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$^{16}O+Ag/Br$ interactions at 2.1 GeV/nucleon and some aspects of intermittency

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Different aspects of scaled factorial moment analysis have been performed with the pion data of $^{16}O+Ag/Br$ interactions at 2.1 GeV/nucleon with the objective of understanding the intermittency effect and its dependence on experimental parameters e.g., incident energy, phase space variable of analysis—its dimension and window size, multiplicity in the window. An overview of analyses carried out, so far, motivates the study. The phase space variable and multiplicity dependences of intermittency have been studied. While compared with other available higher energy data in $^{16}O+Ag/Br$ interactions, the energy dependence of intermittency, though indicative, could not be concluded due to inconsistent values of intermittency exponents in higher energy data of the same projectile-target analyzed by different groups.

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I. INTRODUCTION AND MOTIVATION

The study of intermittency, indicating self-similarity, in the multiproduction of particles in relativistic high-energy interactions began in 1986 with the work of Bialas and Peschanski [1]. They proposed to investigate the scaled factorial moments $F_q(\delta)$, of order q, in small phase space bins of size δ which are smaller than the usual resonance correlation length (say, δ_0). It was suggested that the self-similar "intermittent" fluctuations might lead to a powerlike increase of $F_q(\delta)$ at small δ ($<\delta_0$) range resulting,

$$\ln F_q(\delta) = -a_q \ln\delta + b_q \tag{1}$$

while the standard short range correlation (of order δ_0) study would not appreciate the increase of F_q for $\delta<\delta_0$.

Encouraged by the positive nonzero values of a_q found by the proposers in the cosmic ray event of the JACEE Collaboration [2], the study has already been made on data of pionization from different processes, initiated by leptons, hadrons, and nuclei. In all the analyses carried out, so far, the values of a_q were found to be nonzero positive indicating that intermittency could be a general phenomenon associated with the process of pionization. However, there is no conclusive theoretical interpretation to the effect, as yet. Beside the "progressive" thoughts of "intermittency and fractals" leading to self-similarity, "conservative" interpretations in the framework of conventional approaches also exist as a main source of the effect [3]. Again, different conventional approaches have been countered with observations from the available experimental data [4].

At this stage, for a better understanding of the effect, one needs more experimental data on the intermittency analysis in terms of the strength of the effect, the range of its validity and the kinematic variables in which the scaling holds. Further, major inconsistencies, in the values of the slope parameters from different experiments, demand a study of the dependence of the effect on experimental parameters. Chiu and Hwa, in a model dependent analysis of e^+e^- annihilation, have proposed to study the dependences on the incident energy $\sqrt{s}$, rapidity window Y, and multiplicity N in window—collectively referred as sYN [5]. They showed that intermittency has sYN dependence, a syndrome that must be fully recognized for meaningful comparison of intermittency data. In this article, we analyze different aspects of intermittency of our pionization data of ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon. Our analysis may be helpful in understanding the so-called "sYN syndrome" in high energy ^{16}O + Ag/Br interactions. We do not analyze the rapidity (Y) and the $\sqrt{s}$ energy dependences in a strict sense. Analyses have been carried out in different phase space variables, including (pseudo)rapidity, and in different dimensions. To study the incident energy dependence, we compare our data with other available data of ^{16}O + Ag/Br interactions, in terms of E_{lab}. Further, we do not study the "sYN syndrome" only and we do not restrict our discussion in analyses of heavy-ion data only. In fact, an overview on available experimental data on intermittency analyses motivates the present study and also facilitates a convenient comparison.

In the interpretation of the effect observed in the cosmic-ray JACEE event [2], it was expected that a complex object such as a drop of quark-gluon plasma has been formed and subsequently that drop decays through an intermittent cascade to final-state hadrons [6]. In the accelerator energy range, the effect was first observed by the KLM Collaboration [7] in the pseudorapidity distribution of proton-emulsion and nucleus-nucleus collisions. After that, the effect of intermittency was observed in e^+e^- [8–13], muon-hadron [14], hadron-hadron [15,16], hadron-nucleus [17], and more data on nucleus-nucleus

[17–23] interactions. Thus the intermittency patterns are not the privilege of nucleus-nucleus interactions and neither is their possible description in terms of plasma formation—as one might have thought at the introduction of intermittency through analysis of JACEE events [6].

As the experimental situation stands, intermittency appears to be a *universal feature* in multiparticle production physics. Most of the analyses have been carried out in rapidity (y) or pseudorapidity (η) phase space and increase in moments within the range $1.0 > \delta y$, or $\delta\eta > 0.1$ is well fitted with the power law given by Eq. (1). The slopes of plots of $\ln\langle F_q\rangle$ vs $-\ln\delta y$ (or $-\ln\delta\eta$), i.e., the intermittency exponents (a_q), increase with increasing order (q) of moment.

The general features of analysis in pseudorapidity phase space were found to prevail, while analyses in other variable, such as *azimuthal angle* (ϕ) *phase space*, were carried out [9,14–16,20]. Except for the muon-proton interaction at 280 GeV [14], other data reveal the effect in ϕ space also. Again, except for π^+p and k^+p interaction data [15], the normalized factorial moments in the ϕ space usually show stronger behavior of intermittency than that in (pseudo)rapidity space.

The effect of intermittency in two-dimensional phase space was found to be stronger than that for both the one-dimensional analyses in e^+e^- annihilation [12]. In hadron-hadron interaction data, the effect in two dimensions is stronger than that in one-dimensional analysis in any of the variables, rapidity (y), pseudorapidity (η), and azimuthal angle (ϕ) [15]. In muon-proton [14] and nucleus-nucleus [17,20] interactions, the effect in two-dimensional analysis is stronger compared to that in pseudorapidity space. But compared to azimuthal angle analysis, in nucleus-nucleus collisions [20], the effect is weaker in ^{32}S + Ag/Br interaction at $200A$ GeV and in ^{16}O + Ag/Br interactions at $60A$ and $200A$ GeV for a_2 and a_3. For a higher order of moments, in ^{16}O + Ag/Br data, the two-dimensional intermittency effect is also stronger than the ϕ-distribution analysis.

Multiplicity dependence of intermittency effects has been studied in $p\bar{p}$ collisions by the UA1 Collaboration. For UA1 data, the slopes a_q decreases with increasing multiplicity. However, such a decrease does not agree with different Monte Carlo calculations.

Anomalous fractal dimensions d_q [24] have also been calculated by different experimental groups. The anomalous dimensions have a tendency to grow linearly with increasing rank of the moment [25]. A notable exception was observed in the ^{32}S + Ag/Br central collision data of the KLM Collaboration (EMU07) [17] at $200A$ GeV. Also, the $p\bar{p}$ data at $\sqrt{s} = 630$ GeV [18] show only a small increase of d_q with increasing q. However, heavy-ion data of the same energy and projectile (and with the same selection criteria) of EMU08 experiment [20] contradicts the data of the EMU07 experiment. Of course, the EMU08 data [20] are well supported with the analysis of central ^{16}O + Ag and ^{32}S + Ag interaction data at 200 GeV/nucleon, selected according to high values of E_T by the HELIOS-Emulsion Collaboration [19]. The results from analysis of 200 GeV/nucleon of ^{16}O + (C,Au) interaction data of the WA80 Collaboration [21] cannot be compared, as the correction factor for single-particle rapidity distribution was not considered in the analysis [26].

A detailed phenomenological study on F-moment analysis in nucleus-nucleus interactions at an ultrarelativistic energy range has been carried out by the EMU01 Collaboration [22]. Using data of ^{16}O + Emulsion at $14.6A$, $60A$, and $200A$ GeV, ^{28}Si + emulsion at 14.6 A GeV and ^{32}S + emulsion and gold at $200A$ GeV, the energy, target, projectile, and multiplicity dependence of intermittency effects have been studied in high energy nuclear interactions. Plotting the intermittency indices a_q versus incident energies of ^{16}O + emulsion interactions it was shown in Ref. [22] that the intermittency effect decreases with an increase in energy for the same reactants. Further, the intermittency effect increases with increasing target/projectile mass. The effect decreases with increasing multiplicity. Thus, the dependence of intermittency effect on multiplicity in nuclear interaction is similar to that in hadronic interactions.

The commonly used *models of particle production*, e.g., LUND (JETSET), HERWIG, FRITIOF, PYTHIA, GENCL, VENUS DPM etc, implemented into the well known Monte Carlo codes were tried by different groups. Most groups claim that their intermittency effect cannot be correctly simulated by Monte Carlo generators. However, the recent DELPHI analysis [11] shows that the version 6.2 of LUND parton shower model gives a quite satisfactory description of moments of e^+e^- annihilation at the CERN LEP energy.

II. EXPERIMENTAL DATA

In carrying out the phenomenological analysis on ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon, the data were obtained by irradiating horizontally, an Ilford G5 emulsion stack consisting of pellicles of dimensions $10\times5\times0.06$ cm^3 with the ^{16}O beam of Bevelac, Berkeley, at 2.1 GeV/nucleon incident energy. The flux of incident beam was $\sim10^4$ ions/cm^2. The sensitivity of the emulsion was found to be high enough to give visible tracks of relativistic singly charged particles (6.6 grains per 100 μm).

To obtain minimum-biased statistics, each photoemulsion has been area scanned as well as line scanned under a Leitz-Ortholux microscope provided with a Brower traveling stage. The preliminary scanning was done using a $10\times$ objective in conjunction with a $25\times$ ocular. The events of interactions after 1 cm from the leading edge were identified. The measuring system, connected with the microscope, has along the x and y axes 1 μm resolution, while that along the z axis is 0.5 μm.

Each event, forming a "star" in the emulsion, was scanned at least by two independent observers. Events satisfying the following criteria were recorded for further analysis: (a) The interactions were beyond 20 μm thickness from the top or bottom surface of the processed pellicle. (b) The incident beam must be at angle $< 3°$ (in the projected or in the azimuthal plane) to the mean beam direction within the pellicle. (c) To ensure that the sample of events, finally accepted, contained primary interactions only and not the interactions from the secondary tracks of other interactions in the same pellicle, all the primary tracks were followed back.

Finally each of the selected events was examined under a 100× oil immersion objective for the identification and measurement of angles of the secondary tracks.

In emulsion experiments, the secondary tracks are classified according to their energy characteristics. (i) Black tracks are laid by particles having velocities $\beta \leq 0.2$, range (R) less than 3 mm, and grain density (g) greater than $6g_0$, where g_0 is the plateau grain density. The number of black tracks in an event is denoted by n_b. (ii) Grey tracks are laid by particles with velocities $0.2 \leq \beta \leq 0.7$. The range is usually greater than 3 mm and the grain density $1.4g_0 \leq g \leq 6g_0$. These are often assumed to be knocked out protons with energy range 20–400 MeV. The number of grey tracks emitted in an event is denoted by n_g. N_h denotes the number of heavy tracks, i.e., sum of numbers of black and grey tracks, given by $N_h = n_b + n_g$. A heated target nucleus emits low energy fragments of heavy particles which are identified in emulsion as heavy tracks. (iii) The shower tracks are laid by the relativistic charged particles with $\beta \geq 0.7$, produced in the high energy relativistic interactions. These particles are mostly pions and the grain density of the shower tracks is $g \leq 1.4g_0$. (iv) The projectile fragments with charge greater than 1 constitute a distinct class of tracks, which have constant grain density value over track length of about 2 cm of ionization. These particles, being of relativistic energy, have very long range (generally they are not confined within a plate unless they undergo a secondary interaction). These tracks are confined within a narrow space angle with respect to the incident beam direction. To have an estimate of this space angle we have made use of the relation $\theta_{\mathrm{pf}} < \theta_c$ $(=0.2/P_{\mathrm{lab}})$ where P_{lab} is the laboratory momentum of the incident projectile.

For identification of secondary charged particles, the grain density of tracks was measured with a length >3 μm, near the star, and in at least one point more than 2 cm from the star. For all nonrelativistic tracks, grain density measurements were performed near the star.

Emission angles (θ) and azimuthal angles (ϕ) of all particles were determined by measuring coordinates of the interaction center and of points on the incident and secondary tracks near the point of interaction. The coordinate, normal to the emulsion plane (i.e., along the z axis), was corrected for the reduction of the thickness of the emulsion during processing (i.e., shrinkage factor). The emission angles of tracks due to relativistic particles were more carefully determined by measuring at least three points along the produced particles as well as along the projectile tracks.

In interactions with emulsion nuclei one can divide the targets into three groups: H, C/N/O, and Ag/Br. A target separation into three groups can be made by counting the number of charged particles with $\beta < 0.7$ $(N_h = n_b + n_g)$. Events with $N_h = 1$ are taken to come from colliding with the H target, those with $N_h \leq 8$ are taken to come from colliding with C, N, or O nuclei in emulsion while large N_h values $(N_h > 8)$ are associated with Ag or Br targets. Those events with $N_h > 15$ and no forward projectile fragments of charge $Z \geq 2$ are nucleus + Ag/Br central collisions. Using the criteria for central ^{16}O + Ag/Br collisions, as mentioned above, 731 such events were selected for our analyses.

III. METHOD OF ANALYSIS AND RESULTS

For identification of nonstatistical fluctuations in multiparticle production in our nuclear interaction data of ^{16}O + Ag/Br at 2.1 GeV/nucleon, we chose pseudorapidity (η) phase space as variable, and the scaled factorial moments (SFM) [1], or the F moment,

$$F_q = M^{q-1} \sum_{j-1}^{M} n_j(n_j-1)\cdots(n_j-q+1)/\langle n\rangle^q \qquad (2)$$

for a given positive q, as the tool for analysis of each event. M is the number of bins (of size $\delta\eta$) into which the initial available pseudorapidity phase space $(\Delta\eta)$ is divided. n_j is the number of charged secondaries falling within the jth bin, j running from 1 to M, and $\langle n\rangle$ is the average multiplicity (over all events of the data sample) of charged secondaries within $\Delta\eta$.

For a given q, the horizontally analyzed F moments [given by Eq. (2)], calculated for all events, are vertically averaged over events to obtain $\langle F_q\rangle$. For different q values $\langle F_q\rangle$ are calculated.

Figure 1 presents the whole phase-space pseudorapidity distribution of our data sample of ^{16}O + Ag/Br central and quasicentral events at the incident energy of 2.1 GeV/nucleon. The average multiplicity of produced charged pions of the sample being $\langle n\rangle = 13.00 \pm 0.20$.

The F moments for $q=2$, 3, and 4 have been calculated within the range of pseudorapidity space, $\Delta\eta = 4.0$, around the peak of the pseudorapidity distribution. For

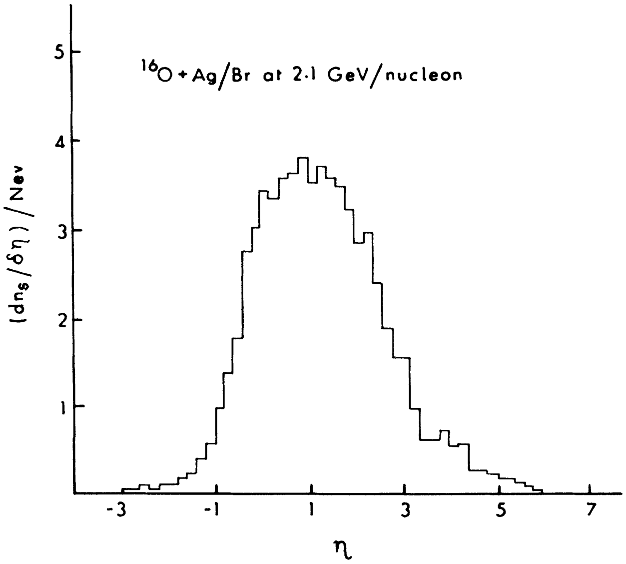

FIG. 1. Pseudorapidity distribution of produced charged pions in ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon.

studying the variation of the $\langle F_q \rangle$ with the size of bin, $\delta\eta$, the range $1.0 \geq \delta\eta \geq 0.1$ was selected by dividing the phase space to a maximum number, $M=40$. The nuclear emulsion, the detector (also the target) in our experiment, covers 4π geometry and provides very good accuracy, even less than 0.1 mrad, in angle measurement of produced particles with respect to the projectile beam axis due to high spatial resolution. Emulsion is thus a suitable detector for studying fluctuations in finely resolved intervals of pseudorapidity phase space. The dependence of $\langle F_q \rangle$ on the size of bin $\delta\eta$ is presented in Fig. 2, plotting $\ln\langle F_q \rangle$ against $-\ln\delta\eta$, for $q=2$, 3, and 4, along with corresponding best fit curves.

The positive slopes of the plots, as evident in Fig. 2, show the qualitative agreement with the power law

$$\langle F_q \rangle \propto (\delta\eta/\Delta\eta)^{-a_q} \propto M^{a_q} , \tag{3}$$

where M is the number of bins, indicating self-similarity in the distribution of produced charged pions.

As can be seen from Fig. 1, the pseudorapidity distribution is not flat within the region $\Delta\eta=4.0$. Therefore the correction needed for a nonflat shape of particle distribution in the phase space by dividing $\langle F_q \rangle$ by the factor [26]

$$R_q = \frac{1}{M} \sum_{j=1}^{M} M^q \langle n_j \rangle^q / \langle n \rangle^q , \tag{4}$$

where

$$\langle n_j \rangle = \frac{1}{N_{\text{event}}} \sum_{i}^{N_{\text{event}}} n_{j,i} ,$$

giving the reduced scaled factorial moments (RSFM)

$$\langle F_{qR} \rangle = \frac{\langle F_q \rangle}{R_q} \tag{5}$$

has also been made.

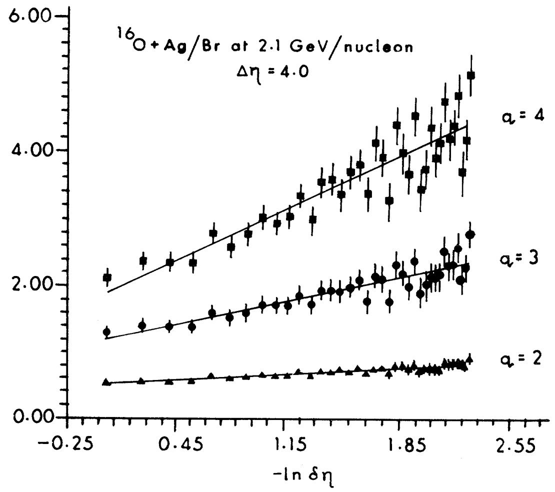

FIG. 2. Plot of $\ln\langle F_q \rangle$ vs $-\ln\delta\eta$ for produced charged pions in ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon in $\Delta\eta=4$.

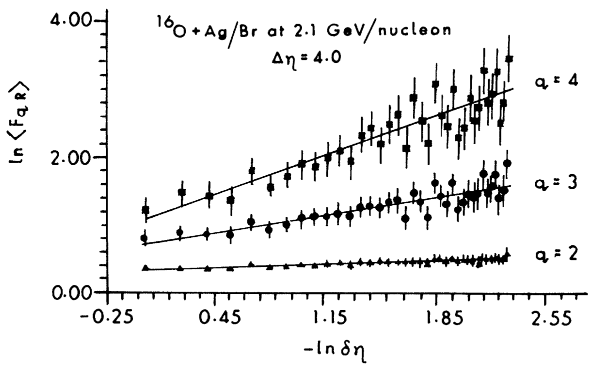

FIG. 3. Plot of $\ln\langle F_{qR} \rangle$ vs $-\ln\delta\eta$ for produced charged pions in ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon in $\Delta\eta=4$.

In Fig. 3, the variations in the values of $\ln\langle F_{qR} \rangle$ with $-\ln\delta\eta$ are plotted for $q=2$, 3, and 4. The slopes, here, are corrected intermittency exponents and, therefore, can be used for quantitative analysis.

Once the power law is observed to be valid in the data, indicating the self-similarity property, the analysis of data in terms of fractal parameters [27] is imperative. The generalized dimensions D_q of fractals and multifractals have been related [24] to the intermittency exponents as

$$D_q = 1 - a_q/(q-1) , \tag{6}$$

where

$$d_q = a_q/(q-1) \tag{7}$$

is known as anomalous fractal dimensions. Although this connection between intermittency and fractality or multifractality has not been fully understood, as yet, attempts have been made to interpret data in terms of anomalous dimensions. As suggested by Bialas and Hwa [25] if a second-order phase transition takes place from a quark-gluon plasma to a hadron phase in the thermodynamic equilibrium, then the produced particles will show intermittency with anomalous dimensions d_q independent of q. On the other hand, if hadronization takes place through the cascading process, d_q is expected to be linear in q.

The anomalous dimensions have been calculated for the data of ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon and are presented in Table I.

TABLE I. Anomalous dimensions d_q calculated from the intermittency exponents a_q for produced pions in pseudorapidity phase of the ^{16}O + Ag/Br interaction at 2.1 GeV/nucleon.

Order of Moment q	Anomalous fractal dimensions d_q in η distribution of ^{16}O + Ag/Br interactions 2.1 GeV/nucleon
2	0.091±0.010
3	0.187±0.020
4	0.276±0.032

We find from the plot of $\ln\langle F_q\rangle$ vs $-\ln\delta\eta$ in Fig. 2 that $\langle F_q\rangle$ exhibits a rise, consistent with the power-law [Eq. (3)] dependence upon $\delta\eta$, which is characteristic of intermittency. The other features, such as an increase in the slope or the intermittency exponents, a_q with increasing order of moment, are also depicted in the figure. After the correction for nonflat single-particle pseudorapidity distribution is made, we observe that (in Fig. 3) the characteristics of intermittency exist, though the effect gets reduced. In any case, the existence of intermittency or the self-similarity in produced pions in the data set is obvious. In terms of speculation by Bialas and Hwa [25], the dependence of values of anomalous dimensions d_q on the order of moment, q, as shown in Table I, indicates the cascading process of hadronization.

Some other aspects of study of intermittency have been considered in the analysis of our data.

A. Analysis of data in different $\Delta\eta$

To see the effect of the size of initially considered phase space $\Delta\eta$ on the scaled factorial moment as well as on the power law given by Eq. (3), in terms of intermittency exponents, we carried out the same analysis with $\Delta\eta=2.0$ around the peak of the distribution, within the same range of $\delta\eta$, i.e., $1.0 \geq \delta\eta \geq 0.1$. Figure 4 represents the power-law behavior through plots of $\ln\langle F_q\rangle$ vs $-\ln\delta\eta$ for $q=2$, 3, and 4 and the corresponding best fits in $\Delta\eta=2.0$. The characteristics of intermittency prevail in $\Delta\eta=2.0$ also. Comparison of Figs. 2 and 4 reveals that the value of moments depend on the considered $\Delta\eta$ interval, with increased values of moments in larger $\Delta\eta$. The slopes a_q are not sensitive to $\Delta\eta$. However, the values of a_q at smaller $\Delta\eta$ within which the distribution is more flat are slightly less.

The effect of $\Delta\eta$ on the moments and intermittency exponents is reduced in the plots of the reduced scaled factorial moments, $\ln\langle F_{qR}\rangle$ vs $-\ln\delta\eta$, which is depicted in a comparison of Figs. 3 and 5, with Fig. 5 representing the behavior of the reduced scaled factorial moments with the size of $\delta\eta$ in $\Delta\eta=2.0$. This nondependence on size of $\Delta\eta$ in the case of F_{qR} calculation, is more clear from Table II, which shows very little difference in the slopes as well as in intercepts.

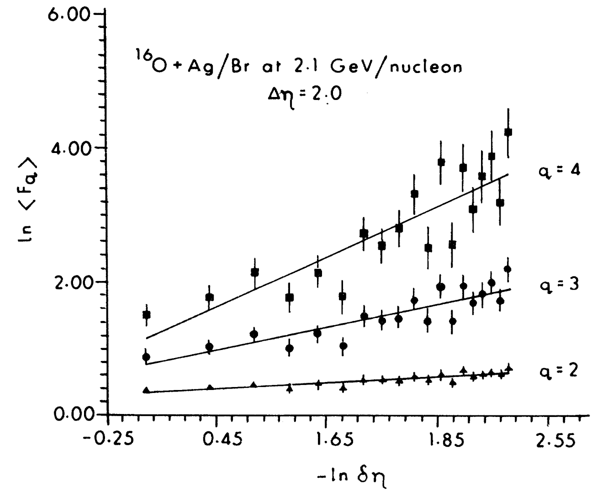

FIG. 4. Plot of $\ln\langle F_q\rangle$ vs $-\ln\delta\eta$ for produced charged pions in ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon in $\Delta\eta=2$.

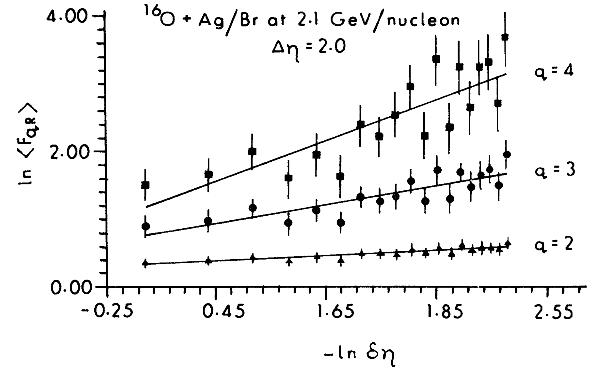

FIG. 5. Plot of $\ln\langle F_{qR}\rangle$ vs $-\ln\delta\eta$ for produced charged pions in ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon in $\Delta\eta=2$.

B. Analysis in azimuthal angle (ϕ) phase space

With our data of the ^{16}O + Ag/Br interaction at 2.1 GeV/nucleon, the intermittency analysis was performed using the azimuthal angle (ϕ) space around the beam axis as the variable for produced charged particles in $\Delta\phi=2\pi$, within the range of $\pi \geq \delta\phi \geq 12°$. Here also, we have used the similar correction factor R_q to eliminate extra M dependence factor.

From the plots of $\ln\langle F_{qR}\rangle$ vs $-\ln\delta\phi$ in Fig. 6, the validity of the power law in ϕ space is obvious, as the general features of plots in Fig. 6 are similar to those of Fig. 2–5. The intermittency effect appears to be stronger in ϕ space than that in η space as can be understood in terms of the slope parameters given in Table II.

C. Analysis in two-dimensional phase space

The concept of multidimensional analysis predicts a more pronounced rise of moments in two-dimensional analysis than that in one-dimensional analysis [28]. We study the power-law behavior of our data in a two-dimensional space (η-ϕ) of pseudorapidity and azimuthal angle.

Scaled factorial moments, in two-dimensions, for cells of areas ($\delta\eta\delta\phi$), are calculated in a similar way as is given by Eq. (2) for one-dimensional analysis. In this case, the summation is extended over $M=M_\eta M_\phi$, where M_η and M_ϕ are respectively the number of divisions the phase spaces pseudorapidity and azimuthal angle are divided. n_j, here, is the number of particles in the jth cell of area ($\delta\eta\delta\phi$). The analysis was performed by subdividing successively and simultaneously each of the two intervals, $\Delta\phi=2\pi$ and $\Delta\eta=4.0$. Thus we divide the $\Delta\eta$-$\Delta\phi$ space into $M=M_\eta M_\phi$ cells of size $\delta\eta\delta\phi$ where $\delta\eta=\Delta\eta/M_\eta=4/M_\eta$ and $\delta\phi=\Delta\phi/M_\phi=2\pi/M_\phi$, with

TABLE II. Values of the intermittency exponents for produced charged pions from the ^{16}O + Ag/Br interaction at 2.1 GeV/nucleon from analyses with different variables and conditions.

Variable (initial size)	Moment	a_2	a_3	a_4
η ($\Delta\eta=4.0$)	SFM	0.141 ± 0.015	0.504 ± 0.045	1.090 ± 0.087
η ($\Delta\eta=4.0$)	RSFM	0.091 ± 0.010	0.373 ± 0.041	0.827 ± 0.098
η ($\Delta\eta=2.0$)	SFM	0.137 ± 0.013	0.494 ± 0.046	1.065 ± 0.093
η ($\Delta\eta=2.0$)	RSFM	0.091 ± 0.011	0.373 ± 0.044	0.830 ± 0.099
ϕ ($\Delta\phi=2\pi$)	RSFM	0.184 ± 0.021	0.607 ± 0.058	0.983 ± 0.078
η-ϕ ($\Delta\eta=4.0, \Delta\phi=2\pi$)	RSFM	0.353 ± 0.035	0.785 ± 0.102	1.088 ± 0.130

maximum values of M_η and M_ϕ being 34.

The dependence of the factorial moments on the area of the cell for our data of ^{16}O + Ag/Br is presented in Fig. 7 by plotting $\ln\langle F_{qR}\rangle$ vs $-\ln(\delta\eta\delta\phi)$. As is evident from the values of the slopes in Table II, the intermittency effect is found to be stronger in two-dimensional analysis than those observed in both the one-dimensional analyses.

D. Scaling law from Central limit theorem

The hadronization process using the self-similar cascade model, approximated by central limit theorem [1], predicts the dependence of the intermittency exponents on the order of moment to scale as

$$a_q = a_2 \frac{q(q-1)}{2} . \tag{8}$$

The scaling law was proposed to test directly for the existence of a cascading process. Further, validity of this scaling law suggests that the cascading is due to the validity of the central limit theorem.

In a later consideration, however, Alberty and Bialas [29] pointed out that the central limit theorem, not being a suitable approximation to a distribution far from its average value, does not generally describe the tails of the distribution. On the other hand, intermittency effects are known to be sensitive to very large deviations from the average.

The validity of the scaling law given by Eq. (8) was tested in our data by calculating $2a_q/q(q-1)$. The values are given in Table III. It is evident from Table III that the scaling [Eq. (8)] does not hold for our multipion production data of 2.1 GeV/nucleon ^{16}O + Ag/Br interactions.

E. Multiplicity dependence of a_q

The multiplicity dependence of the intermittency exponents in terms of particle density $(dn/d\eta)$ are predicted to follow the scaling law, known as the Bialas plot [30],

$$a_q \propto (dn/d\eta)^{-1} \tag{9}$$

In recent studies, however dependence of a_q on multiplicity has also been phenomenologically studied in nucleus-nucleus [23], hadron-hadron [17] interaction data

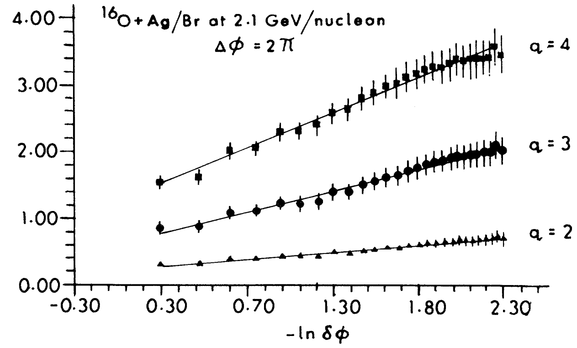

FIG. 6. Plot of $\ln\langle F_{qR}\rangle$ vs $-\ln\delta\phi$ for produced charged pions in ^{16}O + Ag/Br interaction at 2.1 GeV/nucleon data in $\Delta\phi=2\pi$.

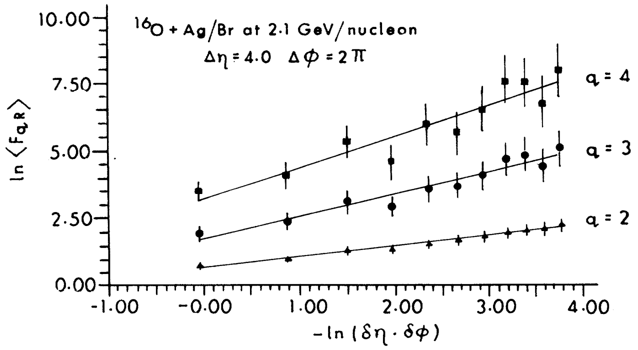

FIG. 7. Plot of $\ln\langle F_{qR}\rangle$ vs $-\ln(\delta\phi\delta\phi)$ for produced charged pions in ^{16}O + Ag/Br interaction at 2.1 GeV/nucleon.

TABLE III. Values of $2a_q/q(q-1)$ for produced charged pions in the ^{16}O + Ag/Br interaction at 2.1 GeV/nucleon.

Order of moment q	Factor $2a_q/q(q-1)$
2	0.091 ± 0.010
3	0.124 ± 0.014
4	0.138 ± 0.016

with selection of different subsamples applying cuts in multiplicity.

To see the multiplicity dependence of intermittency exponents for our data of ^{16}O + Ag/Br interactions at 2.1 GeV/nucleons, we choose different subsamples out of our data sample. Different subsamples with varied $\langle n_s \rangle$ (average multiplicity of charged pions within $\Delta\eta$) were obtained applying different cuts in a number of shower particles. The detail of subsamples are given in Table IV. Figure 8 shows the multiplicity dependence of a_q. As evident from the figure, the values of a_q for subsample with higher $\langle n_s \rangle$ is lower than corresponding values for subsample with lower $\langle n_s \rangle$ value.

The errors bars of F_q, shown in the figures, represent the standard deviations over all events and all bins used. Errors of a_q have been obtained through fitting procedure. However, these are not correct estimation of errors. For a given order, the moment values for different $\delta\eta$ bins are correlated. Also, different orders of moment for same $\delta\eta$ are correlated. So it is difficult to estimate the correct values of errors for intermittency exponents. Though different approaches for error calculations have been prescribed [14,16,30], none of them has been claimed to give a correct estimation.

IV. DISCUSSION

Since production of particles at available energies is finite, the statistical fluctuations due to distribution of particles in large number of bins (as $\delta\eta \rightarrow 0$) are obvious. Using a Bernoulli distribution, it was shown in Ref. [1] that the average scaled factorial moments are equal to the moments of a true probability distribution of particle density, without the expected statistical bias. The study of the scaled factorial moments thus enables one to extract short-range dynamical fluctuations in the distribution of produced particles, disentangling the normal statistical noise. Our analysis in terms of F moments reveals nonstatistical fluctuations in the distribution of produced pions in ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon. Primarily, the observations of our analysis are qualitatively identical with those from analyses of other available data and so comparison with results from other data is meaningful.

(1) From a comparison of the intermittency exponents of earlier data of different types of interactions the hierarchy of $a_q(e^+e^-) > a_q(hh) > a_q(AA)$, where hh is a hadron-hadron collision and AA a nucleus-nucleus collision, appeared to be a general feature of intermittency study. With the availability of our data of ^{16}O + Ag/Br at 2.1 GeV/nucleon, it is indicated that the hierarchy cannot be considered as a general phenomenon, irrespective of incident energy.

(2) The plot of anomalous fractal dimensions (d_q) against the order of moments (q) in Fig. 9 predicts self-similar cascading in particle production in our ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon. The nature of the plot agrees with what is expected at this energy range, when compared with other data [25].

(3) The dependence of the intermittency exponents on the size of the initial phase space ($\Delta\eta$), in our case, is qualitatively identical with those in nucleus-nucleus interaction data of the KLM Collaboration [17] and e^+e^- annihilation data of the HRS Collaboration [9]. However, p + Ag/Br data differ in this respect [17]. Analysis of our data in different $\Delta\eta$ further confirms the necessity for the correction factor, R_q.

(4) In all the available heavy-ion interaction data, the effect of intermittency is stronger in azimuthal angle phase space and that is true for our data at 2.1 GeV/nucleon incident energy, also. However, this increase in the intermittency effect in the ϕ distribution is not always observed in hadron-induced interactions.

(5) The effect of intermittency in two-dimensional phase space is stronger than that in rapidity or pseudorapidity phase space for most of the data, while the same is weaker compared to that in azimuthal angle analysis for heavy-ion data of the EMU08 Collaboration, at incident energies of 60 and 200 GeV/nucleon. In our heavy-ion data at 2.1 GeV/nucleon the effect in two-dimensional space is stronger than the effect in both of the one-dimensional analyses. However, an important aspect regarding two-dimensional analysis needs a mention here. In a recent analysis by the EMU01 Collaboration [30], it has been shown that even a small background of particle pairs with a narrow opening angle can distort the observed effect in two-dimensional analysis. Using FRITIOF simulations, the influence of γ conversion was estimated for ^{32}S + Au and ^{16}O + emulsion interactions at ultrarelativistic energy. While the systematic errors introduced by γ conversion are observed to influence the one-dimensional data partially, the two-dimensional data are strongly influenced.

(6) The scaling law as suggested from the central limit

TABLE IV. Data from study of multiplicity dependence of the intermittency exponents.

Multiplicity cut	$\langle n_s \rangle$	a_2	a_3	a_3
$n_s > 8$	13.38 ± 0.25	0.084 ± 0.011	0.362 ± 0.047	0.793 ± 0.103
$n_s > 12$	15.92 ± 0.32	0.067 ± 0.009	0.334 ± 0.050	0.741 ± 0.111

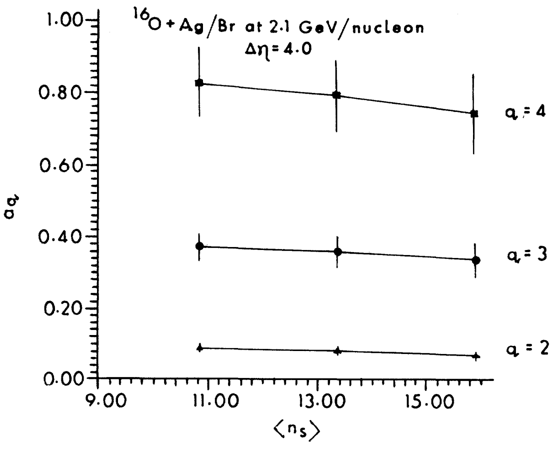

FIG. 8. Plot of a_q vs $\langle n_s \rangle$ showing the dependence of the intermittency exponents on multiplicity.

theorem, with respect to dependence of the intermittency exponents on the order of the moment, while found to agree with the data at energy as high as 200 GeV/nucleon in heavy-ion interactions [20,22], fails with data at low energy and/or low multiplicity. The deviation from the scaling law was observed in EMU01 data [22] of ^{16}O + emulsion interactions at 14.6 and 60 GeV/nucleon and in our data of ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon.

(7) The dependence of the intermittency exponents on the average multiplicity in the considered phase space, i.e., with a sample of different multiplicity ranges in our data of ^{16}O + Ag/Br at 2.1 GeV/nucleon, is similar to such analysis with EMU01 data of ^{32}S + emulsion at 200 GeV/nucleon and with UA1 data $p\bar{p}$ interaction at $\sqrt{s}=630$ GeV. For comparison of multiplicity dependence of a_q of our analysis, in the pseudorapidity variable, with that of other interaction data, we use Bialas Plots [31] in Figs. 10(a) and 10(b) for a_2 and a_3 respectively. Previous analyses revealed that though the scaling law given by Eq. (9) agree well with hadron-hadron data, the a_q values for the heavy-ion reactions (of incident energy $\geq 60A$ GeV) are by a factor ≈ 20 higher than expected (for $dn/d\eta > 10$). It is evident from Figs. 10(a) and 10(b) that the "discrepancy" is observed in our heavy-ion data (at $2.1A$ GeV) also, even though the $dn/d\eta$ for our data is less than 10.

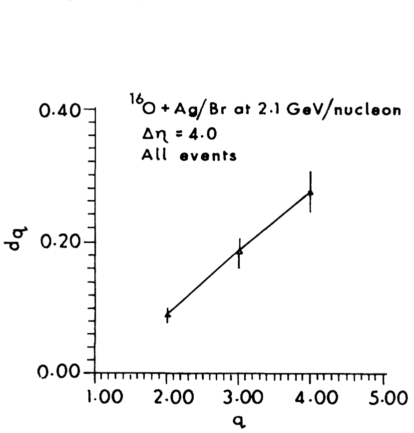

FIG. 9. Plot of anomalous fractal dimensions d_q (obtained from analysis with pseudorapidity variable) versus order of moments q for charged pions produced from ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon.

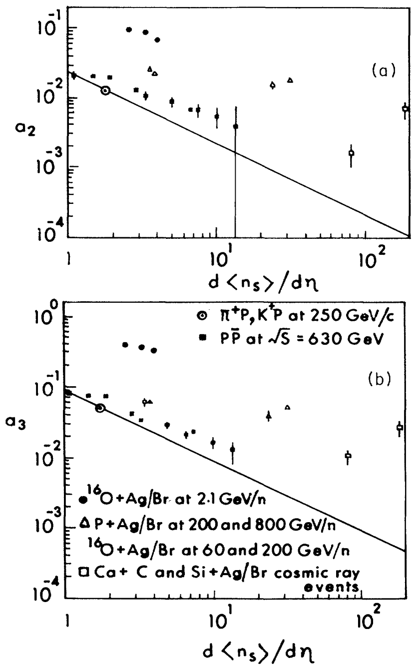

FIG. 10. Bialas plots for $q=2$ and 3, comparing the dependence of the intermittency exponents on multiplicity.

(8) For the energy dependence of the intermittency exponents, in ^{16}O + Ag/Br interactions, we have EMU07, EMU08, and HELIOS Collaboration data to compare with our results. As already discussed, despite identical experimental parameters, the results of EMU07 data do not match those from the other two results for $200A$ GeV ^{16}O + Ag/Br interactions. Further, the EMU08 data show, for ^{16}O + Ag/Br interaction data at energies $60A$ and $200A$ GeV, that the values of intermittency exponents are practically the same at almost every order of q for the η distribution. Analyses in ϕ and $\eta\phi$ variables with the same experiment, however, show higher values for intermittency exponents with higher energy data. This observation in the EMU08 data is in contradiction to the general trend of energy dependence of intermittency exponents in other available data, including the EMU01 Collaboration data of ^{16}O + emulsion interactions. Our data at 2.1 GeV/nucleon, in good agreement with the general trend, give higher values of intermittency exponents compared to those from any higher energy data of any of the experiments with the same reactants. However, nothing conclusive on the energy dependence of the intermittency exponents for ^{16}O + Ag/Br interactions can be drawn with the prevailing anomaly in highest available energy data and without going for the analysis of more data with varied energies.

In this situation, from the cosmic-ray energy range via the highest available laboratory energy range, the study of intermittency in pionization in nuclear interactions has been extended to the comparatively lower energy data. This article primarily concludes with a positive indication of intermittency phenomenon in the distribution of produced pions in ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon, extending the energy range of validity of "intermittency".

Finally, the study of self-similarity or multifractality in high-energy physics is quite promising and is still an open question. There is plenty of work to be done before one can assess the real meaning of the phenomenon of intermittency. This article, by extending the phenomenon in pionization in nuclear interactions in an entirely different energy range, contributes a little towards understanding the phenomenon.

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