## ARTICLES

# Interaction strength of two crossing bands

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The interaction strengths of the ground band with the $S$ band are extracted by using observed states of both bands as well as observed relative $E2$ transition rates for $\gamma$ decay to yrast and nonyrast states. The signature dependence of the strength observed in the negative-parity yrast sequences of $^{157}$Ho is explained, if a relatively strong $n$-$p$ $Q''\cdot Q''$ interaction is introduced in the conventional axially symmetric particle-rotor model. The $Q''\cdot Q''$ interaction is supposed to take care of the possible shape-driving effect due to the presence of aligned quasiparticles.

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## I. INTRODUCTION

In the yrast spectra of deformed nuclei the lowest-lying band crossing, which is the crossing of the ground ($g$) band with the $S$ band, is observed at the rotational frequency of a few hundred keV. The interaction strength $V$ between the two bands is expected to be small when the difference between the alignment of the $g$ band and that of the $S$ band is large, say, $10\hbar$. The expectation can be illustrated, for example, by the fact that the wave functions for the two configurations, as obtained from the cranking model, represent wave packets of states with very different distributions of the total angular momentum [1,2]. The theoretical study of the strength using both the cranking model [3] and the particle-rotor model [4] predicted, furthermore, that $V$ would be an oscillating function of the degree of filling of the unique-parity high-$j$ shell, which is occupied by the strongly aligned two quasiparticles in the $S$ band. The presence of the aligned two quasiparticles in the $S$ band may drive the shape of the nuclear system to deviate from axial symmetry, and the resulting possible triaxial shape depends on the degree of filling of the high-$j$ shell [5,6]. If there is a difference between the effective shape of the $g$ band and that of the $S$ band, the interaction strength $V$ will be affected by the difference. Furthermore, for a given shell filling the triaxial shape preferred by an aligned quasiparticle may depend on the signature of the quasiparticle state. Then, the interaction strength in odd-$A$ nuclei may depend on the signature of the odd quasiparticle, which is equal to the signature of the rotational sequence. A clear signature dependence of $V$ in the negative-parity yrast sequences of the well-deformed nucleus $^{157}$Ho with very good accuracy (namely, 48 and 29 keV for favored and unfavored signatures, respectively) is indeed obtained by using the measured energies of the yrast and yrare states as well as the measured $B(E2)$ values of the yrast-yrast, yrast-yrare, and yrare-yrare states [7]; see also Ref. [8].

The interaction strength between the $g$ band and the $S$ band reflects itself qualitatively in the spectra of $\gamma$-rays from the yrast decay. A strong interaction creates a spectrum of regularly spaced $\gamma$ rays with energies that increase with angular momentum, while weakly interacting bands will show the characteristic backbend [9], namely, a decrease in $\gamma$-ray transition energy with increasing angular momentum. Using observed transition energies in even-even nuclei, various models [10] have been applied to extract the interaction strength, and the values have been found to vary strongly within each isotope chain as a function of the number of neutrons. However, the extraction of an interaction strength by using energies only is not unambiguous, especially when only the yrast states are known both before and after the crossing. In cases where the states of both bands, as well as the relative reduced $E2$ transition rates for the $\gamma$ decay to yrast and nonyrast states, are known in the crossing region, it is possible to extract more accurate values of the size of the interaction strength [11]. Adopting this way of extracting $V$, we have collected the available data to be analyzed in the present paper.

The study of the interaction strengh $V$ in odd-$A$ nuclei using a particle-rotor model is particularly interesting because of the available detailed data [7] for $^{157}$Ho, in which the interaction strength of crossing bands is obtained for several pairs of states with good accuracy. The study is very interesting also because, in the analysis of experimental data performed in the spirit of the so-called cranked shell model (CSM) [1,12], it has been found [13] that the strength $V$ (as well as the frequency of the band crossing) is appreciably different from the one observed in the neighboring even-even nuclei when the odd particle occupies a particular high-$j$ low-$\Omega$ orbit (such as the proton $[541\frac{1}{2}]$ orbit). It is noted that in the CSM, in the absence of residual interactions (or in the absence of an appreciable amount of shape polarization due to the presence of the odd particle [13]), neither the signature dependence of $V$ observed in $^{157}$Ho nor the difference in $V$ between odd-$A$ nuclei and the neighboring even-even nuclei is obtained. The calculation of $V$ between two bands, which have different shapes, has so far not been carried out in the literature. It is suggested in Ref. [14] that an axially symmetric particle-rotor model with a neutron-proton $Q \cdot Q$ interaction be used in odd-$A$ nuclei, in order to take into account effectively the possible shape-driving effect due to the presence of aligned quasiparticles and to simulate the situation in which the effective shape of the $g$ band is different from that of the $S$ band. We make a further application of the model to the present subject.

In Sec. II we show the result of the analysis of available experimental data, which is performed by using the observed states of both bands as well as $E2$ transition rates for the $\gamma$ decay to yrast and nonyrast states. In Sec. III the result of theoretical calculations of $V$ is presented using various models. When we extract the interaction strength $V$ from the calculation of the particle-rotor model, the same method as the one used in Sec. II is employed. The discussion and conclusion are in Sec. IV.

## II. INTERACTION STRENGTH FROM EXPERIMENTAL DATA

We analyze the observed perturbed differences in energy of the nonyrast and yrast states at spin $I$, $\Delta E_I$ and spin $I-2$, $\Delta E_{I-2}$, together with the observed reduced branching ratio $\lambda'$, defined as

$$\lambda' = \frac{B(E2, I_y \to (I-2)_{\text{nony}})}{B(E2, I_y \to (I-2)_y)} = \frac{B(E2, I_{\text{nony}} \to (I-2)_y)}{B(E2, I_{\text{nony}} \to (I-2)_{\text{nony}})} \tag{1}$$

in terms of a simple two-band mixing. Identical diagonal matrix elements ($Q_0$) are assumed for the two bands, and all nondiagonal matrix elements are assumed to be equal to zero. In the following expression for the interaction strength $|V|$, mixing of the wave functions at both spin $I$ and $I-2$ is taken into account:

$$|V| = \frac{\sqrt{\lambda'}}{1+\lambda'} \Delta E_I \left[ (1-R)^2 + \frac{4R\lambda'}{1+\lambda'} \right]^{-1/2}, \tag{2}$$

where $R = \Delta E_I / \Delta E_{I-2}$ and the relation $V(I) = V(I-2)$, expressed as $V$, is assumed. It should be noted that measurements of $\lambda'$ with large errors can still provide a reasonably accurate determination of $|V|$ because of the structure of the mixed wave functions in accordance with the content of Eq. (2). If a pronounced difference in $Q_0$ for the two bands exists, a difference in $\lambda'$ from the yrast and nonyrast states is introduced. The sensitivity depends on the degree to which both initial and final states have mixed wave functions. In the present analysis all possible decay branches have been included in an average value of the interaction strength.

Detailed spectroscopic information, which enables a determination of $|V|$ through Eq. (2), is available in many of the light even-even rare-earth isotopes. The even-even nuclei in this region are most likely axially symmetric, and, therefore, the oscillating behavior of $V$ as a function of the degree of filling of the unique-parity high-$j$ shell (i.e., the neutron $i_{13/2}$ shell in the present nuclei) is expected. This was shown in Refs. [3 and 4].

The energies and relative intensities for $^{156}$Dy, $^{158,160,164}$Er, $^{160,162,164,166}$Yb, and $^{164,166}$Hf are found in Refs. [15,16], [17–20], [11, 21–23], and [24], respectively. In some cases such as $^{156}$Dy and $^{158}$Er, absolute matrix elements are also known [16,17] in the crossing region. The decrease observed in the yrast-to-yrast transtion matrix element is in both cases consistent with the mixing amplitudes derived from the measured branchings.

We show the extracted values of $|V|$ as a function of neutron number in Fig. 1. There are large variations within each isotopic chain, but it is obvious that the neutron number alone does not give a satisfactory measure of the actual degree of the shell filling within the neutron $i_{13/2}$ subshell. An experimental determination of the "absolute" Fermi level has been attempted with so-called "gauge space" analysis techniques, where average values of the sum of the total binding energies (masses) and excitation energies of the adjacent neighbors are evaluated as a function of frequency [13,25]. Such evaluations of the Fermi level include contributions from the symmetry energy, i.e., the change in the neutron potential caused by a change in the number of protons. Therefore, this does not allow a fixed relation between the neutron number and the shell-filling parameter [26] $\lambda/\kappa$ of the neutron $i_{13/2}$ shell in the model calculations.

We therefore choose an approach for determining the *relative* Fermi level within the neutron $i_{13/2}$ subshell for each individual nucleus in the different isotopic chains. For axially symmetric nuclei it is expected that the signature splitting of Routhians, $\Delta e'$, depends smoothly on the occupation number in the subshell. Thus, we use the observed quantity $\Delta e'$ to determine the shell-filling parameter $\lambda/\kappa$ for each nucleus. We take the experimental signature splitting divided by rotational frequency, $\Delta e'/\hbar\omega$, determined at the rotational frequency corresponding to spin $I = \frac{19}{2}$ (well below the crossing) in the positive-parity yrast band in the odd-$n$ neighbors. References to the data are found in Ref. [27]. For each nucleus, an average of the experimental values of $\Delta e'/\hbar\omega$ for its neighbors is converted to relative position in the $i_{13/2}$ subshell, $\lambda/\kappa$, through particle-rotor calculations [14] of this smooth re-

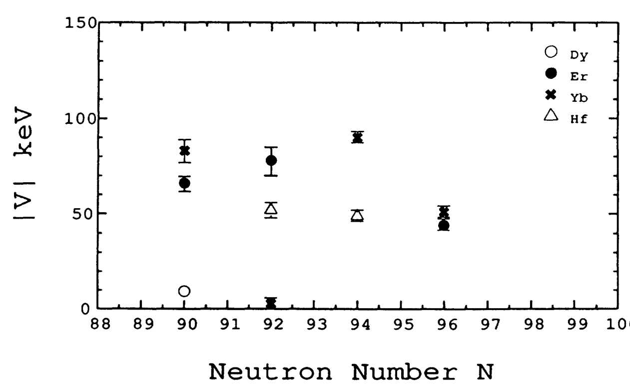


FIG. 1. Experimental values of the interaction strength $|V|$ between the $g$ band and the $S$ band, which are calculated from Eq. (2), are plotted as a function of the number of neutrons.

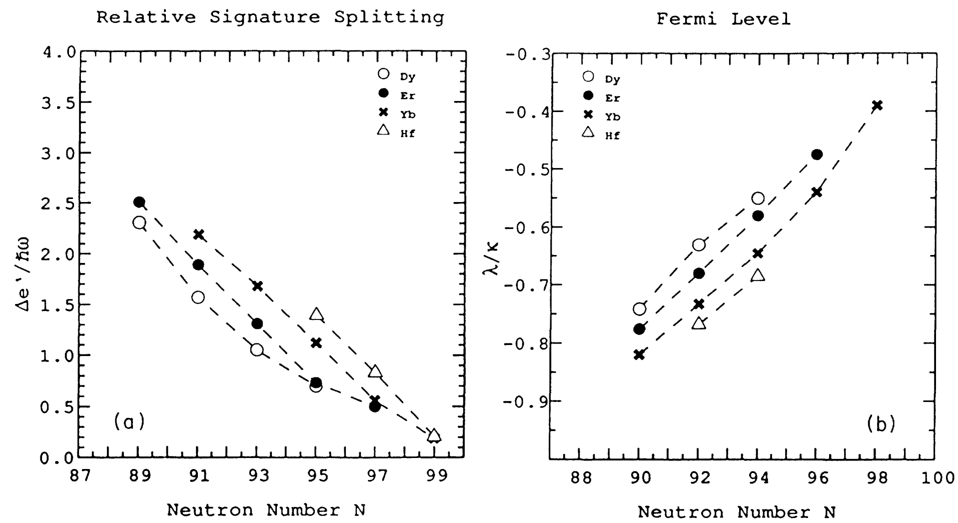


FIG. 2. (a) Experimental relative signature splitting in energy for some odd-$N$ light rare-earth nuclei [28] plotted as a function of the number of neutrons. The values are extracted at a frequency corresponding to $I=\frac{19}{2}$. (b) relative position in the neutron $i_{13/2}$ subshell estimated from the data of part (a) through particle-rotor calculations [14] of the relation between these quantities. In the estimate, using the particle-rotor model with a reduction of the Coriolis interaction by a factor of 0.8, we have used a set of single-particle energies for the $i_{13/2}$ shell ($\epsilon_\Omega/\kappa=-0.99$, $-0.78$, $-0.45$, $-0.02, 0.47, 1.02$, and 1.60 are used for $\Omega=\frac{1}{2},\frac{3}{2},\cdots,\frac{11}{2},\frac{13}{2}$) instead of those obtained from the expression (4) with $\gamma=0$ in Sec. III. The $\epsilon_\Omega$ values used here are more realistic for the $\lambda/\kappa$ values lying low in the $j$ shell [14]. Other parameters used are $\Delta/\kappa=0.30$ and $\theta_{1\kappa}=\theta_{2\kappa}=164$ in Eqs. (3) and (6).

lationship at the same spin value, assuming axial symmetry. The actual transformation is illustrated in Figs. 2(a) and 2(b). From these figures one clearly sees a $Z$ dependence in the value of $\lambda/\kappa$ that includes in an empirical way the effect of both deformation and symmetry energy on the total potential energy of the $i_{13/2}$ subshell. When displayed as a function of these values of $\lambda/\kappa$ [from Fig. 2(b)] the experimentally determined values of the interaction strength show, at least qualitatively, the expected oscillating behavior of $|V|$, as demonstrated in Fig. 3. The parameters used for the calculation of the relative signature splitting (Fig. 2) are the same as these used for the calculated interaction strength shown in the upper part of Fig. 3. The node in the oscillation at $\lambda/\kappa\sim-0.72$ in the calculation seems to be in good agreement with the node seen in the lower part of Fig. 3. The next node is not well established experimentally. As can be seen from the structure of Fig. 2(b), there may not be a nucleus for which $\lambda/\kappa$ corresponds to the expected node.

## III. MODELS AND CALCULATIONS

Our intrinsic Hamiltonian is written as

$$H_{\mathrm{intr}}=\sum_\alpha(\varepsilon_\alpha-\lambda)a_\alpha^\dagger a_\alpha+\tfrac{1}{2}\Delta\sum_{\alpha,\beta}\delta(\bar{\alpha},\beta)(a_\alpha^\dagger a_\beta^\dagger+a_\beta a_\alpha)\ , \tag{3}$$

where $\epsilon_\alpha$ are the one-particle energies for a single particle with angular momentum $j$ moving in a general triaxially deformed quadrupole potential

$$V=\frac{\kappa}{j(j+1)}\{[3j_z^2-j(j+1)]\cos\gamma+\sqrt{3}(j_y^2-j_x^2)\sin\gamma\}\ , \tag{4}$$

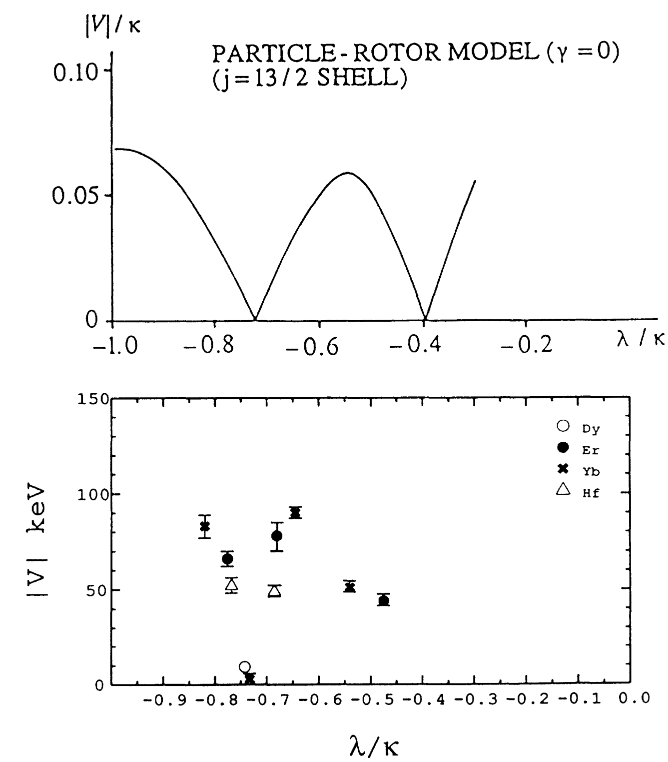


FIG. 3. In the lower part experimental values of the interaction strength $|V|$ calculated from Eq. (2) are plotted as a function of the relative position in the neutron $i_{13/2}$ subshell of Fig. 2(b). In the upper part the $g$-$S$ interaction strength, calculated in the particle-rotor model [14] using the same parameters as those employed in Fig. 2(b), is plotted as a function of relative position in the neutron $i_{13/2}$ subshell.

where $\kappa$ is used as an energy unit [26] in the $j$ shell and is about 2.5–3 MeV in a realistic case. Our cranking Hamiltonian is described as

$$H_{\mathrm{cr}} = H_{\mathrm{intr}} - \omega \sum_{\alpha,\beta} \langle \alpha | j_x | \beta \rangle a_\alpha^\dagger a_\beta \ , \tag{5}$$

while our particle-rotor Hamiltonian is

$$H_{\mathrm{pr}} = H_{\mathrm{intr}} + \sum_k \theta_k^{-1} (I_k - j_k)^2 \ , \tag{6}$$

where $I_k$ is the $k$ component of the total angular momentum in the intrinsic coordinate system.

In the following we use single-particle energies in the potential (4) and do not introduce a reduction factor of the Coriolis interaction in the particle-rotor model, in contrast to the previous section (see caption to Fig. 2). The reason why in the present section we employ a somewhat more schematic parametrization is that, while keeping the Hamiltonian as simple as possible, we aim at making a comparison of the results of model calculations, including the case of $\gamma \neq 0$ in the cranking model. We remark that a certain amount (say, 20%) of the reduction of the Coriolis interaction produces only a minor change in $|V|$ as a function of $\lambda$, though it changes considerably the signature splitting of Routhians.

In Ref. [3], using the cranking model with axially symmetric shape (i.e., $\gamma = 0$), the interaction strength $V$ between the $g$ band and the $S$ band was calculated for a $j = \frac{13}{2}$ shell. The absolute magnitude of $V$ is in fact a sensitive function of the pairing parameter $\Delta$, though the oscillating behavior of $V$ as a function of the shell filling remains unchanged while varying $\Delta$. For example, in a simple model (namely, a model with an equal distance of single-particle levels and a constant matrix element of the Coriolis coupling) by Bohr and Mottelson [28] one obtains that $|V|$ is a decreasing function of $\Delta$, and the $\Delta$ dependence of $V$ can be written as

$$|V| \propto \Delta^{-1} \exp(-c\Delta^{1/2}) \tag{7}$$

in the range of $\Delta$ values of practical interest. In Fig. 4 we show calculated values of $|V|$ for $\gamma = 15°$, 0°, and $-15°$, where $\Delta/\kappa = 0.30$ is used. (Note that in Fig. 2 of Ref. [3] $\gamma = 0$ and the value of $\Delta/\kappa = 0.45$ was used.) We remark that, for example, for $\gamma = -30°$ the peak height of the oscillation of $|V|$ becomes minimum around $\lambda/\kappa = 0$, since around $\lambda/\kappa = 0$ the $S$ band configuration has the maximum alignment at the band-crossing frequency compared with other $\lambda$ values. Though it is very interesting to observe the strong dependence of $V$ on $\gamma$ values exhibited in Fig. 4, the calculations are made assuming the same values of $\gamma$ for both the $g$ band and the $S$ band. In a realistic nucleus one might expect different values of $\gamma$ for the two crossing bands, since two extra strongly aligned high-$j$ quasiparticles are present in the $S$ band configuration. Therefore, the $\gamma$ dependence of the result shown in Fig. 4 may not be immediately applicable to realistic cases.

In Ref. [4], using the particle-rotor model, the value of $V$ was calculated for $\gamma = 0$. In Fig. 5 we show calculated values of $|V|$ for $\gamma = 0$ using $\Delta/\kappa = 0.30$. A comparison between Fig. 5 and the middle graph ($\gamma = 0$) of Fig. 4 shows that using a given intrinsic Hamiltonian there is a non-negligible difference between the values of $V$ calculated in the cranking model and in the particle-rotor model, as was discussed in detail in Ref. [4].

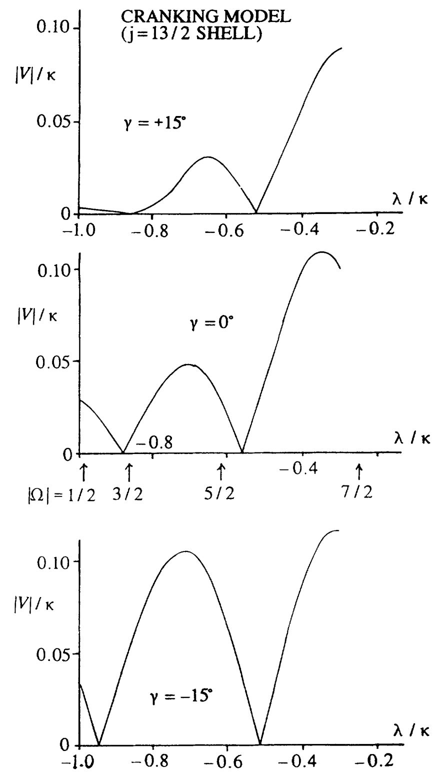


FIG. 4. Interaction strength $V$ between the $g$ band and the $S$ band, which is calculated using a cranking model for a $j = \frac{13}{2}$ shell, as a function of the shell filling. The pairing parameter used is $\Delta/\kappa = 0.30$. In the middle figure (for $\gamma = 0$, an axially symmetric prolate shape) the positions of the single-particle energies $\epsilon_\Omega$ for the $j = \frac{13}{2}$ shell are shown for reference.

Next, we estimate the values of $V$ in odd-$Z$ rare-earth nuclei, and, for simplicity, we choose the case in which the odd proton occupies the unique-parity $h_{11/2}$ shell. In the conventional CSM [1,12] the calculated value of $V$ in

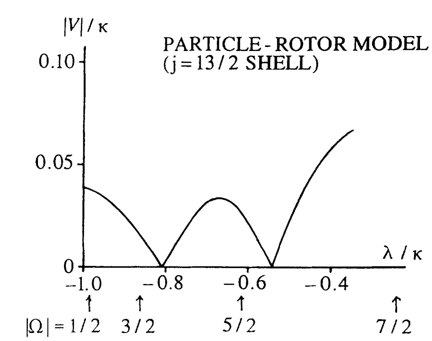


FIG. 5. Same quantity as in Fig. 4, but calculated by using an axially symmetric particle-rotor model for a $j = \frac{13}{2}$ shell. The parameters used are $\Delta/\kappa = 0.30$ and $\theta_1\kappa = \theta_2\kappa = 130$.

odd-$Z$ nuclei is exactly the same as that of neighboring even-even nuclei if one uses the same values of $\Delta_n/\kappa$ and $\lambda_n/\kappa$ for the neutron $i_{13/2}$ shell, since the odd proton is just a spectator in the band crossing. In contrast, in the conventional particle-rotor model [29] the odd proton in odd-$Z$ nuclei is not just a spectator in the band crossing. In the upper part of Figs. 6 and 7 we show values of $|V|/\kappa$ calculated for the proton $h_{11/2}$ shell filling, $\lambda_p/\kappa=-0.90$ and $-0.10$, respectively, assuming axially symmetric shape. In comparison with Fig. 5 we observe a slight change of the calculated $|V|$ values, which comes from the presence of the odd $h_{11/2}$ quaisproton. Specifically, it is seen that a slight signature dependence for $|V|$ appears for larger values of $|V|$. However, when the strength $|V|$ is smaller than about 70 keV, the estimated strength for the favored signature (solid line) is almost identical to that for the unfavored signature (dashed line). Noting that in $^{157}$Ho the measured value [7] of $|V|$ is 48 keV (29 keV) for the favored (unfavored) signature, it does not seem to be possible to explain the observed signature dependence of $|V|$ using the present version of the particle-rotor model. Thus, it seems necessary to take into account the polarization (or shape-driving) effect due to the presence of aligned particles

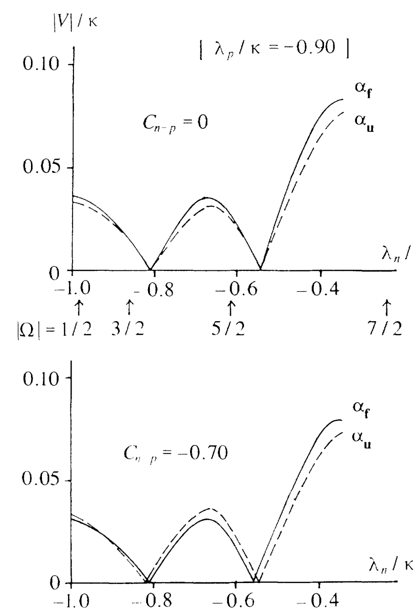


FIG. 6. Interaction strength $V$ between the $g$ band and the $S$ band in odd-$Z$ nuclei with the odd proton in the $h_{11/2}$ shell calculated using an axially symmetric particle-rotor model as a function of neutron shell filling in the $i_{13/2}$ shell. In the lower part of the figure a neutron-proton quadrupole-quadrupole interaction defined in terms of the doubly stretched coordinates is introduced so as to simulate both the configuration dependence and the signature dependence of the deformation using the model of Ref. 13. The solid and dashed lines show the values for the favored and unfavored signatures, respectively. The value of $\lambda_p/\kappa=-0.90$ corresponds to the proton Fermi level lying at the bottom of the $h_{11/2}$ shell. Other parameters used are $\Delta_p/\kappa=0.40$, $\Delta_n/\kappa=0.30$, and $\theta_1\kappa=\theta_2\kappa=130$. See text for details.

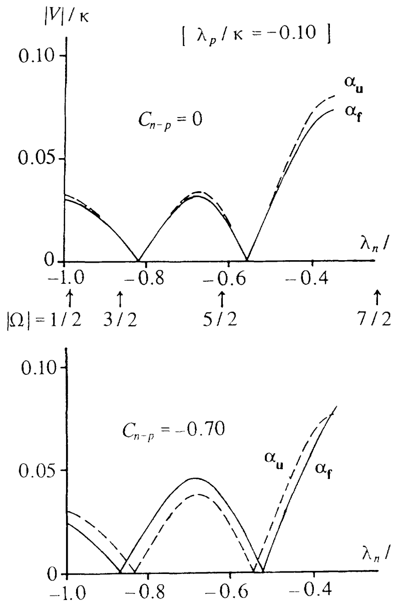


FIG. 7. Same quantity as in Fig. 6, but calculated for the proton Fermi level $\lambda_p/\kappa=-0.10$, which is chosen to be appropriate for the nucleus $^{157}$Ho.

(both the extra two aligned quasineutrons present in the $S$ band and the aligned odd quasiproton) in order to account for the observed signature dependence of $|V|$. To our knowledge the calculation of $V$ between the two bands with two different shapes has so far not been performed.

In Ref. [14] it was shown that it is possible to simulate the shape-driving effect due to the presence of aligned particles by using a particle-rotor model with axial symmetry and introducing a neutron-proton interaction of $Q\cdot Q$ type with its strength adjusted to reproduce the observed signature splitting in energies both before and after the band crossing. In Ref. [14] a usual $Q\cdot Q$ interaction is used; however, in the following we use a quadrupole-quadrupole interaction defined in terms of the doubly stretched coordinates $Q''\cdot Q''$. It has been argued [30] that using the doubly stretched $Q''\cdot Q''$ interaction is more reasonable because the fluctuation about the deformed equilibrium shape thereby can be taken into account. Thus, our Hamiltonian is written as

$$H=H_{\rm pr}+H'_{n\text{-}p}\ , \tag{8}$$

where

$$H_{\rm pr}=H_{\rm intr}+\theta^{-1}\sum_{k=1,2}(I_k-J_{n,k}-J_{p,k})^2 \tag{9}$$

and

$$H'_{n\text{-}p}=x''\sum_{\substack{\nu\\(i,j)=p\\(k,l)=n}}\langle i|(r^2Y^*_{2\nu})''|j\rangle \times\langle k|(r^2Y_{2\nu})''|l\rangle a_i^\dagger a_k^\dagger a_l a_j\ . \tag{10}$$

In our numerical calculation we use a dimensionless parameter defined as

$$C_{n\text{-}p}=\frac{x''}{\kappa}\langle j_p|r^2|j_p\rangle\langle j_n|r^2|j_n\rangle \times\frac{\langle j_p\|Y_2\|j_p\rangle\langle j_n\|Y_2\|j_n\rangle}{(2j_p+1)^{1/2}(2j_n+1)^{1/2}} . \quad (11)$$

In the lower part of Figs. 6 and 7 the estimated values of $|V|$ are shown for the proton $h_{11/2}$ shell filling, $\lambda_p/\kappa=-0.90$ and $-0.10$, respectively. It is seen that introducing the $n$-$p$ interaction, which should simulate both the configuration dependence and the signature dependence of the deformation, leaves the oscillating behavior $V$ as a function of the shell filling unchanged. However, the phase shift of the oscillation now depends on the $\lambda_p/\kappa$ value as well as the signature, and a prominent signature dependence in $|V|$ for a given $\lambda_n/\kappa$ value appears, especially for smaller values of $|V|$ and in the case of $\lambda_p/\kappa=-0.10$. It is noted that the value of $\lambda_p/\kappa=-0.10$ corresponds approximately to the odd-proton Fermi level within the $h_{11/2}$ shell in the observed negative-parity yrast rotational sequences of $^{157}$Ho, in which $|V|=48$ keV for $\alpha_f$ and $|V|=29$ keV for $\alpha_u$ are observed. Thus, using $C_{n\text{-}p}=-0.7$ it seems possible to explain the signature dependence of $|V|$ observed in $^{157}$Ho. However, as was described in Ref. 14, we should remember that the value of $C_{n\text{-}p}=-0.7$ is almost an order of magnitude larger than the value which is obtained from self-consistent coupling constants including the renormalization due to both the isoscalar and the isovector $2\hbar\omega_0$ excitations in the harmonic-oscillator model [29].

The signature dependence of $|V|$ exhibited in the lower parts of Figs. 6 and 7 comes predominantly from the $\nu=2$ component (i.e., fluctuation towards a triaxial shape) of the $n$-$p$ interaction in (10). If we convert the doubly stretched $Q''\cdot Q''$ interaction with a given value of $C_{c\text{-}p}$ into the form of the usual $Q\cdot Q$ interaction, the coupling constant of the $\nu=2$ component becomes stronger by a factor of $(\omega_\perp/\omega_0)^4$, which is, for example, about 1.5 in the case of the quadrupole deformation parameter $\delta=0.3$. This is the reason why in the lower part of Fig. 7 the signature dependence is more prominent than in the lower part of Fig. 9 of Ref. [7], in which the usual $Q\cdot Q$ interaction was used. It is also interesting to observe that the signature dependence of $V$ in the lower part of Fig. 7 is much more prominent than in the lower part of Fig. 6, while the signature splitting of energies (i.e., Routhians) for $\lambda_p/\kappa)=-0.1$ is much smaller than that for $\lambda_p/\kappa=-0.9$, as is well known.

## IV. DISCUSSION AND CONCLUSION

We have extracted the interaction strength between the $g$ band and the $S$ band, choosing the rare-earth nuclei in which the state of both bands, as well as $E2$ transition rates for the $\gamma$ decay to yrast and nonyrast states, are observed. When the obtained interaction strengths are plotted as a function of the number of neutrons, they do not show an oscillating behavior. However, when they were plotted as a function of the degree of the neutron $i_{13/2}$ shell filling, we obtained the expected oscillating behavior of $|V|$.

When the interaction strength $V$ in odd-$Z$ rare-earth nuclei is calculated using the conventional particle-rotor model, it hardly shows an appreciable amount of the signature dependence when the magnitude of $|V|$ is smaller than, say, 70 keV, irrespective of the degree of the proton $h_{11/2}$ shell filling. However, when a neutron-proton $Q''\cdot Q''$ interaction with a coupling constant, which is an order of magnitude larger than the self-consistent value in the harmonic-oscillator model, is introduced in the conventional particle-rotor model, it is possible to explain the signature dependence of $|V|$ observed in $^{157}$Ho. The calculated signature dependence comes from the $Y_{22}$ component (namely, the fluctuation towards a triaxial shape) of the neutron-proton $Q''\cdot Q''$ interaction. Though the introduction of the neutron-proton interaction does produce an appreciable amount of the signature dependence, the oscillating behavior of $|V|$ remains for each signature.

The contribution from the neutron-proton interaction to $|V|$ is found to be rather small (say, less than 60 keV) when the neutron Fermi level lies in the lower part of the $i_{13/2}$ shell (say, $\lambda_n/\kappa<-0.5$), in spite of the fact that we have used a coupling constant which is almost an order of magnitude larger than the self-consistent value. We have also estimated $|V|$ when the Fermi level of an odd proton is placed below a single $h_{9/2}$ shell (instead of $h_{11/2}$ shell) in order to simulate the $[541\frac{1}{2}]$ band in the light rare-earth odd-$Z$ nuclei. It is found that the estimated values of $|V|$ for the favored signature (i.e., $\alpha=\frac{1}{2}$ for an $h_{9/2}$ shell), which is the signature of the rotational sequence observed experimentally, are quite similar to those of $\alpha_f$ shown in Fig. 6. In several odd-$Z$ rare-earth nuclei the frequency of the $g$-$S$ band crossing in the proton $[541\frac{1}{2}]$ configuration is observed to be 20–80 keV larger than the one observed in the $[514\frac{9}{2}]$ or $[404\frac{7}{2}]$ configurations, which usually is close to the crossing frequencies in the even-even neighbors (see, for example, Ref. [13]). To address this problem, using the present calculations in the particle-rotor model of Sec. III we have estimated the $g$-$S$ crossing frequencies for $\lambda_p/\kappa=-0.10$ in the proton $h_{11/2}$ shell (Fig. 7) and for $\lambda_p/\kappa=-1.05$ in, precisely speaking, below the proton $h_{9/2}$ shell. For the neutron $i_{13/2}$ shell filling, $\lambda_n/\kappa=-0.80$ (i.e., $N\simeq 90$), and $C_{n\text{-}p}=-0.7$, the estimated crossing frequency, averaged over the two signatures, for $\lambda_p/\kappa=-0.10$ in the $h_{11/2}$ shell is nearly equal to that of the favored signature for $\lambda_p/\kappa=-1.05$ in the $h_{9/2}$ shell. Bearing in mind the fact that we have used the strength of the neutron-proton interaction, which is stronger than the renormalized self-consistent value by an order of magnitude, it is concluded that one cannot explain the observed peculiar behavior [13] of $|V|$ in the two crossing bands, in which the odd-proton occupies the $[541\frac{1}{2}]$ orbit in odd-$Z$ rare-earth nuclei [31].

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