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Isospin character of the transition to the 0.803-MeV state in ^{206}Pb from $\pi^{\pm}$ scattering at 180 MeV

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Elastic and inelastic $\pi^{\pm}$ scattering by ^{206}Pb has been studied to measure the isospin character of transitions to bound states. The data have been interpreted using both distorted wave impulse approximation and optical model potentials. The data for the collective states at 2.647 MeV (3^-) and 4.111 MeV (2^+) are well reproduced with $\delta_l^+ = \delta_l^- = \delta_l^0$, i.e., assuming that these transitions are isoscalar. For the 0.803-MeV, 2^+ level we deduce $M_n/M_p = 2.6 \pm 0.3$ which is in excellent agreement with a value obtained from inelastic heavy-ion scattering.

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I. INTRODUCTION

The possibility of determining the isospin character of nuclear transitions by utilizing the collective model deformation length (δ_l^h) deduced by the normalization of measured inelastic cross-section data from hadron scattering to distorted wave Born approximation (DWBA) calculations was emphasized by Bernstein, Brown, and Madsen [1]. These authors noted that the deformation length extracted form such measurements is a function not only of the nuclear structure matrix element [i.e., $M_{n,p} = \int \rho_{\rm tr}^{n,p}(r) r^{l+2} dr$, where $\rho_{\rm tr}^{n,p}$ is the neutron (proton) transition density] but of the probe as well [1]. Plots of δ_l^h and M_n/M_p for the first 2^+ state of single-closed-shell (SCS) nuclei versus the ratio of the strength of the interaction of the probe with neutrons and protons were found [1] to be well described by predictions [2] of either a no free parameter (NPSM) or a one free parameter (OPSM) schematic shell model calculation. The data contained contributions from a variety of probes including protons at low and intermediate energies, pions, alpha particles, and neutrons. The deduced M_n/M_p were determined by forming ratios of δ_l's from data obtained with two or more different probes, or with one probe and $\delta_l^{\rm EM}$, where $\delta_l^{\rm EM}$ is calculated using the measured reduced electric transition probability, $B(El)$.

Values of M_n/M_p also have been extracted from single-probe measurements for which the differential cross section exhibits a signature arising from the interference between the Coulomb and nuclear amplitudes. This technique had been suggested by Martens and Bernstein [3]. Rychel *et al.* [4] have used the inelastic scattering of 35.4 MeV alpha particles by ^{90,92,94,96}Zr to deduce M_n/M_p for the first 2^+ and 3^- states. The values of M_n/M_p that they report for the 2^+ states are considerably larger than those deduced from a comparison of inelastic proton and neutron scattering [5], as well as predictions from nuclear structure calculations [6]. On the other hand, the values of M_n/M_p for the giant quadrupole resonance (GQR) in ^{208}Pb, obtained from inelastic heavy-ion scattering [7,8], is lower than that reported from a comparison of π^+/π^- scattering [9]. A similar situation exists in the case of ^{118}Sn [10,11]. It should be noted that the ratios of M_n/M_p obtained using inelastic heavy-ion scattering indicate that the GQR is excited by an isoscalar transition (i.e., $M_n/M_p = N/Z$).

Recently, the excitation of 2^+ and 3^- bound states in ^{204,206,208}Pb by 375-MeV ^{17}O ions was studied as a means to investigate the validity of deducing M_n/M_p from single-probe measurements [12]. Data were obtained over an extended angular region which included angles well inside the grazing angle. The data for the collective 2^+ and 3^- states could be well described by coupled-channels calculations with form factors using the deformed potential model and known $B(El)$'s and $M_n/M_p = N/Z$, in agreement with their isoscalar character. For the first 2^+ states in ^{204,206}Pb, the same procedure gave $M_n/M_p \approx 2.5$, which is in good agreement with predictions of a random phase approximation (RPA) calculation [12]. The mixed isospin character of the first 2^+ state in ^{206}Pb provides a convenient benchmark for comparing the M_n/M_p determined by inelastic heavy-ion scattering and π^+/π^- scattering. In this work, we report the results of π^+/π^- scattering on ^{206}Pb at 180 MeV.

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II. EXPERIMENTAL METHOD

The experiment was performed using the energetic pion channel and spectrometer (EPICS) at the Clinton P. Anderson Meson Physics Facility (LAMPF) with a beam energy of 180 MeV. The target consisted of a self-supporting foil of enriched (99.86%) ^{206}Pb of areal density 112.7 ± 1.1 mg/cm^2. Data were obtained at eight angles between 14° and 48° with the π^- beam, and seven angles between 18° and 42° with the π^+ beam. An overall energy resolution of ~ 120 keV was achieved. Relative cross-section normalizations were obtained using an ion chamber placed in the incident pion beam. Absolute cross sections were determined from measurements of elastic π^+/π^- scattering by hydrogen in a CH_2 target at an angle of 40°. The experimental hydrogen cross sections were normalized to those calculated using pion-nucleon phase shifts [13]. Shown in Fig. 1 are spectra for π^+ and π^- scattering by ^{206}Pb covering an excitation energy region up to ~ 5.0 MeV at a laboratory angle setting of 35°.

III. DATA ANALYSIS

Our primary interest in the data was to obtain differential cross sections for elastic scattering and excitation of the first 2^+ state at 0.803 MeV and the collective 3^- and 2^+ excitations at 2.647 and 4.111 MeV, respectively. The spectra were analyzed using a peak fitting routine FIT [14] in which we fixed the line shapes for all peaks to be that obtained by fitting the elastic peak. We fixed peak positions to correspond with adopted level energies [15]. The underlying background was considered to be constant over the energy interval (0–4.85 MeV) considered here, and was fixed as the average counts/channel for those channels corresponding to negative excitation energy. The results of such an analysis for the spectra taken at $\theta_L = 35°$ are shown in Fig. 1. Differential cross sections for the ground state, first 2^+, and collective 3^- and 2^+ states are given in Figs. 2 and 3.

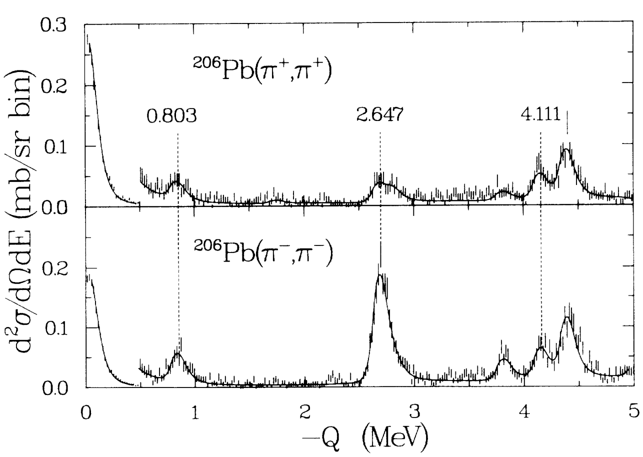

FIG. 1. Spectra from the scattering of 180-MeV pions by ^{206}Pb at a laboratory angle of 35°. The solid lines represent fits to the spectra, and the states of interest in this work are noted by vertical dashed lines.

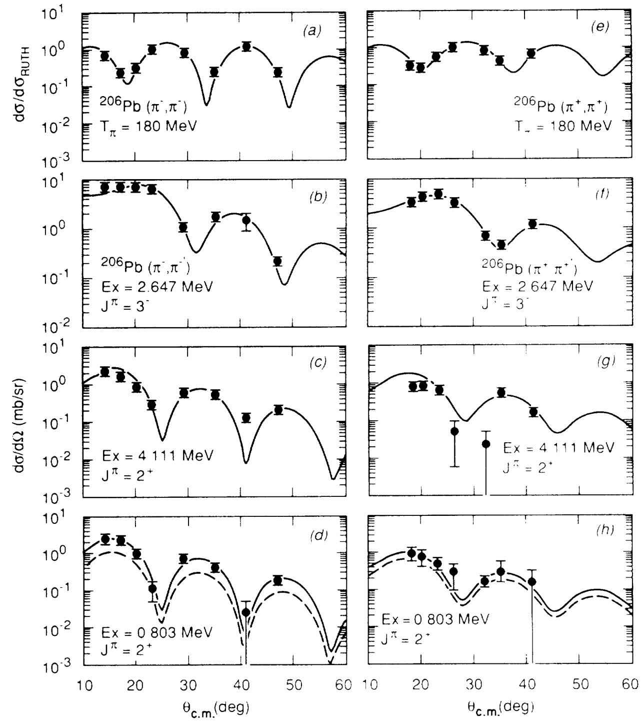

FIG. 2. Comparisons of 180-MeV $\pi^\pm$ differential cross sections with local optical model potential calculations: (a) and (e), elastic scattering; (b) and (f), 2.647-MeV, 3^- state; (c) and (g) 4.111-MeV collective 2^+ state; (d) and (h), 0.803-MeV, 2^+ state. The dashed curves in (d) and (h) are for $M_n/M_p = N/Z = 1.51$, and the solid curves are for $M_n/M_p = 2.5$.

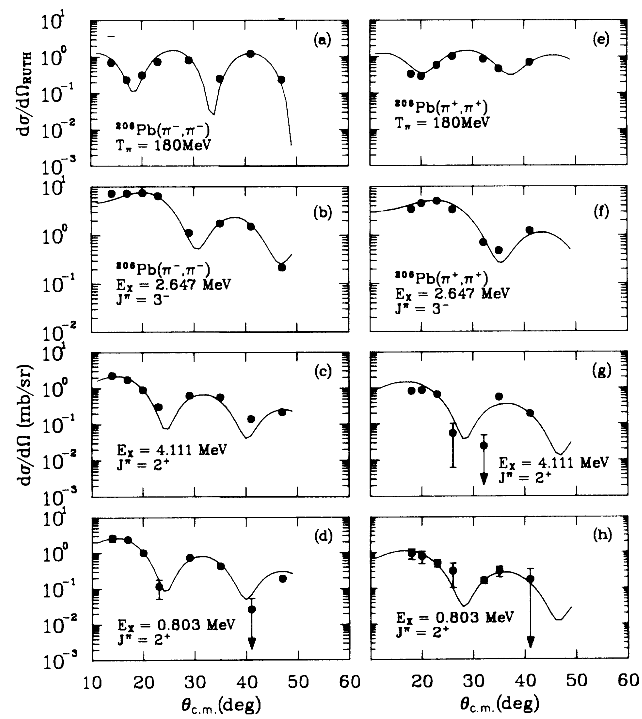

FIG. 3. Comparisons of 180-MeV $\pi^\pm$ differential cross sections with DWIA calculations: (a) and (e), elastic scattering; (b) and (f), 2.647-MeV, 3^- state; (c) and (g), 4.111-MeV, collective 2^+ state; (d) and (h), 0.803-MeV, 2^+ state.

A. Local potential model

Recently, Satchler has analyzed pion scattering data for ^{208}Pb at energies in the vicinity of the (3,3) resonance using a local, Woods-Saxon optical model potential [16]. He was able to obtain good fits to elastic $\pi^{\pm}$ data at incident energies of 116, 162, 180, and 291 MeV, and found pion scattering in this energy region to be similar to scattering for other strongly absorbed hadrons. Because of the limited angular extent of the data, Satchler restricted the nuclear potential to the form

$$U(r)=-Vf(x_v)-Wf(x_W) ,$$
$$f(x)=(e^x+1)^{-1} , \qquad (1)$$
$$x_i=(r-R_i)/a_i ,$$

where $R_i=r_i A_t^{1/3}$. The Coulomb potential was assumed to arise from a uniform charge distribution of radius $R_c=1.2A_t^{1/3}$. The Schrödinger equation was used with the relativistically correct center-of-mass momentum and a reduced mass

$$\mu c^2=E^{\pi}_{\mathrm{c.m.}}/(1+E^{\pi}_{\mathrm{c.m.}}/A_t c^2) ,$$

where $E^{\pi}_{\mathrm{c.m.}}$ is the total energy of the pion in the center-of-mass system.

Several parameter sets, at each incident pion energy, were found to yield equivalent fits to the elastic data. Moreover, the different parameter sets predicted similar differential cross sections for inelastic scattering over the angular range of the measured data when used in deformed potential coupled-channels calculations.

Here, we adopt the 180-MeV, $\pi^{\pm}$ potentials from Satchler (potentials 11 and 14 in his Table 1) [16]. For π^+ scattering, these are $V=45.75$ MeV, $r_v=1.258$ fm, $a_v=0.091$ fm, $W=1000$ MeV, $r_W=0.7654$ fm, and $a_W=0.731$ fm; and for π^- scattering $V=29.3$ MeV, $r_v=1.376$ fm, $a_v=0.093$ fm, $W=1000$ MeV, $r_W=0.9882$ fm, and $a_W=0.511$ fm. The scattering calculations were performed using the computer code PTOLEMY [17].

1. *Elastic scattering*

In Figs. 2(a) and 2(e) the $\pi^{\pm}$ elastic differential cross sections are plotted as a ratio to Rutherford cross sections where the latter are defined as

$$d\sigma_R/d\Omega=(Z_\pi Z_t e^2\mu/2\hbar^2k^2)^2\csc^4(\theta/2) . \qquad (2)$$

Here, $\hbar k$ is the relativistically correct center-of-mass momentum and μ is the reduced mass as given above. As can be seen in Fig. 2, the potentials defined above adequately reproduce the $\pi^{\pm}$ elastic data for ^{206}Pb, as would be expected.

2. *Inelastic scattering*

The inelastic cross sections were calculated using these same potentials to describe the distorted waves, and the deformed potential model was used to generate the nuclear transition potentials, the radial parts of which are given by

$$H_l^h(r)=-\delta_l^h dU(r)/dr , \qquad (3)$$

where δ_l^h represents the deformation length corresponding to the $\pi^{\pm}$ strength factor. To this is added the Coulomb interaction taken as

$$H_l^c=\frac{4\pi Z_\pi e}{2l+1}[B(El)\uparrow]^{1/2}f_l^c(r) , \qquad (4)$$

where

$$f_l^c(r)=\begin{cases}1/r^{l+1}, & r\geq R_c,\\ r^l/R_c^{2l+1}, & r\leq R_c .\end{cases}$$

We assume a uniform charge distribution with $R_c=1.2A_t^{1/3}$ and relate the charge deformation length, δ_l^{EM}, to the $B(El)\uparrow$ by the expression

$$B(El)\uparrow=\left[\frac{3}{4\pi}Z_t e\delta_l^{\mathrm{EM}}R_c^{l-1}\right]^2 . \qquad (5)$$

The δ_l^{EM} are determined for each excitation using known $B(El)\uparrow$. The nuclear deformation lengths, δ_l^h, are then calculated using the relation

$$\delta_l^h/\delta_l^{\mathrm{EM}}=\frac{1+b_n^hM_n/b_p^hM_p}{1+b_n^hN/b_p^hZ} , \qquad (6)$$

where b_n^h and b_p^h represent the strength of the interaction of the pion with neutrons and protons, respectively [1]. Near the (3,3) resonance, the ratio b_n^h/b_p^h takes on the values 3 and $\frac{1}{3}$ for π^- and π^+ scattering, respectively. For an "isoscalar excitation," $M_n/M_p=N/Z$ and $\delta_l^h=\delta_l^{\mathrm{EM}}$. For excitations where $B(El)\uparrow$ is known, the problem reduces to simultaneously fitting the π^+ and π^- inelastic cross sections with a single unknown parameter, i.e., M_n/M_p.

3. *2.647-MeV, 3^- collective excitation*

In a study of the ^{204,206,208}Pb(^{17}O,^{17}O$'$) reaction [12], it was found that the differential cross section for exciting the 2.647-MeV, 3^- state in ^{206}Pb could be well reproduced by assuming that the transition was isoscalar with a $B(E3)\uparrow$ identical to that for exciting the 2.618-MeV, 3^- state in ^{208}Pb, i.e., $B(E3)\uparrow=0.611$ e^2b^3. This is equivalent to a deformation length $\delta_3^{\mathrm{EM}}=\delta_3^h=0.795$ fm. As shown in Figs. 2(b) and 2(f), the calculations using this value for δ_3^+ and δ_3^- are in good agreement with the data for the 2.647-MeV state.

4. *4.111-MeV, 2^+ collective excitation*

The low-lying collective 2^+ strength in ^{204,206}Pb is equivalent to that of the 4.085-MeV state in ^{208}Pb, but is fragmented. The strongest component in ^{206}Pb lies at an excitation energy of 4.107 MeV. The strength to this state has been reported from inelastic ^{17}O scattering [12] as $B(E2)\uparrow\sim 0.20$ e^2b^2 and from inelastic electron scattering [18] as $B(E2)\uparrow=0.25$ e^2b^2. The peak in our spectra at 4.111 MeV is dominated by this fragment of the collective 2^+ excitation, and the cross sections are shown in Figs. 2(c) and 2(g). The calculated curves are for an isoscalar transition with $B(E2)\uparrow = 0.25\ e^2\text{b}^2$, or $\delta_2^{\text{EM}} = \delta_2^h = 0.3604$ fm. Although the quality of the data for this transition is poorer than that for the 3^- state, it is clear from Figs. 2(c) and 2(g) that the cross sections are reproduced about equally well for both the π^+ and π^- scattering.

5. *0.803-MeV, 2^+ state*

The adopted $B(E2)\uparrow = 0.100 \pm 0.002\ e^2\text{b}^2$ gives $\delta_2^{\text{EM}} = \delta_2^+ = \delta_2^- = 0.2279$ fm for an isoscalar transition. Calculations using these δ_2 values are shown in Figs. 2(d) and 2(h) as the dashed curves. As can be seen in this figure, the calculations for both π^+ and π^- scattering grossly underestimate the experimental cross sections. This is not surprising since the transition to this state is predicted to have $M_n/M_p > N/Z$ in both schematic model [19] and quasiparticle RPA model [12] calculations. The solid curves shown in Figs. 2(d) and 2(h) have been calculated using Eq. (5) with $M_n/M_p = 2.5$ to determine $\delta^+ = 0.2788$ fm and $\delta^- = 0.3499$ fm. From simultaneous fits to the $\pi^\pm$ data, we obtain $M_n/M_p = 2.5 \pm 0.3$ which is in excellent agreement with the results found in the $^{204,206}\text{Pb}(^{17}\text{O},^{17}\text{O}')$ measurements as well as the predictions from RPA calculations [12].

B. Distorted wave impulse approximation calculations

In this section we present comparisons of the data with distorted wave impulse approximation (DWIA) calculations for which we used the computer code DWPI [20]. The Kisslinger [21] form for the optical potential was used with an empirically deduced [22] shift in the energy at which the pion-nucleon phase shifts are evaluated to calculate the optical potential. Transition densities were taken as

$$\rho_{\text{tr}}^{n,p}(r)_l = \delta_l^{n,p} \frac{d\rho^{n,p}(r)}{dr} ,$$

where $\rho^{n,p}(r)$ are the ground-state neutron and proton densities, respectively. Here we have chosen the neutron and proton ground-state density distributions to be equal and given by a two parameter Fermi function with radius $c = 6.510$ fm and diffuseness $a = 0.55$ fm.

1. *Elastic scattering*

The DWIA calculations for $\pi^\pm$ elastic scattering are compared with the measured cross sections in Figs. 3(a) and (e). The quality of the DWIA fits to the elastic data in Figs. 3(a) and 3(e) are comparable to those found using the local optical potential model.

2. *Inelastic scattering*

For each excited state, the inelastic $\pi^\pm$ scattering data were fitted simultaneously to deduce δ_l^n and δ_l^p. For the first 2^+ state, use was also made of the adopted values of $B(E2)\uparrow$ to fix δ_2^p and fit the data for M_n/M_p.

3. *2.647-MeV, 3^- collective excitation*

The DWIA fits to the 2.647-MeV, 3^- collective excitation are shown in Figs. 3(b) and 3(f). These give $M_n/M_p = 1.6 \pm 0.3$, and $B(E3)\uparrow = 0.624 \pm 0.087\ e^2\text{b}^3$ in good agreement with the value [12] deduced from heavy-ion scattering.

4. *4.111-MeV, 2^+ collective excitation*

From the DWIA fits to the 4.111-MeV, 2^+ collective excitation shown in Figs. 3(c) and 3(g) we obtain $M_n/M_p = 1.7 \pm 0.3$ and $B(E2)\uparrow = 0.194 \pm 0.027\ e^2\text{b}^2$. The latter value is in somewhat better agreement with the $B(E2)\uparrow$ deduced from heavy-ion [12] scattering than that from (e,e') [18] scattering.

5. *0.803-MeV, 2^+ state*

The DWIA fits to the $\pi^\pm$ data for the 0.803-MeV, first 2^+ state of ^{206}Pb which give $M_n/M_p = 2.8 \pm 0.4$ and $B(E2)\uparrow = 0.090 \pm 0.013\ e^2\text{b}^2$ are shown in Figs. 3(d) and 3(h). The latter is about 10% below the adopted $B(E2)\uparrow = 0.100 \pm 0.002\ e^2\text{b}^2$. Setting $B(E2)\uparrow$ at the adopted value and refitting the data results in $M_n/M_p = 2.7 \pm 0.4$.

IV. SUMMARY

From the results presented here, it is clear that both DWIA and local optical potential model calculations provide equally good fits to the pion scattering data with essentially identical nuclear structure quantities. Furthermore, there is excellent agreement between the values of M_n/M_p deduced from the pion scattering and heavy-ion scattering to the bound states of ^{206}Pb. In particular, the average value of $M_n/M_p = 2.6 \pm 0.3$ for the 0.803-MeV, first 2^+ state of ^{206}Pb obtained here from pion data, agrees well with the value $M_n/M_p = 2.5-3.0$ found for the 0.899-MeV, first 2^+ state of ^{204}Pb (and inferred also for the first 2^+ state of ^{206}Pb) in heavy-ion scattering [12]. This latter result is especially satisfying because of the isospin mixture of the transition to the first 2^+ state. It should also be noted that the M_n/M_p for the three excitation reported here are in good agreement with those predicted by RPA calculations, i.e., $M_n/M_p = 1.50$, ~ 1.68, and 2.2 for the 3^-, collective 2^+, and first 2^+ states, respectively [12].

Hence, we conclude that, at least for bound states, the nuclear structure quantities deduced from pion scattering and single-probe measurements which utilize signatures in the differential cross sections arising from interference effects between the Coulomb and nuclear amplitudes are consistent. The question as to why these methods seem to yield significantly different values of M_n/M_p for the GQR (i.e., states in the continuum) remains unresolved and requires further investigation.

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