3**, 700 (1971).
[45] Z. Zhao *et al.*, Phys. Rev. C **39**, 1985 (1989).
[46] A. G. M. van Hees, A. A. Wolters, and P. W. M. Glaudemans, Nucl. Phys. **A476**, 61 (1988).
[47] C. Gaarde, J. S. Larson, M. N. Harakeh, S. Y. van der Werf, M. Igarashi, and A. Muller-Arnke, Nucl. Phys. **A334**, 248 (1980).
[48] S. Raman and N. B. Gove, Phys. Rev. C **7**, 1995 (1973).
[49] E. K. Warburton, Phys. Rev. C **42**, 2479 (1990).
[50] F. C. Barker, J. Phys. G **2**, L45 (1976).
[51] D. E. Alburger *et al.*, Phys. Rev. C **17**, 1525 (1978).
[52] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).
[53] E. K. Warburton and D. J. Millener, Phys. Rev. C **39**, 1120 (1989).
[54] D. J. Millener and D. Kurath, Nucl. Phys. **A255**, 315 (1975).