Intruder analog states: New classification of particle-hole excitations near closed shells

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Shell-model particle-hole intruder excitations and the collective bands built on them near closed shells are studied and classified, together with low-lying collective bands in adjacent nuclei with the same number of "valence" protons and neutrons, into multiplets. A detailed numerical study within the interacting boson model gives support to such a classification scheme. We introduce the concepts of intruder spin and intruder analog states and show how such a symmetry can be useful to understand nuclear collective structures at and near to closed shells.

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I. INTRODUCTION

The observation of low-lying 0^+ intruder excitations near single closed-shell nuclei has become a widespread phenomenon [1]. It has been pointed out that two-particle–two-hole (2p-2h) excitations across the closed shell are most probably the origin of the formation of 0^+ intruder excitations [1,2]. The low excitation energy is a consequence of the proton-neutron attractive force which results in a large extra binding energy for the intruder configurations relative to the more regular ones. In many cases, e.g., the Sn, Cd, Hg, and Pt nuclei, bands are observed on top of the 0^+ intruding state [1].

A first observation is that the level spacings in the intruder band, which is supposed to be built mainly on a 2p-2h core excited configuration, very closely resemble the level spacings in the two nuclei with the same number of valence neutrons, but having four particles or four holes, relative to the closed shell. This observation has been made for the ^{112}Pd, ^{116}Sn, ^{120}Xe triplet [3]. It hints at the fact that the collectivity determining the band structure reveals an approximate independence of the particle or hole character of the nucleons building up this collectivity, the essential feature being the total number of "active particles" (particles or holes). This can be qualitatively understood because, collectivity, taking the quadrupole residual interaction as the major component of the proton-neutron force results in extra binding energy which is (for a large number of interacting nucleon pairs) proportional to the number of valence protons times the number of valence neutrons [2]. This argument has been developed in some detail in Ref. [4].

It was shown independently by Casten [5,6] that a number of observables $[E_x(4_1^+)/E_x(2_1^+)$, $B(E2;2_1^+ \rightarrow 0_1^+), E_x(2_1^+), \ldots]$ presents an interesting scaling property when plotted according to the variable N_pN_n, where N_p (N_n) denotes the number of valence protons (neutrons)—particles or holes—respectively. These scaling properties clearly reflect the importance of the proton-neutron force acting among proton and neutron pairs, irrespective of their particle or hole character.

In the present paper we discuss in Sec. II, realistic interacting boson model (IBM-2) calculations for the chain of nuclei ^{110}Ru, ^{114}Cd, ^{118}Te, and ^{122}Ba and study the similarities and differences in the intruder band structure. Starting from these more detailed calculations, we pro-

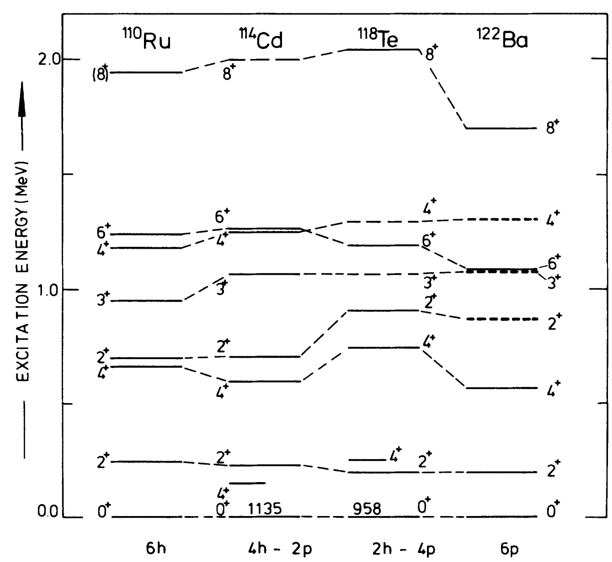

FIG. 1. Low-lying collective bands in ^{110}Ru and ^{122}Ba, connected with the intruder band structure in ^{114}Cd and ^{118}Te built on top of the 0^+ intruder state at 1.135 and 0.958 MeV, respectively. The mh-np structure of the band is also indicated. The thick dashed line results from extrapolations in the Ba nuclei.

TABLE I. IBM-2 parameters (in units of MeV, except for χ_π, χ_ν, which are dimensionless quantities) for the ^{110}Ru, ^{114}Cd, ^{118}Te, and ^{122}Ba nuclei. The proton boson number is indicated by N_π or $\bar{N}_\pi$, indicating a particle or hole character, respectively. FS (FK) denote the parameters of the Majorana force.

		$\epsilon_\pi=\epsilon_\nu$	κ	χ_ν	χ_π	$c_{0,\nu}$	$c_{2,\nu}$	$c_{2,\pi}$	FS/FK[a]
^{110}Ru	($\bar{N}_\pi=3$)	0.61	−0.15	−0.5	0.4	−0.1	−0.05	0.2	0.06/0.12
^{114}Cd	($\bar{N}_\pi=1$)	0.83	−0.14	−0.5	−0.2	−0.2	−0.05	0	0.06/0.12
	($\bar{N}_\pi=3$)	0.40	−0.19	−0.5	0	−0.2	−0.05	0	0.0/0.0
^{118}Te	($N_\pi=1$)	0.85	−0.14	−0.5	−0.45	−0.2	−0.05	0	0.06/0.12
	($N_\pi=3$)	0.40	−0.175	−0.5	−0.45	−0.2	−0.05	0	0.06/0.12
^{122}Ba	($N_\pi=3$)	0.75	−0.14	−0.4	−0.9	−0.2	−0.05	0	0.06/0.12

[a]Reference [16].

pose a classification scheme for intruder excitations, introducing the idea of intruder-spin analog states (Sec. III).

II. STUDY OF THE $N=66$ Ru,Cd,Te,Ba CHAIN

In the comparison of energy spectra with $N=66$ neutrons, ^{110}Ru and ^{122}Ba together with the bands built on the possible 0^+ intruder states in ^{114}Cd and ^{118}Te, a number of important similarities immediately appear (Fig. 1). The yrast 0^+, $2^+,4^+,6^+,8^+$ sequence in the four nuclei indicates a very close set of energy spacings which bring into focus in a clear way the importance of considering the six-proton particles, six-proton holes, and the four-proton-particle–two-proton-hole and two-proton-particle–four-proton-hole distributions of valence nucleons on the same footing in their interaction with the 16 valence neutrons outside the $Z=50$, $N=50$ core. One observes that ^{110}Ru presents more an O(6)-like character [7], whereas ^{122}Ba is approaching an SU(3) structure [8].

In the light of these observations, we have carried out IBM-2 calculations in the four $N=66$ nuclei. For the ^{110}Ru nucleus the parameters from an earlier calculation by Van Isacker and Puddu [7] were used, and for ^{122}Ba parameters from Puddu, Scholten, and Otsuka were taken [8]. For ^{114}Cd and ^{118}Te, parameters were used so as to get a coherent set of those quantities that depend on the neutron variables $(\chi_\nu, c_{0,\nu}, c_{2,\nu})$. These parameters were determined so as to be able to describe the intruder $\bar{N}_\pi(N_\pi)=3$ excitations in ^{114}Cd and ^{118}Te starting from almost the same values (see Table I). In ^{114}Cd these parameters therefore slightly deviate from the ones obtained in the recent study [12] concentrating on that nucleus only. In these four nuclei, since the neutron number remains constant at $N=66$, only small changes will occur in these neutron variables. The parameter set used is given in Table I. The main variation is brought about through the χ_π parameter (Fig. 2), indicating that for ^{110}Ru ($\chi_\pi=0.4$, $\chi_\nu=-0.5$) the structure is near to the O(6) limit and in ^{122}Ba ($\chi_\pi=-0.9$, $\chi_\nu=-0.4$) the structure approaches the SU(3) behavior of deformed nuclei. The results concerning the energy spectra are presented in Fig. 3.

The similarity of the collective band structures in these nuclei goes beyond the excitation energies. We have at the same time calculated the $B(E2;J_i^\pi \rightarrow J_f^\pi)$ reduced $E2$

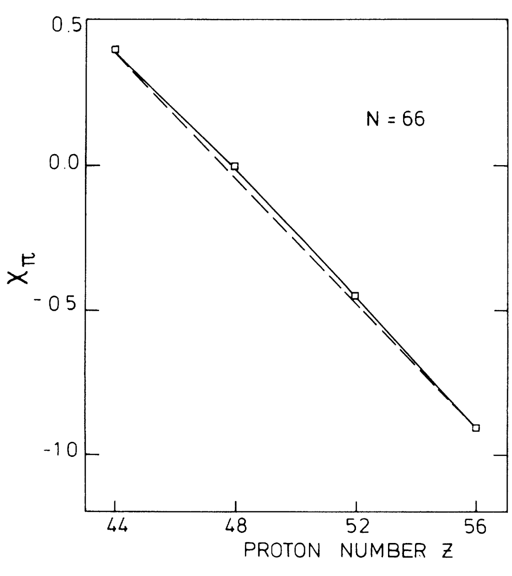

FIG. 2. Variation of the χ_π IBM-2 parameter, describing the major changing parameter in describing the 6h, 4h-2p, 2h-4p, and 6p collective bands in ^{110}Ru, ^{114}Cd, ^{118}Te, and ^{122}Ba, respectively. The variation is almost linear.

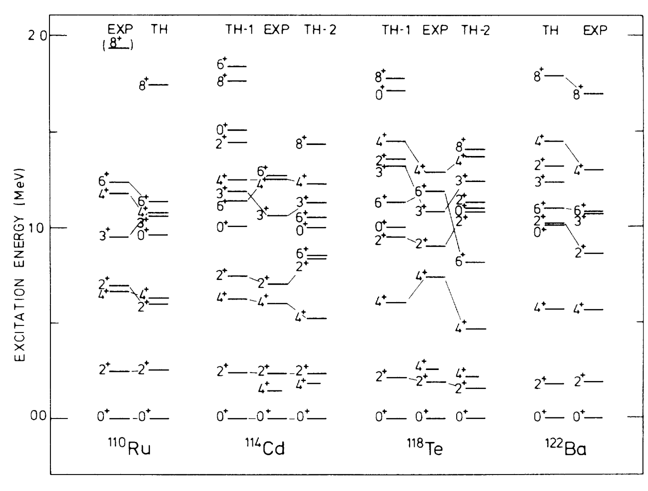

FIG. 3. Results from detailed IBM-2 calculations, using the parameters as given in Table I. Here we present the low-lying regular collective levels in ^{110}Ru and ^{122}Ba and the intruder band structure in ^{114}Cd and ^{118}Te. The calculation denoted with TH-2 corresponds with the realistic choice of parameters as given in Table II. The column TH-1 means that for the intruder structure ($\bar{N}_\pi=3$ or 3) exactly the same IBM-2 parameters were used as in the ^{110}Ru and ^{122}Ba nuclei, respectively.

transition-matrix elements, and here, too, a close resemblance exists, in particular for the $E2$ yrast transition (see Table II). One can even see a number of transition rates approach the various O(6) and SU(3) selection rules. In ^{110}Ru, the $B(E2;2_2^+\to0_1^+)$, $B(E2;3_1^+\to2_1^+)$, $B(E2;4_1^+\to2_2^+)$, $B(E2;4_2^+\to2_1^+)$, $B(E2;4_2^+\to3_1^+)$, and $B(E2;6_1^+\to4_2^+)$ values are almost vanishing ($\Delta\tau=\pm1$ selection rule [9]). For ^{122}Ba one finds very small $B(E2;4_1^+\to2_2^+)$, $B(E2;4_2^+\to2_1^+)$, and $B(E2;6_1^+\to4_2^+)$ values, reflecting the fact that $\gamma\to g$ and $\beta\to g$ band $E2$ transitions are forbidden in the exact SU(3) limit [9]. All these results clearly point to the large similarities in the collective structure for nuclei containing n particle, $n-2$ particle-2 hole, . . . , 2 particle-$n-2$ hole, and n hole configurations.

The analog structure, at least for the $B(E2;2_1^+\to0_1^+)$ matrix elements, can even be shown using a shell-model calculation for the 6p, 4p-2h, 4p-2h and 6h structures. Assuming large j values for the particle-like and hole-like configurations, the different $E2$ transition matrix elements, for $v=2\to v=0$ $E2$ transition, become

$$\langle (j)^n v=2,2^+||\mathbf{M}(E2)||(j)^n v=0,0^+\rangle=\left[\frac{n(2j+1-n)}{2(2j-1)}\right]^{1/2}\langle (j)^2 2^+||\mathbf{M}(E2)||(j)^2 0^+\rangle\ , \tag{2.1}$$

$$\langle (j)^{n-k}v=2,2^+;(j')^{-k}v=0,0^+;2^+||2^+\mathbf{M}(E2)||(j)^{n-k}v=0^+;(j')^{-k}v=0,0^+;0^+\rangle$$
$$=\left[\frac{(n-k)[2j+1-(n-k)]}{2(2j-1)}\right]^{1/2}\langle (j)^2 2^+||\mathbf{M}(E2)||(j)^2 0^+\rangle\ , \tag{2.2}$$

$$\langle (j)^{n-k}v=0,0^+;(j')^{-k}v=2,2^+;2^+||\mathbf{M}(E2)||(j)^{n-k}v=0,0^+;(j')^{-k}v=0,0^+;0^+\rangle$$
$$=\left[\frac{k(2j'+1-k)}{2(2j'-1)}\right]^{1/2}\langle (j')^2 2^+||\mathbf{M}(E2)||(j')^2 0^+\rangle\ . \tag{2.3}$$

In depicting the $2^+\to0^+$ transition as a transition between the intruder 2^+ state, i.e.,

$$\frac{1}{\sqrt{2}}[\,|(j)^{n-k}v=2,2^+;(j')^{-k}v=0,0^+;2^+\rangle+|(j)^{n-k}v=0,0^+;(j')^{-k}v=2,2^+;2^+\rangle\,]\ ,$$

and intruder 0^+, i.e.,

$$|(j)^{n-k}v=0,0^+;(j')^{-k}v=0,0^+;0^+\rangle$$

configurations and for small n and k values ($n,k\ll 2j+1$) with $j\cong j'$, one obtains for the intruder $2^+\to0^+$ $E2$ matrix element the approximate expression

$$\left[\frac{1}{\sqrt{2}}\left[\frac{(n-k)[2j+1-(n-k)]}{2(2j-1)}\right]^{1/2}+\frac{1}{\sqrt{2}}\left[\frac{k(2j+1-k)}{2(2j-1)}\right]^{1/2}\right]\langle (j)^2 2^+||\mathbf{M}(E2)||(j)^2 0^+\rangle\ , \tag{2.4}$$

using the notation $|(j)^{n'}v',J';(j)^{-n''}v'',J'';J\rangle$ for the particle-hole shell-model $v=0$ and $v=2$ configurations in Eqs. (2.2) and (2.3). For small n,k values $n,k\ll 2j+1$, we can put approximately $2j+1-(n-k)\cong 2j+1-k\cong 2j+1$ and using the approximate relation

$$\left[\frac{n-k}{2}\right]^{1/2}+\left[\frac{k}{2}\right]^{1/2}\cong\sqrt{n}\ , \tag{2.5}$$

which holds at the level of $\cong5\%$ up to $n=8$ ($k=0,2,4,6,8$). This matrix element (2.4) approaches the expression of Eq. (2.1) very well for small n values. The above shell-model argument indeed points toward a close similarity between the various $B(E2;2_1^+\to0_1^+)$ transition rates, independent of the repartition of the n "valence particles" in $(n-k,k)$ (particles, holes), respectively.

In Fig. 4 we compare, for the $^{110-120}$Cd nuclei, the experimental data and theoretical spectra, accentuating the region between 1 and 2.5 MeV and concentrating on the two-phonon quadrupole excitations and intruder $0^+,2^+$ excitations. The precise meaning of the symbols is discussed in the caption to this figure. It becomes clear, from the theoretical results, that for the lighter Cd nuclei ($110\le A\le114$), important mixing shows up, in particular for the 0_2^+ and 0_3^+ levels. In Fig. 5, we have also compared the relative 2^+-0^+ intruder excitation energies for a large set of Ru, Cd, and Ba nuclei where experimental evidence exists [10,11] for intruder $0^+,2^+$ excitations. This energy is obtained by normalizing in the Cd nuclei ^{110}Cd$-^{120}$Cd [1] to the 0^+ state that contains the larger part of the intruder configuration [12] and comparing the 2_3^+ theoretical level (mainly the intruder level) with the experimental 2_1^+-0_1^+ energy separation in those even-even nuclei [10,11] with the same neutron number, but now with six proton holes (Ru) and six proton particles (Ba), respectively. The very large similarities indicate extra support for the 4h-2p (six "valence" proton) character of the intruder excitations in the even-even Cd nuclei. Therefore one is tempted to call such states, "intruder analog configurations": here "analog" refers to changing particle pairs into hole pairs, thereby forming the sequence $\pi(6p)\to\pi(4p\text{-}2h)\to\pi(2p\text{-}4h)\to\pi(6h)$. Many-

TABLE II. Reduced $B(E2;J_i^\pi \to J_f^\pi)$ transition probabilities ($e^2\,b^2$) in the ^{110}Ru, ^{114}Cd, ^{118}Te, and ^{122}Ba nuclei. For the ^{110}Ru and ^{122}Ba, the parameters in the IBM-2 Hamiltonian were taken from Refs. [7] and [8], respectively, with charges $e_\pi = e_\nu = 0.1$ e b. For ^{114}Cd and ^{118}Te, the intruder bands are presented with the counting number i,f adjusted so as to indicate the correspondence with the analog states in ^{110}Ru and ^{122}Ba. Here $e_\pi = e_\nu$ $(N_\pi = 1) = 0.1$ e b and $e_\pi = e_\nu$ $(N_\pi = 3) = 0.16$ e b have been used.

	^{11}Ru	^{114}Cd	^{118}Te	^{122}Ba
$2_1^+ \to 0_1^+$	28.99	35.01	61.96	32.75
$2_2^+ \to 0_1^+$	0.02	2.48	1.96	2.67
$2_2^+ \to 2_1^+$	39.81	14.21	3.22	5.49
$3_1^+ \to 2_1^+$	0.06	5.26	2.73	2.18
$\to 2_2^+$	32.69	72.68	63.50	42.26
$4_1^+ \to 2_1^+$	41.08	57.74	91.23	47.64
$\to 2_2^+$	0.09	0.19	0.00	0.05
$\to 3_1^+$	9.90	7.43	3.96	2.43
$4_2^+ \to 2_1^+$	0.01	0.12	0.13	0.19
$\to 2_2^+$	24.37	43.17	24.57	19.33
$\to 3_1^+$	0.85	39.75	54.08	30.36
$\to 4_1^+$	21.55	17.40	9.03	4.89
$6_1^+ \to 4_1^+$	46.75	61.06	87.21	52.75
$\to 4_2^+$	0.12	0.69	0.73	0.10
$8_1^+ \to 6_1^+$	48.83	94.04	121.09	54.24

particle–many-hole configurations (mp-mh) have recently been studied within the interacting boson model (IBM-2) using a configuration mixing calculation [13], with applications to the ^{192}Hg nucleus. At the same time, Barrett and co-workers [14,15] have emphasized the importance of classifying intruder excitations in various nuclei, differing by an α-particle configuration, using symmetry arguments based on F spin [16]. In Sec. III we present a possible classification scheme encompassing both the intruder excitation and regular excitations in many nuclei near closed-shell configurations. The microscopic reason for such analog states should be related to the fact that the strong pairing force in even-even nuclei obscures, for the low-lying states, the fermionic character of the nucleons. The resulting "bosonization" of the paired nucleons then naturally explains the analog nature of these states.

III. INTRUDER ANALOG STATES

The similarities of the collective band structure built on the 0^+ intruder state with ground-state collective bands under the constraint that the number of valence neutrons was identical (see the discussion in Sec. II), suggest that there is an underlying symmetry. Here we study which symmetry could yield a formal classification of these states. This is most clearly done in the interacting boson approximation [16], where the nuclear states are considered to be built from $\ell = 0$ and $\ell = 2$ pairs. These fermion pairs are then replaced by s and d bosons.

For the description of intruder states, one can introduce two sets of bosons $s_p^\dagger, d_p^\dagger$ and $s_h^\dagger, d_h^\dagger$, the first being particle bosons above the closed shell and the second hole bosons made from holes in the filled shell. Neglecting for the moment the particles from the other kind (e.g., neutrons for ^{114}Cd), the dynamical symmetry associated with a certain set of intruder states is spanned in the irreducible representation (irrep) $[N_p] \times [N_h]$ of $U_p(6) \otimes U_h(6)$, where N_p (N_h) are the number of particle (hole) bosons. To relate the intruder analog states to each other, such as discussed above, we embed the $U_p(6) \otimes U_h(6)$ group into a

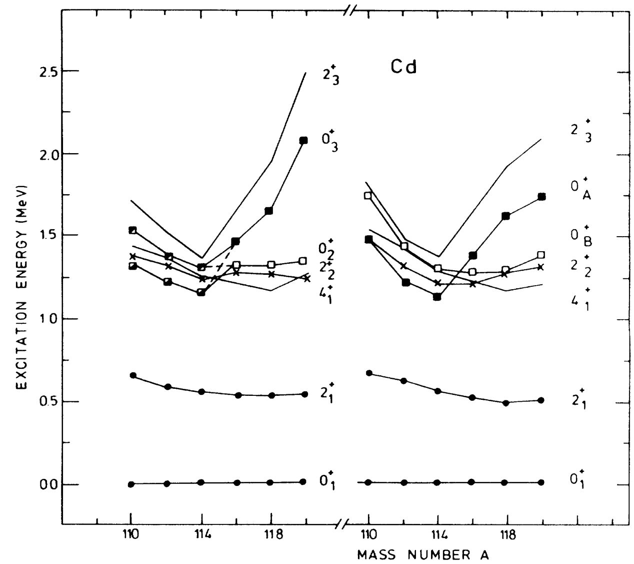

FIG. 4. Theoretical (left-hand part) and experimental (right-hand part) lowest-lying levels in the even-even Cd nuclei with $110 \leq A \leq 120$. In the theoretical spectra, the solid square (open square) denotes the 0^+ intruder and two-phonon 0^+ state, respectively. In case of strong mixing, a mixed square is drawn. The experimental indications for the intruder and two-phonon 0^+ states have been indicated by the 0_A^+ and 0_B^+ levels, respectively (Refs. [10,11]).

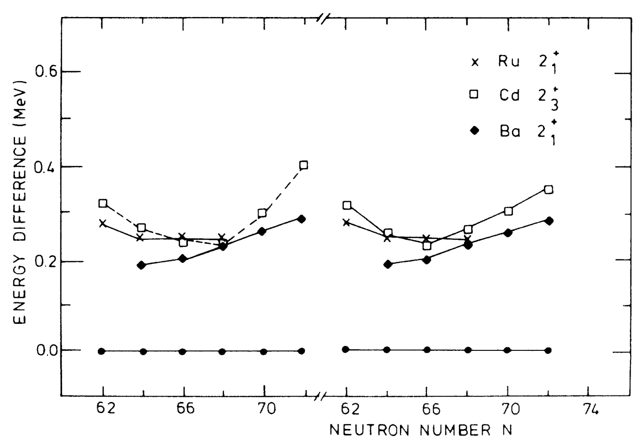

FIG. 5. Experimental (right-hand part) energy differences $E_x(2_3^+) - E_x(0_A^+)$ in $^{110-120}$Cd, together with the $E_x(2_1^+) - E_x(0_1^+)$ energy differences in the Ru and Ba nuclei with the corresponding neutron number. In the left-hand part, the corresponding theoretical energy differences are given (Refs. [10,11]).

larger group $U_{ph}(12)$ via the introduction of the boson generators $(b_p^\dagger \tilde{b}_h)^{(L)}$ and $(b_h^\dagger \tilde{b}_p)^{(L)}$ ($b^\dagger \equiv s^\dagger$ or $d^\dagger$) which are, in addition to the generators $(b_p^\dagger \tilde{b}_p)^{(L)}$ and $(b_h^\dagger \tilde{b}_h)^{(L)}$ forming the $U_p(6)$ and $U_h(6)$ groups. Then we can consider the group chain

$$\begin{array}{ccccc} U_{ph}(12) & \supset & U_p(6) \otimes U_h(6) & \supset & U(6) \\ [N] & & [N_p] \otimes [N_h] & & [N-i,i] \end{array} . \tag{3.1}$$

The last group is obtained by adding up the generators $(b_p^\dagger \tilde{b}_p)^{(L)}$ and $(b_h^\dagger \tilde{b}_h)^{(L)}$.

The reduction rules associated with Eq. (3.1) now provide the connection between the states discussed in Sec. II. They are, for fixed N: $N_p=1,2,\ldots,N$ and $N_h=N-N_p$, meaning that, for example, states with π(6 p), π(4p-2h), π(2p-4h), and π(6h) all belong to the irrep $[N]=[3]$. The second reduction rule is for $[N_p]\otimes[N_h]$, which yields $[N-i,i]$, with $i=0,1,\ldots,\min(N_p,N_h)$ and $N=N_p+N_h$.

In the actual IBM-2 configuration mixing calculations, no distinction is made between particle and hole bosons; this corresponds in (3.1) to the choice of $i=0$. States with i =0 are not considered here; they correspond to a different symmetry character of the wave function with respect to the particle and hole degrees of freedom. (They correspond to the so-called mixed-symmetry states that occur in the neutron-proton interacting boson model [16] and are therefore not related to the intruder analog states discussed here.)

The possibility now exists to reduce the $U_{ph}(12)$ group such that an SU(2) classification of the different members of a $[N]$ irrep becomes possible, i.e., using the chain

$$\begin{array}{ccccc} U_{ph}(12) & \supset & U(6) & \otimes & SU(2) \\ [N] & & [N-i,i] & \otimes & [I] \end{array} . \tag{3.2}$$

For a fixed value of $n=N_p+N_h$, one obtains for the I quantum number the value $I=(N_p+N_h)/2-i$. The full group chain reduction, as discussed before, is illustrated in Fig. 6. For the calculations (IBM-2 configuration mixing), where no distinction between particle and hole bosons is made, $i=0$ results and the I spin always has its maximal value. If the distinction between particle and hole bosons is considered, the I-spin classification label classifies intruder states with different symmetry character $I_{max}, I_{max}-1,\ldots$ in a given nucleus. The possible existence of such states has not been discussed until now. This I-spin classification shows a very strong similarity with isospin, and we call the multiplet of states within a given $[N]$ irrep "intruder analog states." We discuss the similarity between I spin and isospin now in some more detail, with the restriction $i=0$.

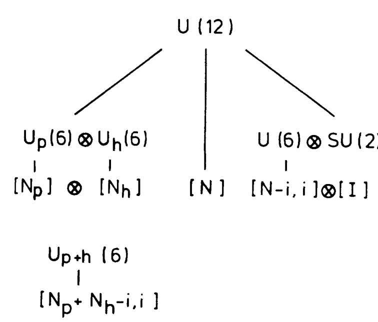

FIG. 6. Group chain sequences in order to classify the intruder analog states according to $U_{ph}(12)$, making use of the $U_p(6)$ and $U_h(6)$ basic building blocks within an extend interacting boson model classification.

In considering the class of states where the total number of proton (neutron) bosons outside a closed shell configuration is a constant value, N, but where the number N can be partitioned into N_p particle bosons and N_h hole bosons, we can denote each pair as a basic spin-$\frac{1}{2}$ system (I spin $\frac{1}{2}$) with projection quantum number $I_z=\pm\frac{1}{2}$ according to the particle or hole character of the bosons. A general state $N(N_p,N_h)$ can then be characterized by the I spin and I_z-projection quantum number in a given multiplet of fixed I spin:

$$|I,I_z\rangle \equiv \left| \frac{N}{2}, \frac{N_p-N_h}{2} \right\rangle . \tag{3.3}$$

The algebra is identical with any spin-$\frac{1}{2}$ algebra and so one can define the step operators $\hat{I}_\pm$ that leave the I spin equal, but change I_z to $I_z\pm 1$, respectively. Thereby one generates from a full particle state with $I_z=(N_p+N_h)/2=N/2$ all $2I+1$ multiplet members using the relation

$$\hat{I}_\pm \left| \frac{N}{2}, \frac{N_p-N_h}{2} \right\rangle = \left[\frac{N}{2}\left[\frac{N}{2}+1\right] - \frac{(N_p-N_h)}{2}\,\frac{(N_p-N_h\pm 2)}{2} \right]^{1/2} \times \left| \frac{N}{2}, \frac{(N_p\pm 1)-(N_h\mp 1)}{2} \right\rangle . \tag{3.4}$$

If we consider the closed shell itself as a vacuum state, the $\hat{I}_\pm$ operators are operators that create (annihilate) a particle boson and annihilate (create) a hole boson. The step operators can now be constructed as

$$\hat{I}_+ = s_p^\dagger s_h + \sum_\mu d_{p,\mu}^\dagger d_{h,\mu} ,$$
$$\hat{I}_0 = \tfrac{1}{2}\left[s_p^\dagger s_p + \sum_\mu d_{p,\mu}^\dagger d_{p,\mu} - s_h^\dagger s_h - \sum_\mu d_{h,\mu}^\dagger d_{h,\mu} \right] , \tag{3.5}$$

with

$$\hat{I}_- = (\hat{I}_+)^\dagger \text{ and } \hat{I}^2 = \hat{I}_+\hat{I}_- + \hat{I}_0^2 - \hat{I}_0 . \tag{3.6}$$

In order to investigate the relation of I spin to the Hamiltonian, we first note that in a given nucleus, we always have $[H,\hat{I}_0]=0$, thus I_0 is a good quantum number. This is not necessarily the case for the total I spin. Invariance of the Hamiltonian with respect to the I spin $[H,\hat{I}^2]=0$ now imposes the use of Hamiltonians that are symmetric under the interchange of $s_p^\dagger, d_p^\dagger$ with the corresponding $s_h^\dagger, d_h^\dagger$ operators. Finally, in order to have intruder analog states and to classify the energies in a given I-spin multiplet, the Hamiltonian needs to fulfill the condition

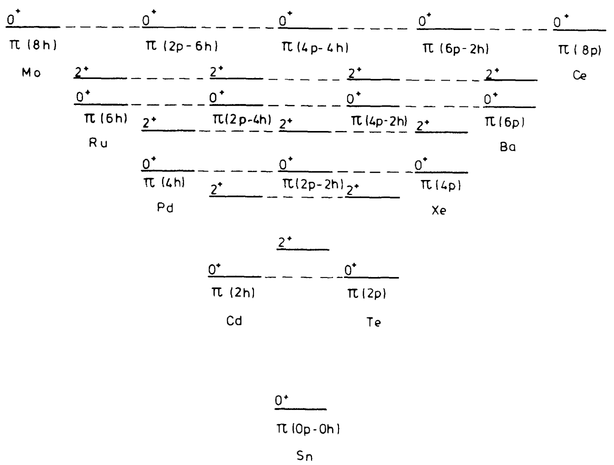

FIG. 7. Schematic representation of the *mp*-*nh* multiparticle-multihole excitations, ordered in corresponding multiplets with constant $m+n$ value and constant *I*- (intruder-) spin quantum number.

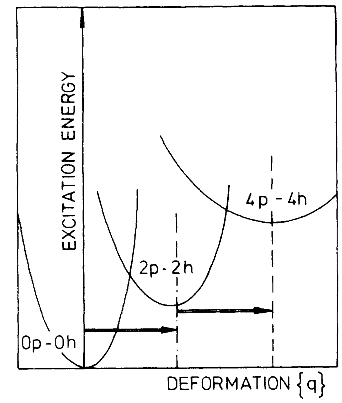

FIG. 8. Possible shape correspondence between increasingly complex *np*-*nh* excitations and the shape representation with collective variables (denoted by $\{q\}$), indicating a collective potential-energy surface with an equilibrium shape which increases in magnitude of $\{q\}$ with increasing *n* value for the *np*-*nh* excitations.

$$[H,\hat{I}_{\pm}]=0 \ . \tag{3.7}$$

The vanishing commutators imply that all members in a given *I*-spin multiplet are degenerate in energy.

In the detailed calculations that were carried out in Sec. II, the relevant states considered are the $|I,I_z\rangle=|\frac{3}{2},-\frac{3}{2}\rangle$, $|\frac{3}{2},-\frac{1}{2}\rangle$, $|\frac{3}{2},+\frac{1}{2}\rangle$, and $|\frac{3}{2},+\frac{3}{2}\rangle$ states, corresponding to levels in the Ru, Cd, Te, and Ba nuclei, respectively. In the Cd and Te nuclei, the regular two-hole (two-particle) excitations are then labeled as the $|\frac{1}{2},-\frac{1}{2}\rangle$, $|\frac{1}{2},+\frac{1}{2}\rangle$ states. Also, the parameters in the Hamiltonians describing the various nuclei of Table II (for the $N_\pi=3$ system) are very close (except for the variation in χ_π, which breaks the *I*-spin symmetry, and the somewhat smaller values of ε_d in ^{114}Cd and ^{118}Te compared with ^{110}Ru and ^{122}Ba). Here the deviations are mainly needed because of the mixing between the $N_\pi=1$ and 3 systems in the latter two nuclei. The parameters used give Hamiltonians that do not deviate significantly between those where one describes regular, low-lying collective bands in ^{110}Ru and ^{122}Ba or collective bands build on a 2p-2h 0^+ intruder excitation in ^{112}Cd and ^{118}Te.

The more ideal situation is the one where the Hamiltonian is fully *I*-spin invariant and where one could classify the various types of intruder excitations into $|I,I_z\rangle$ multiplets. Such a schematic situation is shown in Fig. 7.

If we now restrict to the fully symmetric irrep in the reduction of the $U_{\text{ph}}(12)$ group, which means that $i=0$ so that the *I* spin is always equal to $(N_{\text{p}}+N_{\text{h}})/2$, a two-particle boson, a two-hole boson, and a one-particle–one-hole boson can be classified according to the same scheme. Then the classification allows that for a given nucleus, characterized by the number of valence pairs in the ground-state configuration $N_{\text{p}},N_{\text{h}}$, various more complicated particle-hole excited configurations can occur. Counting with a single pair promotion across the closed shell at the time, the more complicated intruder excitations can be constructed $N_{\text{p}}+1,N_{\text{h}}+1;\ldots;N_{\text{p}}+k,N_n+k$ intruder configurations, now all characterized by the same value of I_z, but with $I\geq|I_z|$.

In a given nucleus, these intruder configurations can be obtained by acting with the operators $(b_{\text{p}}^{\dagger}b_{\text{h}}^{\dagger})^{(L)}$ $(b^{\dagger}\equiv s^{\dagger}$ or $d^{\dagger})$ on the reference state. Extending now the $U_{\text{p}}(6)\otimes U_{\text{h}}(6)$ group by introducing the boson generators $(b_{\text{p}}^{\dagger}b_{\text{h}}^{\dagger})^{(L)}$ and $(\tilde{b}_{\text{p}}\tilde{b}_{\text{h}})^{(L)}$ $(b^{\dagger}\equiv s^{\dagger}$ or $d^{\dagger})$, one obtains the U(6,6) group as the covering group which contains an infinite number of irrep, i.e.,

$$[N_{\text{p}}]\otimes[N_{\text{h}}]\equiv[N]\otimes[0] \oplus[N+1]\otimes[1]\oplus[N+2]\otimes[2]\oplus\cdots \ , \tag{3.8}$$

with $N=N_{\text{p}}+N_{\text{h}}$ and $N_{\text{p}}-N_{\text{h}}=k$ a constant value, which gives a "vertical" intruder state classification in a given nucleus. Starting from the Hamiltonian

$$H=H_{(sd)_{\text{p}}}+H_{(sd)_{\text{h}}}+V_{(sd)_{\text{p}}(sd)_{\text{h}}} \ , \tag{3.9}$$

where $H_{(sd)_{\text{p}}}$ and $H_{(sd)_{\text{h}}}$ describe the full Hamiltonian in the boson particle and boson hole space, respectively, the term $V_{(sd)_{\text{p}}(sd)_{\text{h}}}$ then describes possible interactions coupling the various irrep contained in the infinite-dimensional space of Eq. (3.8).

The study of the energies and eigenvectors of the Hamiltonian given in Eq. (3.9) can be carried out in a number of simple situations [U(5),SU(3)] and using simplified choices of the IBM-2 parameters in the two subspaces. The more general group structure, encompassing both the intruder analog states (horizontal classification) contained within the $U_{\text{ph}}(12)$ group and the *mp*-*nh* intruder excitations in a given nucleus (vertical classification) contained within the U(6,6) group with the subsequent reduction to the U(6) group with its dynamical symmetries [U(5),SU(3),O(6)], including also the "mixed-symmetry" particle-hole irrep, will be studied and work is in progress [17]. One has, moreover, already some ideas about the various energies needed to create 2p-2h, 4p-4h, ... intruder configurations, and the results obtained in Ref. [18] can be used to construct more complex chains of intruder multiplets.

Finally, the connection of the intruding excitations, as studied, using algebraic and/or shell-model techniques with results from studies using collective shape variables and using deformed single-particle potentials has to be pursued. On the connection of 2p-2h 0^+ intruder excitations and the very low total-potential-energy minima, relating to steep up- and down-sloping Nilsson model orbitals, some advancement in understanding equivalences has been made [19]. The np-nh excitations $(n=4,6,\ldots)$ most probably correspond to even larger equilibrium deformations related to the corresponding deformed configurations (Fig. 8). Work in this direction is in progress.

IV. CONCLUSION

We have indicated the possibility of classifying mp-nh intruder excitations, together with regular collective excitations, in multiplets that make use of the intruder spin (I-spin) quantum number.

Application to the Ru, Cd, Te, and Ba nuclei has been carried out, and both the energy spectra and $B(E2)$ values indicate the validity of such a classification scheme, thereby giving support to the idea that it is the number of valence nucleons, counted as particles outside a closed shell and holes inside the closed shell, which determines the level spacing in the collective bands.

We indicate also the possibilities of treating a system of particlelike and holelike bosons ($s_{\rm p},d_{\rm p}$ and $s_{\rm h},d_{\rm h}$ bosons) within an extended interacting boson model description. Further studies along the lines of algebraic model descriptions and the connection to shape variables is needed.

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