"Shadow" properties in sub-barrier fusion

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At energies below the Coulomb barrier the fusion process can be described as the shadow of the Rutherford scattering, but at very low energies it is not valid. However, if we define an effective Coulomb potential, the fusion can be described as the shadow of the elastic scattering relative to this effective potential. Then light systems at very low energy show an anomalous behavior and the effective Coulomb potential is a way to describe it.

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I. INTRODUCTION

The behavior of light systems at very low energies is not known in a satisfactory way. Physical processes such as elastic scattering or nuclear fusion need further investigation at energies far from the Coulomb barrier. Experimental data and theoretical investigations available so far are not able to describe, in a satisfactory way, the properties of light systems at very low energy. Recent investigations on the fusion process pointed out that the nuclear fusion below the Coulomb barrier can be described in the framework of the "shadow" model [1] but light systems at very low energies show anomalous properties so that the "shadow" point of view cannot be used to describe the fusion process [1,2]. In this paper we investigate these anomalous properties, and we show that it is possible to use the "shadow" point of view also at very low energies if we use, for the interacting charged particles, an effective Coulomb potential that is energy dependent.

II. THE "SHADOW" PROPERTIES

In a previous paper we suggested the "shadow" model for sub-barrier fusion [1]. By using this model the fusion process between charged particles can be considered as the shadow of the elastic scattering (Rutherford scattering), so that the fusion cross section can be written

$$\begin{aligned}\bar{\sigma}_f &= 2\pi\int_{\theta_f}^{\pi}\sigma_R(\theta)\sin(\theta)d\theta=\pi(\eta/k)^2\cot^2(\theta_f/2)\\ &=\pi R_f^2[1-2\eta/(kR_f)]\ ,\end{aligned} \tag{1}$$

where $\sigma_R(\theta)$ is a Rutherford differential cross section, the angle θ_f determines the "shadow" region $(\theta_f,\pi),\eta$ is the Coulomb parameter, R_f is the distance of closest approach relative to θ_f, and k is the wave number. By performing a phenomenological [1] analysis we obtained for R_f an analytical expression. In fact, if we define

$$y^{\rm ex}=\ln\left\{\ln\left[-\ln\left[\frac{R_f^{\rm ex}}{2\eta/k}-1\right]\right]\right\}\ , \tag{2}$$

where

$$\frac{R_f^{\rm ex}}{2\eta/k}-1=-\frac{1}{2}+\frac{1}{2}\left[1+\frac{\sigma_f^{\rm ex}k^2}{\pi\eta^2}\right]^{1/2} \tag{3}$$

and $\sigma_f^{\rm ex}$ is the experimental fusion cross section, we obtain that $y^{\rm ex}$ is a linear function of V_B-E, V_B is the Coulomb barrier energy and E is the center of mass energy, so that R_f can be written

$$R_f=(2\eta/k)(1+\exp\{-\exp[\exp(y)]\})\ , \tag{4}$$

$$y=(E_B-E)/E_s\ , \tag{5}$$

E_B,E_S are two parameters to be determined. We found that $E_B\sim V_B$ and E_S is a scale parameter such that

$$y^{\rm ex}\sim y\ . \tag{6}$$

By using Eq. (4), Eq. (1) becomes

$$\bar{\sigma}_f=\pi(2\eta/k)^2[1+G(y)]G(y) \tag{7}$$

with

$$G(y)=\exp\{-\exp[\exp(y)]\}\ . \tag{8}$$

We remark that from Eqs. (4) and (5) it follows that

$$\frac{R_f}{2\eta/k}=1+G(y)\ , \tag{9}$$

$$\frac{\bar{\sigma}_f}{\pi(2\eta/k)^2}=[1+G(y)]G(y)\ , \tag{10}$$

so that the ratios $R_f|(2\eta|k)$, $\bar{\sigma}_f|\pi(2\eta|k)^2$ are universal functions $(2\eta|k)$ is the minimum value of the distance of closest approach, it is obtained when the angular momentum is equal to zero (or $\theta=\pi$). We define the Eqs. (6), (9), and (10) as the "shadow" properties. We note that from the "shadow" properties it follows the validity of the "shadow" model. As shown in previous papers Eq. (7) is not able to reproduce the experimental values of fusion cross section for light systems at very low energies, so that we suggested [2] to modify Eq. (7) as follows:

$$\sigma_f=\bar{\sigma}_f[1-g(y)][1-g_1(y)]\ , \tag{11}$$

where

$$g(y)=\exp\left[-\left[\frac{d-y}{d-y_m}\right]^{\gamma_1'}2.789\right],$$

$$g_1(y)=\exp\left[-\left[\frac{d-y}{d-y_1}\right]^{\gamma_2'}2.789\right], \quad (12)$$

$$d=E_B/E_S,\quad y_m=(E_B-E_m)/E_S,\quad y_1=(E_B-E_1)/E_S .$$

The parameters $y_m, y_1, \gamma_1', \gamma_2'$ can be determined by fitting the experimental values of fusion cross section. In Ref. [2] we showed that Eqs. (11) and (12) are able to reproduce the experimental values of fusion cross section for light systems $A_1+A_2 \geq 3$ at very low energy $E \ll E_B$ in a satisfactory way. Now we want to investigate this reproducibility in the framework of the "shadow" model, or, in general, in the framework of the "shadow" properties, so we assume that the "shadow" point of view is still valid for light systems; i.e., the fusion process is the shadow of the Rutherford scattering, and Eq. (1) is true for the fusion cross section

$$\sigma_f=2\pi\int_{\theta_f}^{\pi}\sigma_R(\theta)\sin(\theta)d\theta=\pi(\eta/k)^2\cot\text{an}^2(\theta_f/2)$$
$$=\pi R_f[1-2\eta/(kR_f)] . \quad (13)$$

Then we determine the parameters $E_B, E_S, E_m, E_1, \gamma_1', \gamma_2'$ by fitting the experimental values of the fusion cross section with σ_f, see Eqs. (11) and (12), so that we can report the values of y^{ex}, see Eqs. (2) and (3), versus $y=(E_B-E)|E_S$. The results for the reactions ^{2}H$(p,\gamma)^3$He, ^{2}H$(d,p)^3$H, ^{2}H$(d,n)^3$He, ^{3}He$(d,p)^4$He, ^{3}He$(^3$He$,2p)^4$He, and ^{7}Li$(p,\alpha)^4$He are reported in Figs. 1–5. By inspection of these figures we can assert that for $E>\{E_1,E_m\}$ the "shadow" properties are fulfilled, so that the "shadow" model is able to describe the fusion process. For $E<\{E_1,E_m\}$ it is

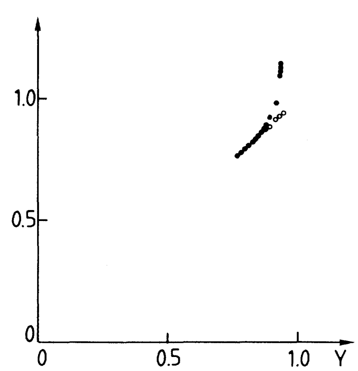

FIG. 1. Full dots: values of y^{ex} defined in Eqs. (2) and (3) versus $y=(E_B-E)/E_S$. The values of E_B and E_S are determined by fitting experimental data. Open dots: values of y'^{ex} defined in Eqs. (29) and (30) versus y. The reaction is ^{2}H$(p,\gamma)^3$He.

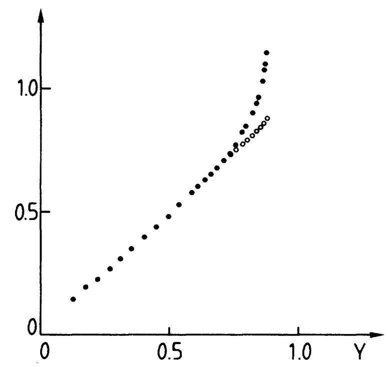

FIG. 2. Same as Fig. 1 for the reactions ^{2}H$(d,p)^3$H, ^{2}H$(d,n)^3$He.

$$y^{\text{ex}}\neq\frac{E_B-E}{E_S}, \quad (14)$$

$$\frac{R_f}{2\eta/k}-1=-\tfrac{1}{2}+\tfrac{1}{2}\{1+4[1+G(y)]G(y)\times[1-g(y)][1-g_1(y)]\}^{1/2}, \quad (15)$$

$$\frac{\sigma_f}{\pi(2\eta/k)^2}=[1+G(y)]G(y)[1-g(y)][1-g_1(y)] \quad (16)$$

so that the "shadow" properties are not fulfilled; in fact, Eq. (6) is not true and the ratios $R_f|(2\eta|k)$, $\sigma_f|\pi(2\eta|k)^2$ are not universal functions because the parameters $E_m, E_1, \gamma_1', \gamma_2'$ are different for different reactions, see Eqs. (15) and (16). Moreover, Eq. (15) gives the critical values of distance of closest approach. It is easy to see that in the framework of Rutherford scattering it is

$$\left.\begin{array}{l}2\eta/k, R_f\to\infty\\ \theta_f\to\pi\end{array}\right\}\ \text{when } E\to 0 , \quad (17)$$

so that the fusion process cannot be interpreted as the shadow of the elastic scattering because the strong interaction is a short range interaction, and the "shadow" point of view it is not able to describe the fusion process at low energies. From the above arguments it follows that Eq. (11) cannot be understood in the framework of the "shadow" model relative to Rutherford scattering.

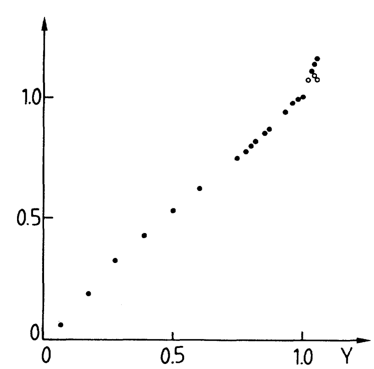

FIG. 3. Same as Fig. 1 for the reaction ^{3}He$(d,p)^4$He.

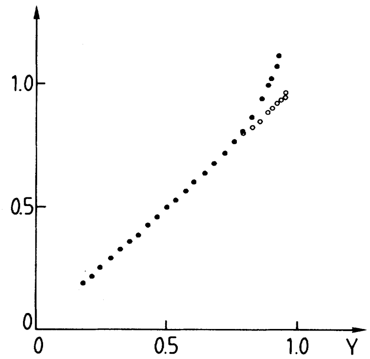

FIG. 4. Same as Fig. 1 for the reaction $^3He(^3He,2p)^4He$.

Now we remind the readers that in Ref. [3] we suggested defining an effective Coulomb potential,

$$V_C'(r,y)=\frac{Z_1Z_2e^2}{r}[1-g(y)]^{1/2}[1-g_1(y)]^{1/2} . \tag{18}$$

This potential is energy dependent; moreover, it is defined as

$$V_C'(r,y)=V_C(r)=\frac{Z_1Z_2e^2}{r} \tag{19}$$

for

$$E>\{E',E_m\}$$

and

$$\lim_{E\to 0} V_C'(r,y)=0 . \tag{20}$$

By using this effective Coulomb potential and the "shadow" point of view we can assert that the fusion process is the shadow of the elastic scattering relative to the potential $V_C'(r,y)$, so that the particles that fuse are those detected in the "shadow" region of the elastic scattering relative to $V_C'(r,y)$ and Eq. (1) becomes

$$\sigma_f=\int_{\theta_f'}^{\pi}\frac{d\sigma}{d\Omega}(\theta,y)\sin\theta\, d\theta , \tag{21}$$

where $d\sigma/d\Omega(\theta,y)$ is the elastic differential cross section relative to $V_C'(r,y)$; it is given [3] by

$$\frac{d\sigma}{d\Omega}(\theta,y)=\left[\frac{Z_1Z_2e^2}{4(E_B-yE_S)}\right]^2[1-g(y)][1-g_1(y)] \times\frac{1}{[\sin(\theta/2)]^4} , \tag{22}$$

θ_f' is the critical value of scattering angle, which defines the "shadow" region. We remind the readers that by using the effective Coulomb potential $V_C'(r,y)$ the singularity of the Rutherford cross section at $E=0$ disappears; in fact, it is

$$\lim_{E\to 0}\frac{d\sigma}{d\Omega}(\theta,y)=0 . \tag{23}$$

Now if $D'(l,y)$ is the distance of closest approach in the elastic scattering relative to the effective Coulomb potential $V_C'(r,y)$, we have

$$\theta=2\arcsin\frac{Z_1Z_2e^2[1-g(y)]^{1/2}[1-g_1(y)]^{1/2}}{2ED'(l,y)-Z_1Z_2e^2[1-g(y)]^{1/2}[1-g_1(y)]^{1/2}} \tag{24}$$

or

$$D'(l,y)=\frac{Z_1Z_2e^2[1-g(y)]^{1/2}[1-g_1(y)]^{1/2}}{2E} \times\left[\frac{1}{\sin(\theta/2)}+1\right] \tag{25}$$

so that Eq. (21) becomes

$$\sigma_f=\pi R_F'^2\left[1-\frac{Z_1Z_2e^2[1-g(y)]^{1/2}[1-g_1(y)]^{1/2}}{ER_f'}\right] , \tag{26}$$

where R_f' is the value of $D'(l,y)$ corresponding to θ_f'. The minimum value of the distance of closest approach $D'(l,y)$ it obtained when $l=0$ ($\theta=\pi$), and from Eq. (25) it follows

$$D'(0,y)=\frac{Z_1Z_2e^2[1-g(y)]^{1/2}[1-g_1(y)]^{1/2}}{E} , \tag{27}$$

so that Eq. (26) can be written

$$\sigma_f=\pi R_f'^2\left[1-\frac{D'(0,y)}{R_f'}\right] . \tag{28}$$

Equation (28) gives an expression of the fusion cross section obtained in the framework of the "shadow" model relative to the effective potential $V_C'(r,y)$. A crucial point of our approach is the determination of the critical value of the distance of closest approach R_f'. Now from Eq. (28) it follows that

$$\frac{R_f'}{D'(0,y)}-1=-\frac{1}{2}+\frac{1}{2}\left[1+\frac{4\sigma_f}{\pi[D'(0,y)]^2}\right]^{1/2} . \tag{29}$$

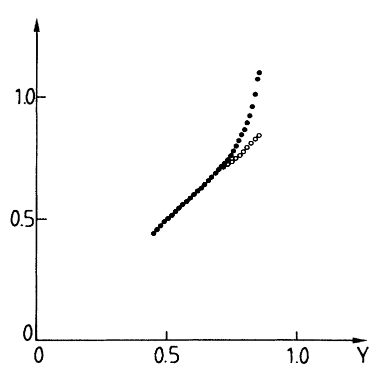

FIG. 5. Same as Fig. 1 for the reaction $^7Li(p,\alpha)^4He$.

Moreover, we define

$$y'^{\rm ex}=\ln\left\{\ln\left[-\ln\left[\frac{R'^{\rm ex}}{D'(0,y)}-1\right]\right]\right\}, \tag{30}$$

where $R'^{\rm ex}$ is obtained from Eq. (28) by setting $\sigma_f=\sigma_f^{\rm ex}$. Using the values of the parameters $E_B, E_S, E_m, E_1, \gamma'_1, \gamma'_2$ obtained by fitting the experimental values of the fusion cross section we report $y'^{\rm ex}$ versus y. The result is shown in Figs. 1–5. Inspection of these figures follows that

$$y'^{\rm ex}=y\ , \tag{31}$$

so that for R'_1 we have

$$\frac{R'_f}{D'(0,y)}=[1+G(y)]\ , \tag{32}$$

and Eq. (28) can be rewritten

$$\frac{\sigma_f}{\pi[D'(0,y)]^2}=[1+G(y)]G(y)\ . \tag{33}$$

From Eqs. (31), (32), and (33) it follows that the fusion process for light systems at very low energies ($E \ll E_B$) can be described in the framework of the "shadow" model relative to the effective Coulomb potential $V'_C(r,y)$ and the "shadow" properties are fulfilled.

III. CONCLUSIONS

From our analysis it stands out that for light systems at energies below the Coulomb barrier the fusion process can be described as the shadow of the Rutherford scattering, but at very low energies it is not valid. However, if we define an effective Coulomb potential, energy dependent, the fusion process can be described as the shadow of elastic scattering relative to this effective potential; moreover, some general properties defined as "shadow properties" are defined at very low energies by using this effective Coulomb potential. From the above arguments it follows that light systems at very low energies show an anomalous behavior and the effective Coulomb potential is a way to describe it. Finally, we believe that it will be of interest, in the future, to investigate processes as nuclear fusion or elastic scattering at very low energies by using experimental procedures or theoretical analyses to obtain information on anomalous behavior of light systems.

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[3] A. Scalia, Nuovo Cimento A **104**, 707 (1991).