Two-to-one relationships between the spectra of selected even-even and even-odd nuclei

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Data are gathered to put to the test of a prediction of the single j shell model, that for selected nuclei the excitation energy of the double isobaric analog state in an even-even nucleus is twice the excitation energy of the single analog state in the adjacent even-odd nucleus. This two-to-one relationship works much better than one would expect. The effects of configuration mixing and the Coulomb interaction are considered.

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Many years ago McCullen, Bayman, and Zamick (MBZ) [1] pointed out that a two-to-one relation exists between the spectra of selected even-even and even-odd nuclei. In a single j shell model calculation ($j=f_{7/2}$) they found that the energy spacings of $J^\pi=0^+$ levels in ^{48}Ti are exactly twice the corresponding spacings of $J^\pi=\frac{7}{2}^-$ levels in ^{49}Ti. A similar relationship holds for the pair ^{44}Ti and ^{43}Ti as well as the pair ^{52}Fe and ^{53}Fe. The elements of the secular matrix for the diagonalization of the $J^\pi=0^+$ levels of ^{48}Ti are two times those for the $J^\pi=\frac{7}{2}^-$ states in ^{49}Ti.

It was further noted by the above authors [1] that the wave functions for $J^\pi=0^+$ states of ^{48}Ti and the $J^\pi=\frac{7}{2}^-$ states of ^{49}Ti are "identical." This means that if the ^{48}Ti wave function is of the form $\psi=\sum_L D[LL][LL]^{J=0}$ and the ^{49}Ti wave function is of the form $\psi=\sum_L D[jL][jL]^{J=j}$ where L is the angular momentum of the two protons, then we have, for any interaction, $D[LL]=D[jL]$. As a consequence it was pointed out that the $l=3$ spectroscopy strength in the reaction ^{48}Ti$(d,p)^{49}$Ti should all be to the ground state with strength 2.

Since that time this observation has been largely ignored despite the fact that some data have accumulated which could put the two-to-one relation to a test. We will discuss this in the next sections.

Note that a necessary condition for getting such relations is that the number of basis states is the same for the even-even and for the even-odd systems. We designate the basis states by $[L_p,L_n]$. For $J^\pi=0^+$ in ^{48}Ti the basis states are [0,0], [2,2], [4,4], and [6,6]. For $J^\pi=\frac{7}{2}^-$ in ^{49}Ti, they are $[0,\frac{7}{2}]$, $[2,\frac{7}{2}]$, $[4,\frac{7}{2}]$, and $[6,\frac{7}{2}]$.

Double analog excitation energies in even-even nuclei versus single analog excitation energies in neighboring even-odd nuclei. A sensible way to test the two-to-one relationship is to consider the excitation energies of the double isobaric analog state in the even-even nucleus (denoted by E_e^*) and the single analog state in the corresponding even-odd nucleus (denoted by E_o^*). For example, in the $f_{7/2}$ model, there are four $J^\pi=0^+$ states in ^{48}Ti. Three of these are $T=2$ states and one is a $T=4$ state. The latter is the double analog of the ground state of ^{48}Ca. In ^{49}Ti, in the same model, there are also four $J^\pi=\frac{7}{2}^-$ states, one of which is the single analog of the $J^\pi=\frac{7}{2}^-$ ground state of ^{49}Sc.

The excitation energy of the $J^\pi=0^+$ double analog state in ^{48}Ti (that is to say, the energy difference of this state and the $J^\pi=0^+$, $T=2$ ground state) should be twice the energy difference of the $J^\pi=\frac{7}{2}^-, T=\frac{7}{2}$ state in ^{49}Ti relative to the $J^\pi=\frac{7}{2}^-, T=\frac{5}{2}$ ground state.

We give the relevant energies for ^{48}Ti, ^{49}Ti, as well as other nuclei in Table I under the column "Expt-data." A particularly useful reference is the work of Kouzes, Kutt, Mueller, and Sherr [2]. By measuring Q values in the reactions (p,t), $(p,^3\mathrm{He})$, and (p,d), this group determined the energies of about 25 isobaric analog states in the $f_{7/2}$ region. The authors found that the $T=4-T=2$ splitting of the $J^\pi=0^+$ states in ^{48}Ti is 17.379 MeV; the $T=\frac{7}{2}-T=\frac{5}{2}$ splitting of the $J^\pi=\frac{7}{2}^-$ states in ^{49}Ti is 8.724 MeV. We use the notation "Δ" to signify the deviation of the excitation energies E_e^* and E_o^* from the two-to-one rule:

$$\Delta=\frac{E_e^*}{2E_o^*}-1\ . \tag{1}$$

For the ^{48}Ti-^{49}Ti pair, the deviation Δ is equal to -0.40%.

Kouzes *et al.* [2] also give the $T=2-T=0, J^\pi=0^+$ splitting in ^{52}Fe, while the $T=\frac{3}{2}-T=\frac{1}{2}, J^\pi=\frac{7}{2}^-$ splitting in ^{53}Fe is given by Suehiro, Finck, and Nolen [3]. The respective numbers are 8.559 and 4.250 MeV. The deviation Δ is 0.69%. There is, however, another measurement by Fortier *et al.* [4] that gives the ^{53}Fe excitation energy as 4.264 MeV. This leads to a deviation of 0.36%.

For ^{44}Ti the excitation energy of the double analog 0^+ state is again given by Kouzes *et al.* [2] as 9.340 MeV. Unfortunately there are no data available on the single analog excitation in ^{43}Ti.

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TABLE I. The experimental and calculated excitation energies (in units of MeV) of the double isobaric analog 0^+ states in selected even-even nuclei and the single isobaric analog $\frac{7}{2}^-$ states in the adjacent even-odd nuclei and the deviations (Δ) of these energies from the two-to-one rule.

Pairs and Dev. Δ	Expt-data	SJ using $A=42$ data ^{42}Sc[a]	CaScTi[b]	SJ and CM using the KB SJ[c]	SJ+C[d]	CM[e]	CM+C[f]
^{44}Ti	9.340	8.284	8.175	5.938	5.901	7.929	7.820
^{43}Ti		4.142	4.052	2.969	2.935	3.643	3.561
Δ		0.00%	0.88%	0.00%	0.53%	8.83%	9.80%
^{44}Ti	9.340	8.284	8.175	5.938	5.901	7.929	7.820
^{45}Ti	4.716	4.112	4.039	2.996	2.957	3.811	3.706
Δ	−0.98%	0.73%	1.20%	−0.90%	−0.22%	4.03%	5.50%
^{46}Ti	14.153	13.204	13.048	9.094	9.037	11.384	11.282
^{47}Ti	7.187	6.590	6.421	4.558	4.514	5.923	5.835
Δ	−1.54%	0.18%	1.60%	−0.24%	0.10%	−3.90%	−3.32%
^{48}Ti	17.379	17.659	17.474	11.549	11.478	14.261	14.148
^{49}Ti	8.724	8.829	8.692	5.775	5.703	7.437	7.340
Δ	−0.40%	0.00%	0.52%	0.00%	0.63%	−4.12%	−3.62%
^{52}Fe	8.559	8.284	8.175				
^{53}Fe	4.250	4.142	4.052				
Δ	0.69%	0.00%	0.88%				

[a]Single $f_{7/2}$ shell calculation using the ^{42}Sc levels.
[b]Single $f_{7/2}$ shell calculation using the ^{42}Ca, ^{42}Sc, and ^{42}Ti levels.
[c]Single $f_{7/2}$ shell calculation using the KB interaction.
[d]Single $f_{7/2}$ shell calculation using the KB+Coulomb interaction.
[e]Extended *fp* space calculation using the KB interaction.
[f]Extended *fp* space calculation using the KB+Coulomb interaction.

Cross conjugates and mirrors. It should be mentioned that there are related data for cross conjugate and/or mirror pairs. The nucleus ^{43}Sc is the mirror of ^{43}Ti. The single analog excitation energy in ^{43}Sc is 4.236 MeV. This gives a large deviation of about 9.4% when compared with ^{44}Ti. However, since the Coulomb interaction is different in the two nuclei, the results are not expected to be so good.

Likewise ^{47}Sc is the cross conjugate of ^{49}Ti. The single analog excitation energies in the two nuclei are, respectively, 8.400 and 8.724 MeV. If we used the ^{47}Sc data to compare with ^{48}Ti we would get about a 3.45% deviation. But as mentioned before, the deviation using the ^{49}Ti data is much smaller (−0.40%). It therefore makes much more sense to make the comparison with nuclei that have essentially the same Coulomb energy.

Lastly, ^{53}Co is the mirror of ^{53}Fe. The single analog excitation energies in these two nuclei are, respectively, 4.390 MeV and (as mentioned before) 4.250 MeV. Again, the deviation with respect to ^{52}Fe is much smaller if we use the ^{53}Fe rather than the ^{53}Co data.

The data which have been referred to in the above are also contained in the Nuclear Data Sheets [5] in volumes 43 and 61 ($A=53$) and 48 ($A=47,49$). In recent years, data on double isobaric analog states are also being accumulated from pion double charge-exchange experiments [6–8].

Other pairs. We may wonder if the values of deviation (Δ) are small only for the special pairs considered above for which the two-to-one relationship holds in the single *j* shell model, or whether the smallness of Δ is a more general phenomenon. We will therefore consider other pairs of nuclei.

Our first example is the pair ^{44,45}Ti. The excitation energy of the $J^\pi=\frac{7}{2}^-, T=\frac{3}{2}$ single analog state in ^{45}Ti, as shown in Table I, is 4.716 MeV, giving a value of Δ equal to −0.98% with respect to ^{44}Ti. The deviation is small.

Another pair of interest is ^{46,47}Ti. The experimental $T=3-T=1, J^\pi=0^+$ splitting in ^{46}Ti is 14.153 MeV while the $T=\frac{5}{2}-T=\frac{3}{2}, J^\pi=\frac{7}{2}^-$ splitting in ^{47}Ti is 7.187 Mev. (Note that for ^{47}Ti, the ground state is a $J^\pi=\frac{5}{2}^-$ state. We must take the excitation energy relative to the lowest $J^\pi=\frac{7}{2}^-$ state which is at an excitation energy of 159 keV.) The value of Δ is −1.54%. This is again very small. The data for $A=45$ and $A=46$ are also contained in Nuclear Data Sheets [5].

It should be noted that in the single *j* shell model, the size of the basis states is different for the odd A and even A systems considered in this section. For example, for ^{46}Ti, there are six 0^+ basis states while for ^{47}Ti there are seventeen $\frac{7}{2}^-$ basis states.

Configuration mixing and charge symmetry violation effects. We would expect the two-to-one relationship to be destroyed by configuration mixing effects and by the Coulomb interaction (or other charge symmetry violation interactions), and indeed it is. The question is by how much.

We first show in Table I a single *j* shell calculation with matrix elements taken from experiment. The

$J^\pi=0^+$ "pairing" energies of ^{42}Ca, ^{42}Sc, and ^{42}Ti are taken from binding energies of Wapstra and Gove [9]. For example, the value of -3.109 MeV for ^{42}Ca is given by $[2E_B(^{41}\text{Ca})-E_B(^{42}\text{Ca})-E_B(^{40}\text{Ca})]$ where E_B is the binding energy. The smaller magnitude for ^{42}Ti (-2.684 MeV) is due mainly to the Coulomb interaction. The fact that the value for ^{42}Sc (-3.182 MeV) is different from that for ^{42}Ca indicates that there are non-Coulombic charge symmetry violation effects. From the compilation of Endt [10] we get the following spectra:

J^π	0^+	1^+	2^+	3^+	4^+	5^+	6^+	7^+
^{42}Ca	0		1.5247		2.7525		3.1893	
^{43}Sc	0	0.6111	1.5863	1.4904	2.8153	1.5101	3.242	0.6163
^{42}Ti	0		1.5549	2.6764		3.0430		

Using the above numbers as input we show also in Table I the results of two single j calculations. In the "^{42}Sc" column, we use the levels of ^{42}Sc and maintain charge independence by setting all the $T=1$ matrix elements to be independent of T_z. In the "CaScTi" column we use the ^{42}Ca levels for the nn interaction, the ^{42}Sc levels for the np interaction, and the ^{42}Ti levels for the pp interaction.

The ^{42}Sc calculation verifies that the value of Δ is zero for special pairs ^{43,44}Ti, ^{48,49}Ti, and ^{52,53}Fe. When the charge symmetry is broken (column "CaScTi"), one gets positive values of Δ for all pairs, ranging from 0.52% to 1.60%. However, experimentally for all *known* cases except for ^{52,53}Fe, the value of Δ is negative (see column "Expt-data" in Table I).

We next consider the effects of configuration mixing and present the results again in Table I. We use the modified Kuo-Brown (KB) [11] interaction. The matrix diagonalization is performed with the OXBASH shell model code [12]. We present the results progressively as follows: Column "SJ" gives the single j results using the KB interaction; column "SJ+C" gives single j results using the KB plus Coulomb interaction (this should be compared with column "CaScTi" in the same table); column "CM" gives the configuration mixing results using the KB interaction; and column "CM+C" gives the configuration mixing results using the KB plus Coulomb interaction. In the configuration mixing case, for ^{43}Ti and ^{44}Ti, all nucleons could be excited into the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ orbits while for heavier titanium isotopes only two nucleons (due to space-time limitations) were allowed to be excited to higher shells.

We see that in the full calculation (CM+C), the values of $|\Delta|$ are much larger than experiment. The values of Δ for the pairs ^{43,44}Ti, ^{44,45}Ti, ^{46,47}Ti, and ^{48,49}Ti are, respectively, 9.80%, 5.50%, -3.32%, and -3.62%, as compared with experimental values, (unknown), -0.98%, -1.54%, and -0.40%. In other words, the two-to-one relation holds much better empirically than calculations with configuration mixing would suggest. Since the ^{43}Ti data are not available we consider the pair ^{43}Sc-^{44}Ti. The "CM+C" calculation gives an excitation energy of 3.632 MeV for the single analog $J^\pi=\frac{7}{2}^-$ state in ^{43}Sc. This gives a calculated value of Δ of 7.65%. The experimental value for this pair is 9.4%.

Note that the deviation due to the Coulomb interaction is always positive, whereas the configuration mixing effects are positive for ^{43,44}Ti and ^{44,45}Ti pairs, but negative for ^{46,47}Ti and ^{48,49}Ti pairs. Thus for the two heavier pairs the smallness of the experimental deviation can be explained partly by the cancellation of the Coulomb and the configuration mixing effects. In detail, however, the empirical deviation is smaller. Comparing columns "CaScTi" and "SJ+C," one might deduce that the other charge symmetry violation effects add constructively with the Coulomb effects to give a larger, positive contribution to Δ than the Coulomb alone.

For ^{43,44}Ti the calculations indicate that the Coulomb and CM effects will add up constructively to give a *larger* deviation from the two-to-one rule. For ^{44,45}Ti the Coulomb and CM also add constructively to give a larger deviation, 5.5%, whereas the empirical deviation is only -0.98%.

It has been suggested to us by Sherr [13] that a simple interaction $b\mathbf{t}_1\cdot\mathbf{t}_2$ will give a two-to-one ratio for the nuclear pairs considered. For ^{43,44}Ti, ^{48,49}Ti, and ^{52,53}Fe, this has to be true because any charge independent interaction will give the two-to-one ratio. However, Sherr's suggestion widens the scope of pairs to be considered. It includes the cases ^{44,45}Ti and ^{46,47}Ti for which the single j shell model does not predict a precise two-to-one ratio, but which nevertheless obey the rule to better than 1.6%.

In closing we note that we have found a two-to-one relationship between selected even-even and adjacent even-odd nuclei which works much better than one could expect, given that it is based on the single j shell model. We strongly suggest that a much wider search be made for more examples of what was discussed here (double and single analog excitation energies) and that we look for other such interesting relations. Experiments to fill in the missing data, e.g., the single analog excitation energy in ^{43}Ti, would clearly be of great value.

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