Relativity and the enhancement of the weak axial-charge matrix elements in the lead region

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We present a calculation of first-forbiden β-decay transitions in the lead region using relativistic mean-field and relativistic Hartree approximations to the quantum hadrodynamic-II model of Serot and Walecka. A formalism for the response of the core to a valence particle or hole in the case of weak transitions is developed and studied for the π- and ρ-meson fields present in the model. Two applications are shown for which the mean-field results show a sizable enhancement of the matrix element. This enhancement is partly quenched by the inclusion of the vacuum corrections. The core response for these cases turns out to be negligible in the long wavelength limit. Altogether, it is concluded that the relativistic effects discriminate against nonrelativistic calculations but, however, are not enough to account for the recently reported discrepancy between shell model analysis of beta-decay rates and the (nonrelativistic) impulse approximation and meson-exchange-current calculations.

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I. INTRODUCTION

It has been known for quite some time that the extracted value of the effective axial-vector coupling constant *in nuclei* differs appreciably from what is obtained in free space. Particularly, studies of axial-charge transitions in light nuclei [1] have shown an enhancement of the axial coupling constant of roughly 50% with respect to the experimental value. In other words, the matrix elements calculated in the nuclear medium using the impulse approximation (IA) (nonrelativistic) and the axial coupling constant determined from *free* neutron decay ($g_A \approx 1.26$) have turned out to be, typically, 50% smaller in nuclei than what is observed experimentally. Simultaneously, theoretical studies of meson-exchange-current (MEC) effects during the past decade or so [2–6] have predicted a strong renormalization of the axial-charge transition matrix elements in nuclei by roughly 40% to 60%. This enhancement was sufficient to account for the missing strength in the matrix elements. In particular, a calculation conducted on the lead region—the one of interest in this paper—by Kirchbach and Reinhardt [7] anticipated a 40% enhancement of the corresponding matrix element due to the MEC contribution.

Recently, however, careful shell-model analyses of first-forbidden β-decay (FFBD) rates in nuclei in the region $A = 208 - 212$ [8] suggested the existence of an anomalous enhancement of the *effective* axial-vector coupling constant of about 100% with respect to the free-space value. This enhancement which cannot be accounted for by the existing MEC calculations shows that an underprediction of the data of the order of 40% to 50% still remains. Kubodera and Rho have recently [9] ascribed this excess enhancement to an in-medium renormalization of the pion decay constant and nucleon mass that appear as parameters in the pion exchange part of the axial-charge operator. This idea has been discussed by Kirchbach, Riska, and Tsushima [10] who claim that this enhancement is not an effect that depends strongly on the medium but a natural consequence of the large attractive central force component of the nucleon-nucleon interaction, which can be represented as an effective scalar-meson-exchange component. Since this is one of the key elements in relativistic mean-field theories it becomes of interest to examine whether the use of strong relativistic dynamics might help to explain the missing strength on the determination of the IA matrix elements. As a matter of fact, for the case of transitions that proceed via the axial charge, relativity is expected to produce a strong enhancement since the operator driving these transitions ($\gamma_0\gamma_5$) is off diagonal in Dirac space thus connecting upper-to-lower components of the medium-modified Dirac spinors [11]. The enhanced lower components would, *a priori*, enhance the current matrix elements (a true statement if initial and final states coincide). This occurs much the same way in which isoscalar magnetic moments, deriving from the Dirac (nonanomalous) coupling (γ), are enhanced over the Schmidt values [12]. However, it is also known that this enhancement is compensated by a medium-induced current driven by new meson fields whose source is the extra valence particle or hole [13–15].

Relativistic effects in weak transitions have been explored before in the literature [14, 16, 17]. To our knowledge, all calculations were conducted, exclusively, for the $0^+ \leftrightarrow 0^-$ transitions in the $A = 16$ system. After some initial controversy on the importance of the relativistic effects for axial-charge transitions (see, for example, Refs. [14] and [18]), the current understanding is that β-decay matrix elements *do* get enhanced in the nuclear medium with respect to their nonrelativistic counterparts [the amount of this enhancement depending on the $(2s_{1/2}1p^{-1}_{1/2})$ — $(1d_{3/2}1p^{-1}_{3/2})$ mixing in the 0^-, $T = 1$ first-excited state in ^{16}N]. For the inverse process $^{16}O(0^+) \rightarrow ^{16}N(0^-)$ through muon capture, however, the capture rate turns out to be almost insensitive to the relativistic dynamics with the proviso that pseudo-vector (PV) coupling be used at the πN vertex [which satisfies partial conservation of the axial current (PCAC) at the operator level] in order to obtain the induced pseudoscalar contribution to the axial-vector current. This term becomes more important as the energy transfer increases and tends to cancel the enhancement of the axial charge. Despite that the relativistic description improves on the standard nonrelativistic method these calculations have not provided a categorical answer as to the necessity of using relativistic dynamics to understand nuclei. The fact is that the unexplained territory between IA and experiment in the $A = 16$ system can be covered simultaneously by relativity, MEC, and configuration mixing or any combination of the three. The FFBD rates in the $A \approx 208$ system appear to be somewhat different from those in $A = 16$. The gap between IA and experiment is broader in the former, which may indicate a medium-dependent effect of the sort found in relativistic mean-field theories.

So far, all relativistic calculations in the weak sector have been carried out at a phenomenological level. In this paper we conduct a model-consistent relativistic calculation in which vacuum corrections and the influence of isovector core-polarization effects are included. We present results for the mean-field and the relativistic Hartree approximations to the quantum hadrodynamic (QHD-II) relativistic model of Serot and Walecka [19] which includes, besides the baryons and the mesons σ and ω, the charged meson fields π and ρ, and the photon, as the dynamical degrees of freedom. The idea is to give an account of the full one-body contribution to the weak current and to give a quantitative idea of the effects introduced by the relativistic treatment. It should be borne in mind that for a heavy system the initial and final wave functions may differ not only in isospin but in their spatial content, thus requiring numerical results before drawing any conclusion. The two-body contribution to the weak current is not considered here and will be examined in a future paper [20].

The paper is divided into three sections. In Sect. II we describe the general formalism we use to calculate relativistic matrix elements. We present first some general considerations on the structure of the axial current and give an expression for the β-decay rate in terms of multipole operators for the long-wavelength approximation. The restrictions imposed by parity selection rules between nuclear states for the different multipole operators are summarized in the same section. In Sect. III we study the changes that the addition of one nucleon to a filled Fermi sphere introduces in the single-particle core wave functions—and consequently their matrix elements. The extra particle (hole) gives rise to additional meson fields which modify the core. It is known that *isoscalar electromagnetic* transitions for the odd-A nucleus are strongly renormalized by this core response to the added valence particle (hole). For *isovector electromagnetic* transitions the medium response driven by the isovector ρ-meson field is weak [15] (and known, from nonrelativistic calculations, to be affected by nuclear structure effects and meson-exchange currents). Thus, for *isovector weak* currents—those of interest in this paper—one may also anticipate a weak response of the core. However, this is not straightforward. Weak currents have a vector and an axial piece and since the pion, which is also an isovector meson present in QHD–II, carries axial charge it might act as a meson source for an extra axial-vector response from the core. As it turns out, however, we show in this section that the pion contribution to the axial current is vanishingly small in the $q \rightarrow 0$ limit. Since in our applications we work in this limit, there is no pion core response to the valence particle. To calculate the core response we resort to perturbation theory in nuclear matter and derive a general expression for the matrix element of a current operator between initial and final states sharing the same core but which differ by the valence particle. The perturbative series is summed to all orders in the random-phase approximation (RPA), and the polarization insertions to the static π- and ρ-meson fields are calculated.

In Sect. IV we apply the developments of the preceding sections and present results for two transitions in the lead region: the $9/2^+ \rightarrow 9/2^-$ β transition of $^{209}Pb \rightarrow ^{209}Bi$ and the $1/2^+ \rightarrow 1/2^-$ transition of $^{207}Tl \rightarrow ^{207}Pb$ where we have one nucleon (hole) outside the double closed shell of ^{208}Pb. These transitions offer *a priori* the best possibilities to investigate the implications that the relativistic model might bring about. They involve only one nucleon (particle or hole) outside the double closed shell of ^{208}Pb and have been extensively studied in the nonrelativistic form (see, for example, Chap. 14, Sect. 3 in Ref. [21]). Finally in the same section we discuss the results and give some conclusions.

II. RELATIVISTIC CURRENT AND MULTIPOLE OPERATORS

A. General considerations

Throughout the paper we use the formalism of Walecka [22] to calculate semileptonic weak processes. It is an alternative approach to the more traditional one of Behrens and Bühring [21] but has the advantage of being more amenable to the use of the relativistic dynamics than the traditional approach. A one-to-one relationship between the two formalisms has been established and can be found in Ref. [21]. We begin with the basic $V - A$ current-current form of the charge-changing weak interaction with an effective hadronic current density operator of the form

$$J_\mu^{\mathrm{wk}} = J_\mu^{0,\mathrm{wk}} + J_\mu^{5,\mathrm{wk}}. \tag{2.1}$$

In the following we drop all isospin dependence for convenience. With the assumptions of the impulse approximation, the conserved-vector-current hypothesis (CVC), and the absence of second-class currents (G invariance), the one-body weak vector current operator is given by

$$J_\mu^{0,\mathrm{wk}} = \left[F_1(q_\mu^2)\gamma_\mu + iF_2(q_\mu^2)\sigma_{\mu\nu}q^\nu\right]. \tag{2.2}$$

Here, q_μ is the four-momentum transfer and F_1 and F_2 are form factors which, via CVC, coincide with the electromagnetic Dirac and Pauli form factors [$F_1(0) \equiv g_V$]. Likewise, with the same assumptions as above the general form of the one-body axial-vector weak current is expressed as

$$J_\mu^{5,\mathrm{wk}} = g_A\left[F_A(q_\mu^2)\gamma_5\gamma_\mu + F_P(q_\mu^2)\gamma_5 q_\mu\right], \tag{2.3}$$

where $g_A = -1.261 \pm 0.004$, F_A is a form factor such that $F_A(q^2 \to 0) = 1$, and F_P is the induced pseudoscalar (pseudovector) form factor,

$$F_P(q_\mu^2) = g_P(q_\mu^2)\frac{2m_N}{m_\pi^2 - q^2} \qquad \text{(pseudoscalar)},$$

$$F_P(q_\mu^2) = -g_P(q_\mu^2)\frac{q_\nu\gamma^\nu\gamma_5}{m_\pi^2 - q^2}; \qquad \text{(pseudovector)}$$

with $g_P(m_\pi^2) \approx 1$. There is, clearly, an ambiguity as to the choice of pseudoscalar or pseudovector coupling for the pion—giving rise to this contribution—which can be lifted, however, by invoking partial conservation of the axial current at the operator level [14],[17]. As we mentioned in the introduction, this will be important for μ capture but not for the FFBD processes that we study in this paper.

Since we are interested in transitions between nuclear states with good angular momentum and definite parity, we perform a multipole expansion of the Fourier transform of the timelike and spacelike pieces of the weak hadronic current. We follow closely the prescriptions of Ref. [23] with only minor modifications due to different phase conventions. Thus, we first define the following scalar and vector functions in terms of spherical Bessel functions and scalar and vector spherical harmonics:

$$M_J^M(q\mathbf{r}) = j_J(qr)Y_J^M(\hat{\mathbf{r}}), \tag{2.4}$$

$$\mathbf{M}_J^M(q\mathbf{r}) = j_L(qr)\mathbf{Y}_{JL}^M(\hat{\mathbf{r}}), \tag{2.5}$$

where

$$\mathbf{Y}_{JL}^M = \sum_{m\mu} C_{m\mu M}^{L1J} Y_L^M(\hat{\mathbf{r}})e_\mu. \tag{2.6}$$

The e_μ are the spherical unit vectors

$$\left[e_{\pm 1} = \mp(e_x \pm i e_y)/\sqrt{2}, \quad e_0 = e_z\right] \tag{2.7}$$

and q denotes the magnitude of the three-momentum transfer to the nucleus. The multipole operators of the vector and axial-vector current densities are

$$C_{JM}^{(0,5)}(q) = \int d^3r M_J^M(q\mathbf{r})J_0^{(0,5)}(\mathbf{r}), \tag{2.8a}$$

$$L_{JM}^{(0,5)}(q) = \int d^3r\left[\frac{i}{q}\nabla M_J^M(q\mathbf{r})\right]\cdot\mathbf{J}^{(0,5)}(\mathbf{r}), \tag{2.8b}$$

$$T_{JM}^{\mathrm{el}(0,5)}(q) = \int d^3r\left[\frac{1}{q}\nabla\times\mathbf{M}_J^M(q\mathbf{r})\right]\cdot\mathbf{J}^{(0,5)}(\mathbf{r}), \tag{2.8c}$$

$$T_{JM}^{\mathrm{mag},(0,5)}(q) = \int d^3r\mathbf{M}_J^M(q\mathbf{r})\cdot\mathbf{J}^{(0,5)}(\mathbf{r}), \tag{2.8d}$$

which correspond to the Coulomb, longitudinal, transverse electric, and transverse magnetic multipoles, respectively. The superscript "(0,5)" indicates that the above expressions apply to both vector and axial-vector multipoles. Note that for a conserved vector current, the longitudinal and Coulomb multipole are related by

$$L_{JM}^0(q) = \frac{q_0}{q}C_{JM}^0(q) \tag{2.9}$$

The β-decay rate in the threshold $|\beta_e| \to 0$ form, i.e., in the limit of low electron momentum, is given by (Ref. [22])

$$\omega_{\beta\pm} = \frac{2G^2}{3\pi^2}\int_{m_e}^{W_0=Q_\beta+m_e} \beta_e\epsilon^2(W_0^\pm - \epsilon)^2F^\pm(Z,\epsilon)\,d\epsilon\frac{1}{2J+1} \times\left\{\sum_{J=0}^{\infty}|\langle J_F||\hat{C}_J(0) - \hat{L}_J(0)||J_I\rangle|^2 + \sum_{J\geq 1}^{\infty}|\langle J_F||\hat{T}_J^{\mathrm{el}}(0) \mp \hat{T}_J^{\mathrm{mag}}(0)||J_I\rangle|^2\right\}. \tag{2.10}$$

Here the electron energy ϵ goes from the electron mass m_e to the maximum energy $W_0^\pm$ and the factor $F^\pm(Z,\epsilon)$ is the Fermi function which accounts for Coulomb effects and is the ratio of the electron density at the nucleus to that at infinity. Also $\beta_e \equiv [1 - (m_e/\epsilon)^2]^{1/2}$ and Z is the charge of the final nucleus. The overall weak coupling constant is taken to be $GM_p^2 = 1.023 \times 10^{-5}$, where M_p is the proton mass.

B. Parity selection rules

In the long-wavelength approximation that we employ in Eq. (2.10), i.e., the $q \to 0$ limit, only the multipoles $J = 0$ and $J = 1$ will be different from zero. Below, we list the multipole operators with their parity selection rules. Thus, $C_0^{(5)}$ and $L_0^{(5)}$ are the two operators allowed by parity for $\Delta J = 0$ transitions in FFBD's, and $C_1^{(0)}$, $L_1^{(0)}$, $T_1^{\mathrm{el},(0)}$, and $T_1^{\mathrm{mag},(5)}$ are those allowed by parity for $\Delta J = 1$ transitions.

$\Delta J=0$	$C_0^{(0)}$	$L_0^{(0)}$	$T_{J\geq 1}^{\mathrm{el},(0)}$	$T_{J\geq 1}^{\mathrm{mag},(0)}$	$C_0^{(5)}$	$L_0^{(5)}$	$T_{J\geq 1}^{\mathrm{el},(5)}$	$T_{J\geq 1}^{\mathrm{mag},(5)}$
Parity	+	+	no	no	–	–	no	no
$\Delta J=1$	$C_1^{(0)}$	$L_1^{(0)}$	$T_1^{\mathrm{el},(0)}$	$T_1^{\mathrm{mag},(0)}$	$C_1^{(5)}$	$L_1^{(5)}$	$T_1^{\mathrm{el},(5)}$	$T_1^{\mathrm{mag},(5)}$
Parity	–	–	–	+	+	+	+	–

In the strict $q=0$ limit the contribution from the matrix elements of the $L_0^{(5)}$, $C_1^{(0)}$, $L_1^{(0)}$, and $T_1^{\mathrm{mag},(5)}$ become vanishingly small and we are left essentially with the $C_0^{(5)}$ and $T_1^{\mathrm{el},(0)}$ operators.

C. Nuclear model

The relativistic wave functions for the nucleons in the nucleus are calculated in the standard Hartree approximation for a doubly-closed shell nucleus (^{208}Pb in our case). In calculating the β-decay rates we considered the QHD–II relativistic model of Serot and Walecka [19]. It includes, besides the σ and ω mesons, the ρ meson, the pion and the photon. The Lagrangian is given by

$$\begin{aligned}\mathcal{L} = \bar{\psi}\left[\gamma_\mu\left(i\partial^\mu - g_\omega\omega^\mu + \frac{1}{2}g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu - \frac{1}{2}e(1+\tau_3)A^\mu\right) - (m_N - g_\sigma\sigma)\right]\psi \\ -ig_\pi\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\psi - A_\mu\left[(\boldsymbol{\rho}_\nu\times\mathbf{B}^{\mu\nu})_3 + \{\boldsymbol{\pi}\times[\partial^\mu\boldsymbol{\pi} + g_\rho(\boldsymbol{\pi}\times\boldsymbol{\rho}^\mu)]\}_3\right] \\ +\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) + \frac{1}{2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}) - \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ -\frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \mathcal{L}_{ct},\end{aligned} \tag{2.11}$$

where

$$\mathbf{B}^{\mu\nu} \equiv \partial^\mu\boldsymbol{\rho}^\nu - \partial^\nu\boldsymbol{\rho}^\mu - g_\rho(\boldsymbol{\rho}^\mu\times\boldsymbol{\rho}^\nu) \tag{2.12}$$

is the ρ-meson field strength. Equation (2.11) omits terms containing Higgs mesons and nonlinear terms which vanish in the mean–field approximation. Though this applies also to the pion, it is kept in the Lagrangian since it will be considered later for the core-response calculation [24].

The equations of motion are solved in the Hartree approximation (MFT) with the parameters of the model fixed to nuclear matter saturation properties and the rms radius of ^{40}Ca [25]. The Hartree solutions were also calculated including the vacuum contributions to the energy and the source densities (RHA). This vacuum-corrected solutions were calculated using what Furnstahl and Price [26] call RHA/EP (RHA/effective potential); i.e., the effects of mean fields on the states in the Dirac sea were taken into account in local density approximation (LDA) [15]. The corrections due to the addition of one particle (hole) to a doubly-closed shell core will be addressed in the next section.

III. LINEAR RESPONSE CALCULATION

In this section we study the changes that the addition of one nucleon to a filled Fermi sphere introduces in the single-particle core wave functions—and consequently their matrix elements. We are interested in obtaining a general expression, in linear response theory, for the matrix element of a current operator of the form $J_{\Gamma_A}(x) = \bar{\psi}(x)\Gamma_A\psi(x)$, between given initial and final states. Γ_A is an arbitrary matrix in spin-isospin space. The initial and final states must have in common the same core (the filled Fermi sphere) and may differ only by the addition of a particle to an unoccupied state with the quantum numbers (k_i, s_i, τ_i) for the initial and (k_f, s_f, τ_f) for the final state. The derivation of this result is left for the appendix. Here we quote the final expression

$$\begin{aligned}j_{\Gamma_A}(x) \cong \langle\phi_f|J_{\Gamma_A}(x)|\phi_i\rangle = \frac{1}{\Omega}\exp\{-i(k_i-k_f)x\} \\ \times\Big\{\bar{u}(\mathbf{k}_f,s_f,\tau_f)\Gamma_A u(\mathbf{k}_i,s_i,\tau_i) + (2\pi)^3\delta^3(\mathbf{k}_f-\mathbf{k}_i)\delta_{s_f,s_i}\delta_{\tau_f,\tau_i}B^{\Gamma_A} \\ -\bar{u}(\mathbf{k}_f,s_f,\tau_f)\Gamma_B^a u(\mathbf{k}_i,s_i,\tau_i)\Delta^{ab}(k_i-k_f)\Pi^{\Gamma_A\Gamma_B^b}(k_i-k_f)\Big\},\end{aligned} \tag{3.1}$$

where $\Delta^{ab}(q)$ is the meson propagator, $\Pi^{\Gamma_A\Gamma_B}(q)$ is the dressed polarization insertion, which can be calculated (see Fig. 1) from the "bare" polarization $\Pi_0^{\Gamma_A\Gamma_B}$,

$$\Pi_0^{\Gamma_A\Gamma_B}(q) = i\int \frac{dk^4}{(2\pi)^4}\exp(ik^0\eta) \times \mathrm{Tr}\left[\Gamma_A G(k+q)\Gamma_B G(k)\right], \quad (3.2)$$

and B_A^Γ is given by

$$B^{\Gamma_A} = -i\int \frac{dk^4}{(2\pi)^4}\exp(ik^0\eta)\ \mathrm{Tr}\left[\Gamma_A G(k)\right]. \quad (3.3)$$

In the above formulas Γ_B is a matrix in Dirac space in terms of which the interaction Hamiltonian is expressed as

$$H_I(\psi) = \bar{\psi}(x)\Gamma_B^a\Pi^a(x)\psi(x) \quad (3.4)$$

with $\Pi(x)$ a generic meson field operator.

The three terms in Eq. (3.1) can be schematically depicted as shown in Fig. 2. The first term corresponds to the valence-particle current interaction with the external field; the second to the "core" interaction with the field (which vanishes in the case that Γ_A corresponds to the axial-vector isovector current), and the third to the "backflow" or relativistic random-phase-approximation current. We shall use this expression to calculate the core-polarization correction to the single-particle matrix element.

We now calculate the bare polarization insertion $\Pi_0^{\Gamma_A\Gamma_B}(q)$ of Eq. (3.2) for the case where Γ_A is any of the matrix operators of the isovector axial-vector current. Hence, we only need to consider for Γ_B those operators stemming from the isovector baryon-meson interaction terms in the Lagrangian, namely the ρ- and π-meson fields. Using the analytic form of the Hartree propagator in nuclear matter (see appendix), the polarization insertion can be written, after integrating over the zeroth component, in the form,

$$\Pi^{\Gamma_A\Gamma_B}(q) = \int\frac{dk^3}{(2\pi)^3}\Bigg\{\mathrm{Tr}\left[\Gamma_A\Lambda_+(\mathbf{k}+\mathbf{q})\Gamma_B\Lambda_+(\mathbf{k})\right]\left[\frac{n_{\mathbf{k}}(1-n_{\mathbf{k}+\mathbf{q}})}{q^0-E^*(\mathbf{k}+\mathbf{q})+E^*(\mathbf{k})+i\delta}+\frac{n_{\mathbf{k}+\mathbf{q}}(1-n_{\mathbf{k}})}{q^0-E^*(\mathbf{k}+\mathbf{q})+E^*(\mathbf{k})-i\delta}\right]$$
$$+\mathrm{Tr}\left[\Gamma_A\Lambda_+(\mathbf{k}+\mathbf{q})\Gamma_B\Lambda_-(-\mathbf{k})\right]\frac{1-n_{\mathbf{k}+\mathbf{q}}}{q^0-E^*(\mathbf{k}+\mathbf{q})-E^*(\mathbf{k})+i\delta}$$
$$-\mathrm{Tr}\left[\Gamma_A\Lambda_-(-\mathbf{k}-\mathbf{q})\Gamma_B\Lambda_+(\mathbf{k})\right]\frac{1-n_{\mathbf{k}}}{q^0+E^*(\mathbf{k}+\mathbf{q})+E^*(\mathbf{k})-i\delta}\Bigg\}, \quad (3.5)$$

where the first term corresponds to particle-hole excitations and the second and third to particle-antiparticle excitations. For the case of FFBD transitions the four-momentum transfer q^μ is small compared to κ_{Fermi}, which in first approximation serves as a justification for calculating the polarization insertion in the limit of q^μ going to zero. Different results are obtained by making $\mathbf{q}$ and q^0 go to zero in different order [15, 27]. In our case, $\mathbf{q}$ is strictly different from zero, which unlike the strict $q = 0$ uniform case, allows for the excitation of particle-hole pairs besides the particle-antiparticle excitations which are the ones allowed at zero momentum transfer. If we take first the $\mathbf{q} \to 0$ limit with finite q^0, the term corresponding to particle-hole excitations vanishes. Conversely, taking $q^0 \to 0$ followed by $\mathbf{q} \to 0$, this term gives rise, in general, to a nonzero contribution. This last limit is appropriate for our case. Keeping only the density-dependent terms, the results we obtain are shown in Tables I and II (all isospin dependence has been dropped for convenience).

Notice that, as advertised, the pion contribution to the axial response of the core vanishes in the $q \to 0$ limit (left column in Table I), and is strictly null when considered for the polar-vector response of the core (left column in Table II). Regarding the ρ contribution to the core response it is worth noticing that Π^{00} is notably enhanced with respect to that obtained taking first the $\mathbf{q} \to 0$ limit with finite q^0. This contribution is however irrelevant for our calculation since, as shown in Sect. II B, the zeroth component of the vector part of the weak *single-particle* current vanishes in the long-wavelength limit. The other

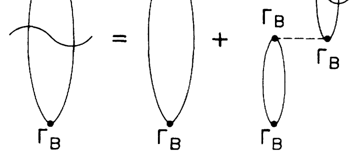

FIG. 1. Diagrammatic representation of the Bethe-Salpeter equation that gives the "dressed" polarization insertion $\Pi^{\Gamma_A\Gamma_B}$ in terms of the "bare" polarization $\Pi_0^{\Gamma_A\Gamma_B}$.

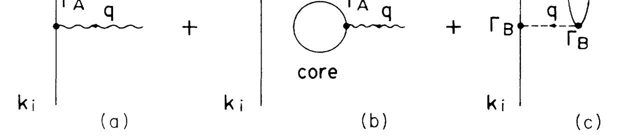

FIG. 2. Schematic representation of the three terms of Eq. (3.1) in the text: (a) valence-particle current interaction with the external field, (b) "core" interaction with the field, and (c) the "backflow" or relativistic random-phase-approximation current.

TABLE I. The polarization insertion of Eq. (3.5) for Γ_A corresponding to the first term in Eq. (2.3) for the axial-vector current.

$\Gamma_A = g_A\gamma^5\gamma^\mu$	
$\Gamma_B = ig_\pi\gamma^5$ (πNN vertex)	$\Gamma_B = \frac{1}{2}g_\rho\gamma^\nu$ (ρNN vertex)
$\lim_{q\to 0}\Pi^{\Gamma_A\Gamma_B}(q) = 0$	$\Pi^{\Gamma_A\Gamma_B}(q) = 0$

components of the polarization vanish regardless of the order chosen for the limits. As expected, current conservation requires that the longitudinal part of the vector polarization vanish in $q \to 0$ limit. In this limit these results agree with those of Chin [28].

In view of the results obtained for nuclear matter, we do not expect strong "backflow" effects from the core when dealing with finite nuclei. Thus, for the applications considered in the next section this contribution will be neglected.

IV. RESULTS AND DISCUSSION

In Table III we show the results for the $^{209}\mathrm{Pb} \to ^{209}\mathrm{Bi}$ transition and in Table IV the results corresponding to the $^{207}\mathrm{Tl} \to ^{207}\mathrm{Pb}$ β-decay. In the tables the first column corresponds to the nonrelativistic result, the second to the fully relativistic calculation using the MFT wave functions, and the third is also a fully relativistic calculation using the RHA/EP wave functions. Between brackets we quote the percentile increase with respect to the nonrelativistic result. The latter ones were obtained as follows. In order to use the computed relativistic matrix elements we substituted the Hartree wave functions for the solutions of the Dirac equation obtained using only one potential V (of timelike vector character) with a strength given by the sum of the scalar and vector Hartree potentials ($V \approx -90$ MeV). We tested this solution against standard nonrelativistic results (energy levels and magnetic moments) using Woods-Saxon potentials, with satisfactory agreement (one misses, of course, the spin-orbit splittings).

As expected the $\Delta J = 0$ matrix elements are strongly enhanced in both cases. In the decay of ^{209}Pb the matrix element of the $\hat{C}_0^{(5)}$ multipole operator is 60% larger than its nonrelativistic counterpart, whereas for the $1s_{1/2} \to 1p_{1/2}$ transition in ^{207}Tl the enhancement amounts to 40%. In the case of the $\Delta J = 1$ transitions the matrix elements of the $\hat{T}_1^{\mathrm{el}(0)}$ operator are suppressed with respect to the axial-charge $\Delta J = 0$ contribution, in both nuclei. For ^{209}Pb they are negligible ($\langle\hat{T}_1^{\mathrm{el}(0)}\rangle \approx \langle\hat{C}_0^{(5)}\rangle/20$) and for ^{207}Tl they amount to roughly 50% of the contribution of the axial charge. For this transition the relativistic calculation shows an enhancement of 51% with respect to the nonrelativistic result, whereas the former is quenched 22%. This is due to the different structure of the *Coulomb* and *transverse electric* operators, the latter one being very sensitive to the spatial difference between initial and final states.

The third column in the tables shows the results of the same calculation as above where the vacuum contributions to the mean fields have been considered. The presence of the vacuum quenches noticeably the enhancement obtained in the MFT case. Thus, for the $\Delta J = 0$ matrix elements of the $\hat{C}_J^{(5)}$ operator, the increment with respect to the nonrelativistic result is now 28% in the decay of ^{209}Pb and only 17% for the $^{207}\mathrm{Tl} \to ^{207}\mathrm{Pb}$ transition. As for the $\Delta J = 1$ transitions, the only relevant one—that in ^{207}Tl—gets its contribution also reduced with respect to the MFT calculation. The matrix element of the $\hat{T}_1^{\mathrm{el}(0)}$ operator is now 23% larger than the nonrelativistic result.

V. CONCLUSIONS

We have conducted relativistic mean-field calculations of first–forbidden β decays in the lead region using the relativistic Hartree approximation to QHD–II. We took into account the corrections due to the presence of the vacuum of the theory and the response of the core originating in the additional meson fields generated by an extra particle. The main conclusion is that in MFT the axial-charge matrix elements get notably enhanced. The response of the core in the long-wavelength limit (and in LDA) does not alter this result. However, the presence of the vacuum serves to quench this enhancement and to bring it to about 25% of the nonrelativistic result. This suggests that the use of relativity is not enough to account for the difference between the experimental and the IA values of the matrix elements. However, one has

TABLE II. The polarization insertion of Eq. (3.5) for Γ_A corresponding to the first term in Eq. (2.2) for the weak vector current.

$\Gamma_A = g_V\gamma^\mu$	
$\Gamma_B = ig_\pi\gamma^5$ (πNN vertex)	$\Gamma_B = \frac{1}{2}g_\rho\gamma^\nu$ (ρNN vertex)
$\Pi^{\Gamma_A\Gamma_B}(q) = 0$	if $\mu = \nu = 0$, $\lim_{q\to 0}\Pi^{\Gamma_A\Gamma_B}(q) = \frac{1}{2\pi^2}g_\rho g_V \kappa_{\mathrm{Fermi}} E^*(\kappa_{\mathrm{Fermi}})$.
	if $\mu = i \neq \nu = j$, $\Pi^{\Gamma_A\Gamma_B}(q) = 0$.
	if $\mu = 3$ or 0 and $\nu = 0$ or 3, longitudinal term, $\lim_{q\to 0}\Pi^{\Gamma_A\Gamma_B}(q) = 0$.
	if $\mu = \nu = 3$, $\lim_{q\to 0}\Pi^{\Gamma_A\Gamma_B}(q) = 0$.
	if $\mu = \nu = 1, 2$, transverse term, $\lim_{q\to 0}\Pi^{\Gamma_A\Gamma_B}(q) = 0$.

TABLE III. Contribution of the multipole operators $\hat{C}^{(5)}_{J=0}$ and $\hat{T}^{\mathrm{el}(0)}_{J=1}$ to the ^{209}Pb $\to ^{209}$Bi transition (ν : $2g_{9/2}$ $\to$ π : $1h_{9/2}$) in the nonrelativistic, mean-field (MFT) and RHA/EP cases (see text). Shown between parentheses is the percentile increase with respect to the corresponding nonrelativistic entry.

	Nonrelativistic	MFT	RHA/EP
$\langle\hat{C}^{(5)}_{J=0}\rangle$	-1.233×10^{-1}	-1.978×10^{-1}(+59.8%)	-1.579×10^{-1}(+27.7%)
$\langle\hat{T}^{\mathrm{el}(0)}_{J=1}(J=1)\rangle$	-8.821×10^{-3}	-7.780×10^{-3}(−22.3%)	-7.752×10^{-3}(−22.4%)

TABLE IV. Contribution of the multipole operators $\hat{C}^{(5)}_{J=0}$ and $\hat{T}^{\mathrm{el}(0)}_{J=1}$ to the ^{207}Tl $\to ^{207}$Pb transition [π : $(3s_{1/2})^{-1}$ $\to$ ν : $(3p^{-1}_{1/2})$] in the nonrelativistic, mean-field (MFT) and RHA/EP cases (see text). Shown between parentheses is the percentile increase with respect to the corresponding nonrelativistic entry.

	Nonrelativistic	MFT	RHA/EP
$\langle\hat{C}^{(5)}_{J=0}\rangle$	8.303×10^{-2}	1.156×10^{-1}(+39.2%)	9.125×10^{-2}(+17.0%)
$\langle\hat{T}^{\mathrm{el}(0)}_{J=1}(J=1)\rangle$	3.089×10^{-2}	4.662×10^{-2}(+50.9%)	3.796×10^{-2}(+22.8%)

to keep in mind the following observations. For the two examples that we showed above the experimental $\log(ft)$ values are in both cases slightly larger than those obtained in IA showing no need for an enhancement and, if anything, the need for a small quenching of the nonrelativistic result. This may be interpreted as the inability of our approach to describe correctly the nuclear structure. If this problem is temporarily ignored, there remains the fact that we have used typical nonrelativistic meson-exchange results which, of course, were not intended to be consistent with the model we employed above. Though we do not anticipate large discrepancies in this regard, a calculation of this sort is necessary before concluding whether relativity is relevant to understand axial charge transitions. Work in this direction is currently in progress.

APPENDIX

In this appendix we derive a general expression for the matrix element of a current operator of the form $J(x) = \bar{\psi}(x)\Gamma_A\psi(x)$, as described in Sect. III. The initial and final states have in common the same core (the filled Fermi sphere) and may differ only by the addition of a particle to an unoccupied state with the quantum numbers $(\mathbf{k}_i, s_i, \tau_i)$ for the initial and $(\mathbf{k}_f, s_f, \tau_f)$ for the final state. Thus, in the single-particle approximation we can write

$$|\phi_i\rangle = a^\dagger_{\mathbf{k}_i,s_i,\tau_i}|\phi_{\mathrm{core}}\rangle \quad \text{and} \quad \langle\phi_i| = \langle\phi_{\mathrm{core}}|a_{\mathbf{k}_f,s_f,\tau_f}, \tag{A1}$$

$$\begin{aligned} j_{\Gamma_A}(x) &= \langle\phi_f|J_{\Gamma_A}(x)|\phi_i\rangle \\ &= \langle\phi_{\mathrm{core}}|a_{\mathbf{k}_f,s_f,\tau_f}\bar{\psi}(x)\Gamma_A\psi(x)a^\dagger_{\mathbf{k}_i,s_i,\tau_i}|\phi_{\mathrm{core}}\rangle. \end{aligned} \tag{A2}$$

If we now write the operators corresponding to the creation or the annihilation of the added particle in terms of the field operators,

$$\psi(x) = \frac{1}{\sqrt{\Omega}}\sum_{\mathbf{k},s,\tau}\Big[a_{\mathbf{k},s,\tau}u(\mathbf{k},s,\tau)\exp\{-i[\epsilon^+(\mathbf{k})t-\mathbf{k}x]\} + b^\dagger_{\mathbf{k},s,\tau}v(\mathbf{k},s,\tau)\exp\{i[\epsilon^-(\mathbf{k})t+\mathbf{k}x]\}\Big], \tag{A3a}$$

$$\bar{\psi}(x) = \frac{1}{\sqrt{\Omega}}\sum_{\mathbf{k},s,\tau}\Big[a^\dagger_{\mathbf{k},s,\tau}\bar{u}(\mathbf{k},s,\tau)\exp\{i[\epsilon^+(\mathbf{k})t-\mathbf{k}x]\} + b_{\mathbf{k},s,\tau}\bar{v}(\mathbf{k},s,\tau) \times\exp\{-i[\epsilon^-(\mathbf{k})t+\mathbf{k}x]\}\Big], \tag{A3b}$$

we get

$$a_{\mathbf{k}_f,s_f,\tau_f} = \int\frac{dx^3_f}{\sqrt{\Omega}}\,\bar{u}(\mathbf{k}_f,s_f,\tau_f)\gamma^0\exp\{ik_fx\}\psi(x), \tag{A4a}$$

$$a^\dagger_{\mathbf{k}_i,s_i,\tau_i} = \int\frac{dx^3_f}{\sqrt{\Omega}}\,\bar{\psi}(x)\gamma^0u(\mathbf{k}_i,s_i,\tau_i)\exp\{-ik_ix\}, \tag{A4b}$$

and after substituting in Eq.(A2) we end up with

$$\begin{aligned} j_{\Gamma_A}(x) = \frac{1}{\Omega}\int dx^3_f\,dx^3_i\,\exp\{ik_fx\}\,\exp\{-ik_ix\}\,\bar{u}_\Omega(\mathbf{k}_f,s_f,\tau_f)\gamma^0_{\Omega,\alpha}\Gamma_{A\,\gamma,\delta} \\ \times\langle\phi_{\mathrm{core}}|\psi_\alpha(x_f)\bar{\psi}_\gamma(x)\psi_\delta(x)\bar{\psi}_\beta(x_i)|\phi_{\mathrm{core}}\rangle\gamma^0_{\beta,\epsilon}u_\epsilon(\mathbf{k}_i,s_i,\tau_i). \end{aligned} \tag{A5}$$

In the above we used a normal-mode expansion for the field operators with periodic boundary conditions in a large box of volume Ω. The u and v are spinors of positive- and negative-energy, respectively, solutions to the Euler-Lagrange equations for the fermionic fields of the Lagrangian of Eq. (2.11) in nuclear matter. The normalizations are such that

$$u^\dagger(\mathbf{k},s,\tau)u(\mathbf{k},s',\tau') = v^\dagger(\mathbf{k},s,\tau)v(\mathbf{k},s',\tau') = \delta_{s,s'}\delta_{\tau,\tau'}. \tag{A6}$$

We have also used that

$$\epsilon^{\pm}(\mathbf{k}) = g_\omega\omega_0 \pm \left[\mathbf{k}^2 + m_N^{*2}\right]^{1/2} = g_\omega\omega_0 \pm E^*_{\mathbf{k}} \tag{A7}$$

and

$$m_N^* = m_N - g_\sigma\sigma. \tag{A8}$$

Finally, in Eqs. (A4a)–(A4b) and (A5) we have used the definitions $k_f^0 = \epsilon^+(\mathbf{k}_f)$ and $k_i^0 = \epsilon^+(\mathbf{k}_i)$.

Since the calculation of the matrix element of Eq. (A5) cannot depend on the choice of the initial and final times and in order to introduce a time ordering in the matrix element we take

$$\langle\phi_{\rm core}|\psi_\alpha(x_f)\bar\psi_\gamma(x)\psi_\delta(x)\bar\psi_\beta(x_i)|\phi_{\rm core}\rangle = \lim_{\substack{t_i\to-\infty\\ t_f\to+\infty}} \lim_{x'\to x^+} \langle\phi_{\rm core}|\mathrm{T}\left[\psi_\alpha(x_f)\bar\psi_\gamma(x')\psi_\delta(x)\bar\psi_\beta(x_i)\right]|\phi_{\rm core}\rangle. \tag{A9}$$

The latter matrix element can be calculated using standard perturbation theory in terms of the unperturbed (by the extra particle) field operators. In general the interaction Hamiltonian may be written in the following fashion:

$$H_I(\psi) = \bar\psi(x)\Gamma^a_B\Pi^a(x)\psi(x), \tag{A10}$$

where Γ_B is a matrix in Dirac space and $\Pi(x)$ is a meson field operator. With "a" we denote the set of indices that run in spin and isospin. Next, we sum the perturbative series to all orders in the RPA approximation and, upon using the analytic form of the Hartree propagator in nuclear matter [19],

$$G^H(k) = (1-n_{\mathbf{k}})\frac{\Lambda_+(\mathbf{k})}{k^0 - E^*_{\mathbf{k}} + i\delta} + n_{\mathbf{k}}\frac{\Lambda_+(\mathbf{k})}{k^0 - E^*_{\mathbf{k}} - i\delta} - \frac{\Lambda_-(-\mathbf{k})}{k^0 + E^*_{\mathbf{k}} - i\delta}, \tag{A11}$$

where $\Lambda_+(k)$ and $\Lambda_-(k)$ are the standard projection operators for the core system, we obtain

$$j_{\Gamma_A}(x) = \frac{1}{\Omega}\exp\{-i(k_i-k_f)\cdot x\}\left\{\int\frac{dk^0}{2\pi}F^{\Gamma_A}_{fi}(k^0) + (2\pi)^3\delta^3(\mathbf{k}_f-\mathbf{k}_i)\delta_{s_fs_i}\delta_{\tau_f\tau_i}B^{\Gamma_A} - \int\frac{dk^0}{2\pi}F^{\Gamma^a_B}_{fi}(k^0)\Delta^{ab}(\mathbf{k}_i-\mathbf{k}_f,k^0)\Pi^{\Gamma_A\Gamma^b_B}(\mathbf{k}_i-\mathbf{k}_f,k^0)\right\}, \tag{A12}$$

where $\Delta^{ab}(q)$ is the meson propagator, $\Pi^{\Gamma_A\Gamma^b_B}(q)$ is the dressed polarization insertion, and B^Γ_A and $F^{\Gamma^a_B}$ are given by

$$B^{\Gamma_A} = -i\int\frac{dk^4}{(2\pi)^4}\exp\{ik^0\eta\}\,\mathrm{Tr}\left[\Gamma_A G(k)\right], \tag{A13a}$$

$$F^{\Gamma^a}_{fi} = 2\exp\{i(k_i^0 - k_f^0 - q^0)x_0\}\,\bar u(\mathbf{k}_f,s_f,\tau_f)\Gamma^a u(\mathbf{k}_i,s_i,\tau_i) \times\left[\frac{\exp\{i(q^0-k_i^0+k_f^0)t_f\} - \exp\{i(q^0-k_i^0+k_f^0)t_i\}}{2i[q^0-k_i^0+k_f^0]}\right]. \tag{A13b}$$

Given the freedom of choice for the limits $t_{i,f}\to\infty$ we take these limits symmetrically by putting $t_f = -t_i = -t'$ and taking $t'\to+\infty$. Further use of the result

$$\lim_{t'\to+\infty}\frac{\sin[(q^0-k_i^0+k_f^0)t']}{q^0-k_i^0+k_f^0} = \pi\,\delta(q^0-k_i^0+k_f^0) \tag{A14}$$

gives

$$F^{\Gamma^a}_{fi}(q^0) = 2\pi\delta(q^0-k_i^0+k_f^0)\,\bar u(\mathbf{k}_f,s_f,\tau_f)\Gamma^a u(\mathbf{k}_i,s_i,\tau_i). \tag{A15}$$

Finally substituting this in the expression for $j_{\Gamma_A}(x)$ of Eq. (A5) we obtain Eq. (3.1) in the text,

$$j_{\Gamma_A}(x) \cong \langle \phi_f | J_{\Gamma_A}(x) | \phi_i \rangle = \frac{1}{\Omega} \exp\{-1(k_i - k_f)x\} \times \Big\{ \bar{u}(\mathbf{k}_f, s_f, \tau_f)\Gamma_A u(\mathbf{k}_i, s_i, \tau_i) + (2\pi)^3 \delta^3(\mathbf{k}_f - \mathbf{k}_i)\delta_{s_f,s_i}\delta_{\tau_f,\tau_i} B^{\Gamma_A} - \bar{u}(\mathbf{k}_f, s_f, \tau_f)\Gamma^a_B u(\mathbf{k}_i, s_i, \tau_i)\Delta^{ab}(k_i - k_f)\Pi^{\Gamma_A \Gamma^b_B}(k_i - k_f) \Big\}. \quad \text{(A16)}$$

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