Quadrupole amplitude in the $\gamma N \leftrightarrow \Delta$ transition

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The tensor part of the color hyperfine interaction between quarks leads to a small electric quadrupole amplitude in the $\gamma N \leftrightarrow \Delta$ excitation. It is demonstrated that pion photoproduction data available previously have a low sensitivity to the (isospin $\frac{3}{2}$) electric quadrupole amplitude, $E_{1+}(\frac{3}{2})$. An additional difficulty is that there is an appreciable background contribution to the total E_{1+} amplitude. The consequent difficulties in extracting the resonant part of this small amplitude from experiment are discussed. It is shown that $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ cross sections near 0° and 180°, and also those near 90° with polarized γ rays, both with and without polarized targets, have the greatest sensitivity to the $E_{1+}(\frac{3}{2})$ amplitude. Since the anticipated observable effects of the resonant $E2$ amplitude are expected to be $\sim 10\%$ to 20% in magnitude, the required accuracy to extract a precise $E2/M1$ ratio is $\sim 1\%$ for both experiment and theory.

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I. INTRODUCTION

In analogy with the nucleon-nucleon and atomic hyperfine interactions, the interaction between quarks is believed to have a tensor component [1,2]. This gives a d-state admixture in the predominantly s-state quark wave functions for the nucleon and Δ. This also leads to important predictions [2] about hadron structure including mass splitting, decay probabilities nonzero quadrupole moments of Δ and Ω^-, and a nonzero electric form factor for the neutron [2,3]. The tensor interaction between quarks also leads to a resonant ($I=\frac{3}{2}$) electric quadrupole amplitude[1] $E_{1+}(\frac{3}{2})$ in the $\gamma N \leftrightarrow \Delta$ transition, which is primarily an $I=\frac{3}{2}$ magnetic dipole $M_{1+}(\frac{3}{2})$ transition. An accurate measurement of the $E_{1+}(\frac{3}{2})$ amplitude is therefore of great importance in testing nucleon models.

Multipole analyses [4] of the $N(\gamma,\pi)$ reactions constitute a first step in determining the resonant. $E_{1+}(\frac{3}{2})$ amplitude; or, equivalently, the ratio $R_{\rm EM} = E_{1+}(\frac{3}{2})/M_{1+}(\frac{3}{2})$ (or $E2/M1$) for the resonant amplitudes. The determination of the resonant $E_{1+}(\frac{3}{2})$ amplitude is difficult for several reasons. First, it is small compared to the dominant $M_{1+}(\frac{3}{2})$ amplitude. Second, the relative magnitude of the background is large for this amplitude. Therefore, it is difficult to avoid a model dependence in separating the background contribution from the entire $E_{1+}(\frac{3}{2})$ amplitude to get the resonant part. Previous empirical attempts [5–10] obtained a range of values from 0 to -5% for $R_{\rm EM}$ using available data. Since these analyses were based on essentially the same data, the spread in the values reflects a systematic error in the analysis. In order to understand the reason for this systematic variation, we have made, for the first time, a quantitative estimate of the effect of the resonant $E_{1+}(\frac{3}{2})$ amplitude on the observables. This paper is primarily motivated by the fact that new experimental facilities and techniques have made a more accurate measurement of the quantities which are sensitive to the quadrupole amplitude in the $\gamma N \leftrightarrow \Delta$ transition feasible. However, as will be discussed below, before these measurements can be properly interpreted, one must be able to distinguish between the signal and the background.

[1]The amplitudes are denoted by $E_{l+}(I)$ and $M_{l+}(I)$, where l is the orbital angular momentum of the photoproduced pion, the $\pm$ sign refers to the total pion-nucleon angular momentum $j=l\pm\frac{1}{2}$, and I is the isospin of the πN system. Thus, $E_{1+}(\frac{3}{2})$ is the resonant electric quadrupole amplitude (E2) and $M_{1+}(\frac{3}{2})$ is the resonant magnetic dipole amplitude (M1).

II. GENERAL CONSTRAINTS ON RESONANT MULTIPOLES

Since the $E_{1+}(\frac{3}{2})$ amplitude is small and very likely to have a large background component in addition to the resonant part [11–13], it is important to discuss the basic quantum mechanics of resonance amplitudes [14]. First consider resonances in πN scattering. These are most gradually defined as poles in the S matrix. However, for a strong resonance with a smooth background (e.g., Δ), one can describe the phase shift δ (in the p_{33} channel) as

$$\tan\delta(W)=\frac{\Gamma}{2}(W_r-W)^{-1}+A(W) , \quad (1)$$

where Γ is the full width at half maximum, W is the total c.m. energy, W_r is the resonance energy, and $A(W)$ is a slowly varying background term [14]. With Eq. (1) and $A=0$, one obtains the usual Breit-Wigner resonance formula. At $W=W_r$, $\delta(W)=\pi/2$ so that the real part of the scattering amplitude goes to zero and the imaginary part goes through a maximum.

For the photoproduction amplitudes, the main constraint comes from the Fermi-Watson theroem [15]. This states that the multipoles M_α can be written in the form $M_\alpha(W)=|M_\alpha(W)|\exp i\delta_\alpha(W)$, where $\delta_\alpha(W)$ is the πN phase shift in the quantum state $\alpha=l,j,I$. For the 3,3 channel at resonance, one notes that $\mathrm{Re}[M_\alpha(W_r)]=0$. This is the only general constraint on the $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ multipoles.

We can now write the resonant multipole amplitudes (or equivalently the t-matrix elements) in the form [11,12]

$$\begin{aligned} M(W)&=M_\Delta(W)+M_{\mathrm{VR}}(W)+M_B(W)\\ &=M_R(W)+M_B(W) , \end{aligned} \quad (2)$$

where $M_\Delta(W)$ is the bare resonant amplitude, $M_{\mathrm{VR}}(W)$ is the "vertex renormalization" (due to πN rescattering before Δ formation). $M_B(W)$ is the background amplitude, and $M_R(W)=M_\Delta(W)+M_{\mathrm{VR}}(W)$ is the dressed resonant amplitude. In Fig. 1, the diagrams showing the interactions which are included in each of these terms [12,16] are indicated. The background term [Fig. 1(b)] has final-state interactions in nonresonant states and is unitary. We have

$$M_B(W)=|M_B(W)|e^{i\delta_B} , \quad (3)$$

where $\delta_B(W)$ is the background phase shift in πN scattering. In Fig. 1(c), we see that the dressed resonance is composed of the bare Δ plus the vertex renormalization. The bare Δ has the final-state interaction in the resonant p_{33} channel and is unitary; i.e.,

$$M_\Delta(W)=|M_\Delta(W)|e^{i\delta_{p_{33}}} . \quad (4)$$

The vertex renormalization term in Fig. 1(c) has the initial-state interaction in the resonant p_{33} channel.

Although the entire amplitude $M(W)$ is unitary, the Δ resonance dressed amplitude itself is not [11,12]. One way to enforce unitarity for the dressed amplitude is to write it in terms of the bare Δ amplitude, via [13,18]

$$M_R(W)=M_\Delta(W)e^{i\phi} . \quad (5)$$

Here ϕ is an empirically determined phase, whose dynamic origin is not clear at this stage. Noting that the resonant amplitude is the sum of the bare delta and vertex renormalization amplitudes, and using Eq. (5), one can write

$$M_R(W)=M_\Delta(W)+M_{\mathrm{VR}}(W)=M_\Delta(W)e^{i\phi} . \quad (6)$$

From Eq. (6), it is clear that the multiplicative factor $e^{i\phi}$ essentially represents the additive vertex renormalization [16].

The electromagnetic ratio R_{EM} characterizes the $E2/M1$ ratio at resonance. Since these quantities are complex we define

$$R_{\mathrm{EM}}=\mathrm{Re}(E_{1+}/M_{1+})=\frac{\mathrm{Re}(E_{1+}M_{1+}^*)}{|M_{1+}|^2} . \quad (7)$$

One of the advantages of using a dynamical model is that we can separately demonstrate the effects of the dressed and bare $E_{1+}(\frac{3}{2})$ amplitude on the observables. We therefore define two electromagnetic ratios: $R^R_{\mathrm{EM}}=E^R_{1+}/M^R_{1+}$, the "dressed" $E2/M1$ ratio; and $R^\Delta_{\mathrm{EM}}+E^\Delta_{1+}/M^\Delta_{1+}$, the "bare" $E2/M1$ ratio.

One difficulty in interpreting the R_{EM} value is that the quadrupole amplitude is calculated in the framework of quark or soliton models. But a phenomenological hadronic model, which takes πN final-state interactions into account, must be used to extract the quadrupole amplitude from experiment. The connection between the extracted amplitude and that of the quark or soliton model is not entirely clear. It is probably most appropriate to compare quark models (e.g., the nonrelativistic quark model with harmonic-oscillator wave functions) to the bare Δ amplitude, and models which have pion clouds (e.g., the cloudy bag, chiral bag, or soliton models) to the dressed Δ amplitude. We stress that this identification is intuitive and remains to be demonstrated by more realistic identifications.

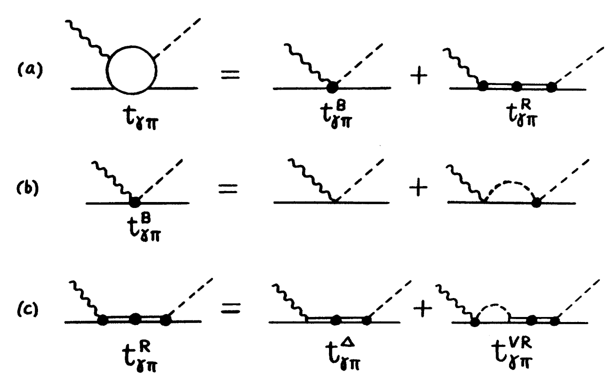

FIG. 1. Graphical representation of the t matrix for pion photoproduction. (a) The total amplitude, (b) the background term, and (c) the dressed resonance term in terms of the bare Δ and the vertex renormalization. For a more detailed discussion see Refs. [12,16].

III. RESONANT $M1$ AND $E2$ AMPLITUDES

We now present results for the $E_{1+}(\frac{3}{2})$ and $M_{1+}(\frac{3}{2})$ amplitudes calculated with the model of Nozawa, Blankleider, and Lee (NBL) [11]. This model gives reasonable agreement with experimental data and has several attractive features. It is gauge invariant, preserves unitarity, and takes into account the off-shell final-state πN interactions (FSI). The πN interaction used in the model reproduces the phase shift data. In this model, the value of $R^{\Delta}_{EM} \cong -3\%$ has been obtained from a fit to the data [11]. It can arbitrarily be set to zero to study the sensitivity of the observables to the $E_{1+}(\frac{3}{2})$ amplitude.

We stress that our primary use of this model here is to determine the sensitivity of the observables to the quadrupole amplitude and to discuss the general question of how this amplitude can be obtained from experiment. The model employed is sufficiently realistic to accomplish this goal, since it is in reasonable agreement with the data.

In Fig. 2, we show the calculated $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ amplitudes along with two empirical (energy-dependent) values [4,17]. There is reasonable agreement for both $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ amplitudes. The $M_{1+}(\frac{3}{2})$ multipole has a typical Breit-Wigner resonance shape [Figs. 2(a) and 2(b)]. As is required for a resonance, the real part of $M_{1+}(\frac{3}{2})$ goes through zero at the resonance energy ($E_\gamma=338$ MeV, $W=1232$ MeV). However, it can be seen that there is a significant background contribution in the real part of the $M_{1+}(\frac{3}{2})$ amplitude. The parameters of the NBL model were chosen to fit the Berends-Donnachie multipoles [4]. There are small differences between the Berends-Donnachie and Arndt *et al.* [17] $M_{1+}(\frac{3}{2})$ multipoles, as can be seen in Figs. 1(a) and 1(b).

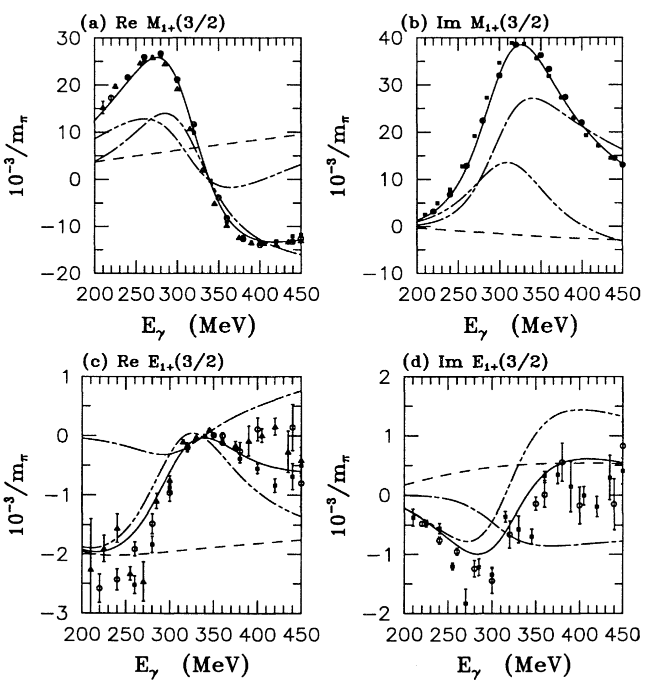

FIG. 2. The $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ multipole amplitudes (in units of $10^{-3}/m_\pi$) as a function of the photon laboratory energy E_γ. The four sections of the figure are (a) Re $M_{1+}(\frac{3}{2})$, (b) Im$M_{1+}(\frac{3}{2})$, (c) Re$E_{1+}(\frac{3}{2})$, (d) Im$E_{1+}(\frac{3}{2})$. The curves are full calculation ——; background, - - - - - - - -; bare △, —-— ; and vertex renormalization plus background, —- -—. The points with the error bars are the empirical (energy-dependent) multipole results of Pfeil and Schwela [4], ○; Berends and Donnachie [4], ▲; and Arndt *et al.* [17], ■.

Qualitatively, the shape of $E_{1+}(\frac{3}{2})$ [Figs. 2(c) and 2(d)] indicates that it is not a simple resonance like $M_{1+}(\frac{3}{2})$. The fact is that the $E_{1+}(\frac{3}{2})$ amplitude goes through zero near the Δ resonance. This was first confirmed by Berends and Donnachie [4] and was subsequently demonstrated by Jurewiez [4] to be predicted by dispersion relations. It remained, however, for the recent theoretical models [9–13] to show physically that this unusual shape was due to a cancellation between the dressed resonant amplitude and the background. One obtains Re$E_{1+}(\frac{3}{2})=0$ at resonance ($E_\gamma=338$ MeV, $W=1232$ MeV) as required by the Fermi-Watson theorem. One obtains Im$E_{1+}(\frac{3}{2})=0$ slightly above resonance; in the NBL model this occurs at $E_\gamma=342$ MeV, $W=1234$ MeV. The fact that these zeros are so close to each other is a "dynamical accident." There are two important consequences of this background cancellation: (1) the observable effects of the $E2$ amplitude are reduced, and (2) it is important to separately determine the resonance and background contribution.

From the shapes of the resonant multipoles one can discuss the determination of R_{EM} [Eq. (7)] in more detail. For the unitary amplitudes (total and bare Δ) the real parts of the amplitudes are zero at resonance and therefore R_{EM} is the ratio of the imaginary parts of each amplitude. For the M_{1+} amplitude the imaginary part has a maximum at resonance. For the E_{1+} amplitude there is a strong cancellation between the resonant and background contributions making the total amplitude even smaller. For the dressed amplitude, which is not unitary, the ratio of E_{1+}/M_{1+} is complex at resonance. However since R_{EM} is defined as the real part of the ratio, we obtain a real number.

IV. SENSITIVITY OF THE CROSS SECTION TO THE $E2$ AMPLITUDE

We now study the sensitivity of the cross sections for the $p(\gamma,\pi^0)$ reaction to the resonant $E_{1+}(\frac{3}{2})$ amplitude with both polarized and unpolarized photons. Calculations have been performed for the cross sections for unpolarized photons ($\sigma_{\rm unpol}$), photons polarized parallel to the production plane ($\sigma_{\parallel}$), and photons polarized perpendicular to the production plane ($\sigma_{\perp}$). They are related to each other by

$$\frac{d\sigma_{\rm unpol}(\theta)}{d\Omega}=\frac{1}{2}\left[\frac{d\sigma_{\perp}(\theta)}{d\Omega}+\frac{d\sigma_{\parallel}(\theta)}{d\Omega}\right], \quad (8)$$

where θ is the pion production angle. It should be pointed out that cross sections $\sigma_{\rm unpol}$. $\sigma_{\parallel}$, and $\sigma_{\perp}$ become identical at $\theta=0$ and π, where they are equally sensitive to the $E_{1+}(\frac{3}{2})$ amplitude.

We now present numerical results obtained by the NBL model [11]. First we define the ratio of the cross sections with and without the resonant $E2$ amplitude as

$$R_\alpha = \frac{d\sigma_\alpha(\text{with } E2)}{d\Omega} \Big/ \frac{d\sigma_\alpha(\text{without } E2)}{d\Omega} , \quad (9)$$

where $\alpha \equiv$ unpolarized, $||$, and $\perp$. These ratios are convenient to show the sensitivity of the observables to the $E2$ amplitude (they are not directly observable); typical sensitivities are of the order of 5–20 % for $E2/M1$ ratios of a few percent. The ratios are useful to indicate which observables to study and can be used to estimate the required experimental accuracy. To determine which observables are sensitive to the resonant $E2$ amplitude only requires a reasonably realistic model. Once the pertinent observables are measured the requirements for the theoretical model are more stringent. It will be shown that the accuracy which is required to measure accurate $E2$ amplitudes needs to be $\sim 1\%$ for both theory and experiment.

For the $\gamma p \to \pi^0 p$ reaction we show the calculated results for R_α for the bare Δ in Fig. 3(a) and for the dressed Δ in Fig. 3(b). Note how different these three sensitivity curves are. For the bare Δ, the curves are symmetric about 90°; for the dressed Δ, they are not. For the entire $E_{1+}(\frac{3}{2})$ amplitude, the effect is negligible (and is therefore not plotted), indicating the large effect that the background has in canceling the resonant signal. In all cases, there is the greatest sensitivity for parallel polarized photons. For $R^{\Delta}_{\mathrm{EM}} \cong -3\%$, there is a 15% increase in $R_{\|}$ for the bare Δ near $\theta = 90°$. We conclude that the measurement of the cross section for parallel polarized photons provides the most sensitive measurement of the $E2$ amplitude. The first polarized photon data were recently taken at the L.E.G.S facility at Brookhaven [22] with $\sim 5\%$ errors (which is approaching the required accuracy). These data indicate the shortcoming of current models and multipole solutions [22].

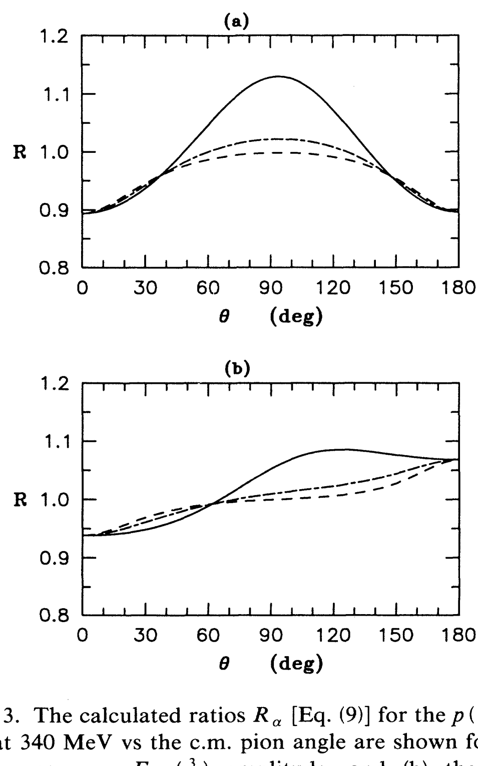

FIG. 3. The calculated ratios R_α [Eq. (9)] for the $p(\gamma, \pi^0)$ reaction at 340 MeV vs the c.m. pion angle are shown for (a) the bare Δ resonance $E_{1+}(\frac{3}{2})$ amplitude, and (b) the dressed resonance $E_{1+}(\frac{3}{2})$ amplitude. The curves correspond to R_{unpol}, — - —; $R_{\|}$, ——; and $R_{\perp}$, - - - - - - - -.

The maximum sensitivity to the $E2$ resonance occurs for parallel polarized photons near $\theta = 90°$. This can be seen by examining the multipole expressions for the polarized cross sections. Keeping s-, p- and d-wave multipoles, one can write the cross sections at $\theta = \pi/2$ as

$$\frac{d\sigma_\perp(\pi/2)}{d\Omega} = \frac{k}{\omega_q} \{ |E_{0+} - D_\perp|^2 + |P_\perp|^2 \} , \quad (10a)$$

$$\frac{d\sigma_\|(\pi/2)}{d\Omega} = \frac{k}{\omega_q} \{ |E_{0+} - D_\||^2 + |P_\||^2 \} . \quad (10b)$$

Here k and ω_q are the pion momentum and the photon energy in the c.m. system, respectively. In Eq. (9), $P_\perp$ and $P_\|$ are p-wave amplitudes given by

$$P_\perp = 2M_{1+} + M_{1-} \quad (11a)$$

$$P_\| = 3E_{1+} - M_{1+} + M_{1-} . \quad (11b)$$

Similarly, the d-wave amplitudes are

$$D_\perp = \tfrac{3}{2}E_{2+} - E_{2-} + 3M_{2+} - 3M_{2-} , \quad (12a)$$

$$D_\| = \tfrac{9}{2}E_{2+} + 2E_{2-} . \quad (12b)$$

It is evident from Eqs. (10)–(12) that at $\theta = 90°$, $\sigma_\|$ has a *maximum* sensitivity to the $E_{1+}(\frac{3}{2})$ amplitude, whereas $\sigma_\perp$ has no sensitivity.

We compare in Fig. 4(a) the predicted unpolarized cross section with the Bonn data [19]. Other data sets [19] are in reasonable agreement with those shown. The full calculation (solid curve) is in fair agreement with the data. This figure indicates the need for more accurate data and theoretical calculations. In order to demonstrate the importance of the background $E2$ amplitude, we have added two curves in Fig. 4(a). These are the cross sections without the bare and without the dressed $E_{1+}(\frac{3}{2})$ amplitude. As can be seen in Fig. 4(a), the maximum sensitivity of the unpolarized cross section to the $E2$ amplitude is near 0° and 180°. As is shown in Figs. 3 and 4(a), there is an interesting difference between the forward and backward cross section with respect to the bare, dressed, and total $E_{1+}(\frac{3}{2})$ amplitudes. In particular, for the dressed amplitude, the effect of the $E2$ resonance is to lower the cross section at forward angles and to increase it at large angles. For the bare $E2$ resonance, the cross section is lowered at both forward and backward angles. For the total $E_{1+}(\frac{3}{2})$ amplitude, the effect is rather small and is almost independent of angle; dramatically showing the cancellation of the background and resonance amplitudes. The data [as illustrated in Fig. 4(a)] look in somewhat better agreement at backward angles. A new data set which should be an improvement for the entire angular region was recently taken at Mainz [20]. This should give valuable information on the relative contributions of resonance and background contributions to the quadrupole transition amplitude.

In Fig. 4(b), we show the calculated polarized cross sections for the $\gamma p \rightarrow \pi^0 p$ section at $E_\gamma = 340$ MeV. It can be seen that the parallel cross section is about four times smaller near 90° and not as angular dependent as the perpendicular cross section. In Fig. 4(c), we show the effect of the bare and dressed $E2$ amplitudes on the cross section for photons polarized in the reaction plane. The same information can be obtained by multiplying the cross section shown in Fig. 4(b) with the ratios shown in Fig. 3. We believe that it is useful in one case to show the effect of the $E2$ amplitudes in both ways.

In order to make an isospin decomposition to obtain the resonant $I = \frac{3}{2}$ amplitudes we need measurements of the charged as well as the neutral pion channels. In Fig. 5, we show the sensitivity of the $\gamma p \rightarrow n \pi^+$ reaction to the $E2$ amplitude. In Figs. 5(a) and 5(b) the sensitivity of the polarized and unpolarized cross sections to the bare and dressed $E2$ amplitudes are shown. These results are similar to the corresponding ones for the $\pi^0 p$ channel shown in Figs. 3(a) and 3(b). In Fig. 5(c), we show the predicted cross sections for the parallel and perpendicular polarization for the $\pi^+ n$ channel. The cross section is larger for the perpendicular case. The corresponding cross sections for the $\pi^0 p$ channel are shown in Fig. 4(b).

The double polarization observables, i.e., both target and photon polarized, in the (γ, π) reaction [10,21] have similar sensitivities (up to $\approx 20\%$ for $E2/M1 = -3\%$) to those we presented for the polarized photon cross sections. These measurements involve a polarized nucleon target polarized perpendicular to the reaction plane and photons polarized either parallel or perpendicular to the reaction plane. As was the case for unpolarized targets there is more sensitivity to the $E2$ amplitude for photons polarized parallel to the reaction plane; therefore the results for this case will be presented here. In Fig. 6, we present the calculated results for the double polarization observables for the $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ reactions with photons polarized parallel to the reaction plane. In Figs. 6(a)and 6(b) we show the sensitivity ratios R_α [Eq.

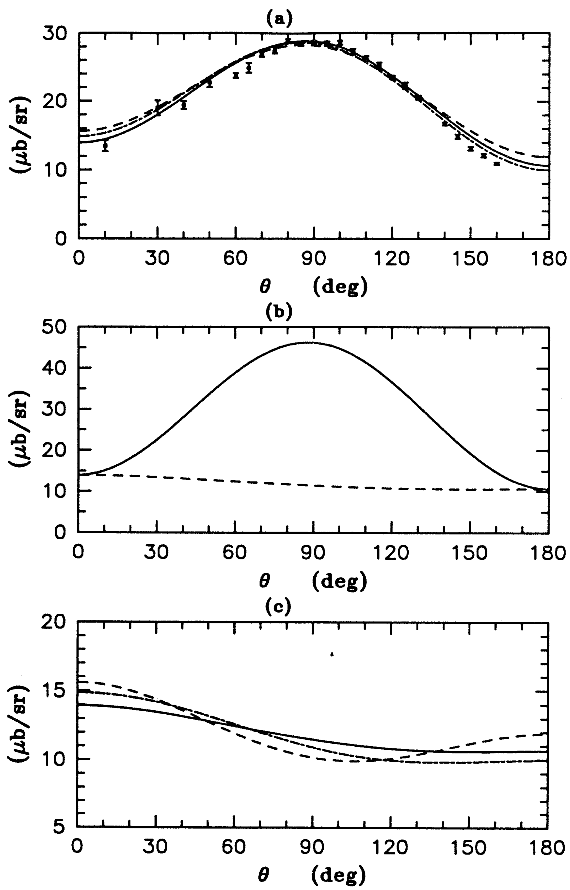

FIG. 4. Cross sections for the $p(\gamma, \pi^0)$ reaction at $E_\gamma = 340$ MeV vs the c.m. pion angle. (a) shows the unpolarized cross sections. The curves are ——, full calculation; - - - - - - - -, no bare Δ resonant $E_{1+}(\frac{3}{2})$ amplitude; and —-—, no dressed $E_{1+}(\frac{3}{2})$ resonance amplitude. Experimental data are taken from Ref. [19]. (b) shows $\sigma_\perp$ —— and $\sigma_\parallel$, - - - - - - - -. Curves in (c) present the polarized cross section $\sigma_\parallel$ for $E_\gamma = 340$ MeV [the curves are labeled as in (a)].

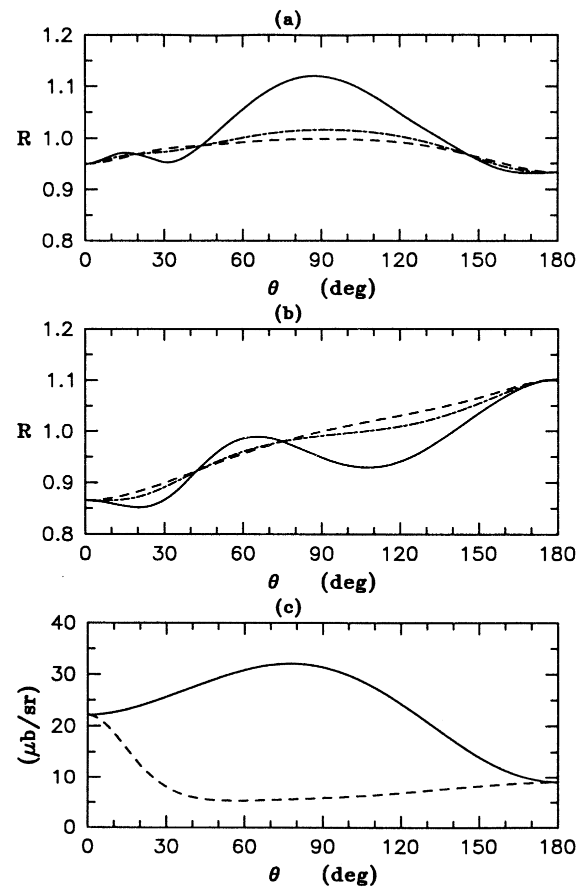

FIG. 5. Calculated ratios R_α [Eq. (9)] and cross sections vs pion c.m. angles for the $\gamma p \rightarrow n \pi^+$ reaction at $E_\gamma = 340$ MeV. (a) shows the sensitivity to the bare Δ resonance $E_{1+}(\frac{3}{2})$ amplitude; (b) for the dressed Δ resonance $E_{1+}(\frac{3}{2})$ amplitude. The curves are ——, $R_\parallel$; - - - - - - - -, $R_\perp$; and - - - - - - - -, R_{unpol}. (c) shows the calculated magnitudes of $\sigma_\perp$, —— and $\sigma_\parallel$, - - - - - - - -.

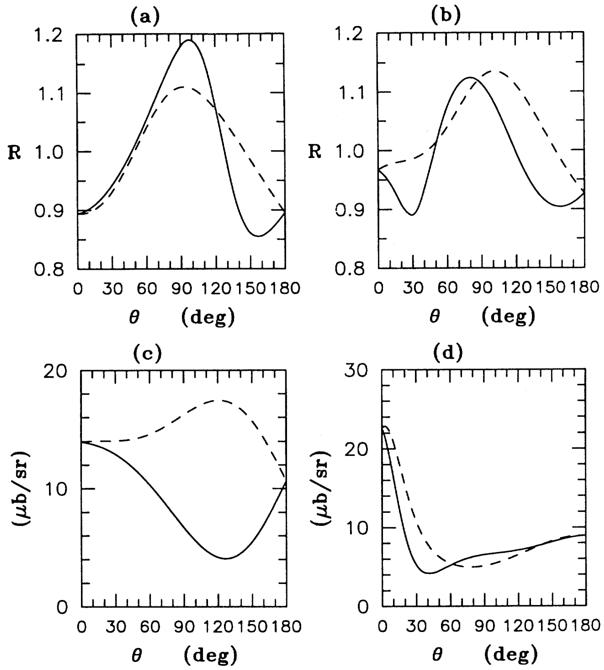

FIG. 6. Double polarization observables calculated at $E_\gamma = 340$ MeV vs the c.m. angle. All figures are for the photons polarized in the reaction plane and the dashed (solid) lines are for the proton polarized up (down) relative to the reaction plane. (a) and (b) show the ratios R_α [Eq. (9)] to the bare Δ amplitudes for the $\gamma p \to \pi^0 p$ and $\gamma p \to \pi^+ n$ reactions, respectively. (c) and (d) show the cross sections for $\gamma p \to \pi^0 p$ and $\gamma p \to \pi^+ n$ reactions, respectively.

(9)] for the bare Δ coupling for the proton spin oriented up and down relative to the reaction plane. As was the care for unpolarized targets the effects are predicted to be up to 20% for an assumed $E2/M1 = -3\%$. It can be seen that the sensitivity to the $E2$ amplitude depends on the target spin orientation. The calculated cross sections for the full amplitude are shown in Figs. 6(c) and 6(d) for the two target spin orientations. These experiments are within the capabilities of present electromagnetic facilities. For example, at Bonn University, the $p(\gamma,\pi^+)n$ reaction was studied with unpolarized gamma rays using unpolarized [23] and polarized [24] targets, and also double polarization experiments with polarized beam and target are planned [24]. Associated recent data [25] on the $\pi^- \vec{p} \to \gamma n$ reaction will also be valuable for understanding the amplitudes.

V. PREVIOUS DETERMINATIONS OF THE $E2/M1$ RATIO

Having presented the results of the $E2/M1$ sensitivity on the observables, we are now in a position to discuss the $E2/M1$ ratios obtained in the literature. The most recent version of the Review of Particle Properties [26] lists four values of the $E2/M1$ ratio $-1.1\pm0.4\,\%$ and $-1.5\pm0.2\,\%$ from the two papers of Davidson, Mukhopadhyay, and Wittman [7], $+3.7\pm0.4\,\%$ from Tanabe and Ohta [12], and $-1.3\pm0.5\,\%$ from the last analysis performed by the Particle Data Group [6]. These values, although not all that are found in the literature, have been obtained with quite differing assumptions and actually represent different quantities.

The analysis of the particle data group is based on the two helicity amplitudes $A_{1/2}$ and $A_{3/2}$. The relationship between the resonant and helicity amplitudes is

$$M1 = -1/2(A_{1/2} + \sqrt{3}A_{3/2}) ,$$
$$E2 = -1/2(A_{1/2} - A_{3/2}/\sqrt{3}) . \quad (13)$$

The $E2/M1$ ratio of $-1.3\pm0.5\,\%$ is then obtained from the measured helicity amplitudes [6]. This means that no background contribution has been subtracted. The quoted error is based on the measured uncertainties in the helicity amplitudes and does not reflect any systematic errors in the extraction of the $E2/M1$ ratio.

The most ambitious effort to determine the $E2/M1$ ratio from the multipoles is due to Davidson and Mukhopadhyah [7]. They assumed K-matrix forms for the photopion production and pion-nucleon scattering ($K_{\gamma\pi}$ and $K_{\pi\pi}$, respectively):

$$K_{\gamma\pi} = A/(W_R - W) + B ,$$
$$K_{\pi\pi} = C/(W_R - W) + D . \quad (14)$$

Here A, B, C, and D are smoothly varying functions of W; in practice, they were assumed to be constant near resonance. The t-matrix elements (multipoles) are calculated from the K matrix. At $W = W_R$, the resonance energy, one obtains[7] $\mathrm{Re}\,t_{\gamma\pi} = 0$, in agreement with the Fermi-Watson theorem; and $\mathrm{Im}\,t_{\gamma\pi} = A/C$, the ratio of the K-matrix residues for the photoproduction and pion scattering. Note that the K-matrix background term B does not contribute at resonance. This assumption therefore represents the strong model-dependent choice for the t matrix (multipoles), that there is no background contribution. As shown in Sec. III and also by Davidson *et al.* [13] using an effective Lagrangian, and also by others [11,12], there is a large background term which is comparable to the resonant amplitude for the $E_{1+}(\frac{3}{2})$ multipole. We therefore conclude that the "model-independent" method of Davidson and Mukhopadhyay [7] is in effect a highly restrictive (no background) determination of the $E2/M1$ ratio. Once that is understood, the results are interesting. A number of multipole solutions to the data were analyzed with a uniform procedure, and the $E2/M1$ ratios were obtained; the results varied from $-0.6\pm1.0\,\%$ to $-2.3\pm1.0\,\%$. Since the multipole analyses were based on essentially the same database, this spread in the values represents a systematic uncertainty in the $E2/M1$ ratio. The values should not be combined statistically as if they were independent measurements of the same quantity. In fact, it is a triumph of the multipole analyses that given the relative lack of sensitivity of the data to the $E2$ amplitude (as shown in Sec. III), that the results of the different multipole analyses are so consistent. Finally we note that the $E2/M1$ ratios obtained by Davidson, Mukhopadhyay, and Wittman [7] can be interpreted as being for the bare coupling [16]. The

TABLE I. The helicity amplitudes (10^{-3} $\text{GeV}^{-1/2}$).

	PDG (Ref. [6])	DMW (Ref. [13])	NBL (Ref. [11])	Quark models
$A_{1/2}$	-145 ± 5	-136 ± 16	-97	-102 [35], -91 [36]
$A_{3/2}$	-260 ± 8	-256 ± 34	-191	-177 [35], -159 [36]

reason is that the unitarization procedure used in Ref. [7] is effectively equivalent to the vertex renormalization discussed in Sec. II.

There have been several empirical attempts to subtract a background contribution in the $E_{1+}(\frac{3}{2})$ amplitude [8,10]. The results are -0.6 and -1.9%, respectively, for the dressed amplitude. A third approach uses a model to calculate the background amplitude, then determines the resonance contribution by fitting the empirical $E_{1+}(\frac{3}{2})$ amplitude. The results for the bare Δ amplitude are $-(3.1\pm1.3)\%$ for the model presented here [11], $-(1.5\pm0.72)\%$ [13], -4% [18], 0% [9], and $+4\%$ [12] (Tanabe and Ohta). The results for the dressed Δ amplitude are -2.2% for the model presented here [11]. It is clear that there is a significant model dependence for the extracted $E2/M1$ ratio; much of this is probably due to the different off shell treatment of the πN scattering in the final state.

Finally, we show in Table I the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ of Davidson, Mukhopadhyay, and Wittman, and of Nozawa, Blankleider, and Lee. Using Eq. (13) and the results of Table I, one can calculate the $M1$ and $E2$ amplitudes. For example, $M1(\text{DMW})=289\pm37$, $E2(\text{DMW})=-5.7\pm2.7$, whereas $M1(\text{NBL})=214$, $E2(\text{NBL})=-6.6$. All amplitudes are in units of 10^{-3} $\text{GeV}^{-1/2}$. There is a strong disagreement between the two models for the dominant $M1$ multipole. The NBL model value is much closer to the quark model values, as discussed in Ref. [27].

VI. CONCLUSIONS

In summary, we have quantitatively investigated the resonant electric quadrupole $E_{1+}(\frac{3}{2})$ sensitivity of the observables for the (γ,π) reaction for the first time. It was shown that the spread in the values for $E2/M1$ obtained in previous analyses is due to the model dependencies of the analyses, and to the fact that they are based on data which do not have the angular coverage or the polarization data to be very sensitive to the resonant $E2$ amplitude. In addition, there is a sizable background contribution to the $E_{1+}(\frac{3}{2})$ amplitude which has been neglected in several analyses [6,7]. This background contribution almost cancels out the resonant amplitude and makes the $E2$ amplitude more difficult to observe.

We have shown that the $\gamma p\to\pi^0 p$ and $\gamma p\to\pi^+ n$ reactions are sensitive to the resonant $E2$ amplitude for photons polarized in the reaction plane or for unpolarized photons producing pions near 0° and 180°. We have shown similar sensitivities for the double polarization observables (i.e., with both the photon and target proton polarized). To estimate which observable is sensitive to the resonant $E2$ amplitude requires a reasonable realistic model (such as the one employed here); to extract the $E2$ amplitude from experiment will need a model accuracy at the 1% level.

An accurate determination of the $E2/M1$ ratio will need new data from dedicated experiments; the required experimental accuracy will be $\sim1\%$ since the predicted effects are 10–20% for the assumed value of $R_{\text{EM}}^{\Delta}=-3\%$. Experiments which employ different polarization observables are important to perform because they all have different relative sensitivities to the $E_{1+}(\frac{3}{2})$ and the other multipoles. Compton scattering [28] is also important since it depends differently on the resonant and background amplitudes.

To extract the $E2/M1$ ratio from these reactions will require more accurate models than are presently available. Multipole analyses are important but are of limited help, because they cannot distinguish between the resonant and background contributions to the $E_{1+}(\frac{3}{2})$ amplitude; such a breakdown is model dependent.

In addition to the photoreactions, the d-state contribution to the $N\leftrightarrow\Delta$ transition can be measured in the $e\vec{p}\to e'p\pi^0$ reaction [29–33]. In particular, the interference term between the longitudinal and transverse virtual photon contributions is sensitive to the d-state components in the nucleon wave function. The "fifth" or polarization-dependent structure function is the imaginary part of the same amplitude and is sensitive to the interference between the background and resonant amplitudes. These experiments will provide valuable information about the d-state nucleon amplitudes. Of related interest are planned [34] measurements of $E2/M1$ ratios for the analogous radiative transitions with Σ and Ξ hyperons.

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