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Spin dependent spectral function of ^{3}He and the asymmetry in the process $^3\vec{\text{He}}(\vec{e},e')X$

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The asymmetry for quasielastic inclusive scattering of polarized electrons by polarized ^{3}He has been computed for the first time in terms of a spin dependent spectral function obtained from a realistic three-body wave function without using the closure approximation. It is shown that a proper choice of kinematics minimizes the proton contribution to the asymmetry, which therefore becomes very sensitive to different models of the electromagnetic form factors of the neutron. The calculated asymmetry qualitatively agrees with the available experimental data.

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Quasielastic (QE) electron scattering by nuclei has proven to be a very useful tool for investigating both nucleon and nuclear properties. The advent of new experimental facilities, allowing systematic measurements with polarized targets and beams, will provide more detailed information on hadronic systems than the measurement of spin averaged cross sections [1]. Of particular relevance are experiments on ^{3}He, since this nucleus could be viewed as an effective neutron target. Indeed, the proton pair in ^{3}He is mainly in an $^1S_0$ state with opposite proton spins and therefore the proton contribution to the spin dependent part of the cross section largely cancels out [2,3]. Polarization experiments on ^{3}He have already been performed [4,5] or are being planned [6] in order to study extremely relevant physical quantities, like the spin dependent structure functions of the nucleon and the neutron electromagnetic form factors.

In the seminal work by Blankleider and Woloshin [2] the asymmetry for the reaction $^3\vec{\text{He}}(\vec{e},e')X$ was computed by using a *spin dependent momentum distribution,* i.e., adopting the closure approximation. In order to improve the description of polarized cross sections, one has to go beyond the closure approximation by properly taking into account both the removal energy and momentum distributions of the nucleon, which requires the knowledge of the *spin dependent spectral function.* We have obtained the latter quantity and used it to calculate the cross sections of the process $^3\vec{\text{He}}(\vec{e},e')X$. The aim of this Rapid Communication is to illustrate to what extent the asymmetry is affected by (i) the closure approximation, (ii) the proton contribution, and (iii) different models for the nucleon electromagnetic form factors. A comparison with available experimental data at low momentum transfer will be presented.

In the one-photon-exchange approximation, the QE inclusive scattering cross section of linearly polarized electrons by polarized ^{3}He is given by

$$\sigma(\omega,q,h,\mathbf{S}_A)=\sigma_M\sum_{j=p,n}N_j\left\{W_2^j+2\tan^2\frac{\theta}{2}W_1^j+2h\tan^2\frac{\theta}{2}\left[\frac{G_1^j}{M_A}(\mathbf{k}_i+\mathbf{k}_f)+2\frac{G_2^j}{M_A^2}(\varepsilon_i\mathbf{k}_f-\varepsilon_f\mathbf{k}_i)\right]\cdot\mathbf{S}_A\right\}, \tag{1}$$

where ω and $q=|\mathbf{q}|$ are the energy and momentum transfers, respectively, h the incident electron helicity, $\mathbf{S}_A$ the target polarization vector, σ_M the Mott cross section, $N_{p(n)}$ the number of protons (neutrons), W_1^j and W_2^j the usual unpolarized nuclear structure functions, G_1^j and G_2^j the polarized structure functions, $(\varepsilon_i,\mathbf{k}_i)$ and $(\varepsilon_f,\mathbf{k}_f)$ the four-momentum of the incident and final electrons, respectively, and θ the scattering angle. If the vector $\mathbf{S}_A$ is taken to lie within the scattering plane and the target polarization angle is measured with respect to the direction of the incident electron ($\cos\beta=\hat{\mathbf{k}}_i\cdot\mathbf{S}_A$), the right-hand side of Eq. (1) reduces to the expression given in Ref. [2], whereas, if the polarization angle is measured with respect to the momentum transfer ($\cos\theta^*=\hat{\mathbf{q}}\cdot\mathbf{S}_A$), the expression of Refs. [1,4,5] is recovered. In the following, the vector $\mathbf{S}_A$ is always assumed to lie within the scattering plane.

In plane-wave impulse approximation (PWIA) the polarized structure functions $G_1^j(q,\omega)$ and $G_2^j(q,\omega)$ are given by

$$\frac{G_1^j}{M_A}=\pi\int_{E_{\min}}^{E_{\max}}dE\int_{k_{\min}}^{k_{\max}}kdk\frac{(F_1^j+F_2^j)F_1^j}{qE_k}\left[\left(\frac{\omega}{q}-\frac{k\cos\alpha}{E_k+M}\right)k\mathcal{P}-MP_{\parallel}^j\right], \tag{2}$$

$$\frac{G_2^j}{M_A^2}=\pi\int_{E_{\min}}^{E_{\max}}dE\int_{k_{\min}}^{k_{\max}}kdk\frac{(F_1^j+F_2^j)}{qE_k}\left[\left[E_kP_\parallel^j-\frac{k^2\cos\alpha}{E_k+M}\mathcal{P}\right]\frac{F_2^j}{2M}-k\mathcal{P}\frac{F_1^j}{q}\right], \tag{3}$$

where the limits of integration, $E_{\min}$, $E_{\max}(q,\omega)$, $k_{\min}(q,\omega,E)$, $k_{\max}(q,\omega,E)$, and $\cos\alpha=\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}$ are obtained from the energy conservation (see, e.g., Ref. [7]), E is the removal energy, $E_k=[M^2+k^2]^{1/2}$, and $F_{1(2)}^j$ the Dirac (Pauli) nucleon form factor. In obtaining Eqs. (2) and (3) the relativistic scattering of a polarized electron by an on-shell, polarized nucleon with momentum $\mathbf{k}$ has been assumed. The real functions $P_\parallel^j(k,E,\alpha)$ and

$$\mathcal{P}(k,E,\alpha)=[\sin\alpha P_\perp^j(k,E,\alpha)+\cos\alpha P_\parallel^j(k,E,\alpha)]$$

can be expressed in terms of the spin dependent spectral function $P_{\sigma,\sigma',M}^j(\mathbf{k},E)$ as follows:

$$P_\parallel^j=P_{1/2,1/2,M}^j(\mathbf{k},E)-P_{-1/2,-1/2,M}^j(\mathbf{k},E),$$

$$P_\perp^j=2P_{1/2,-1/2,M}^j(\mathbf{k},E)e^{i\phi},$$

where ϕ is the polar angle, which describes the rotation of $\mathbf{k}$ around $\mathbf{q}$ [the factor $e^{i\phi}$ cancels the ϕ dependence of the nondiagonal part of $P_{\sigma,\sigma',M}^j(\mathbf{k},E)$]. The spin dependent spectral function can be cast in the following form

$$P_{\sigma,\sigma',M}^j(\mathbf{k},E)=\sum_{f_j}\langle\mathbf{k},\sigma|\langle\Psi_{A-1}^f|\Psi_A^{JM}\rangle(_j\langle\mathbf{k},\sigma'|\langle\Psi_{A-1}^f|\Psi_A^{JM}\rangle)^*\delta(E-E_{A-1}^f+E_A)\quad(j=p,n), \tag{4}$$

where Ψ_A^{JM} and Ψ_{A-1}^f are the intrinsic wave functions of the target nucleus in the ground state and of the $(A-1)$ nucleon system in the state f, with eigenvalues E_A and E_{A-1}^f, respectively, f denotes the complete set of quantum numbers of the $(A-1)$ nucleon system, and $|\mathbf{k},\sigma\rangle_j$ is the eigenstate of the free jth nucleon with momentum $\mathbf{k}$ and spin σ. It can be seen that, unlike the spin averaged spectral function, the spin dependent spectral function $P_{\sigma,\sigma',M}^j(\mathbf{k},E)$ depends on σ, σ', M, as well as on the direction of $\mathbf{k}$. The overlap integrals $_j\langle\mathbf{k},\sigma|\langle\Psi_{A-1}^f|\Psi_A^{JM}\rangle$ appearing in Eq. (4) have already been employed for the calculation of the spin averaged spectral function [8] and correspond to the Reid soft-core interaction [9].

If the closure approximation is adopted [$E_{\max}=\infty$ and $E=\bar{E}$ in the integration limits $k_{\min}(q,\omega,E)$ and $k_{\max}(q,\omega,E)$ in Eqs.(2) and (3)], only the spin dependent nucleon momentum distribution is needed and the expressions for G_1 and G_2 given in Ref. [2] can be recovered.

In this Rapid Communication, the longitudinal asymmetry $A=(\sigma^+-\sigma^-)/(\sigma^++\sigma^-)$, with $\sigma^\pm=\sigma(\omega,q,h=\pm1,\mathbf{S}_A)$, is calculated by using Eqs. (1)–(3). Figures 1–3 show the dependence of the asymmetry upon the closure approximation, the proton contribution, and different models of the nucleon form factors. Figure 1(a) illustrates that, at low values of the momentum transfer ($q=400$–460 MeV/c, corresponding to the available experimental data [4,5]), the closure approximation does not give an accurate description of the asymmetry; at the top of the QE peak (corresponding to the nuclear scaling variable $y=0$ [7]), it can differ up to 15% from the full calculation depending on the value of the target polarization angle (e.g., for $20°<\theta^*<100°$), whereas for $|y|>50$ MeV/c large differences (of the order of 100%) can be found. At higher momentum transfers ($q>1$ GeV/c), these differences persist for $|y|\neq0$, whereas they disappear at the top of the QE peak ($y=0$), where the closure approximation becomes a very accurate one.

In Refs. [4,5] the asymmetry was measured around the top of the QE peak in the θ^* kinematics (i.e., assuming $\hat{\mathbf{q}}$ as the reference direction) at $\theta^*\approx0$ and $\theta^*\approx90°$, with the aim of obtaining information on G_M^{n2} and on $G_M^nG_E^n$, respectively. The experimental data, listed in Table I, agree with our theoretical results. Unfortunately, the present large experimental uncertainties do not allow one to discriminate between different nucleon form factors. It should be pointed out that a quantitative comparison between experimental data and theoretical calculations, at low momentum transfer [$Q^2=q^2-\omega^2<0.3$ (GeV/c)^2] makes necessary the evaluation of the final-state interaction [13]. Calculations are in progress and will be reported elsewhere [14].

The proton contribution to the asymmetry is illustrated in Fig. 1(b). Although the contribution of the protons, which is one order of magnitude larger than the neutron contribution in the total cross section, substantially cancels out in the asymmetry, it is not negligible even at the top of the QE peak and can be very large for $y\neq0$: for example, in the case $\varepsilon_i=1500$ MeV, $\theta=60°$,

TABLE I. The values of the asymmetry (%), measured in Refs. [4] and [5], compared with the results of the present calculations [see Eqs. (1)–(3)], obtained with the form factors of Refs. [10] (GK), [11] (H), and [12] (BZ), respectively.

Q^2 (GeV/c)^2	θ^*	Expt. (%)	Ref.	GK (%)	H (%)	BZ (%)
0.16	$\approx104°$	$2.38\pm1.27\pm0.44$	[4(b)]	2.7	2.5	2.4
0.20	$\approx6°$	$-3.49\pm1.23\pm0.54$	[4(a)]	-4.1	-4.0	-3.9
0.20	$\approx90°$	$1.75\pm1.22\pm0.35$	[5]	2.6	2.3	2.1
0.20	$\approx3°$	$-2.6\pm0.9\pm0.5$	[5]	-3.8	-3.8	-3.7

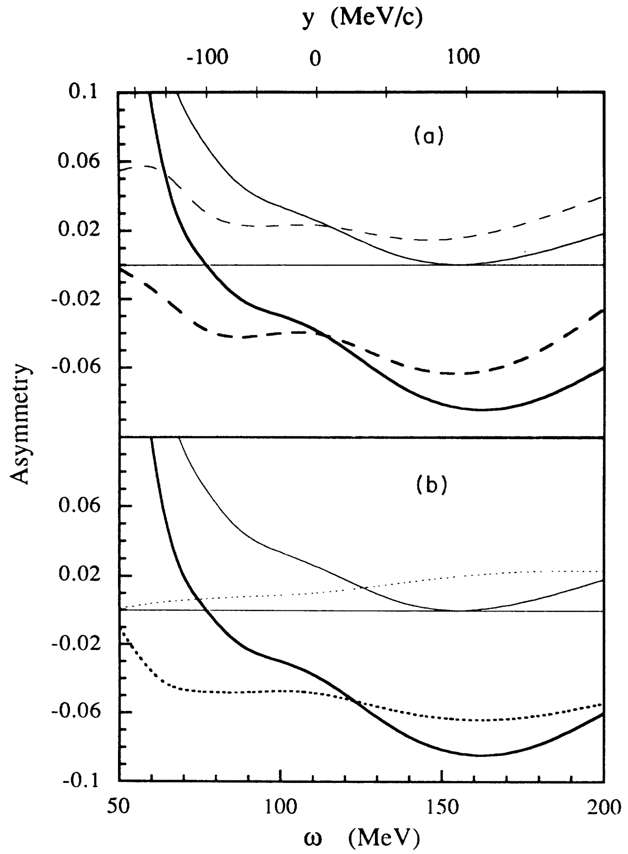

FIG. 1. The asymmetry for $\varepsilon_i = 574$ MeV and $\theta = 51.1°$, calculated at $\theta^* = 0$ (thick lines) and $\theta^* = 90°$ (thin lines), vs the energy transfer ω and the scaling variable y. The solid lines in (a) and (b) represent the full calculation, based on Eqs. (2) and (3), for the total asymmetry. The dashed lines in (a) represent the total asymmetry in closure approximation, whereas the dotted lines in (b) represent the full calculation for the neutron contribution. The form factors of Ref. [10] have been used.

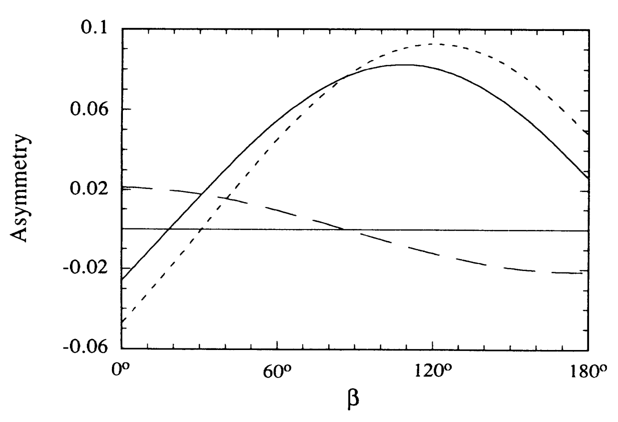

FIG. 2. The asymmetry [evaluated using Eqs. (2) and (3)] at the top of the QE peak ($y = 0$), for $\varepsilon_i = 1000$ MeV and $\theta = 60°$, vs the target polarization angle β. Solid line: total asymmetry; long-dashed line; proton contribution; short-dashed line: neutron contribution. The form factors of Ref. [10] have been used.

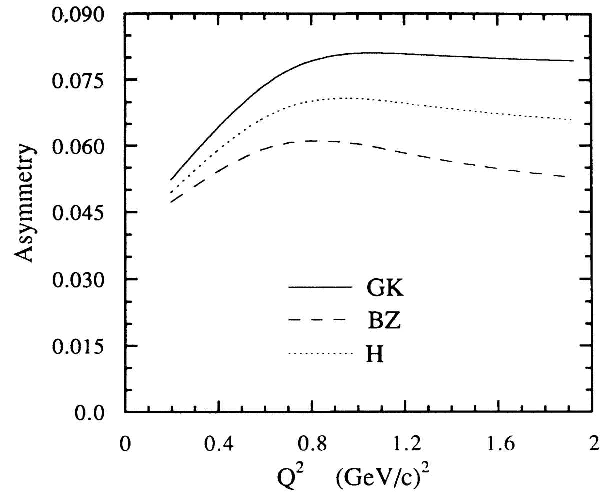

FIG. 3. The asymmetry [evaluated using Eqs. (2) and (3)] at the top of the QE peak ($y = 0$) for $\theta = 60°$ and $\beta = 85°$ with the form factors of Ref. [10] (solid line), Ref. [11] (dotted line), and Ref. [12] (dashed line) vs Q^2.

and $\theta^* = 0$, the absolute value R of the ratio of the proton and neutron contributions is larger than 5 for $y < -350$ MeV/c and $y > 300$ MeV/c, independently of the different form factors. This behavior is clear, since the S state in the ^{3}He wave function is no longer the dominant component in the wings of the QE peak, where high momentum components play an important role. It should be pointed out that by using the spin dependent spectral function the proton contribution to the polarized structure functions does not vanish even considering only the most likely $^1S_0$ state for the proton pair in ^{3}He. Moreover, we have found [14] an overall underestimation of the proton contribution within the closure approximation. However, the proton contribution to the asymmetry at $y = 0$ can be minimized by a proper choice of kinematics and polarization. In Fig. 2 the asymmetry at the top of the QE peak, calculated with the electromagnetic form factors of Ref. [10], is shown, together with the proton and neutron contributions, vs the polarization angle β. The β kinematics, where $\hat{\mathbf{k}}_i$ is assumed as the reference direction, has the remarkable feature that at $\beta \approx 85°$ there is essentially no proton contribution, and, more important, such a pattern holds for any value of the momentum transfer in the range 0.5 GeV/$c < q < 2$ GeV/c, independently of the different models of the form factors that we have investigated (Refs. [10]–[12]). On the contrary, in θ^* kinematics, there is a remarkable change with q of the polarization angle where the proton contribution becomes negligible.

In Fig. 3 the asymmetry at the top of the QE peak is shown vs the four-momentum transfer for $\beta = 85°$ (no proton contribution) and for the form factors of Refs. [10]–[12]. A sensible dependence upon the neutron form factors can be observed for $Q^2 > 0.6$ (GeV/c)^2.

In summary, the spin dependent spectral function of ^{3}He corresponding to the Reid soft-core interaction has been obtained and used for a detailed investigation of the asymmetry for the reaction $^3\vec{\mathrm{He}}(\vec{e},e')X$. Our results show that (i) the closure approximation, as expected, is a poor one for $y\neq0$; (ii) although the polarization of ^{3}He is mostly due to the neutron, the contribution of the proton to the asymmetry can be very large and it is underestimated by the closure approximation; (iii) the choice of the β kinematics allows one to select a value of the polarization angle β for which the proton contribution at the top of the QE peak can be made negligible in a large kinematical region, so that the asymmetry is entirely governed by the neutron form factors, and therefore, valuable information on the latter could be obtained by $^3\vec{\mathrm{He}}(\vec{e},e')X$ experiments.

[1] T.W. Donnelly and A. S. Raskin, Ann. Phys. (N.Y.) **169**, 247 (1986)

[2] B. Blankleider and R. M. Woloshyn, Phys. Rev. C **29**, 538 (1984).

[3] J. L. Friar, B. F. Gibson, G. L. Payne, A. M. Bernstein, and T. E. Chupp, Phys. Rev. C **42**, 2310 (1990).

[4] (a) C. E. Woodward *et al.*, Phys. Rev. Lett. **65**, 698 (1990); (b) C. E. Jones-Woodward *et al.*, Phys. Rev. C **44**, R571 (1991).

[5] A. M. Bernstein, Few-Body Systems Suppl. **6**, 485 (1992).

[6] J. Beaufays *et al.*, Report No. CERN/SPSC 88-47 (1988); SPSC/P242 (1988); R. Arnold *et al.*, SLAC Proposal E142 (1989); K. Coulter *et al.*, Report No. DESY/PRC 90/1, (1990); L. Dick *et al.*, Report No. CERN/LEPC 88-16 (1988); Report No. LEPC/89-10 (1989).

[7] C. Ciofi degli Atti, E. Pace, and G. Salmè, Phys. Rev. C **43**, 1155 (1991).

[8] C. Ciofi degli Atti, E. Pace, and G. Salmè, Phys. Lett. **141B**, 14 (1984).

[9] R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).

[10] M. Gari and W. Krumpelmann, Z. Phys. A **322**, 689 (1985); Phys. Lett. B **173**, 10 (1986).

[11] G. Hoehler *et al.*, Nucl. Phys. **B114**, 505 (1976).

[12] S. Blatnik and N. Zovko, Acta Phys. Austriaca **39**, 62 (1974).

[13] J. M. Laget, Phys. Lett. B **199**, 493 (1987); E. Van Meijgaard and J. A. Tjon, *ibid.* **228**, 307 (1989).

[14] C.Ciofi degli Atti, E. Pace, and G. Salmè, in preparation.