^{11}Li+p elastic scatterings in a four-body model with the eikonal approximation

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Elastic differential cross sections of ^{11}Li on a proton target recently observed at $E_{\rm lab}/A=62$ MeV are analyzed in a (^{9}Li+n+n)-p four-body model to take the halo structure of ^{11}Li into consideration. The eikonal and adiabatic approximations are employed to derive the optical potential that includes the breakup effect of halo neutrons to continuum states. It is shown that both the breakup of ^{11}Li and the exchange force of the n-p interaction lead to the reduced cross sections that are in satisfactory agreement with experiment with no adjustable parameters.

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Very recently, elastic differential cross sections of ^{11}Li and ^{9}Li on a proton target have for the first time been measured in the range of the center-of-mass angle $\theta_{\rm c.m.}$ of 26°–62° at the incident energy of about $E_{\rm lab}=60$ MeV/nucleon [1]. The result is quite impressive. Although the ^{9}Li+p scattering follows the systematics of other lighter Li isotopes, that is, the ^{6}Li+p and ^{7}Li+p scatterings, the ^{11}Li+p elastic scattering shows a remarkable reduction in the cross sections compared with those of the other isotopes. Reflecting its very small two-neutron separation energy of about 200 keV, ^{11}Li is well described with a three-body model of ^{9}Li+n+n [2,3], where two neutrons have a spatially extended density distribution forming the so-called neutron halo. When a loosely bound nucleus is involved in heavy-ion scatterings, the folding model is known to fail to obtain the real part of the optical potential [4], but instead the breakup process of the nucleus to the continuum becomes very important, as shown in the analysis of deuteron scatterings [5–7]. The reduction in the cross sections is apparently related to the breakup process of ^{11}Li. We have recently presented a four-body treatment [8,9] for the ^{11}Li+target reaction at intermediate energies with the use of the eikonal and adiabatic approximations in order to elucidate the halo-neutron wave function. The approximations are found to work quite successfully for describing various reaction mechanisms including the breakup process and in addition have the advantage that they easily lead to the construction of the optical potential which includes the breakup effect. We have predicted that the elastic cross section of ^{11}Li-^{12}C is much smaller than that of ^{9}Li-^{12}C. Since this provides us with the first opportunity to test the usefulness of the four-body model, we will apply the model to the ^{11}Li+p elastic scattering in this Brief Report and show that the anomalous behavior of the cross sections can basically be understood by considering the breakup of ^{11}Li *and* the exchange force of the p-n interaction.

Because of its unusual structure, the elastic scattering of ^{11}Li has attracted considerable theoretical attention [10–12]. A study [11] of the ^{11}Li+p elastic scattering has already been undertaken in the Glauber model [13] and compared to the ^{9}Li+p and ^{12}C+p scatterings. Our approach is akin to Refs. [11,12], but exploits the halo-neutron wave function as well as the ^{11}Li-target and neutron-target optical potentials. In addition, the four-body approach enables us to calculate the phase shift function without recourse to the optical limit approximation which is not very accurate for the ^{11}Li reactions as shown in Ref. [14].

Following the formalism presented in Refs. [8,9], the ^{11}Li+p elastic scattering is described in the four-body model as

$$\left\{\frac{\mathbf{P}^2}{2\mu}+h_0+U(\mathbf{R},\mathbf{r}_1,\mathbf{r}_2)\right\}\Psi(\mathbf{R},\mathbf{r}_1,\mathbf{r}_2)=E\Psi(\mathbf{R},\mathbf{r}_1,\mathbf{r}_2)\ , \quad (1)$$

$$U(\mathbf{R},\mathbf{r}_1,\mathbf{r}_2)=U_{^9\mathrm{Li}}(\mathbf{R})+U_n(\mathbf{R}+\mathbf{r}_1)+U_n(\mathbf{R}+\mathbf{r}_2)\ . \quad (2)$$

Here $\mathbf{R}$ and $\mathbf{P}$ denote the relative coordinate and momentum between ^{11}Li and the proton, respectively, and $\mathbf{r}_i$ is the radius vector of ith neutron with respect to ^{9}Li. μ is the reduced mass of the relative motion. h_0 is the internal Hamiltonian of ^{11}Li as a three-body system. $U_{^9\mathrm{Li}}(R)$ denotes the ^{9}Li-p interaction potential. Its nuclear part can be taken as the p-^{9}Li optical potential and may include the spin-orbit component. U_n is the p-n interaction potential. For simplicity, the difference between the center of masses of ^{11}Li and ^{9}Li is ignored.

The wave function $\Psi(\mathbf{R},\mathbf{r}_1,\mathbf{r}_2)$ describes various reaction processes where ^{9}Li remains in the ground state. By using the eikonal and adiabatic approximations, the elastic scattering amplitude for the momentum transfer $\mathbf{q}$ is given by

$$f_{\alpha\beta}(\mathbf{q})=\frac{iK}{2\pi}\int d\mathbf{b}\, e^{-i\mathbf{q}\cdot\mathbf{b}}\{\delta_{\alpha\beta}-\langle\phi_0\eta_\beta|\exp[i\chi(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)]|\phi_0\eta_\alpha\rangle\}\ , \quad (3)$$

where K is the wave number of the relative motion, $\mathbf{b}$ the impact parameter, $\mathbf{s}_i$ the component of $\mathbf{r}_i$ perpendicular to the incident beam direction, which is taken to be parallel to the z axis, and α,β denote the spin orientations of the proton spin function η. ϕ_0 represents the ground-state halo-neutron wave function. The phase shift function can in general be obtained by [13,15]

$$\exp[i\chi(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)] = \sum_{n=0}^{\infty}\frac{1}{n!}\left[-\frac{i}{\hbar v}\right]^n \int_{-\infty}^{\infty}dz_1\int_{-\infty}^{\infty}dz_2\cdots\int_{-\infty}^{\infty}dz_n P\{U(\mathbf{b}+z_1\mathbf{e}_z,\mathbf{r}_1,\mathbf{r}_2)U(\mathbf{b}+z_2\mathbf{e}_z,\mathbf{r}_1,\mathbf{r}_2)\cdots U(\mathbf{b}+z_n\mathbf{e}_z,\mathbf{r}_1,\mathbf{r}_2)\}$$
$$= \left\{\exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty}dz U(\mathbf{b}+ze_z,\mathbf{r}_1,\mathbf{r}_2)\right]\right\}_+ , \quad (4)$$

where $v=\hbar K/\mu$ and P, Dyson's ordering operator, arranges the U's in the brackets in increasing order of the arguments of the z's from right to left. When U's with different arguments commute with each other, the ordering is irrelevant and the phase shift function reduces to the well-known form.

It is possible to calculate the cross sections directly from Eq. (3). In this study the eikonal scattering amplitude of Eq. (3) is employed to derive the phase-equivalent optical potential that enables us to obtain more accurate cross sections as described below. We first define the optical phase shift function $\chi_{\rm opt}(\mathbf{b})$ by

$$\exp[i\chi_{\rm opt}(\mathbf{b})] = \langle\phi_0|\exp[i\chi(\mathbf{b},\mathbf{s}_1,\mathbf{s}_2)]|\phi_0\rangle . \quad (5)$$

As shown by Glauber [13], the local and energy-dependent optical potential for the ^{11}Li$+p$ scattering can then be constructed from $\chi_{\rm opt}(\mathbf{b})$ by

$$U_{\rm opt}(\mathbf{R}) = \frac{\hbar v}{\pi}\int_0^{\infty}dx\frac{\chi'_{\rm opt}[(R^2+x^2)^{1/2}]}{(R^2+x^2)^{1/2}} , \quad (6)$$

where $\chi_{\rm opt}(\mathbf{b})$ is assumed to depend on only b. By definition, $U_{\rm opt}(R)$ gives the same phase shift function as $\chi_{\rm opt}(b)$ in the eikonal approximation. The eikonal approximation has been tested to be sufficiently accurate at intermediate energies [8,9]. However, once we get $U_{\rm opt}(R)$, it will be more appropriate to solve a Schrödinger equation quantum mechanically without recourse to the eikonal approximation, particularly at the energy of the present investigation.

To proceed further we need to specify $U_{^9{\rm Li}}$ and U_n. The p-^{9}Li optical potential is assumed to have the standard form

$$Vf(r,R_v,a_v) + i\left[W_v f(r,R_w,a_w) - 4a_w W_s\frac{d}{dr}f(r,R_w,a_w)\right] + V_{ls}\left[\frac{\hbar}{m_\pi c}\right]^2\frac{d}{dr}f(r,R_{ls},a_{ls})\mathbf{L}\cdot\boldsymbol{\sigma} , \quad (7)$$

where $f(r,R,a)$ is the Woods-Saxon form with $R=r_0A^{1/3}$. The parameters of the potential are $V=-30.55$, $r_{0v}=1.16$, $a_v=0.76$, $W_v=-8.55$, $r_{0w}=1.21$, $a_w=0.52$, $W_s=-0.684$, $V_{ls}=4.24$, $r_{0ls}=1.42$, and $a_{ls}=0.81$, where the well depth parameters are in units of MeV and the r_0's and a's are in units of fm. This potential is a slightly modified version of the potential used in Ref. [1]. It reproduces the experimental angular distribution at $E_{\rm lab}=60$ MeV/nucleon fairly well, as shown by the dashed curve in Fig. 1. The data include the inelastic scattering cross sections to the 2.69-MeV state of ^{9}Li, which are considered to be small. The discrepancy between experiment and theory becomes conspicuous at forward angles of less than 30°. For the n-p interaction, we must recall that a neutron impinging on a proton is scattered backward with high probability in the n-p center-of-mass system. This was a clear indication of the presence of exchange force. A simple force was proposed by Serber to explain the nearly equal

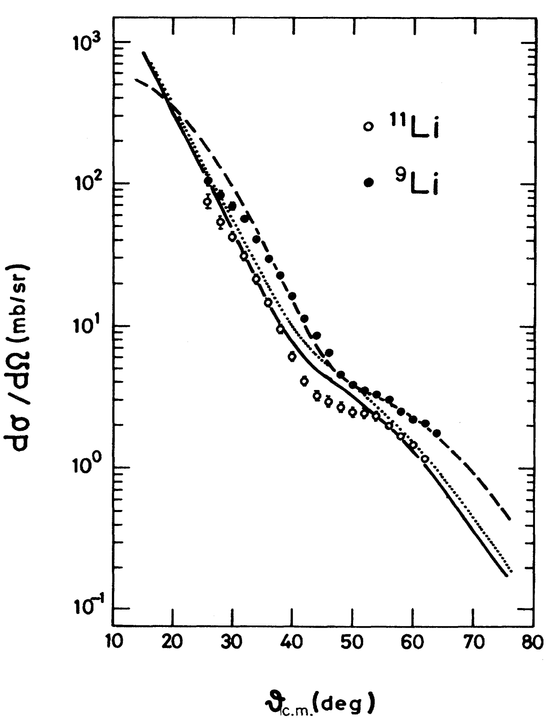

FIG. 1. Elastic differential cross sections for the ^{11}Li$+p$ system at $E_{\rm lab}(^{11}{\rm Li})/A=62$ MeV and for the ^{9}Li$+p$ system at $E_{\rm lab}(^9{\rm Li})/A=60$ MeV. The dashed curve denotes the ^{9}Li$+p$ cross sections calculated with the optical potential of Eq. (7), while the dotted and solid curves denote the ^{11}Li$+p$ cross sections obtained with the folding and optical potentials derived in our model, respectively. The data are from Ref. [1].

differential cross sections at forward and backward angles,

$$v(r)(1+P_M)\ , \tag{8}$$

with the Majorana exchange operator P_M. It should be noted that the potential of type (8) does not fit in the eikonal approximation because the Majorana force produces a large momentum transfer. The principal role of the Majorana force in the ^{11}Li$+p$ scattering is considered to lead to a channel of $n+{}^{11}$Be (isobaric analog state of ^{11}Li). The state of ^{11}Be contains a neutron-proton halo structure surrounding the ^{9}Li core, as discussed in Ref. [16]. We thus assume that the Majorana force makes a negligible contribution to the ^{11}Li$+p$ elastic scattering and therefore assume that the n-p potential of type (8) may effectively be replaced with a complex potential

$$v(r)(1+i\gamma)\ , \tag{9}$$

provided that we attempt to describe the ^{11}Li$+p$ elastic scattering in the single-channel four-body model. This is our crucial assumption. We confirmed that no attractive and purely real n-p potentials reproduced such cross sections that were smaller than the ^{9}Li cross sections. The presence of the imaginary potential gives rise to the total reaction cross section for the n-p scattering even below the pion threshold. The calculated total reaction cross section is interpreted as representing the elastic cross section contributed from the backward scattering. We require that the elastic cross section and the total reaction cross section calculated theoretically should be approximately equal to each other and that their sum should be close to the experimental n-p elastic cross section at the relevant energy. We also require that the angular distribution of the n-p elastic scattering calculated with the potential (9) reproduce experiment at very small scattering angles. These requirements enable us to determine the strength of γ and $v(r)$. For example, a one-range Gaussian potential

$$-45\exp[-0.51r^2](1+0.45i)$$

meets these conditions fairly well around the incident energy of 60 MeV. We have found that the ^{11}Li$+p$ elastic cross sections are rather insensitive to a precise form of the n-p potential. In the following $v(r)$ is taken from the Minnesota potential [17]

$$v(r)=120\exp[-1.487r^2]-53.4\exp[-0.639r^2]-27.55\exp[-0.465r^2]\ ,$$

and $\gamma=0.4$.

We assume a simple $(p_{1/2})^2$ configuration for the halo-neutron wave function

$$\phi_0(\mathbf{r}_1,\mathbf{r}_2)=[\varphi_{p_{1/2}}(\mathbf{r}_1)\times\varphi_{p_{1/2}}(\mathbf{r}_2)]_{J=0}\ , \tag{10}$$

where the single-particle wave function $\varphi_{p_{1/2}}$ is generated from the Woods-Saxon potential whose depth is chosen to set the single-particle energy equal to 0.1 MeV. Figure 1 displays the differential cross sections for the ^{11}Li$+p$ elastic scattering at $E_{\rm lab}=62$ MeV/nucleon. The dotted curve denotes the cross sections that are obtained by combining the p-^{9}Li optical potential with the n-p potentials folded with the wave function of Eq. (10). Since we take account of the exchange force as discussed above, the cross sections calculated with the folding potential are smaller than those of the ^{9}Li$+p$ scattering. They are not, however, small enough to reproduce the data. The solid curve represents the cross sections that include the breakup effect of halo neutrons to the continuum. It is clearly seen that the breakup effect plays the role of reducing the cross sections further and reproducing experiment fairly well. The agreement between theory and experiment is satisfactory when we consider that our theory has essentially no adjustable parameters. The breakup effect depends on the property of the wave function ϕ_0. We changed the single-particle energy to 0.3 MeV and repeated the calculation. The differential cross sections did not change very much. Increasing the single-particle energy to 8 MeV gave differential cross sections that are larger at $\theta_{\rm c.m.}<60°$ than the folding model and even larger at $45°<\theta_{\rm c.m.}<55°$ than the ^{9}Li$+p$ differential cross sections. Although our theory explains the basic features of the ^{11}Li data, the discrepancy in the angular range of 40°–50° may indicate that something is missing in our theory.

The p-^{11}Li optical potential can be expressed in our treatment as a sum of the folding potential and the dynamical polarization potential due to the breakup of ^{11}Li [9]. Figure 2 displays the real and imaginary parts of the p-^{9}Li and p-^{11}Li optical potentials with the spin-orbit force of Eq. (7) ($A=9$ for both) subtracted. The long tail of the p-^{11}Li optical potential is due to the spatially extended density distribution of the halo neutrons. The characteristics of the obtained dynamical polarization potential are the same as those of the ^{11}Li-^{12}C case [9]: The dynamical polarization potential gives a repulsive effect to the real part of the optical potential and an absorptive effect to the imaginary part.

To conclude, the ^{11}Li$+p$ elastic scattering has been studied in a (^{9}Li$+n+n$)-p four-body model. The breakup effect of the halo neutrons on the differential cross sec-

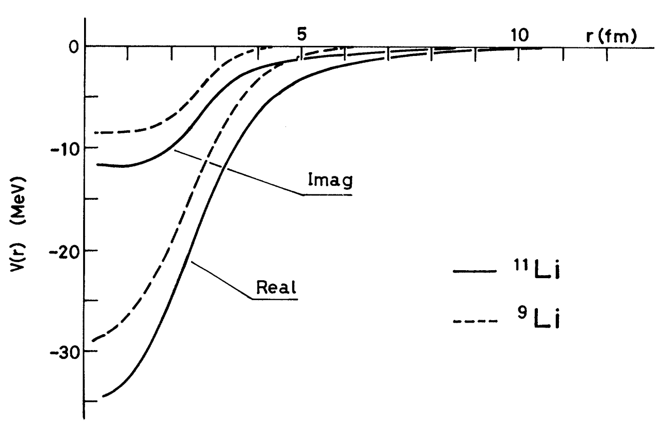

FIG. 2. Comparison of the p-^{9}Li and p-^{11}Li optical potentials. The spin-orbit potential of Eq. (7) is not included in this figure for both the cases.

tions has been examined on the basis of the eikonal and adiabatic approximations presented in Refs. [8,9]. A complex effective n-p potential is introduced to simulate the exchange force of the n-p interaction, which becomes important in coupling the $p+{}^{11}$Li channel with the $n+{}^{11}$Be (isobaric analog state of ^{11}Li) channel. We have shown that to realize both the breakup effect and the n-p exchange force is important in obtaining a good agreement with experiment. The p-^{11}Li optical potential derived has much longer tail than the p-^{9}Li optical potential, as expected. The four-body model has been shown to be quite useful for the ^{11}Li reaction. It is easy to extend the model to more general cases involving exotic nuclei.

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