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Observation of identical bands in superdeformed nuclei with the cranked Hartree-Fock method

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We describe identical, or twinned bands, within the cranked Hartree-Fock model, with a Skyrme effective interaction. We show that the filling of specific orbitals can lead to bands with deexcitation γ-ray energies differing by at most 2 keV in nuclei differing by two mass units and over a range of angular momenta comparable to the experimental one. We show also that the continuous change of the mean field with angular momentum is a key ingredient for twinning.

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Several superdeformed bands (SD) have been found experimentally in the last ten years (for a review, see Ref. [1]). These experiments have revealed striking properties which have been a puzzle for nuclear theory. The highly deformed configurations which characterize these SD states are related to shell effects associated with new "magic" numbers, different from those observed at normal deformation. A particularly striking feature has been the observation of bands with nearly identical deexcitation γ-rays energies in nuclei differing by one or two mass units. These bands are commonly referred to as identical bands (IB) or twin bands. Differences of 1–3 keV have been observed for more than ten consecutive transitions in nuclei both in the Dy and Hg regions. Since then, it has been shown [2–4] that such twinned bands had already been found in the rare-earth and the actinide region at normal deformation, though this was not explicitly pointed out in the original analysis of the data.

A simple $A^{5/3}$ rule for quadrupole moments and moments of inertia provides a rough estimate of the energy changes expected between bands belonging to nuclei differing by one or two mass unit. Such a simple even though *a priori* reasonable calculation gives numbers nearly one order of magnitude larger than the observed ones for ID bands. The puzzle comes partly from the fact that many theoretical models include an intrinsic scaling with mass. In models based on oscillators, the volume conservation law depends upon the mass number. Similarly, the Strutinsky method relies partly on a liquid drop formula. Despite these uncertainties, detailed analyses using parametrized mean fields have been performed. Dudek *et al.* [5] were able to derive analytical expressions for the contributions of each single-particle orbital to the moment of inertia within the rotating oscillator model. Ragnarsson [6] studied the contribution to the moment of inertia due to the alignment of valence particle using either modified oscillator or Woods-Saxon single-particle wave functions. Both studies arrive at the conclusions that the contribution to the moment of inertia of the single-particle orbitals depends very much both on deformation and on the Nilsson quantum numbers of the orbitals, some orbitals bringing nearly no contributions to the moment of inertia. Another attempt to explain this phenomenon [7] relies on the strong coupling limit of the particle rotor model. Again, the filling of specific orbitals favors the appearance of identical bands in nuclei differing by one mass unit. This last result assumes that the moment of inertia of the even-even core is not affected by the extra particle, a hypothesis which is not easy to justify.

Our goal is to check the above ideas within a parameter free model. Furthermore, we want to see whether this twinning phenomenon is within the scope of a mean field description of superdeformation. For that purpose, a method based neither on a scaling law nor on an inert core assumption is necessary. The cranked Hartree-Fock method (CHF) with Skyrme-like interactions meets these two requirements. Indeed, its sole ingredient, the effective interaction, is not specially designed to fit the properties of SD states; it does not make any use of a scaling law and no inert core approximation is made. Within this

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spirit, Girod *et al.* [8] have constructed three SD bands in ^{192}Hg, two of them being nearly identical. However, variation of the mean field with rotation was not included in their model. Therefore, this calculation was limited to low angular momentum (below $22\hbar$) for the sole ^{192}Hg nucleus where only one SD band has been observed.

We have applied our fully self-consistent CHF method which we have developed for the study of the rotation of ^{24}Mg [9] and ^{80}Sr [10] to study superdeformation in the Hg region. In this region of the mass table, bands twinned with the SD band of the "magic" SD nucleus ^{192}Hg have been detected in two even-even nuclei: ^{194}Hg [11, 12] and ^{194}Pb [13–15]. Thus we avoid the complications and ambiguities intrinsic to odd nuclei.

All our calculations[1] have been performed with the Skyrme parametrization SkM*, which we have used in all our previous studies of superdeformation [16, 17]. Pairing correlations have been neglected, which simplifies greatly the numerical effort. Although the variation of the moment of inertia of SD bands is certainly related to the gradual disappearance of pairing correlations with rotation, we do not believe that pairing is responsible for the very existence of twinned bands in even-even nuclei. Moreover, the strength of the pairing force is not well known at such large deformations: calculations based on a Woods-Saxon potential and a seniority pairing interaction require an *ad hoc* renormalization of the pairing strength in order to reproduce the experimental increase of the dynamical moment of inertia $\mathcal{J}_2$ [12]. Finally, pairing is known not to affect in a significant manner the bulk of the mean field, especially near shell closure. We expect that pairing will not change the basics of the mean field shape of ^{192}Hg which is magic when superdeformed, so that the resulting single-particle orbitals will remain unchanged.

Figure 1 shows the neutron and proton Routhians for ^{192}Hg SD ground-state band as a function of J_z. Nilsson quantum numbers have been tentatively assigned to some relevant orbitals. As discussed by Meyer *et al.* [18], HF orbitals are a mixture of many different Nilsson orbitals so that this identification is only a means to compare with other works. Most orbitals do not display large signature splitting and their Routhians are essentially independent of J_z. A few intruder orbitals are rapidly downsloping, but not in the range of angular momentum of interest. The Routhians of these orbitals behave similarly to those obtained in Ref. [12] using a Woods-Saxon potential. However, the neutron gap at $N = 112$ is much less pronounced in our calculation.

Experimentally, one SD band has been observed in ^{192}Hg [19, 20]. Bands with γ rays identical to those of ^{192}Hg have been seen in ^{194}Pb [13–15] and in ^{194}Hg [11, 12]. Only one band has been found in ^{194}Pb; its γ rays have identical energies with those of ^{192}Hg to better than a couple of keV over angular momentum ranging from $12\hbar$ to $30\hbar$. Three bands were observed in ^{194}Hg, one of which, labeled the second excited band or band 3, also "twins" with the same ^{192}Hg band for 15 consecutive transitions.

We have constructed two ^{194}Pb bands, which we will refer to as ^{194}Pb and ^{194}Pb$^{(*)}$ by filling either the $6_{5/2}$ or the $[514]\frac{9}{2}$ proton orbitals. Similarly, we have calculated two ^{194}Hg bands referred to as ^{194}Hg and ^{194}Hg$^{(*)}$ by filling either the $[512]\frac{5}{2}$ or the $[624]\frac{9}{2}$ neutron orbitals. Table I summarizes our results at $20\hbar$ for these five bands. The excitation energy of the ^{194}Pb$^{(*)}$ band relative to the ^{194}Pb band is 2.6 MeV, while it is only 0.33 MeV for the ^{194}Hg$^{(*)}$ band.

The excitation energies for these five bands vs J_z are shown in Fig. 2. They are drawn relative to a rigid-body reference so that comparison is easier. Three of the bands (^{192}Hg, ^{194}Pb$^{(*)}$, and ^{194}Hg$^{(*)}$) have a remarkably close behavior, contrary to the ^{194}Pb and ^{194}Hg bands. The correspondence between the two bands ^{194}Hg$^{(*)}$ and ^{194}Pb$^{(*)}$ is even more striking. These two nuclei have the same mass number; no scaling law is expected. Further comparison with the ^{192}Hg band, which is two mass units away, shows that this global law plays no role on such a small mass difference. To support the interpretation of

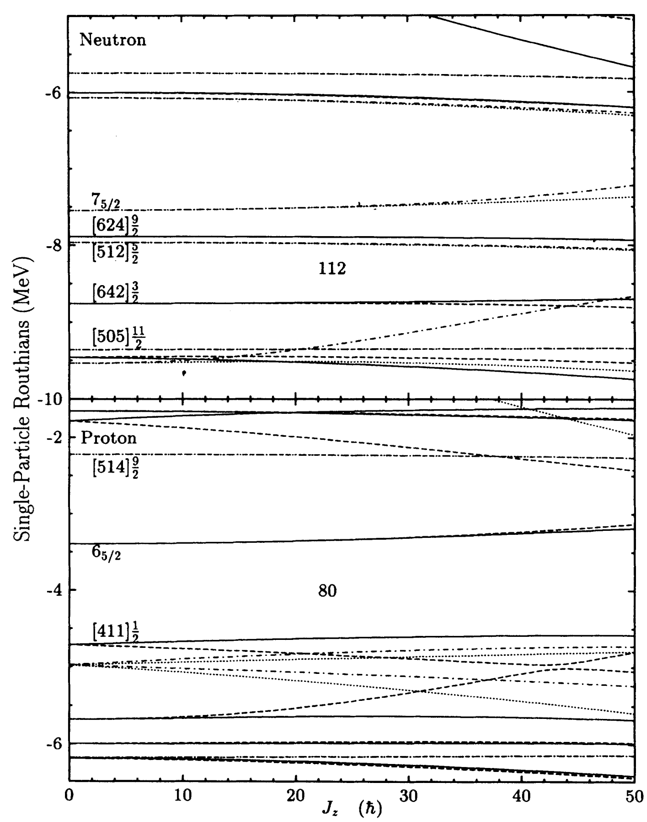

FIG. 1. Neutron and proton Routhians for ^{192}Hg. The parity and signature (P, S) of each individual orbital are indicated by solid $(+,+)$, dashed $(+,-)$, dot-dashed $(-,+)$, and dotted $(-,-)$ lines.

[1]Our cranked HF code has been transposed onto the massively parallel Intel DELTA and GAMMA computers at Caltech, where most of the calculations have been done. The details of the parallelization procedure will be presented elsewhere.

TABLE I. Energies, quadrupole moments, dynamical $\mathcal{J}_2$, and rigid-body moments of inertia $\mathcal{J}_0$ and angular frequencies ω obtained at $20\hbar$ for the five SD bands studied in this work.

Band	E (MeV)	E_{rot} (MeV)	Q_0 (fm^2)	$\mathcal{J}_2$ (MeV^{-1})	$\mathcal{J}_{\mathrm{rig}}$ (MeV^{-1})	ω (MeV)
^{192}Hg	-1507.734	1.74468	4446.009	115.407	119.319	0.17392
^{194}Pb	-1513.777	1.81887	4644.611	110.681	122.365	0.18151
^{194}Pb$^{(*)}$	-1511.611	1.73897	4383.145	116.009	120.133	0.17318
^{194}Hg	-1523.683	1.71442	4393.146	117.925	120.300	0.17056
^{194}Hg$^{(*)}$	-1523.350	1.73655	4446.266	116.279	120.709	0.17285

these bands as identical, or twinned, bands, we have plotted the differences in γ-ray energies between the $A = 194$ bands and the ^{192}Hg one taken as a reference on Fig. 3. The four bands show deviations with respect to the ^{192}Hg reference proportional to J_z. For both ^{194}Pb and ^{194}Hg nuclei, the deexcitation energies of their excited band differ by 1–3 keV with that of ^{192}Hg over the experimentally observed range of angular momentum. Our calculation predicts that these differences may be somewhat larger at higher angular momentum, say above $32\hbar$. The other bands in ^{194}Pb and ^{194}Hg do not resemble each other at all. Neither of them twins with the ^{192}Hg band.

Figure 4 shows the dynamical moments of inertia $\mathcal{J}_2$ as a function of J_z for the five SD bands calculated by differentiating the total angular momentum with respect to ω. This plot supports the conclusion that the ^{194}Hg$^{(*)}$ and ^{194}Pb$^{(*)}$ bands are identical to each other and to that of ^{192}Hg, whereas no such conclusion can be drawn for the two other bands. The overall trends of these moments are quite similar except that the two excited bands display slightly larger $\mathcal{J}_2$ slopes than the three SD ground-state bands. All five bands display a continuous rise of $\mathcal{J}_2$ with the increasing angular momentum. This result contradicts other studies based on either Woods-Saxon [21] or rotating oscillator [6] potentials and where pairing effects have also been neglected. However, the rise that we obtain is much smaller than what is observed experimentally. A large part of the experimental variation is most probably related to the variation of pairing with rotation.

Table I shows that the overall changes in the quadrupole moments from one band to the other are not related to the changes in dynamical moments of inertia. The ^{194}Pb band has a larger quadrupole moment and a smaller moment of inertia, while the situation is inversed for the ^{194}Hg band. These features illustrate that neither the dynamical moments of inertia nor the quadrupole moments obey a $A^{5/3}$ rule. Indeed, as expected, the relative values of the rigid-body moments of inertia follow the same trends as the quadrupole moments. They are all significantly different from the $\mathcal{J}_2$ values, the largest differences being obtained for the ^{194}Pb SD ground-state band.

To understand the origin of twinning, it is important to examine how the two extra particles added to the ^{192}Hg core contribute to the rotation. Table II shows the single-particle Routhian energies at $20\hbar$ and $40\hbar$ for the eight orbitals filled in the various bands we have constructed here together with their single-particle contributions to the angular momentum J_z and to $\mathcal{J}_2$. This last contribution is calculated as $dj_z/d\omega$. These orbitals show small signature splitting, especially the ones leading to identical bands. They have negative alignments along the rotational axis and negative contributions to the moment of inertia. The extra orbitals filled in ^{194}Hg$^{(*)}$ and ^{194}Pb$^{(*)}$ bring very similar contributions to the moment of inertia. Although small, these negative contributions are not negligible; they represent 1.5% of the total angular mo-

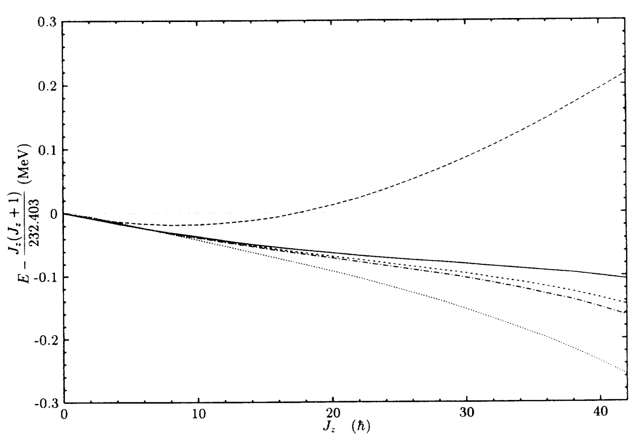

FIG. 2. Rotational energies (relative to a rigid rotor reference) versus spin for the various bands constructed here. The bandhead energies have been subtracted. The bands are represented by solid (^{192}Hg), long-dashed (^{194}Pb), short-dashed (^{194}Pb$^{(*)}$), dotted (^{194}Hg), and dot-dashed curves (^{194}Hg$^{(*)}$).

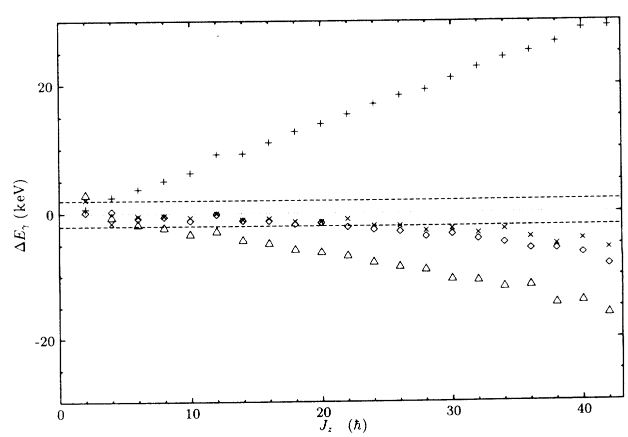

FIG. 3. Differences in γ-ray energies ΔE_γ between the $A = 194$ bands and the ^{192}Hg band. The bands are represented by + (^{194}Pb), $\times$ (^{194}Pb$^{(*)}$), $\triangle$ (^{194}Hg), and $\diamond$ (^{194}Hg$^{(*)}$). The two dashed lines at ± 2 keV represent the accuracy required experimentally for twinning.

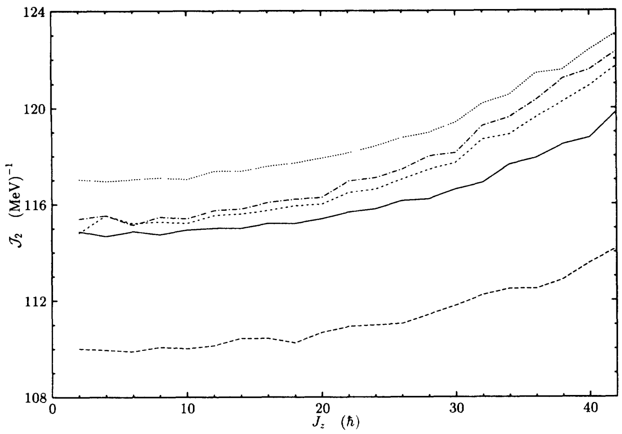

FIG. 4. Dynamical moments of inertia $\mathcal{J}_2$ for the five bands constructed here. The same line types as in Fig. 2 are used here.

mentum. The extra orbitals of ^{194}Pb have more negative angular momenta and contributions to $\mathcal{J}_2$, leading to a lower dynamical moment of inertia than in ^{192}Hg, while the situation is reversed for ^{194}Hg. In this last case, the extra two particles bring very small contributions to the angular momentum and to the moment of inertia, but do not lead to twinning. This indicates that the core polarization of ^{192}Hg induced by the added particles is by no means negligible. "Twinning" thus results from a delicate balance between the contribution of the extra particles and the self-consistent response of the mean field.

The tiny variation of the single-particle Routhians with rotational frequency indicates that the single-particle angular momenta are not equal to the derivatives of the single-particle Routhians with respect to the rotational frequency, as they would be if the mean field Hamiltonian does not depend on ω. We have verified this by taking the derivatives of the Routhians, which leads to values of j_z smaller by at least a factor of 5 than those obtained from the mean values. This shows that the main contribution to the single-particle angular momenta is coming from the variation of the mean field with angular momentum. This variation has two origins. The first is the dependence of the nuclear deformation on rotation. Such dependence is also included in calculations based on parametrized mean fields and leads to very small effects. The second effect is dominant in our calculation and is due to terms appearing in microscopically determined mean fields when time reversal invariance is broken (see Ref. [22] and the Appendix A of Ref. [9] for their explicit form). These terms arise from the velocity dependent terms of the effective interaction and are of the order of 1 MeV at 40$\hbar$.

Our results present some similarity with those obtained by Ragnarsson [6] and by Dudek [5], who have also shown that the filling of specific single-particle orbitals leads to twinning. However, our calculation points out the importance of the dependence of the mean field on the rotation, an effect not included in previous calculations. In our study, twinning does result from a precise balance between the changes of the mean field and the behavior of single-particle states.

Let us summarize the results of our calculations. (1) Within a fully self-consistent CHF, the filling of specific orbitals leads to identical bands with an accuracy well within the experimental data. (2) Changes of deformation with increasing mass are not simply related to changes in the dynamical moment of inertia. (3) Scaling of the moment of inertia by a $A^{5/3}$ rule is not correct for SD bands in ^{192}Hg, ^{194}Hg, and ^{194}Pb. The generality of this rule must be reexamined. (4) Continuous changes of the self-consistent mean field with angular momentum, which invalidate the usual relation between single-particle energies and angular momenta, is a key ingredient for twinning. Further investigations of the terms of the Skyrme functional appearing when time reversal invariance is broken are necessary to obtain a better understanding of the phenomena. One consequence of these changes is the continuous rise in the dynamical moment of inertia that we obtain and which is absent from previous studies.

Many features remain to be explained and demonstrated. Most important is to show that pairing will not invalidate the conclusions drawn from our simple CHF picture. The answer to this question will necessitate the introduction of a pairing force with nonconstant matrix elements between the orbitals. Another important question is the choice of orbitals to construct the SD bands in the three isotopes [23]. Other choices could

TABLE II. Angular momenta j_z, contributions to $\mathcal{J}_2$, and single-particle Routhian energies of the eight orbitals added to the ^{192}Hg orbitals.

	^{194}Pb		^{194}Pb$^{(*)}$		^{194}Hg		^{194}Hg$^{(*)}$	
	$6^+_{5/2}$	$6^-_{5/2}$	$[514]\frac{9}{2}^+$	$[514]\frac{9}{2}^-$	$[512]\frac{5}{2}^+$	$[512]\frac{5}{2}^-$	$[624]\frac{9}{2}^+$	$[624]\frac{9}{2}^-$
$20\hbar$ j_z	-0.241	-0.260	-0.174	-0.175	-0.077	-0.049	-0.165	-0.164
$\frac{dj_z}{d\omega}$	-1.247	-1.739	-1.013	-1.014	-0.373	-0.331	-0.993	-0.993
ϵsp	-2.927	-2.927	-1.870	-1.870	-8.120	-8.125	-7.989	-7.989
$40\hbar$ j_z	-0.357	-0.625	-0.349	-0.350	-0.130	-0.105	-0.345	-0.344
$\frac{dj_z}{d\omega}$	-0.062	-2.148	-1.048	-1.054	-0.322	-0.310	-1.129	-1.124
ϵsp	-2.821	-2.798	-1.883	-1.883	-8.173	-8.184	-8.103	-8.103

be investigated for the sake of completeness. Lastly, one should note that the one SD band observed in ^{194}Pb twins both experimentally and theoretically to the only band observed in ^{192}Hg. In ^{192}Hg, our calculated band is the SD band built upon the SD minimum, whereas in ^{194}Pb it is an excited band. This raises several questions. Although the single-particle level schemes calculated with the Skyrme interaction SkM* and with the most frequently used Woods-Saxon parametrization are very similar [23], is the quadrupole deformation at which the $6_{5/2}$ and $[514]\frac{9}{2}$ states cross underestimated? The inclusion of a seniority pairing interaction, although decreasing the quadrupole deformations of the SD bands, does not modify the order of the ^{194}Pb bands, at least at zero spin. Assuming the validity of our calculations, is it possible that the SD band built upon the ^{194}Pb SD minimum has not yet been observed? This would question the assumption frequently made by experimenters that the most intense band observed is the lowest one. Although we have demonstrated that twinning arises within fully self-consistent CHF calculations without further assumptions, there still exists the question: is there a hidden symmetry?

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