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Low-spin identical bands in the N_pN_n scheme

M. Saha and S. Sen
Saha Institute of Nuclear Physics, Calcutta—700064, India
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The occurrence of low-spin identical bands in the rare-earth region is shown to be the manifestation of a more general property of nuclear excitation mechanism, i.e., a smooth dependence of the moment of inertia on a very simple function of the valence proton and neutron numbers.

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Recent discovery [1–4] of superdeformed rotational bands in adjacent even and odd mass nuclei having almost identical gamma transition energies has generated considerable interest. Several explanations were put forward [3–6] assuming the occurrence of such identical bands to be a specific property of the superdeformed states of nuclei. However, careful and systematic analysis of available experimental data on high- and low-spin excited states of normally deformed even and odd mass nuclei has shown [7–10] that occurrence of such identical bands is not necessarily related to the phenomenon of superdeformation or excitation of very-high-spin states in nuclei. Casten *et al.* [10] have recently shown that low-spin identical bands occur in sets of widely dispersed nuclei, such as ^{156}Dy-^{180}Os, i.e., spanning as much as 24 mass units. Moreover, it is also found that a simple correlation exists between the nuclei showing identical excitation spectra and their valence proton (N_p), neutron numbers (N_n). In fact, the similarity between the excitation spectra of pairs of rare-earth nuclei having identical N_pN_n products as well as $|N_p-N_n|$ values like ^{160}Er-^{168}Hf, ^{158}Dy-^{170}Hf, etc. is so striking that it is a bit surprising that the phenomenon has not received sufficient attention earlier. Another interesting aspect of the problem is the recent discovery of the existence of low-spin identical bands in neighboring odd-A and even-even nuclei. Baktash *et al.* [9] have shown that in normally deformed rare-earth nuclei, around 30% of odd-Z nuclei at low spin have moments of inertia nearly identical to that of the neighboring even-even nucleus with one less nucleon. The traditional picture of nuclear pair correlations predicts variations of about 15% in the moments of inertia of configurations differing by one unit of seniority. In short, the phenomenon of identical bands has assumed a more general character spanning a very wide region of nuclear deformation and spin.

The purpose of the present article is to show that the existence of low-spin identical bands in the well deformed rare-earth region is a manifestation of a more general phenomenon, i.e., almost linear dependence of the moment of inertia on a very simple function of the valence proton and neutron number. Casten [11] has shown earlier that a simple and economical classification of the changes in nuclear structure in the transitional region can be obtained in terms of the product of valence proton and neutron numbers (N_pN_n). It is generally assumed that the moment of inertia has a mass dependence, although the dependence may not be as simple as that expected in case of a rigid rotor moment of inertia. The existence of identical bands in nuclei differing as much as 24 mass units obviously points out that the low-spin moment of inertia cannot be directly related to nuclear masses. Moreover, it is also expected on the basis of the conventional nuclear models that the moment of inertia of any nucleus will be determined by the detailed nature of residual interactions (both pairing and long range) between the valence nucleons of that nucleus and the nature of occupation of single particle orbitals involved. This expectation also appears to be belied to a great extent by the observed phenomena. Since the nuclei having identical N_pN_n and $|N_p-N_n|$ values are found to have identical moment of inertia, the function

$$f(N_pN_n)=N_pN_n(N_p+N_n)$$

naturally appears to be a parameter which is related not only to the absolute value of ground state moment of inertia but also to its angular momentum dependence. In Table I, we have listed a number of even rare-earth nuclei having $Z=66$–76 and $N=90$–106 and the calculated $f(N_pN_n)$ values for each of them. In the sixth column, the excitation energies of the 2^+ states have been calculated (with ^{164}Yb as standard) assuming a simple linear dependence of the moment of inertia ($\mathcal{J}$) on the factor $f(N_pN_n)$, i.e.,

$$\mathcal{J}\propto f(N_pN_n)\ .$$

It is interesting to note that for all nuclei having $f(N_pN_n)<5000$, the experimental excitation energies of the first 2^+ states [12] are nicely reproduced assuming such a simple relationship. In fact, the deviation is found to be well within 10% of the experimental energy. For nuclei having $f(N_pN_n)>5000$, the calculated excitation energies are found to be systematically lower than their corresponding experimental energies. In fact the higher the value of $f(N_pN_n)$, the greater the deviation. It shows that these nuclei are gradually approaching the region of saturated collectivity [13]. Even this gradual approach of saturated collectivity can be suitably parametrized in terms of the factor $f(N_pN_n)$. It can easily be seen that of all the rare-earth nuclei considered here, the nucleus $^{170}_{68}$Er$_{102}$ has the largest value of this factor (referred to as structure factor, SF, in subsequent discussion), i.e., $f(N_pN_n)_{max}=9520$. If we assume a linear dependence of the moment of inertia on $f(N_pN_n)$ over the entire region of nuclei under study, then the calculated E_{2^+} for ^{170}Er nucleus comes out to be 45 keV. On the other hand, the minimum value for E_{2^+} in this region has been found to be equal to 73 keV in ^{164}Dy. If we take this fact into account, it will appear that the moments of inertia of these nuclei are reduced approximately by a factor

$$\left[1+\frac{\mathrm{SF}}{(\mathrm{SF})_{max}}\right]^{-1},$$

which we denote as saturation parameter (SP). We have calculated the E_{2^+} excitation energies of these nuclei (with ^{174}Yb as standard) by assuming a functional dependence of the moment of inertia

$$\mathcal{J}\propto f(N_pN_n)\left[1+\frac{\mathrm{SF}}{(\mathrm{SF})_{max}}\right]^{-1},$$

and obtained good agreement with the experimental values (shown in the seventh column of Table I). The E_{2^+} excitation energies of all the nuclei under study can be calculated assuming this functional dependence. This will naturally alter to a certain extent the nature of agreement, but still in more than 70% cases, the agreement will remain well within 10%. However, so far as the correlation between the occurrence of identical bands and their N_pN_n values are concerned, certain differences can be pointed out between these two groups of nuclei. Therefore, we would prefer to classify these nuclei into two broad categories and discuss the phenomenon of identical bands separately for these groups. It may be mentioned here that if instead of the factor $f(N_pN_n)$, we use the product N_pN_n as a parameter and calculate the moment of inertia, then the agreement becomes somewhat worse. On the other hand, comparable agreement can be achieved by using a modified function like

$$f'(N_pN_n)=(N_pN_n)^{0.5}(N_p+N_n)^{1.66}.$$

We think that these details may be of some interest for future studies.

TABLE I. Calculated and experimental 2^+ excitation energies. Theo. I and II denote energies calculated without and with saturation parameter, SP (defined in text), respectively.

				E_{2^+} (keV)		
					theo.[b]	
Nucl.	A	SF[a]	SP	expt	I	II
Dy	156	3072		138	140	
Dy	158	4160		99	103	
Er	158	2464		192	174	
Er	160	3360		126	128	
Yb	160	1920		243	223	
Er	162	4368		102	98	
Yb	162	2640		166	162	
Yb	164	3456		124	124	
Yb	166	4368		102	98	
Hf	166	2640		159	162	
Hf	168	3360		124	128	
Hf	170	4160		100	103	
W	174	3744		112	114	
W	176	4480		109	96	
Os	178	3120		132	137	
W	180	4480		104	96	
Os	180	3696		132	116	
Os	182	3120		127	137	
Dy	160	5376	0.640	87	80	102
Dy	162	6720	0.586	80	64	89
Dy	164	8192	0.537	73	52	80
Er	164	5488	0.634	91	78	101
Er	166	6720	0.586	81	64	89
Er	168	8064	0.541	80	53	80
Yb	168	5376	0.640	88	80	102
Yb	170	6480	0.595	84	66	91
Er	170	9520	0.500	78	45	74
Yb	172	7680	0.553	79	56	83
Hf	172	5040	0.654	95	85	106
Yb	174	8976	0.515	76	48	76
Hf	174	6000	0.613	91	71	95
Yb	176	7680	0.553	82	56	83
Hf	176	7040	0.573	88	61	87
Hf	178	6000	0.613	93	71	95
W	178	5280	0.643	106	81	103

[a]$\mathrm{SF}=f(N_pN_n)$.
[b] ^{164}Yb and ^{174}Yb are used as standards for sets I and II, respectively.

It has already been pointed out by Casten *et al.* [10] that not all the rare-earth N_pN_n multiplets show identical bands. On the other hand, several nuclei are found to exhibit identical bands which are not expected to behave as such if one sticks strictly to this N_pN_n scheme of classification. Nuclei belonging to the first group are found to strictly follow this scheme, i.e., the nuclei having nearly equal values of SF show identical excitation spectra. In fact, although Casten *et al.* have considered ^{180}Os in conjunction with ^{156}Dy for identification of identical bands, according to the present classification the excitation spectrum of the nucleus ^{178}Os should be closer to that of ^{156}Dy as their SF values are nearly equal. In Table II, we have listed the first few transition energies of ^{156}Dy, ^{178}Os, and ^{180}Os and it can be seen that they are almost identical in ^{156}Dy and ^{178}Os. The transition energies of ^{158}Dy, ^{166}Yb, and ^{180}W are also listed in Table II to demonstrate a close correlation between the SF values and the excitation spectra. In fact, in this category, only the nucleus ^{172}W is found to deviate to a certain extent from this general behavior because although the nuclei ^{156}Dy and ^{172}W have identical SF values, their excitation spectra are not as identical as observed in other pairs in this region like ^{160}Dy-^{168}Hf, ^{158}Dy-^{170}Hf, ^{162}Er-^{166}Yb, and ^{160}Dy-^{168}Yb.

In the second group of nuclei, such strict adherence to the N_pN_n scheme is not always found. For example, the nuclei ^{164}Dy and ^{168}Er have almost identical SF values but there is a considerable difference (compared to the ^{166}Yb-^{180}W pair) between their transition energies. On the other hand, although the SF values of ^{164}Dy and ^{174}Yb differ to a considerable extent, their transition energies are quite close to each other. Similarly, if one looks

TABLE II. Experimental transition energies in some selected nuclei.

Nucl.	A	$f(N_pN_n)$	Transition energies (keV)[a] 2→0	4→2	6→4	8→4
Dy	156	3072	137.8	266.3	366.2	445.4
Os	178	3120	132.1	266.3	363.1	432.9
Os	180	3696	132.0	276.5	386.6	462.2
Dy	158	4160	98.9	218.3	320.6	406.2
Yb	166	4368	102.4	228.1	337.5	430.3
W	180	4480	103.6	234.0	350.9	450.0
Dy	164	8192	73.4	168.9	259.0	342.4
Er	168	8064	79.8	184.3	284.6	379.6
Yb	174	8976	76.5	176.6	272.9	363.5
Dy	162	6720	80.6	185.0	282.8	372.8
Yb	170	6480	84.2	193.2	296.1	390.0
Yb	172	7680	78.7	181.6	279.5	371.7
Yb	176	7680	82.1	189.6	293.1	389.3

[a]Reference [12].

at the SF values of the nuclei ^{162}Dy,^{170,172,176}Yb, one would expect the pairs (^{172} Yb,^{176}Yb) and (^{162}Dy, ^{170}Yb) to show identical excitation spectra. Contrary to expectation, the pairs (^{162}Dy, ^{172}Yb) and (^{170}Yb,^{174}Yb) show almost identical transition energies, although according to the criterion given by Casten *et al.* [10], all the four bands can be considered to be identical.

A systematic analysis of the available experimental data [12] in the rare-earth region will show that, in general, the percentage change in transition energies in neighboring even isotopes decreases with increasing angular momentum. It is found that the low-spin dynamical moments of inertia of these rare-earth nuclei, defined as

$$\frac{\hbar^2}{2\mathcal{J}_2}=\frac{E_\gamma(4^+\rightarrow 2^+)-E_\gamma(2^+\rightarrow 0^+)}{8},$$

show an approximate functional dependence

$$\mathcal{J}_2 \propto [f(N_pN_n)]^{1/2}.$$

The calculated values of $\hbar^2/2\mathcal{J}_2$ for several rare-earth nuclei belonging mainly to the first group are listed in Table III and compared with their corresponding experimental values. It can be seen that this functional dependence gives quite close agreement for the Dy, Er, Yb, and Hf isotopes. In W and Os nuclei the agreement is not so good. This observation can be related to the existence of strikingly similar bands in Dy, Er, Yb, and Hf nuclei having identical or nearly identical SF values.

In conclusion, it may be emphasized that the occurrence of identical bands in widely dispersed nuclei in the rare-earth region can be considered to be the manifestation of a more general property of nuclear excitation mechanism in this region, i.e., a smooth dependence of the moment of inertia on a simple function of the valence proton and neutron numbers. It appears as if the detailed microscopic structure of the ground state configurations, which is expected to vary considerably as one goes from isotope to isotope, has very little influence in determining the magnitude of moment of inertia in the low-spin region. If this is true, then the observation of almost identical moments of inertia in many adjacent odd- and even-A nuclei, contrary to the expected 15% variation due to difference in pair correlation, may not be completely unexpected. In fact if the dynamical moments of inertia of the even as well as odd rare-earth nuclei have the same functional dependence on N_p and N_n values, it can be easily seen that the difference in moments of inertia of adjacent even- and odd-A nuclei in many cases will be less than 5%. The product N_pN_n embodies a first order estimate of the integrated strength of the valence p-n interaction [11]. Therefore, this product is in some way related to the deformation parameter of the nucleus. On the other hand, the sum (N_p+N_n) may be assumed to play the role of "effective mass" of the rotating system. Another interesting feature of the problem is that the identical nature of bands observed in a region of shape transition is relatively more pronounced than that observed in the region of saturated collectivity. The present work lends additional support to an earlier observation [10] that the N_pN_n scheme [11] and the F spin formalism [14] may provide the basis for systematizing the available data on identical bands and for further theoretical studies.

TABLE III. Calculated and experimental dynamical moments of inertia.

Nucl.	A	$f(N_pN_n)^{1/2}$	$\hbar^2/2\mathcal{J}_2$ (keV) expt[a]	calc.[b]
Dy	156	55.4	16.0	18.3
Dy	158	64.5	15.0	15.8
Er	158	49.6	17.9	20.5
Dy	160	73.3	13.8	13.9
Er	160	58.0	17.3	17.5
Yb	160	43.8	25.4	23.2
Er	162	66.1	15.7	15.4
Yb	162	51.4	19.3	19.8
Er	164	74.1	14.6	13.7
Yb	164	58.8	17.3	17.3
Yb	166	66.1	15.7	15.4
Hf	166	51.4	19.1	19.8
Yb	168	73.3	13.9	13.9
Hf	168	58.0	17.3	17.5
Hf	170	64.5	15.1	15.8
Hf	172	71.0	14.9	14.3
W	174	61.9	16.4	16.6
W	178	72.7	16.4	14.0
Os	178	55.8	16.8	18.2
W	180	66.9	16.3	15.2
Os	180	60.8	18.1	16.7

[a]Reference [12].
[b] ^{164}Yb used as standard.

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